

# 1 FINANCIAL MATHEMATICS

**1A** Review of percentages

**1B** Financial applications of percentages

**1C** Understanding simple interest

**1D** Working with simple interest

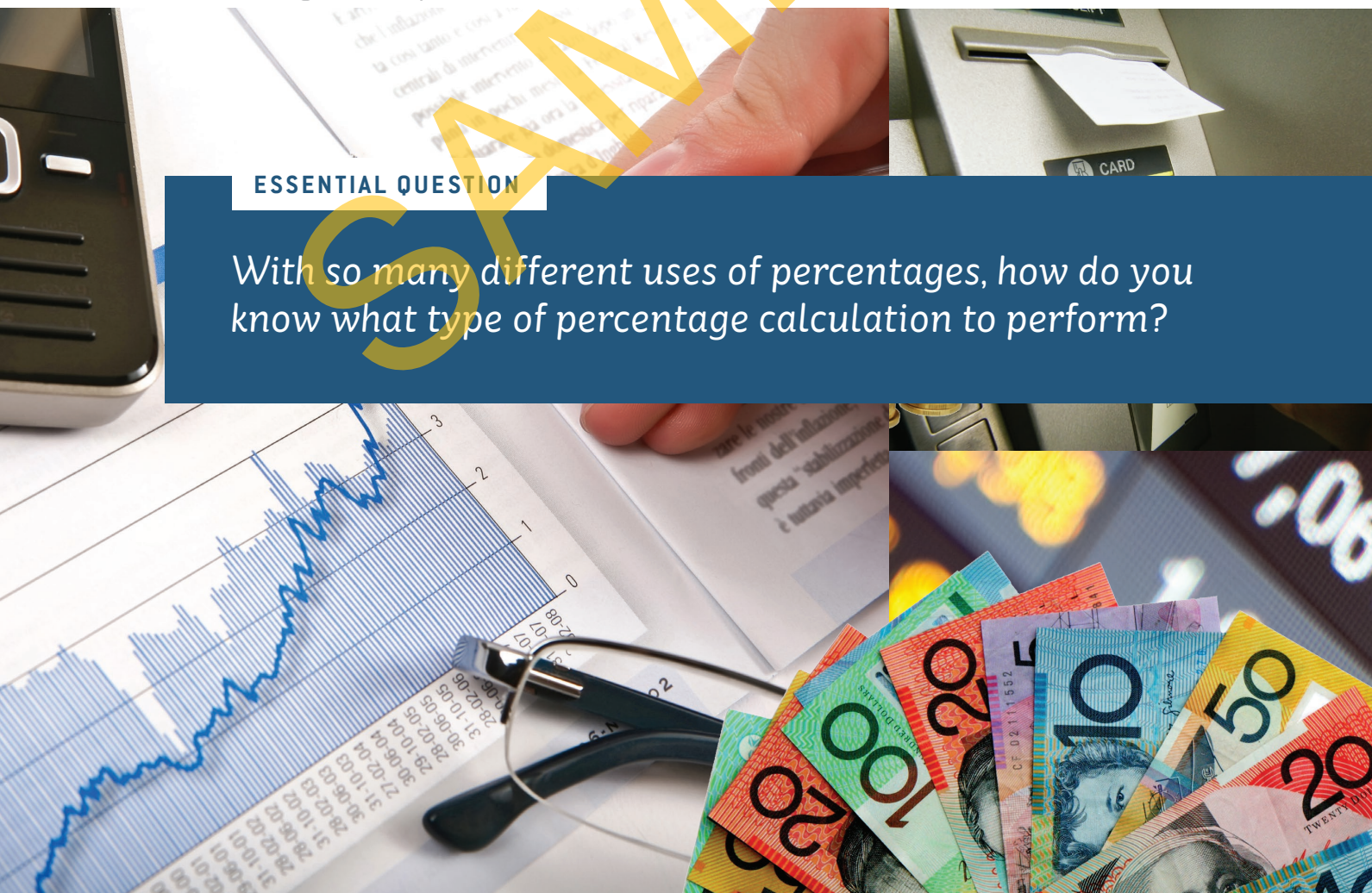
**1E** Understanding compound interest

**1F** Working with the compound interest formula

**1G** Working with compound interest

## ESSENTIAL QUESTION

With so many different uses of percentages, how do you know what type of percentage calculation to perform?



1A ▶ 1 What is  $\frac{3}{4}$  as a percentage?

1A ▶ 2 The decimal 0.65 is equivalent to which percentage?

- A 65%                      B 0.0065%  
C 0.65%                    D 0.65

1A ▶ 3 Which represents 35% of \$200?

- A  $35 \times \$200$             B  $\frac{35}{200} \times 100$   
C  $0.35 \times \$200$         D  $\frac{65}{100} \times \$200$

1A ▶ 4 What is 22% of \$440?

1A ▶ 5 Jackson scored 21 out of a possible 24 marks on his Maths test.

- a Write Jackson's marks as a fraction of the total marks in simplest form.  
b What is Jackson's score as a percentage?

1B ▶ 6 A book store offers a 15% discount on all their books. What is the sale price of a book originally priced at \$25.40?

- A \$3.81                      B \$21.59  
C \$10.40                    D \$29.21

1C ▶ 7 \$2000 is invested for 15 months at an interest rate of 6.4% p.a. What are the values for the variables of the simple interest formula?

- A  $P = \$2000$ ,  $R = 6.4$  and  $T = 15$   
B  $P = \$2000$ ,  $R = 6.4$  and  $T = 1.25$   
C  $P = \$2000$ ,  $R = 0.064$  and  $T = 1.25$   
D  $P = \$2000$ ,  $R = 0.064$  and  $T = 15$

1F ▶ 8 A How many months are in 3 years?  
B How many quarters are in 2 years?  
C How many weeks are in 4 years?  
D How many quarters are in 7 years?

1F ▶ 9 What is the percentage  $10\frac{1}{2}\%$  equivalent to as a decimal?

1G ▶ 10 What does  $1.015^4$  mean?

- A  $1.015 \times 4$   
B  $1 + 0.015^4$   
C  $1.015 + 1.015 + 1.015 + 1.015$   
D  $1.015 \times 1.015 \times 1.015 \times 1.015$

# 1A Review of percentages

## Start thinking!

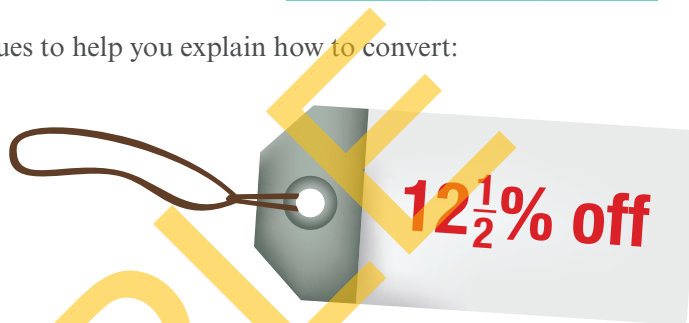
**Percentages**, **fractions** and **decimals** are different forms that can be used to represent the same amount. Consider this table displaying the equivalent forms of three different amounts as percentages, fractions and decimals.

Percentage	Fraction	Decimal
19%	$\frac{19}{100}$	0.19
10%	$\frac{1}{10}$	0.10
80%	$\frac{4}{5}$	0.80

- Consider each row of the table and use the values to help you explain how to convert:
  - a percentage to a fraction
  - a percentage to a decimal.
- Reconsider each row of the table and use the values to help you explain how to convert:
  - a fraction to a percentage
  - a decimal to a percentage.
- The value given in the price tag above advertises a **discount** amount written as a percentage.
  - Use your explanations from question 1 to write this percentage as a fraction and a decimal.
  - What was the main difference between the calculations you performed in question 1 and the calculations performed in part a?
- Copy and complete the missing values in this table.  
Remember to write the fractions in their simplest form.

The skills you use to convert between percentages, fractions and decimals are widely used in financial calculations.

Complete **1A DISCOVER** task to learn more.



Percentage	Fraction	Decimal
35%		0.35
	$\frac{2}{5}$	
		0.12
	$\frac{1}{8}$	
$15\frac{1}{2}\%$	1	

## KEY IDEAS

- Fractions, decimals and percentages are all closely related. To convert:
  - ▷ a percentage to a decimal, divide the percentage by 100
  - ▷ a percentage to a fraction, write the percentage with a denominator of 100 and simplify the fraction if possible
  - ▷ a decimal to a percentage, multiply the decimal by 100
  - ▷ a fraction to a percentage, multiply the fraction by 100.
- To calculate a percentage of an amount, write the percentage as a decimal and multiply by the amount.
- To write one quantity as a percentage of another, write the first value as a fraction of the second value and multiply by 100.

## EXERCISE 1A Review of percentages

A &lt;numbers to come&gt;

B &lt;numbers to come&gt;

C &lt;numbers to come&gt;

UNDERSTANDING AND FLUENCY

1 Write each percentage as a decimal.

- a 15%      b 90%      c 112%      d  $75\frac{1}{2}\%$       e 18.8%      f 0.25%

2 Write each percentage as a fraction in simplest form.

- a 21%      b 85%      c 225%      d 64%      e 38%      f 80.2%  
 g 110%      h  $5\frac{1}{4}\%$       i  $15\frac{3}{4}\%$       j 0.55%      k  $8\frac{2}{5}\%$       l 0.004%

3 Write each fraction as a percentage.

- a  $\frac{3}{5}$       b  $\frac{3}{8}$       c  $\frac{7}{10}$       d  $\frac{13}{20}$       e  $\frac{11}{16}$       f  $\frac{15}{24}$

4 Calculate each of these.

- a 12% of \$480      b 20% of \$3000  
 c 35% of \$567      d 150% of \$888  
 e 65% of \$1920      f 5% of \$14 000  
 g 40% of \$10 500      h 225% of \$4880

### EXAMPLE 1A-1

#### Calculating a percentage of an amount

Calculate  $17\frac{1}{2}\%$  of \$124.95 and round the answer to two decimal places.

#### THINK

- Write the percentage as a decimal and multiply by the amount. Remember that  $17\frac{1}{2}\%$  is the same as 17.5.
- Round the answer to two decimal places.
- Write your answer.

#### WRITE

$$\begin{aligned}
 17\frac{1}{2}\% \text{ of } \$124.95 &= \frac{17.5}{100} \times \$124.95 \\
 &= 0.175 \times \$124.95 \\
 &= \$21.866\ 25 \\
 &= \$21.87 \\
 17\frac{1}{2}\% \text{ of } \$124.95 &= \$21.87
 \end{aligned}$$

5 Calculate each of these, rounding each answer to two decimal places.

- a 11% of \$145.95      b  $42\frac{1}{2}\%$  of \$180.99  
 c  $12\frac{1}{2}\%$  of \$95.55      d 45.5% of \$852  
 e 17.5% of \$1550.25      f  $7\frac{1}{4}\%$  of \$39.95  
 g 2.75% of \$29.45      h  $10\frac{1}{8}\%$  of \$455.95



**EXAMPLE 1A-2****Writing one amount as a percentage of another**

Write \$70 as a percentage of \$280.

**THINK**

- 1 Write the first quantity (\$70) as a fraction of the second (\$280).
- 2 Convert the fraction to a percentage by multiplying it by 100%.
- 3 Cancel common factors to the numerators and denominators and simplify.

**WRITE**

$$\begin{aligned}
 &= \frac{70}{280} \\
 &\frac{70}{280} \times 100\% \\
 &\frac{1}{4} \times \frac{100}{1}\% \\
 &\frac{100}{4}\% \\
 &25\%
 \end{aligned}$$

- 6 Write these amounts as percentages, rounding your answers to two decimal places where appropriate.

- a \$90 as a percentage of \$450
- b \$80 as a percentage of \$130
- c \$29 as a percentage of \$150
- d \$36.50 as a percentage of \$145
- e \$29.95 as a percentage of \$58
- f \$152.50 as a percentage of \$65
- g \$12.85 as a percentage of \$94.50
- h \$140.20 as a percentage of \$255.95
- i \$345.60 as a percentage of \$180.75

- 7 Complete the missing values in this table. Remember to write all amounts as exact values and fractions in their simplest form.

	Percentage	Fraction	Decimal
a		$\frac{1}{6}$	0.1 $\dot{6}$
b	46 $\frac{2}{3}$ %		
c		$\frac{2}{3}$	0.6
d	12 $\frac{1}{3}$ %		
e		$\frac{7}{6}$	
f		$\frac{11}{12}$	
g	85 $\frac{1}{6}$ %		
h		$\frac{35}{9}$	



- 8 Shayden bought this scarf and at the end of winter sold it on eBay for \$6.55.

- a Write Shayden's selling price as a fraction of the initial price in simplest form.
- b Write the fraction in part a as a percentage and a decimal, both rounded to two decimal places.



- 9 A store recorded the payment methods used by customers, and found that 35% of payments were in cash and the rest were electronic. Of the electronic payments, five-eighths were payments by credit card, three-tenths were electronically withdrawn from savings accounts and the rest were debit card purchases.

- a What fraction of the total payments were in electronic form?
- b Of all the electronic payments, what fraction describes the payments made with a debit card?
- c What percentage of electronic payments were made with a debit card?
- d Show that the purchases made by withdrawals from savings accounts represent 19.5% of the store's total sales.
- e What percentage of the store's total sales do each of the other forms of electronic payments represent?

- 10 Two-thirds of all customers expressed satisfaction with their regular supermarket.

- a What is this fraction as a decimal, correct to two decimal places?
- b Explain how the decimal from part a can be written as an exact number, rather than an approximated number.
- c Write the fraction as an exact decimal.
- d Represent the fraction as an exact percentage.
- e What two different forms can the exact percentage in part d take?

- 11 A bookstore offers an out-of-print novel for sale with a discount of \$3.50. The novel was originally priced at \$12.50.

- a Write the discount amount as a percentage of the original price.
- b Briefly explain how you would write the discount amount as a fraction of the original price. Both the numerator and the denominator must be whole number values and the fraction must be in simplest form.
- c Write the discount amount as a fraction of the original price.

- 12** Bruno buys raw coffee beans and roasts them to sell to customers. A 60-kg sack of raw beans has a wholesale price of \$600. After roasting them, Bruno sells the beans for \$14 per 250-g bag.

- a** What is the wholesale price of the raw beans per 250 g?
- b** Compare the wholesale price per 250-g bag with the selling price of the 250-g bag of roasted beans.
- c** Write Bruno's selling price as a fraction of the wholesale price using the 250-g prices.
- d** Convert the fraction to a percentage and briefly explain what this percentage means to Bruno's business.

- 13** A camping store offers a 20% discount to general customers and a 26.5% discount to VIP customers.

- a** How much discount would a non-VIP customer receive on the tent shown in the photo?
- b** How much discount would a VIP customer receive on the tent?
- c** What fraction of the original price does each discount represent?



- 14** **a** Calculate  $33\frac{1}{3}\%$  of \$400. **b** Calculate  $\frac{1}{3}$  of \$400.  
**c** What do you notice about the answers to parts **a** and **b**? Why do you think this is so?

- 15** Calculations with percentages, fractions and decimals can be simplified using some basic number facts.

- a** Calculate each of these.
  - i** 10% of \$356.00
  - ii** 5% of \$356.00
  - iii** 1% of \$356.00
- b** Consider part **a i**. Describe a shortcut to calculating 10% of any amount without using a calculator.
- c** How does your answer to part **a ii** compare with **a i**? Describe a shortcut to calculating 5% of any amount without using a calculator.
- d** How does your answer to part **a iii** compare with **a i**? Describe a shortcut to calculating 1% of any amount without using a calculator.

- 16** Test your methods from question **15** with each of these. Check with a calculator that each answer is correct.

- |                          |                          |                          |
|--------------------------|--------------------------|--------------------------|
| <b>a</b> 10% of \$438.50 | <b>b</b> 5% of \$438.50  | <b>c</b> 1% of \$438.50  |
| <b>d</b> 1% of \$229.85  | <b>e</b> 5% of \$1458.99 | <b>f</b> 10% of \$148.35 |
| <b>g</b> 1% of \$295.10  | <b>h</b> 10% of \$95.66  | <b>i</b> 5% of \$2458.45 |

- 17** **a** Explore how you can extend your findings from question **15**. Calculate each value.

- i** 15% of \$356.00
- ii** 20% of \$356.00
- iii**  $\frac{1}{2}\%$  of \$356.00

- b** Consider each percentage value in part **a**. Briefly describe a shortcut to calculate each percentage of an amount without using a calculator. (Hint: how does each percentage relate to 10%?)

- 18** Test your methods from question **17** with each of these. Check with a calculator that each answer is correct.

- |                           |                                       |                                      |
|---------------------------|---------------------------------------|--------------------------------------|
| <b>a</b> 15% of \$845.80  | <b>b</b> 20% of \$845.80              | <b>c</b> $\frac{1}{2}\%$ of \$845.80 |
| <b>d</b> 15% of \$336.90  | <b>e</b> $\frac{1}{2}\%$ of \$1254.00 | <b>f</b> 20% of \$265.00             |
| <b>g</b> 20% of \$4780.95 | <b>h</b> $\frac{1}{2}\%$ of \$45.50   | <b>i</b> 15% of \$384.40             |

- 19** Calculations involving a percentage of an amount where the original amount is not known can be solved using the **unitary method**. Consider a laptop that is discounted by 12.5%, which amounts to \$56.25.

- a** Let  $x$  represent the original (pre-discount) price of the laptop. The discount calculation is:

$$12.5\% \text{ of } x = \$56.25.$$

For the unitary method, find how much 1% represents (one unit). Calculate 1% of the original price. (Hint: divide the amount for 12.5% by 12.5.)

- b** The original price of the laptop represents the full amount, or 100%. Use your answer to part **a** to calculate 100% of the original price. (Hint: multiply the amount for 1% by 100.)
- c** What is the pre-discount price of the laptop?

- 20** You can use an alternative method to the one described in question **19** to obtain the same result.

- a** Reconsider the initial calculation  $12.5\% \text{ of } x = \$56.25$ . Write the percentage as a decimal and show that the equation simplifies to  $0.125x = \$56.25$ .
- b** Look at the equation formed in part **a**. What operation can you perform to solve the equation for  $x$ ?
- c** Solve the equation to determine the value for  $x$ .
- d** How does the value you obtained in part **c** compare with the value obtained in **19b**?
- e** Consider the methods used in this question and question **19**. Briefly explain why they produce the same result.

- 21** Calculate the pre-discount price in each situation. Where necessary, round each answer to the nearest cent.

- a** A football is discounted 15%, which amounts to \$12.60.
- b** A mobile phone is discounted 10.5%, which amounts to \$5.90.
- c** Make-up is discounted \$3.20 or 8.5%.
- d** A furniture store offers a discount of \$145.50 on a bedroom suite, which is equivalent to 17.5%.
- e** A supermarket discounts their excess chocolates by  $27\frac{1}{2}\%$  or \$5.50.
- f** Sporting equipment is discounted 9.75%, which amounts to \$85.30.

### Reflect

How does converting percentages to decimals help to simplify a percentage of an amount calculation?



# 1B Financial applications of percentages

## Start thinking!

Chelsea works part-time job in a clothing store. Her duties include pricing the clothes. The dress shown is currently priced at \$84.95, but will go on sale with a  $22\frac{1}{2}\%$  discount.



- 1 The amount of the discount in dollars can be found using a percentage of an amount calculation. Determine this amount, correct to two decimal places.
- 2 Briefly explain how you can now determine the discounted price of the dress, and calculate the price.
- 3 Percentage discount is an example of a **percentage decrease**. The selling price of the dress can be determined by subtracting the percentage amount from 100% and finding this new percentage of the original price.
  - a A discount of  $22\frac{1}{2}\%$  means you pay  $22\frac{1}{2}\%$  less: new selling price =  $(100 - 22\frac{1}{2})\%$  of original price. What percentage of the original price does this discount represent?
  - b Show this method produces the same selling price as question 2.

Chelsea's duties also include marking up retail prices based on the wholesale prices the store pays for its goods. The store has a **mark-up** of 110% on all items.

- 4 A new jacket has a wholesale price of \$38. Find the actual amount of the mark-up by performing a percentage of an amount calculation (in a similar way to the discount calculation performed earlier).
- 5 Knowing the mark-up amount, briefly explain how you could determine the new selling price for the jacket, and calculate this selling price.
- 6 Percentage mark-up is an example of a **percentage increase**. The selling price of the jacket can be determined by adding the percentage amount to 100% and finding this new percentage of the wholesale price.
  - a A mark-up of 110% means you pay 110% more: selling price =  $(100 + 110)\%$  of wholesale price. What percentage of the wholesale price does this mark-up represent?
  - b Show this method produces the same selling price as question 5.

Complete **1B DISCOVER** task to learn more.

## KEY IDEAS

- ▶ Selling price following a percentage discount can be calculated using the rule:  
selling price =  $(100 - \text{percentage discount})\% \times \text{original price}$
- ▶ Selling price following a percentage mark-up can be calculated using the rule:  
selling price =  $(100 + \text{percentage mark-up})\% \times \text{original price}$
- ▶ A **profit** occurs when the selling price is higher than the original price and a loss occurs when the selling price is lower than the original price.
- ▶ Percentage profit (or loss) on the original price =  $\frac{\text{profit (or loss)}}{\text{original price}} \times 100\%$ .

## EXERCISE 1B Financial applications of percentages

A &lt;numbers to come&gt;

B &lt;numbers to come&gt;

C &lt;numbers to come&gt;

- 1 Calculate each of these, rounding to two decimal places where appropriate.
  - a 15% of \$150
  - b 35% of \$3200
  - c 8.5% of \$957.65
  - d  $122\frac{1}{2}\%$  of \$400

### EXAMPLE 1B-1

#### Calculating a percentage discount

Calculate the selling price after a  $17\frac{1}{2}\%$  discount on a necklace originally priced at \$229.95.

#### THINK

- 1 Write a calculation for the selling price.  
A percentage discount of  $17\frac{1}{2}\%$  means you pay  $(100 - 17\frac{1}{2})\%$  of the original price.
- 2 Write the percentage as a decimal and multiply by the amount.
- 3 Round the amount to the nearest cent (two decimal places).
- 4 Write your answer.

#### WRITE

$$\begin{aligned}
 \text{selling price} &= (100 - 17\frac{1}{2})\% \text{ of } \$229.95 \\
 &= 82\frac{1}{2}\% \text{ of } \$229.95 \\
 &= \frac{82.5}{100} \times \$229.95 \\
 &= 0.825 \times \$229.95 \\
 &= \$189.70875 \\
 &= \$189.71
 \end{aligned}$$

The selling price after a  $17\frac{1}{2}\%$  discount is \$189.71.

- 2 Calculate the selling price for each of these.
  - a 25% discount on \$185
  - b  $17\frac{1}{2}\%$  discount on \$9200
  - c 31.5% discount on \$650.95
  - d 12.5% discount on \$165.50
  - e 28.5% discount on \$325.99
  - f  $45\frac{1}{2}\%$  discount on \$499.45

**EXAMPLE 1B-2****Calculating a percentage mark-up**

Calculate the selling price after a  $55\frac{1}{2}\%$  mark-up is applied to sporting equipment with a wholesale price of \$600.

**THINK**

- 1 Write a calculation for the selling price.  
A percentage mark-up of  $55\frac{1}{2}\%$  means you pay  $(100 + 55\frac{1}{2})\%$  of the original price.
- 2 For the rate to be in simplest form, the second quantity needs to be 1. To achieve this, divide both quantities by 9.
- 3 Write your answer.

**WRITE**

$$\begin{aligned}\text{selling price} &= (100 + 55\frac{1}{2})\% \text{ of } \$600 \\ &= 155\frac{1}{2}\% \text{ of } \$600\end{aligned}$$

$$\begin{aligned}&= \frac{155.5}{100} \times \$600 \\ &= 1.555 \times \$600 \\ &= \$933\end{aligned}$$

The selling price after a  $55\frac{1}{2}\%$  mark-up is \$933.

- 3 Calculate the selling price for each of these.
  - a 35% mark-up on \$340
  - b  $15\frac{1}{2}\%$  mark-up on \$18 600
  - c 40.5% mark-up on \$844.20
  - d 125% mark-up on \$350.50
  - e 265% mark-up on \$1420.90
  - f  $165\frac{1}{2}\%$  mark-up on \$680.95
- 4 For each of these, determine:
  - i the selling price
  - ii the mark-up or discount amount.
  - a A smart phone is bought for \$89.90 and sold later at a mark-up of 155%.
  - b A jewellery piece originally marked at \$1495.80 is offered for sale at a discount of 12.5%.
  - c Sport shorts marked at \$49.90 are offered for sale with a 35% discount.
  - d Network cabling purchased for \$2.85 per metre is marked up by 480%.
- 5 For each situation:
  - i state if a profit or loss has been made and determine the amount
  - ii write the profit or loss amount as a percentage of the original price, correct to two decimal places where appropriate.
  - a Clothing is bought for \$84.95 and later sold for \$53.60.
  - b A market stallholder buys fruit for \$1.70 per kilogram and sells it for \$5.95 per kilogram.
  - c A new car is purchased for \$37 935 and sold later for \$17 390.
  - d A laptop is bought for \$548 and sold for \$652.
  - e A bookstore buys a book for \$32.95 and sells it for \$47.80.
  - f Computer games are bought at a market for \$42.95 and sold later for \$16.

- 6 The following represent the original prices and the percentage discount or percentage mark-up amounts offered on some goods. In each case, calculate:
- the selling price after the discount or mark-up
  - the discount or mark-up amount.

Where appropriate, round answers to the nearest cent.

- |  |                                      |
|--|--------------------------------------|
| a \$1285; 20% discount                 | b \$1800; $15\frac{1}{2}\%$ discount |
| c \$540; 185% mark-up                  | d \$450; 125.5% mark-up              |
| e \$346.95; 17.5% discount             | f \$850.40; 62.5% discount           |
| g \$3458.99; $87\frac{1}{2}\%$ mark-up | h \$6240.55; 255% mark-up            |
- 7 A school fundraiser collected \$2915.65 for charity. Compare this with the \$2584.80 collected last year.
- How much extra money was raised this year?
  - Write the increase as a percentage of last year's amount, correct to two decimal places.
- 8 Diana buys a digital radio for \$141.45 and sells it later for \$99.
- What is the amount of the loss Diana has made?
  - Write the loss amount as a fraction of the price Diana originally paid.
  - State the percentage loss on the original price, rounded to two decimal places.
- 9 For each of these:
- state the value of the profit or loss
  - write the profit or loss amount as a fraction of the original price
  - write the profit or loss as a percentage of the original price (rounded to the nearest 1%).
- original price \$68, selling price \$92
  - original price \$30 000, selling price \$12 850
  - original price \$145.50, selling price \$99
  - original price \$37.95, selling price \$12.50
  - original price \$699.45, selling price \$1225.50
  - original price \$294.58, selling price \$346.99
- 10 A soccer club decided to purchase new match balls at a sale. The balls were originally priced at \$84.95 each.
- Based on the advertised discount, what percentage of the original price will the new selling price be?
  - Calculate the selling price of each soccer ball.
  - What is the amount saved following the discount?
- If ordering more than 20 balls, the soccer club is eligible for an additional  $5\frac{1}{2}\%$  discount. The club decides to order 25 balls.
- How much does the club pay for the 25 balls?
  - Following the bulk purchase, what is each ball worth?



- 11** Bonnie is planning to hire a dress for her school formal. When she first enquired about the hire costs, the total hire price was quoted as \$85 plus a deposit of \$50. When she next contacted the store, she learned that the cost of hire and the deposit had increased by 12.5%.

- What percentage of the original hire price will the new hire price be?
- What amount will Bonnie pay to hire the dress, including the deposit?
- By how much has the hire charge increased, to the nearest cent?

- 12** Ricardo needs to purchase a new camera for his photography course. His local camera store is offering cameras for sale at a discount of 18%. Ricardo is interested in purchasing the camera shown in the photo.



- What is the selling price of the camera following the discount?
  - As a valued customer of the store, Ricardo receives an additional 4.5% discount. Apply this discount to the answer in part a to determine the amount Ricardo will pay for the camera.
  - Is the answer to part b the same as receiving a 22.5% ( $18\% + 4.5\%$ ) discount on the original price?
  - What was the total discount amount Ricardo received on this sale?
  - Write the discount amount as a percentage of the original price.
  - The percentage in part e represents the successive discounts as a single percentage amount. How does this single amount compare with the two successive percentage discounts?
- 13** Calculate the original price in each situation. Where necessary, round each answer to the nearest cent.
- A smart phone sells for \$297.50 following a discount of 15%.
  - A sports skirt sells for \$37.50 after a discount of 17.5%.
  - Jewellery sells for \$229.95 after a mark-up of 245%.
  - An artist sells her t-shirt prints for \$20.95 following a 135.5% mark-up.
- 14** Dennis plans to buy a new computer game for sale at a 15% discount. He is about to pay when he learns that a further 4.25% has been taken off from the price.
- Dennis expected to pay \$74.46 for the game. What is the selling price after the additional discount?
  - What was the original price of the game?
  - What single calculation can be applied to the original price to determine the selling price after two successive discounts?



- 15** James works in telephone sales and receives 2.75% commission on the total value of all his sales plus a retainer of \$450 as his gross weekly income. In a week when James' sales total \$12 452, what is his gross weekly income?
- 16** Laura earns a weekly retainer of \$925.50 and 4.125% commission of the total value of her weekly sales. Calculate her gross income for a week with each total sales value.
- |                    |                    |
|--------------------|--------------------|
| <b>a</b> \$0       | <b>b</b> \$15 000  |
| <b>c</b> \$175     | <b>d</b> \$21 587  |
| <b>e</b> \$1648.90 | <b>f</b> \$6381.45 |
| <b>g</b> \$489     | <b>h</b> \$8468.20 |
- 17** Harry applies a mark-up of 165% to the wholesale price of all items he sells in his clothing store. The goods and services tax (GST) of 10% must also be added to the marked-up price. Harry's newly arrived men's trousers have a wholesale price of \$68.
- What is the mark-up amount Harry needs to add to the wholesale price?
  - Calculate the selling price of the trousers after GST is included in the price.
  - Show that the calculation for the selling price is not the same as increasing the wholesale price by 175% ( $165\% + 10\%$ ).
  - Write the difference between the selling price and the wholesale price as a percentage of wholesale price.
  - What single calculation will give the selling price of the trousers?
  - These prices represent the wholesale prices of different clothing items. Increase the prices by Harry's mark-up amount and add the GST to determine each selling price.
    - shirts \$45
    - jackets \$87.50
    - belt \$29.40
    - suit \$189.95

## CHALLENGE

**Reflect**

Why are percentages so important in financial calculations?

# 1C Understanding simple interest

## Start thinking!

Last year, you were introduced to simple interest and its formula,  $I = P \times R \times T$ .

1 Briefly explain what interest means in finance.

When you borrow money from a bank, you generally repay more than you borrow. This additional repayment is known as interest.

When you invest (deposit money into a bank or other institution) rather than borrow money, interest can be paid to you on your investment.

2 How does interest on a loan differ from interest received for an investment?

3 Consider the simple interest formula,  $I = P \times R \times T$ .

- What do the variables in this formula represent?
- How must the value for  $R$  be written in the formula?
- What unit must be used for  $T$  in the formula?

4 Kiara's school provides a laptop costing \$1200 for each student. The laptops may be bought on a 3-year purchase plan, with interest of 6.5% p.a. for the 3 years.

- What does the abbreviation p.a. represent?
- Is this plan for a loan or an investment?
- Identify the values for  $P$ ,  $R$  and  $T$ .
- Use the simple interest formula to calculate  $I$ .
- What does your answer to part d represent?

Complete **1C DISCOVER** task to learn more.

## KEY IDEAS

- Interest can be an additional charge on a loan or a bonus payment on an investment.
- Interest can be calculated using the formula  $I = P \times R \times T$ , where:
  - ▷  $I$  = interest
  - ▷  $P$  = **principal** (the amount borrowed or invested)
  - ▷  $R$  = interest rate. The rate must be converted to a fraction or decimal.  
For example, 5% would be substituted as  $\frac{5}{100}$  or 0.05.
  - ▷  $T$  = time of the loan or investment in years.

## EXERCISE 1C Understanding simple interest

A &lt;numbers to come&gt;

B &lt;numbers to come&gt;

C &lt;numbers to come&gt;

UNDERSTANDING AND FLUENCY

1 Calculate the simple interest in each case.

a  $P = \$5000$ ,  $R = 6\%$ ,  $T = 4$  years

b  $P = \$850$ ,  $R = 12\%$ ,  $T = 2$  years

c  $P = \$125\,000$ ,  $R = 8.4\%$ ,  $T = 3$  years

d  $P = \$64\,000$ ,  $R = 3.6\%$ ,  $T = 5$  years

e  $P = \$9550$ ,  $R = 10\%$ ,  $T = 2.5$  years

f  $P = \$22\,000$ ,  $R = 12.5\%$ ,  $T = 3$  years

2 Convert each time to years.

a 24 months

b 48 months

c 27 months

d 45 months

e 7 months

f 15 months

g 19 months

h 1 month

i 52 weeks

j 65 weeks

k 4 weeks

l 312 weeks

### EXAMPLE 1C-1

#### Working with the simple interest formula

For a loan of \$12 000 at an interest rate of 8.4% p.a. for 30 months, calculate:

a the amount of simple interest charged

b the total amount repaid at the end of the loan

c the amount of each repayment, if equal repayments are made monthly.

#### THINK

a 1 Write the formula and identify the key terms. Rate must be written as a fraction or a decimal and time must be written in years.

2 Substitute the values and find the answer.

b 1 The total repaid at the end of the loan is the interest added to the principal.

2 Write the answer.

c 1 The amount of each repayment is found by dividing the total amount to be paid by the number of instalments.

2 Write the answer.

#### WRITE

a  $I = P \times R \times T$

$P = \$12\,000$

$R = 8.4\% = \frac{8.4}{100} = 0.084$

$T = 30 \text{ months} = \frac{30}{12} \text{ years} = 2.5 \text{ years}$

$I = \$12\,000 \times 0.084 \times 2.5 = \$2520$   
simple interest charged = \$2520

b amount =  $\$12\,000 + \$2520$   
= \$14 520

total amount repaid = \$14 520.

c amount to be paid = \$14 520  
number of instalments = 30  
amount of each instalment =  $\$14\,520 \div 30$   
= \$484

amount of each repayment = \$484

- 3 For each loan, calculate the:
- i amount of simple interest charged
  - ii total amount repaid at the end of the loan
  - iii amount of each repayment if equal repayments are made monthly.
- a \$8000 at an interest rate of 4% p.a. over 2 years
  - b \$15 000 at an interest rate of 10% p.a. over 4 years
  - c \$1250 at an interest rate of 5.4% p.a. over 3 years
  - d \$14 000 at an interest rate of 8.2% p.a. over 30 months
  - e \$25 000 at an interest rate of 6.8% p.a. over 18 months
  - f \$6550 at an interest rate of 4.5% p.a. over 104 weeks
- 4 For each investment, calculate the:
- i amount of simple interest earned
  - ii value of the investment at the end of the term.
- a \$12 000 at an interest rate of 3% p.a. for 3 years
  - b \$66 000 at an interest rate of 5% p.a. for 2 years
  - c \$125 000 at an interest rate of 6% p.a. for 2.5 years
  - d \$5500 at an interest rate of 4.2% p.a. for 12 months
  - e \$8250 at an interest rate of 8.5% p.a. for 15 months
  - f \$4580 at an interest rate of 3.6% p.a. for 36 months
- 5 Calculate the simple interest, given each of these.
- a  $P = \$8550$ ,  $R = 3.6\%$  p.a.,  $T = 2.5$  years
  - b  $P = \$9500$ ,  $R = 3.7\%$  p.a.,  $T = 1.5$  years
  - c  $P = \$22\ 000$ ,  $R = 8\%$  p.a.,  $T = 42$  months
  - d  $P = \$8490$ ,  $R = 8.4\%$  p.a.,  $T = 12$  weeks
  - e  $P = \$11\ 650$ ,  $R = 9.6\%$  p.a.,  $T = 5.5$  years
  - f  $P = \$155\ 000$ ,  $R = 12.9\%$  p.a.,  $T = 5$  months
- 6 An investment of \$12 000 is made at a bank that offers an annual interest rate of 6.25% p.a. The interest is calculated as simple interest and the money is invested for 3 years.
- a Identify the values for  $P$ ,  $R$  and  $T$ .
  - b How much simple interest is earned on the investment?
  - c What is the total value of the investment after 3 years?
- 7 A loan of \$12 000 is taken at an interest rate of 6.25% p.a. for 3 years, charged as simple interest.
- a Identify the values for  $P$ ,  $R$  and  $T$ .
  - b How much simple interest is charged to the loan?
  - c What is the total amount to be repaid at the end of the loan?

- 8 Consider each situation and answer in questions 6 and 7. Use your responses to explain how you can apply and interpret simple interest calculations in loans and investments.

- 9 For each of the values in the table on the right, calculate:

- the amount of simple interest
- the total amount at the end of the term.

	Principal	Rate (p.a.)	Time
a	\$14 000	3.8%	3.5 years
b	\$8 475	6.25%	12 months
c	\$12 390	$9\frac{1}{2}\%$	34 months
d	\$115 000	$8\frac{3}{4}\%$	120 days
e	\$250 000	2.95%	4.75 years
f	\$234 580	15.85%	17 months

- 10 An electronics store offers a purchase plan on the smart television shown, in which simple interest is charged at 12.75% p.a. for 20 months.

- What is the total amount of interest charged for the term?
- Using this purchase plan, what is the total cost of the television?
- If the total amount to be repaid is divided into equal monthly instalments for the term of the plan, what is the value of each instalment?

The store offers another option for customers paying cash, which involves a discount of 2.5% off the advertised price.

- What is the selling price for cash customers?
  - Calculate the difference in the television's price using the two options.
- 11 Jake plans to invest his savings in a term deposit at his bank. The bank offers interest of 4.8% p.a. for any period from 6 to 15 months and pays interest at maturity. Jake invests \$1500 for 14 months.
- Identify the values for  $P$ ,  $R$  and  $T$ .
  - How much interest does Jake earn on his investment?
  - What is the total value of Jake's investment at the end of his term?





- 12** A bank offers an interest rate of 1.4% p.a. on its everyday accounts, with interest calculated on the daily balances. Consider the account balances shown here.

Date	Transaction	Amount	Balance
01/05	Opening balance		\$985.90
09/05	Withdrawal at ATM	\$60.00	\$925.90
11/05	Deposit (Pay)	\$875.55	\$1801.45
20/05	EFTPOS Purchase	\$185.95	
22/05	Gym membership	\$69.50	
25/05	Deposit (Pay)	\$875.55	
31/05	Interest for May		

- The account balances following three of the transactions are not shown. State the missing account balances.
  - The opening balance of \$985.90 applies for the first eight days of the month as each new balance applies on the date the transaction is made. How many days does each balance on this account apply for?
  - For each new balance on the account, calculate the simple interest based on the number of days each balance applies. Remember that 8 days is equivalent to  $\frac{8}{365}$  years.
  - Add all the amounts from part c to calculate the total interest for the month.
  - What is the account balance at the end of May?
- 13** A bank is offering these interest rates to attract investment in term deposits. Interest is calculated at the end of the investment term.

Term	Investment amount			
	\$5 000 up to \$10 000	\$10 000 up to \$20 000	\$20 000 up to \$75 000	\$75 000 up to \$250 000
1 up to 2 months	2.5%	2.5%	2.5%	2.5%
2 up to 6 months	6.25%	6.25%	6.25%	6.25%
6 up to 8 months	3.25%	3.25%	3.25%	3.25%
8 up to 12 months	5.35%	5.35%	5.35%	5.35%
12 up to 24 months	5.4%	5.4%	5.4%	5.4%

- Answer these using the information in the table.
  - What is the minimum amount that can be invested in a term deposit?
  - Do the interest rates change as the amount invested increases?
  - The bank appears to have a special rate for a period of time. What investment term receives this rate?
- What interest rate will be offered on an investment of \$45 000 for 12 months?
- Calculate the amount of simple interest earned on an investment of \$45 000 for 12 months.
- Compare your answer to part c with the option of investing the same amount for two 6-month terms. (Assume the interest rate will stay the same for the second term.)

- e Reconsider the option of investing \$45 000 for two 6-month terms as calculated in part d. If, at the end of the first 6 months, the interest was added to the investment amount for the second term, how does the amount of interest for the second term change?
- f Compare all the investment option described. Which earned the greatest amount of interest?
- 14 Some savings accounts offered by banks include bonus interest rates if certain conditions are met. These bonus rates are seen as a reward to encourage people to save and earn more interest. Consider Maddy's account balance for November, shown here. Her account offers interest at 1.2% p.a. (calculated daily) plus a bonus 4.55% p.a. if no withdrawals are made in the month and the balance increases by at least \$250 each month.

- a Will this account receive the bonus interest rate? Provide a reason to support your answer.

Date	Transaction	Amount	Balance
01/11	Opening balance		\$8400.90
15/11	Deposit – Pay	\$655.80	
19/11	Deposit – at branch	\$240.00	
29/11	Deposit – Pay	\$655.80	
31/11	Interest		

- b Calculate the total interest earned for the month.  
(Hint: first determine the account balances following each transaction).
- c State the final account balance for the month.
- d On the final day of the month, Maddy made an EFTPOS purchase at a clothing store for \$100 and the transaction did not appear on her statement. How would the balances and interest calculations have differed if the withdrawal had appeared on the statement?

<insert new AW N01017\_29432>

- 15 Kyna invests her savings in a term deposit at her bank. The bank offers her an interest rate of 4.64% p.a. for a period of 3 months and will pay the interest at maturity. Kyna invests \$4250 for the 3 months.

- a How much interest does Kyna earn on her investment?
- b What is the value of Kyna's investment at the end of the term?
- c What single calculation can be performed to determine the final value of Kyna's investment?
- d Use your method from part c to determine the final value for each of these.
- an investment of \$3500, interest rate 4.8% p.a., 2 years
  - an investment of \$1800, interest rate 8.4% p.a., 4.5 years
  - an investment of \$12 000, interest rate 6% p.a., 15 months
  - an investment of \$25 000, interest rate 4.2% p.a., 4 months

### Reflect

Why must interest rates be written as an annual rate and the term of an investment be in years when using the simple interest formula?

# 1D Working with simple interest

## Start thinking!

When the value for  $I$  is known, the simple interest formula can be used to find the value for one of the other variables; that is,  $P$ ,  $R$  or  $T$ .

For example, Kim is considering some different purchase options for a new music stereo system. One store offers the system shown for sale at an interest rate of 12.5% p.a. charged as simple interest. Kim is informed that the total amount of interest charged for the term of this option is \$375 and that the total amount is repaid with equal-sized monthly repayments.



- 1 **a** From the simple interest formula, which variable do you *not* know the value of?  
**b** What variable does each of the given values represent?
- 2 Substitute the values into the simple interest formula and show that it simplifies to  $375 = 150T$ .
- 3 Solving the equation from question 2 will enable you to determine the value for  $T$ .  
**a** Explain what method you can use to find the value for  $T$ .  
**b** Use your method to find the value for  $T$ . What does this value mean in relation to Kim's purchase?
- 4 The selling price plus the interest amount represents the total Kim must pay for the stereo in this payment option. What is the size of each monthly repayment?

Kim notices another stereo system advertised for \$1100. The purchase option for this stereo also has equal-sized monthly repayments and charges \$352 interest for the term of 24 months.

- 5 **a** From the simple interest formula, which variable do you not know the value of?  
**b** What variable does each of the given values represent?
- 6 **a** Substitute the values into the simple interest formula and show that it simplifies to  $352 = 2200R$ .  
**b** Solve the equation in part a to determine the value for  $R$ .  
**c** The answer in part b represents the interest rate written as a decimal. What needs to be done to this decimal so that the value is written as a percentage amount?  
**d** State the annual interest rate (p.a.) that is applied to this purchase option.

Complete **1D DISCOVER** task to learn more.

## KEY IDEAS

- The simple interest formula is  $I = P \times R \times T$  or  $I = PRT$ .
- Strategies for solving equations can be used to find any of these quantities when the other three are known.

## EXERCISE 1D Working with simple interest

A &lt;numbers to come&gt;

B &lt;numbers to come&gt;

C &lt;numbers to come&gt;

UNDERSTANDING AND FLUENCY

- 1 Calculate the simple interest in each case.
  - a  $P = \$500$ ,  $R = 12\%$ ,  $T = 4$  years
  - b  $P = \$3500$ ,  $R = 9.5\%$ ,  $T = 3$  years
  - c  $P = \$850$ ,  $R = 2.5\%$ ,  $T = 1$  year
  - d  $P = \$44\,000$ ,  $R = 8.4\%$ ,  $T = 2.5$  years
- 2 Find the value for  $T$  in each of these.
  - a How long will it take for an investment of \$10 000 with simple interest rate 3.5% p.a. to earn \$1400?
  - b How long does a loan of \$1600 at an interest rate of 8% p.a. take to earn \$320 in simple interest?
  - c How long will it take for an investment of \$8500 with a simple interest rate of 5.6% p.a. to earn \$1428?
- 3 Find the value for  $P$  in each of these.
  - a How much needs to be invested at an interest rate of 7% p.a. for 4 years to earn \$4200 in simple interest?
  - b How much is borrowed at an interest rate of 8.8% p.a. over 2.5 years to earn \$1430 in simple interest?
  - c How much is borrowed at an interest rate of 4.75% p.a. over 3 years to earn \$2850 in simple interest?

### EXAMPLE 1D-1 Using the simple interest formula to find $R$

A loan of \$12 500 is taken over 36 months. The amount of simple interest charged on the loan is \$2475. What is the annual interest rate that is applied to this loan?

#### THINK

- 1 Write the simple interest formula and identify the variables. Remember that time must be written in years.
- 2 Substitute the values into the formula and simplify.
- 3 Use the balance method to find the value for  $R$ .
- 4 Multiply the decimal by 100 to write as a percentage amount. Write the answer.

#### WRITE

$$\begin{aligned}
 I &= PRT \\
 I &= \$2475 \\
 P &= \$12\,500 \\
 T &= 36 \text{ months} = \frac{36}{12} \text{ years} = 3 \text{ years} \\
 2475 &= 12\,500 \times R \times 3 \\
 2475 &= 37\,500 \times R \\
 \frac{2475}{37\,500} &= \frac{37\,500 \times R}{37\,500} \\
 R &= 0.066 \\
 &= 0.066 \times 100\% = 6.6\% \\
 \text{The annual interest rate is } &6.6\% \text{ p.a.}
 \end{aligned}$$

- 4 Find the value for  $R$  in each of these.
- What annual interest rate is applied to a loan of \$8000 over 2 years if the simple interest charged is \$704?
  - What annual interest rate is applied to a loan of \$16 000 over 4.5 years if simple interest charged is \$7560?
  - What annual interest rate is applied to a \$25 000 investment over 30 months if simple interest earned is \$2812.50?
- 5 Find the missing value in each of these.
- $I = \$1000$ ,  $P = \$5000$ ,  $R = 5\%$ ,  $T = ?$
  - $I = \$4680$ ,  $P = ?$ ,  $R = 6.5\%$ ,  $T = 4$  years
  - $I = \$8160$ ,  $P = \$64\,000$ ,  $R = ?$ ,  $T = 18$  months
  - $I = \$3864$ ,  $P = \$9200$ ,  $R = 8.4\%$ ,  $T = ?$
  - $I = \$4320$ ,  $P = ?$ ,  $R = 4.8\%$ ,  $T = 4.5$  years
  - $I = \$675$ ,  $P = \$22\,500$ ,  $R = ?$ ,  $T = 15$  months
- 6 Leanne invests a sum of money into a term deposit with her bank. Her investment will earn her \$2304 at maturity. The interest rate is 6.4% p.a. and Leanne invests the money for 36 months.
- From the simple interest formula, which variable do you not know the value of?
  - What variable does each of the given values represent?
  - How much money does Leanne invest? Hence, state the value at the end.
- 7 Kevin needs to borrow the entire cost to attend an end-of-school year trip. A friend loans him the money and charges him 8.5% p.a. simple interest over an agreed term. In total, the simple interest charged totals \$828.75.
- <insert new AW N01016\_29432><label to read 'Explore Machu Picchu for only \$6500'>
- From the simple interest formula, which variable do you not know the value of?
  - What variable does each of the given values represent?
  - Using the given values, how many months has the interest charge been based on?
  - Kevin agrees to repay his friend with equal monthly instalments for the term of the loan. What is the amount of each instalment?



- 8 Use the simple interest formula to determine the value for the missing amounts. For the missing  $T$  values, state the time in years.

	$I$	$P$	$R$ (% p.a.)	$T$
a		\$35 000	9.5	4.5 years
b	\$2180.90	\$9 650	5.65	
c	\$15.93		2.95	3 years
d	\$39 809	\$110 000		5.5 years
e	\$269.90		4.85	7 months
f	\$137.15		21.5	31 days
g	\$1890	\$15 000		18 months
h	\$3163.50	\$8 450	29.95	

- 9 If the  $P$  values in the table in question 8 represent amounts borrowed, what is the total amount to be repaid in each case?
- 10 A term deposit advertised on radio offers an interest rate of 6.8% p.a. fixed for 3 months.
- What do you think the expression 'fixed for 3 months' means?
  - How much interest is earned on an investment of \$12 000 in this term deposit?
  - What amount of money must be invested to earn \$500 in interest?
- At the end of the term, the rate of interest decreases to a new amount of 6.15% p.a. fixed for 100 days.
- What amount of money must be invested to earn the same amount as you calculate in part b?

- 11 Jacqui is saving money to purchase a racing bike like the one shown. She has saved \$1980 and wishes to earn the remaining amount in interest on an investment. The best simple interest rate she can find is 7.2% p.a.



- For how long must Jacqui invest the money in order to earn the extra amount she needs?
- Jacqui wants to investigate ways to earn the money more quickly through a shorter investment. What interest rate does she need if she wishes to have the money in 18 months?
- What will the interest rate be if Jacqui wishes to have the money in 8 months?
- Consider the rates you found in parts b and c. What issues do you feel Jacqui may have in achieving these goals?

- 12** Banks offers various interest rates depending on the length of the investment and the frequency of the interest payment. Consider these options.

Option 1: 5.4% p.a. with interest paid at maturity

Option 2: 5.2% p.a. with interest paid monthly

- a** Consider how much interest is earned in each option if \$10 000 is invested for 1 month.
- b** Now consider how much interest is earned in each option if \$10 000 is invested for 3 months. For option 2, the monthly interest amounts are passed on to the investor, not added to the investment.

Now explore what happens to the value of investment in option 2 if the monthly interest amounts are added to the investment at the end of each month.

- c** What is the value of the investment after the first month's interest calculation? This value becomes the principal for the second month.
  - d** Using the answer from part **c**, how much interest is earned in the second month and what is the new value of the investment?
  - e** Using the new value from part **d**, how much interest is earned in the third month and what is the final value of the investment?
  - f** What is the total amount of interest earned on the investment using the methods explored in parts **c–e**?
  - g** Compare the amount of interest from part **f** with the total amount of interest you calculated for option 2 in part **b**. Are the amounts the same or different? Provide a brief explanation as to why this is the case.
- 13** In question **12**, the interest rates for the two options were different. Reconsider the calculations in parts **c–f** using the same interest rate as option 1, that is, 5.4% p.a.
- a** If the monthly interest amounts are added to the investment at the end of each month, what will the final value of the investment be after 3 months? (You may wish to use the parts of question **12** to assist you with this calculation.)
  - b** How does the total amount of interest using this method compare to the method of interest calculation at maturity? (Compare your answer with option 1 in **12b**.)
  - c** Consider the final value of the investment you calculated in part **a**. If planning to have your interest paid at maturity, what amount of money needs to be invested so that the interest earned is the same?
  - d** If the total amount of interest from part **a** is to be earned on an investment of \$10 000 for 3 months, with interest paid at maturity, what interest rate should be applied?

- 14** Consider each short-term investment. For each, calculate the total amount of interest earned if interest is calculated:
- i** at maturity
  - ii** monthly, with interest added to the investment at the end of each month.
- a** \$10 000 at 3.6% p.a. for 2 months
  - b** \$15 000 at 8.4% p.a. for 3 months
  - c** \$35 000 at 12.75% p.a. for 4 months
  - d** \$8400 at 21.5% p.a. for 3 months
- 15** Reconsider the amount of interest earned in each investment in question **14** with the monthly interest amounts added to the investment (part **ii**). To earn the same amount of interest, what interest rate should be applied to the principal if interest is calculated at maturity?

**Reflect**

Why are interest rates so important in financial calculations?

# 1E Understanding compound interest

## Start thinking!

For investments that receive simple interest, the investor is given the interest at the end of the investment term or at maturity. The principal and interest are combined to give the total value of the investment at the end. Loans involving simple interest calculations are performed in a similar way. The combined amount of the principal and interest is the total amount to be repaid.

- 1 Consider \$5000 invested at 6% p.a. for 3 years.
  - a Calculate the amount of simple interest that is earned on this investment.
  - b What is the value of the investment after 3 years?
- 2 What is the amount of interest earned on an investment of \$5000 at 6% p.a. for 1 year?
- 3 Consider the amount of simple interest earned in each of the 3 years of the investment.

Use your results from questions 1 and 2 to complete these.

- a Copy and complete this table.
  - b If you add the individual  $I$ -values for each year in the table, is this sum the same as the value you calculated in question 1a?
  - c What do your results show you about the amount of simple interest earned in each year of an investment?
- | Year | $P$ | $R$ | $I$ |
|------|-----|-----|-----|
| 1    |     |     |     |
| 2    |     |     |     |
| 3    |     |     |     |
- 4 Consider the investment of \$5000 at 6% p.a. for 3 years, but with interest calculated *during* the investment period at yearly intervals and added to the principal.
    - a How much interest is earned in the first year?
    - b Add the interest from part a to the principal. This new amount is the principal for the second year of the investment.
    - c Use the new principal value to calculate how much interest is earned during the second year.
    - d Add the interest from part c to the principal for the second year. This new amount is the principal for the third year.
    - e Use the new principal value to calculate how much interest is earned during the third (final) year of the investment.
    - f Add the interest from part e to the principal for the third year. This new amount is the final value of the investment.

- 5 Use your question 4 results to complete this table.

The method of interest calculation used in question 4 is known as **compound interest** and is commonly used in financial interest calculations. The regular intervals at which the interest is calculated are known as **compounding periods**.

Complete **1E DISCOVER** task to learn more.

Year	Principal at start of year	Interest rate (p.a.)	Interest earned	Principal at end of year
1				
2				
3				

<SEE OVERMATTER FOR KEY IDEAS>

## EXERCISE 1E Understanding compound interest

A &lt;numbers to come&gt;

B &lt;numbers to come&gt;

C &lt;numbers to come&gt;

UNDERSTANDING AND FLUENCY

- 1 For each investment, state the:
  - i number of compounding periods
  - ii time length of each compounding period.
- a \$5000 is invested at 4.5% p.a. for 2 years with interest calculated at the end of each year.
- b \$12 000 is invested at 9% p.a. for 3 years with interest calculated at the end of each year.
- c \$8500 is invested at 5% p.a. for 4 years with interest calculated at the end of each 6 months.
- d \$25 000 is invested at 8.4% p.a. for 2 years with interest calculated at the end of each month.

### EXAMPLE 1E-1

#### Calculating compound interest

\$15 000 is invested at 5% p.a. for 3 years with interest calculated at the end of each year and added to the principal amount. Calculate the total amount of interest compounded during this time.

#### THINK

- 1 This is compound interest, as interest is calculated and added to the principal at regular intervals.
- 2 Calculate the interest earned in the first year. Add this to the principal to determine the principal for the second year.
- 3 Calculate the interest earned in the second year. Add this amount to the principal to determine the principal for the third year.
- 4 Calculate the interest earned in the third year. Add this amount to the principal to determine the final value of the investment.
- 5 The amount of interest compounded is found by subtracting the initial principal from the final value.

#### WRITE

$$P = \$15\,000$$

$$R = 5\% \text{ p.a.} = 0.05$$

Interest is compounded yearly.

$$I = PRT = \$15\,000 \times 0.05 \times 1 = \$750$$

$$\begin{aligned} \text{new principal} &= \$15\,000 + \$750 \\ &= \$15\,750 \end{aligned}$$

$$\begin{aligned} I &= PRT = \$15\,750 \times 0.05 \times 1 \\ &= \$787.50 \end{aligned}$$

$$\begin{aligned} \text{new principal} &= \$15\,750 + \$787.50 \\ &= \$16\,537.50 \end{aligned}$$

$$\begin{aligned} I &= PRT = \$16\,537.50 \times 0.05 \times 1 = \$826.88 \\ \text{final value} &= \$16\,537.50 + \$826.88 = \$17\,364.38 \end{aligned}$$

$$\begin{aligned} \text{The total amount of interest compounded} \\ &= \$17\,364.38 - \$15\,000 = \$2\,364.38 \end{aligned}$$



- 2 In each investment, interest is calculated at the end of each year and added to the principal amount. Calculate the total amount of interest compounded during this time.

- a \$10 000 is invested at 5% p.a. for 2 years.
- b \$5000 is invested at 6% p.a. for 3 years.
- c \$25 000 is invested at 8% p.a. for 3 years.
- d \$40 000 is invested at 5% p.a. for 4 years.
- e \$6000 is invested at 4.8% p.a. for 3 years.
- f \$18 000 is invested at 7.2% p.a. for 4 years.

- 3 For each investment, calculate the amount of:

- i simple interest earned over the term of the loan
- ii compound interest earned if interest is compounded each year.

- a \$12 000 is invested at 4% p.a. for 2 years.
- b \$10 500 is invested at 5% p.a. for 3 years.
- c \$45 000 is invested at 6.2% p.a. for 3 years.
- d \$21 900 is invested at 8.5% p.a. for 4 years.

- 4 Mac is investigating ways to earn more interest on an investment. His bank offers him an interest rate of 5.2% p.a. with interest compounded annually. Mac has \$8500 to invest for 3 years.

- a From the given information:
  - i how many compounding periods will there be in Mac's investment
  - ii how long is each compounding period?
- b How much interest will Mac earn in the first year?
- c Add the interest from part b to the principal for the first year to determine the new amount for the principal for the second year.
- d How much interest will Mac earn in the second year?
- e Add the interest from part d to the principal for the second year to determine the new amount for the principal for the final year.
- f How much interest will Mac earn in the final year of his investment?
- g What will be the final value of Mac's investment?

- 5 For each of the investments shown in the table on the right, calculate the amount of interest earned if interest is compounded each year.



	Principal	Interest rate (p.a.)	Term
a	\$8 400	12%	3 years
b	\$12 000	5.4%	4 years
c	\$25 000	3.75%	3 years
d	\$4 590	9.5%	4 years
e	\$150 000	6.25%	2 years
f	\$35 500	15.8%	5 years

- 6** Pitia plans to invest his savings from a part-time job. He invests \$5000 with his bank at 4.65% p.a. compounded annually for 3 years.
- Calculate the interest earned in the first year of the investment and use this to determine Pitia's principal amount for the second year of the investment.
  - Calculate the interest earned in the second year of the investment and use this to determine the principal for the third year of the investment.
  - Calculate the interest earned in the third year of the investment and use this to determine the final value of the investment.
- 7** Reconsider the information from question **6**. If Pitia had taken a loan for \$5000 from his bank with interest charged at 4.65% p.a. and compounded annually for 3 years, how would this new situation compare?
- 8** To support her new tennis-coaching business, Lois borrows \$80 000 from her bank. She agrees to an interest rate of 6.5% p.a. compounded annually for 4 years.
- State the number of compounding periods and their length.
  - What is the total amount of money that Lois must repay on her loan?
  - What is the total amount of interest?
  - If Lois agrees to repay the amount in equal monthly instalments, how much does she pay each month?
- 9** Laura is investigating the best option for her savings plan. She plans to invest \$14 500 for 2 years and wants to find out whether she would be better off investing it with interest calculated as simple interest or compound interest. The rates are:
- Simple interest: 4.75% p.a. interest paid at maturity  
Compound interest: 4.75% p.a. interest paid annually.
- For the simple interest option:
    - how much interest is earned on the investment?
    - what is the final value of the investment?
  - For the compound interest option:
    - how much interest is earned on the investment?
    - what is the final value of the investment?
  - Laura's friend tells her that the interest for both options should be the same because they both have the same interest rate. How accurate is this claim?
  - Consider the interest calculations for both options in parts **a** and **b**. Compare the amount of interest for the two options for each year. Predict what will happen to the interest earned if the investment continues for a third year at the same rate.
  - Which option should Laura be advised to take?
- 10** Reconsider the information provided in question **9**. If Laura's principal now represents a loan amount, which option would you advise Laura to take? Provide a brief reason.

- 11 Frank plans to invest his money for the next 3 years, then use the final amount as a deposit for a house. He has \$13 500 to invest and is considering these two options.

Option 1: Interest rate of 6.7% p.a. interest calculated at maturity

Option 2: Interest rate of 6.5% p.a. interest compounded annually

- What will the value of Frank's investment be at the end of the term using option 1?
- What will the value of Frank's investment be at the end of the term using option 2?
- If the compounded rate remains the same, what rate must be applied to option 1 to make this the best option? Provide working to support your answer.



- 12 Banks sometimes advertise their investment rates according to the amount of money invested, length of the investment or the frequency of the interest payments. These tables display interest rates offered by a bank on term deposits of any size.

Interest paid monthly	
Term	Interest rate on amount invested (p.a.)
1 to less than 2 months	2.35%
2 to less than 3 months	2.35%
3 to less than 4 months	4.6%
4 to less than 5 months	3.05%
5 to less than 6 months	5.0%

Interest paid at maturity	
Term	Interest rate on amount invested (p.a.)
1 to less than 2 months	2.55%
2 to less than 3 months	2.55%
3 to less than 4 months	4.8%
4 to less than 5 months	3.25%
5 to less than 6 months	5.2%

- a Which table appears to use compound interest as the interest calculation?
  - b Using the tables, what interest rate is earned on an investment for:
    - i 3 months with interest compounded monthly
    - ii 4 months with interest paid at maturity only.
  - c A bank customer wishes to invest \$7500 for 4 months. Calculate the amount of interest earned on the investment with interest:
    - i paid at maturity only
    - ii compounded monthly.
  - d Which option would you advise this customer to take? Explain your reasoning.
- 13** Consider an investment of \$10 000 for 2 years at 4.4% p.a. compounded annually.
- a What is the total value of the investment at the end of the term?  
Imagine the terms of the investment were altered so the interest was compounded 6-monthly instead of annually.
  - b How many compounding periods are there now?
  - c Calculate the total value of the investment at the end of the term?  
(Hint: there are four calculations, and use  $\frac{1}{2}$  as the  $T$  value.)
  - d Compare the final value of the investment using the two methods of calculation. Which method resulted in the greater value? Why do you think this is so?

**Reflect**

What is the difference between simple interest and compound interest?

# 1F Working with the compound interest formula

## Start thinking!

Leroy has decided to invest the money saved from his part-time job. He has \$16 000 and decides to invest it for 3 years in the hope it will grow enough to enable him to buy a new car. Leroy is investigating whether it is best to have the interest paid at maturity or at regular intervals during his investment.

- 1 The first investment option offers Leroy an interest rate of 6.5% p.a. with interest paid at maturity.
  - a Why is this an example of simple interest rather than compound interest?
  - b Calculate the amount of interest Leroy will earn.
  - c What will the value of the investment be at the end of the term?
- 2 The second investment option also offers Leroy an interest rate of 6.5% p.a., but with interest compounded annually.
  - a Why is this an example of compound interest rather than simple interest?
  - b Calculate the interest earned in the first year of the investment and use this to determine the principal for the second year of the investment.
  - c Calculate the interest earned in the second year of the investment and use this to determine the principal for the third year of the investment.
  - d Calculate the interest earned in the third year of the investment and use this to determine the final value of the investment.
  - e What is the total interest earned for this option? Briefly explain how it can be calculated.
- 3 The compounding calculations completed in question 2 follow a pattern that can assist you to develop a rule used for compound interest calculations.
  - a This table shows how the compounded values for each year are obtained. Copy and complete the table. The first row has been completed for you.
  - b Describe the pattern formed by the values in the table. How does each value relate to the information given in the compound interest option?

End of year	Compounded value at the end of each year	Alternative form	Value
1	Increase \$16 000 by 6.5% $\$16\,000 \times 1.065$	$\$16\,000 \times 1.065^1$	\$17 040
2	Increase the first year's compounded value by 6.5% $(\$16\,000 \times 1.065) \times 1.065$	$\$16\,000 \times 1.065^2$	
3	Increase the second year's compounded value by 6.5% $(\$16\,000 \times 1.065 \times 1.065) \times 1.065$	$\$16\,000 \times 1.065^3$	

Complete **1F DISCOVER** task to learn more.

## KEY IDEAS

- Compound interest can be calculated using the formula:  $A = P(1 + R)^n$ , where
  - ▷  $A$  = compounded value of the investment (the final amount)
  - ▷  $P$  = principal (the initial investment amount or amount borrowed)
  - ▷  $R$  = interest rate for the compounding period, written as a decimal
  - ▷  $n$  = number of compounding periods.
- The compound interest is the difference between the final amount and the initial amount:
 
$$\text{compound interest} = A - P.$$

<5 LINES  
TOO DEEP>

## EXERCISE 1F Working with the compound interest formula

A &lt;numbers to come&gt;

B &lt;numbers to come&gt;

C &lt;numbers to come&gt;

UNDERSTANDING AND FLUENCY

- 1 For each investment, state the number of compounding periods,  $n$ .
  - a \$5000 is invested at 6.7% p.a. for 4 years with interest calculated annually.
  - b \$8450 is invested at 3.9% p.a. for 3 years with interest calculated at the end of each year.
  - c \$25 000 is invested at 8.5% p.a. for 2 years with interest calculated at the end of each 6 months.
  - d \$35 000 is invested at 9.6% p.a. for 4 years with interest calculated at the end of each month.
- 2 Use the compound interest formula to find the value of  $A$  in each of these.
  - a  $A = ?$ ,  $P = \$10\ 000$ ,  $R = 0.05$ ,  $n = 4$
  - b  $A = ?$ ,  $P = \$5000$ ,  $R = 0.06$ ,  $n = 3$
  - c  $A = ?$ ,  $P = \$600$ ,  $R = 0.08$ ,  $n = 4$
  - d  $A = ?$ ,  $P = \$44\ 000$ ,  $R = 0.038$ ,  $n = 5$
  - e  $A = ?$ ,  $P = \$100\ 000$ ,  $R = 0.015$ ,  $n = 8$
  - f  $A = ?$ ,  $P = \$19\ 500$ ,  $R = 0.0025$ ,  $n = 10$

### EXAMPLE 1F-1

#### Using the compound interest formula to find $A$

For an investment of \$12 000 at an interest rate of 4.8% p.a. for 4 years with interest compounded annually, calculate:

- a the value of the investment after 4 years
- b the amount of compound interest earned.

#### THINK

- a 1 Write the formula to use and identify the key terms. Interest is compounded annually, so there are four compounding periods.
- 2 Substitute the values into the formula and calculate the result. Round the answer to two decimal places.
- 3 Write the answer.
- b The amount of compound interest earned is the difference between the final amount and the initial amount.

#### WRITE

$$\begin{aligned}
 \text{a } A &= P(1 + R)^n \\
 A &= ? \\
 P &= \$12\ 000 \\
 R &= 4.8\% \text{ p.a.} = \frac{4.8}{100} = 0.048 \\
 n &= 4
 \end{aligned}$$

$$\begin{aligned}
 A &= \$12\ 000(1 + 0.048)^4 \\
 &= \$12\ 000 \times 1.048^4 \\
 &= \$14\ 475.260\ 12 \\
 &= \$14\ 475.26
 \end{aligned}$$

The value of the investment after 4 years is \$14 475.26.

$$\begin{aligned}
 \text{b } \text{compound interest} &= A - P \\
 &= \$14\ 475.26 - \$12\ 000 \\
 &= \$2475.26
 \end{aligned}$$



- 3 For each investment, calculate:
- the value of the investment at the end
  - the amount of compound interest earned.
- an investment of \$7500 at 5% p.a. for 3 years with interest compounded annually
  - an investment of \$9000 at 4.5% p.a. for 3 years with interest compounded annually
  - an investment of \$22 500 at 6.8% p.a. for 5 years with interest compounded annually
  - an investment of \$40 000 at 7.3% p.a. for 4 years with interest compounded annually

- 4 For each investment, use the compound interest formula to calculate the final value of the investment if interest is compounded annually.

- 5 Calculate the amount of interest earned in each investment listed in the table in question 4.

	Principal	Interest rate (p.a.)	Term
a	\$3 200	6.4%	4 years
b	\$4 590	9.8%	5 years
c	\$16 000	2.85%	2 years
d	\$25 900	10.5%	3 years
e	\$100 000	4.99%	5 years
f	\$255 000	12.15%	6 years

- 6 For each investment:
- write the given interest rate so that it matches the compounding period
  - state the value for  $R$  and  $n$  to be used in the compound interest formula.
- \$15 000 is invested at 4.8% p.a. for 3 years with interest calculated quarterly.
  - \$22 000 is invested at 5.9% p.a. for 4 years compounded 6-monthly.
  - \$3500 is invested at 8.4% p.a. for 2 years with interest calculated monthly.
  - \$14 500 is invested at 9.6% p.a. for 5 years with interest calculated at the end of each month.

- 7 Use the compound interest formula to determine the value for  $A$  given the information provided in the table.

	Principal	Interest rate (p.a.)	Term	Compounding period
a	\$4800	8.15%	3 years	annually
b	\$9500	8.8%	4 years	6-monthly
c	\$18 000	4.84%	5 years	quarterly
d	\$35 000	16.8%	4 years	monthly
e	\$50 000	6.95%	3 years	6-monthly
f	\$150 000	8.16%	1 year	quarterly
g	\$500 000	4.98%	2 years	4-monthly
h	\$750 000	10.26%	3 years	2-monthly

- 8 Maria and her family begin long-term planning for an overseas holiday. They invest their savings of \$16 000 for 3 years in an account with their bank, which offers them 5.75% p.a. interest compounded annually.
- a How many compounding periods will there be in Maria's investment?
  - b What is the length of each compounding period?
  - c Identify the values for  $P$ ,  $R$  and  $n$ .
  - d Use the compound interest formula to calculate the final value of the investment.
  - e How much interest will be earned on this investment?
  - f Maria estimates that she will need \$20 000 to fund the entire overseas holiday. If she invests the money for another year, will she achieve this goal?
- 9 Carolyn borrows \$45 000 to set up a dance studio for her daughter. The loan is over 4 years with interest charged at 8.95% p.a. compounding annually. The terms of the loan require Carolyn to repay the total amount in equal monthly instalments.
- a Use the compound interest formula to calculate the total amount that Carolyn must repay.
  - b How much of the amount in part a represents the interest charge?
  - c What is the value of each of Carolyn's monthly repayment?
- 10 While investigating different investment options with his bank, Bailey decided on two possibilities to further explore. The two options are as displayed.
- Interest rate of 5.35% p.a. with interest paid at maturity
- Interest rate of 4.95% p.a. interest compounded annually
- Bailey has \$9000 to invest and starts out planning to invest the money for 3 years.
- a What would be the value of the investment at the end of the term for each option?
  - b Which option would Bailey be advised to take for a 3-year investment? Provide a supporting reason with your answer.
  - c How do the answers to parts a and b change if Bailey invests the money for 5 years?

- 11** Sheetal is exploring the benefits of investing with an online bank. She finds the bank will pay 8.6% p.a. interest on investments over 36 months with interest compounded quarterly. Sheetal decides to invest her savings of \$8400 for 5 years.



- a** Write the annual interest rate as a rate per quarter.
  - b** Use your answer from part **a** to state the value for  $R$  in the compound interest formula.
  - c** How many compounding periods,  $n$ , will there be in this investment?
  - d** Use the compound interest formula to determine the final value of the investment.
  - e** What is the total amount of interest Sheetal will expect to earn on this investment?
- 12** An investment of \$10 000 is made for 4 years with a bank that offers an annual interest rate of 8.4%. Explore how the frequency of the compounding periods affects the final value of the investment.
- a** Calculate the final value of the investment if the interest is calculated at the end of the term, that is, at maturity.
  - b** Calculate the final value of the investment if the interest is compounded:
    - i** 6-monthly
    - ii** quarterly
    - iii** 2-monthly
    - iv** monthly
  - c** Use your results from part **b** to briefly explain what happens to the final value of an investment as the number of compounding periods increases. Will this always be the case?
- 13** Through a store purchase plan, Troy is informed that interest is calculated as compound interest based on the amount borrowed. In this situation, would Troy be better off with a plan involving a higher frequency of compounding periods or fewer compounding periods?
- 14** Erica knows she can earn 6.8% p.a. for her investments with her bank. She plans to invest some money for 2 years with interest compounded quarterly.
- a** For this investments, state:
    - i** the number of compounding periods,  $n$
    - ii** the value for  $R$  to be used in the compound interest formula.
  - b** At the end of her 2-year investment, Erica hopes to have an investment that has grown to \$6500. Which variable of the compound interest formula does this value represent?
  - c** Substitute the known values into the compound interest formula and solve the equation for the unknown variable.
  - d** To the nearest dollar, how much should Erica invest at the start of her investment?

- 15** Consider an investment of \$5000 at 5% p.a. for 5 years with interest compounded annually.
- a** How much interest is earned in each year of the investment?
  - b** What is the value of the investment at the end of each year during this term?
  - c** Present the growth of the investment as a line graph displaying the years on the horizontal axis and the value of the investment on the vertical axis. You may wish to use digital technology to present your graph.
  - d** Repeat parts **a–c** with the same values, but for these compounding periods:
    - i** quarterly      **ii** 6-monthly      **iii** monthly.
  - e** If the interest for this investment was calculated using simple interest, briefly explain how the value of the investment could be shown on a graph.
  - f** How will a graph displaying the investment's growth using simple interest compare with the compound interest graphs? You may wish to present all graphs on the one set of axes.

**Reflect**

Why is it important that the interest rate used in the compound interest formula matches the compounding period?

# 1G Working with compound interest

## Start thinking!

Manisha received a flyer from her bank encouraging her to invest in a savings account so that her money would increase in value more quickly. Manisha agreed to invest \$14 000 in an account for 2 years at an interest rate of 4.8% p.a. with interest compounded every 3 months.

- 1
  - a How many compounding periods will there be in each year of Manisha's investment?
  - b How many compounding periods will there be for the entire term of Manisha's investment?
  - c The answer obtained in part **b** represents the value for  $n$  in the compound interest formula. Develop a rule that can be used to determine the value for  $n$  for any compound interest situation.
- 2 The value for  $R$  must match the compounding period before it can be used in the compound interest formula.
  - a What is the annual interest rate that is applied to this investment?
  - b As the interest is compounded every 3 months (quarterly), what needs to be done to the annual interest rate to match the compounding period?
  - c Use your response from part **b** to write the interest rate as a percentage per quarter.
  - d Convert the percentage to a decimal value.
  - e The answer obtained in part **d** represents the value for  $R$  in the compound interest formula. Develop a rule that can be used to determine the value for  $R$  for any compound interest situation.
- 3
  - a From the compound interest formula, which variable do you not know the value of and what is the value for each of the known variables?
  - b Use the formula to calculate the value of the missing variable.
  - c What does your answer from part **b** represent in relation to Manisha's investment?
  - d How much compound interest is to be earned over the term of the investment?
- 4 How could you use the compound interest formula to find the value of  $P$  (when given  $A$ ,  $R$  and  $n$ ) or to find  $R$  (when given  $A$ ,  $P$  and  $n$ )?

Complete **1G DISCOVER** task to learn more.



## KEY IDEAS

- ▶ The compound interest formula is  $A = P(1 + R)^n$ .
- ▶ The amount of compound interest =  $A - P$ .

## EXERCISE 1G Working with compound interest

A &lt;numbers to come&gt;

B &lt;numbers to come&gt;

C &lt;numbers to come&gt;

UNDERSTANDING AND FLUENCY

- 1 For each investment, state:
    - i the number of compounding periods,  $n$
    - ii the value for  $R$  to be used in the compound interest formula.
  - a \$12 000 is invested at 3.8% p.a. for 4 years with interest calculated annually.
  - b \$9500 is invested at 6.4% p.a. for 3 years with interest calculated each 6 months.
  - c \$30 000 is invested at 12.8% p.a. for 2 years with interest calculated quarterly.
  - d \$42 000 is invested at 8.4% p.a. for 4 years with interest calculated each month.
- 2 Calculate the value of  $A$  for each investment in question 1.

### EXAMPLE 1G-1

Using the compound interest formula to find  $P$ 

How much must be invested at an interest rate of 6.8% p.a. in order for the value to grow to \$5000 over 3 years, with interest compounded 6-monthly?

#### THINK

- 1 Write the compound interest formula and identify the variables. You need to find the initial value (principal,  $P$ ) that will grow to \$5000 (final amount,  $A$ ).
- 2 Substitute the values into the formula and simplify.
- 3 Solve for  $P$  by dividing both sides of the equation by  $1.034^6$ .
- 4 Write the answer.

#### WRITE

$$\begin{aligned}
 A &= P(1 + R)^n \\
 A &= \$5000 \\
 n &= 2 \times 3 = 6 \\
 R &= 6.8\% \text{ p.a.} = 3.4\% \text{ per 6 months} = 0.034 \\
 P &= ?
 \end{aligned}$$

$$\begin{aligned}
 5000 &= P(1 + 0.034)^6 \\
 &= P \times 1.034^6
 \end{aligned}$$

$$\begin{aligned}
 \frac{5000}{1.034^6} &= \frac{P \times 1.034^6}{1.034^6} \\
 P &= 4091.16
 \end{aligned}$$

\$4091.16 will grow to \$5000 over 3 years at 6.8% p.a. compounded 6-monthly.



- 3 Find the value for  $P$  in each of these.
  - a How much must be invested at an interest rate of 10.5% p.a. in order for the value to grow to \$30 000 over 4 years with interest compounded annually?
  - b How much must be invested at an interest rate of 5.4% p.a. in order for the value to grow to \$10 000 over 3 years with interest compounded 6-monthly?
  - c How much must be invested at an interest rate of 6.4% p.a. in order for the value to grow to \$5500 over 5 years with interest compounded quarterly?
  - d How much must be invested at an interest rate of 7.2% p.a. in order for the value to grow to \$8000 over 2 years with interest compounded every month?
  
- 4 Find the missing value in each of these.
 

a $A = ?$ , $P = \$20\,000$ , $R = 0.08$ , $n = 4$	b $A = \$30\,000$ , $P = ?$ , $R = 0.045$ , $n = 6$
c $A = \$15\,000$ , $P = ?$ , $R = 0.034$ , $n = 8$	d $A = ?$ , $P = \$6000$ , $R = 0.0175$ , $n = 4$
  
- 5 Angelina has invested \$5800 into a savings account for 3 years with her bank at an interest rate of 6.2% p.a. with interest compounded quarterly.
  - a How many compounding periods will there be?
  - b What is the interest rate per compounding period?
  - c Identify the values for  $P$ ,  $R$  and  $n$ .
  - d Use the compound interest formula to calculate the final value of the investment.
  - e How does the final value of Angelina's investment compare with the same investment given the same rate of simple interest?
  
- 6 Determine the number of compounding periods,  $n$ , if interest is compounded:
 

a annually for 3 years	b 6-monthly for 5 years
c quarterly for 4 years	d monthly for 5 years
e quarterly for 4 years and 6 months	f 6-monthly for $3\frac{1}{2}$ years
  
- 7 Determine the value for the interest rate per compounding period, if the annual percentage rate (p.a.) is:
  - a 8.4% with interest compounded quarterly
  - b 9.06% with interest compounded monthly
  - c 7.5% with interest compounded every 4 months
  - d 4.62% with interest compounded 6-monthly
  - e 3.5% with interest compounded monthly
  - f 9.15% with interest compounded 6-monthly
  
- 8 Briefly explain how you can use the compound interest formula to determine the value for  $P$ , when given the value for  $A$ ,  $R$  and  $n$ .

- 9 Use the compound interest formula to determine the value for  $P$ , to the nearest dollar, given the information provided in the table.

	Final value (compounded value)	Interest rate (p.a.)	Term	Compounding period
a	\$5 200	6.75%	3 years	annually
b	\$10 000	4.2%	6 years	monthly
c	\$22 000	12.5%	4 years	6-monthly
d	\$16 650	15.5%	3 years	quarterly
e	\$225 000	9.95%	$4\frac{1}{2}$ years	6-monthly

- 10 How much interest is earned in each case in question 9?

- 11 Consider an investment of \$12 000 that grows to \$15 000 over a 3-year period with interest compounded annually.

- From the compound interest formula, which variable do you not know the value of?
- Substitute the known values into the compound interest formula and show that it simplifies to  $\frac{5}{4} = (1 + R)^3$ .
- To solve the equation for  $R$ , you can perform inverse operations to both sides of the equation. (Note that the inverse operation to cubing a number is to take the cube root of a number.)

Copy and complete the following to solve the equation for  $R$ .

$$\frac{5}{4} = (1 + R)^3$$

$$1.25 = (1 + R)^3$$

$$(1 + R)^3 = 1.25$$

$$\sqrt[3]{(1 + R)^3} = \sqrt[3]{\quad}$$

$$1 + R = \sqrt[3]{\quad}$$

$$1 + R - 1 = \sqrt[3]{1.25} - \quad$$

$$R = 0.077\ 217 \dots$$

- The value for  $R$  in part c is the rate per compounding period as a decimal. State this as an annual rate.

- 12 Jade's parents invested \$5500 for 4 years into a savings account and it grew to \$8000, with interest compounding annually.

- State the values for  $A$ ,  $P$  and  $n$ .
- Use the compound interest formula to solve for  $R$ .
- What annual interest rate was applied to Jade's savings account?

- 13** Use the compound interest formula to determine:

- i** the value for  $R$  (as a decimal)
- ii** the annual interest rate for the information given in the table.

	Principal	Final value	Term	Compounding period
<b>a</b>	\$1 000	\$1 500	3 years	annually
<b>b</b>	\$8 000	\$12 000	4 years	annually
<b>c</b>	\$16 000	\$19 000	2 years	6-monthly
<b>d</b>	\$3 500	\$4 000	1 year	quarterly
<b>e</b>	\$12 000	\$16 500	3 years	6-monthly
<b>f</b>	\$25 000	\$29 500	2 years	quarterly

- 14** Riley aims to have \$15 000 by the time he turns 18 so that

he can buy a car. He opens a savings account with his bank that pays interest at a rate of 7.2% p.a. compounded monthly. Riley plans to keep the money invested on these terms for 3 years and needs to know how much he should invest.

- a** How many compounding periods,  $n$ , will there be in Riley's investment?
- b** What is the interest rate per compounding period? Hence, state the value of  $R$ .
- c** From the compound interest formula, which variable do you not know the value of?
- d** Use the compound interest formula to calculate the amount of money Riley must invest in order for it to grow to \$15 000 over the 3-year term.

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- 15** When the compounding periods are shorter than usual, what must be done to the  $R$  value to determine the annual rate of interest?

- 16** An investment of \$25 000 grows to \$30 000 over a period of time in a savings account that earns 5.0% p.a. with interest compounded annually.

- a** From the compound interest formula, which variable do you not know the value of?
- b** Substitute the known values into the compound interest formula and show that it simplifies to  $1.2 = (1.05)^n$ .
- c** One strategy for solving the equation for  $n$  is the guess, check and improve method. Substitute different values for  $n$  and evaluate the right side until the right side value is greater than the left side value. This value for  $n$  represents the solution to the equation.

Value for $n$	Left side	Right side	Comment
1	1.2	$1.051 = 1.05$	Right side value is too small
2	1.2	$1.052 = 1.1025$	
3	1.2	$1.053 =$	
4	1.2	$1.054 =$	

- d** The value for  $n$  obtained in part **c** is the number of compounding periods for the investment. Briefly explain how this relates to this investment.

- 17** For a period of time, Yousef watched his savings grow from \$850 to \$1100 in a savings account that earns 4.2% p.a. with interest compounded annually.
- Use the given information to state the values for  $A$ ,  $P$  and  $R$ .
  - Substitute the given values into the compound interest formula and simplify.
  - Use the guess, check and improve method to solve the equation for  $n$ .  
You may wish to use a similar table to the one used in the previous question.
  - How many compounding periods did it take for Yousef's savings to grow from \$850 to \$1100?
- 18** For the information given in the table, state the number of compounding periods,  $n$ , for the initial principal to grow into the final value.

	Principal	Final value	Interest rate (p.a.)	Compounding period
<b>a</b>	\$900	\$1200	5.0%	annually
<b>b</b>	\$12 000	\$14 000	3.5%	annually
<b>c</b>	\$15 000	\$18 500	4.8%	6-monthly
<b>d</b>	\$650	\$900	10.5%	quarterly
<b>e</b>	\$35 000	\$42 000	6.8%	6-monthly
<b>f</b>	\$50 000	\$55 590	7.2%	quarterly

- 19** Considering each investment in question 18, write each compounding period in years.
- 20** In question 16 you were introduced to a method of guess, check and improve as a strategy to determine the value of  $n$  in the compound interest formula. Investigate other strategies that may be used to solve for  $n$ . You may wish to re-use the information in questions 16–18 as part of your investigations.  
You may wish to write a brief report on your investigations using examples to demonstrate your findings.  
Hint: equations such as  $1.2 = (1.05)^n$ , where the pronumeral is in the power, are known as exponential equations.

**Reflect**

What other areas in mathematics can the compound interest formula be applied to?

# CHAPTER REVIEW

## SUMMARISE

Create a summary of this chapter using the key terms below. You may like to write a paragraph, create a concept map or use technology to present your work.

percentage	discount	interest	loan
fraction	percentage decrease	simple interest	compound interest
decimal	mark-up	principal	compounding period
percentage of an amount	percentage increase	interest rate	compounded value
one amount as a	profit	time	
percentage of another	loss	investment	

## MULTIPLE-CHOICE

- 1A** 1 What is  $22\frac{1}{2}\%$  of \$284.95, rounded to the nearest cent?  
 A \$63.26      B \$64.10  
 C \$64.11      D \$262.45
- 1B** 2 What is the selling price following a discount of 10.5% on \$120?  
 A \$12.60      B \$107.40  
 C \$109.50      D \$132.60
- 1C** 3 An amount of \$1500 is borrowed at 6.75% p.a. simple interest for 24 months. Which set of values should be substituted into the simple interest formula?  
 A  $P = 1500, R = 6.75, T = 24$   
 B  $P = 1500, R = 6.75, T = 2$   
 C  $P = 1500, R = 0.0675, T = 2$   
 D  $P = 1500, R = 0.0675, T = 24$
- 1D** 4 What is the annual interest rate that is applied to a loan of \$20 000 taken over 3.5 years if the amount of simple interest charged on the loan is \$2975?  
 A 0.0425%      B 1.92%  
 C 4.25%      D 14.875%
- 1E** 5 An investment of \$10 000 for 4 years earns interest at 8.4% p.a. compounded annually. What is the value of the investment after 2 years?  
 A \$11 750.56      B \$11 680  
 C \$12 737.61      D \$13 807.57
- 1F** 6 For an investment of \$4800 for 3 years at 3.6% p.a. with interest compounded monthly, what is the number of compounding periods?  
 A 3      B 12      C 15      D 36
- 1G** 7 Over a 4-year period, an investment grew from \$18 500 to \$27 000 with interest compounded quarterly. From the compound interest formula, which variable do you not know the value of?  
 A  $A$       B  $P$       C  $R$       D  $n$
- 1G** 8 The annual interest rate on an investment compounded quarterly is 8.25% p.a. What is the interest rate per compounding period?  
 A 8.25%      B 2.0625%  
 C 0.0825%      D 0.020 625%

## SHORT ANSWER

- 1A** ▶ **1** Write each percentage as a decimal.  
**a** 24% **b** 35.8%  
**c**  $16\frac{1}{4}\%$  **d** 0.272%
- 1A** ▶ **2** Write each percentage as a fraction in simplest form.  
**a** 86% **b** 235% **c**  $15\frac{1}{5}\%$   
**d** 8.65% **e**  $13\frac{2}{3}\%$  **f** 0.0512%
- 1A** ▶ **3** Calculate each of these.  
**a** 15% of \$685 **b** 170% of \$9500  
**c**  $17\frac{1}{2}\%$  of \$98.50  
**d** 18.65% of \$1340.80
- 1A** ▶ **4** **a** Write \$85 as a percentage of \$680.  
**b** Write \$39.95 as a percentage of \$72.50, rounding your answer to two decimal places.
- 1A** ▶ **5** A CD is originally priced at \$23.95 and sold at a sale for \$12.45. Write the selling price as an exact percentage of the original price.
- 1B** ▶ **6** The following parts **a–d** list the original prices and the percentage discount or percentage mark-up on some goods. In each case, calculate:  
**i** the selling price after the discount or mark-up  
**ii** the discount or mark-up amount, rounding to the nearest cent.  
**a** \$2550;  $15\frac{1}{2}\%$  discount  
**b** \$899.50; 125% mark-up  
**c** \$18 045; 38.4% mark-up  
**d** \$1945.90; 37.5% discount
- 1B** ▶ **7** For each of these:  
**i** state the value of the profit or loss  
**ii** write the profit or loss as a percentage of the original price (rounded to the nearest 1%).  
**a** original price \$139.85, selling price \$171.30  
**b** original price \$27 490, selling price \$9813.95
- 1B** ▶ **8** A furniture store offers a lounge suite for sale at a discount of  $25\frac{1}{2}\%$  to encourage a quick sale. After the discount, the sale price was \$1847.60. What was the original price of the lounge suite prior to the discount?
- 1C** ▶ **9** Calculate the amount of simple interest if:  
**a**  $P = \$2500$ ,  $R = 6\%$ ,  $T = 3$  years  
**b**  $P = \$28\ 000$ ,  $R = 4.25\%$ ,  $T = 5$  years  
**c**  $P = \$13\ 500$ ,  $R = 8.2\%$  p.a.,  $T = 32$  months
- 1C** ▶ **10** For each investment, calculate:  
**i** the amount of simple interest earned  
**ii** the value of the investment at the end.  
**a** An investment of \$150 000 at an interest rate of 4.8% p.a. for  $3\frac{1}{2}$  years.  
**b** An investment of \$9450 at an interest rate of 2.95% p.a. for 5 months.
- 1C** ▶ **11** To fund the purchase of a new laptop, a customer agrees to a loan of \$1985 at 10.25% p.a. simple interest over 20 months.  
**a** How much interest is charged on the loan?  
**b** If the total amount is to be repaid in equal monthly instalments, how much is each payment?
- 1D** ▶ **12** Find the missing value in each of these. Remember to write the value for  $R$  as a percentage, and time in years.  
**a**  $I = \$308$ ,  $P = \$3850$ ,  $R = 6.4\%$ ,  $T = ?$   
**b**  $I = \$765$ ,  $P = ?$ ,  $R = 4.25\%$ ,  $T = 12$  months  
**c**  $I = \$9843.75$ ,  $P = 35\ 000$ ,  $R = ?$ ,  $T = 2.5$  years

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## MIXED PRACTICE

- What is  $25\frac{1}{2}\%$  written as a decimal?
- The percentage value  $12\frac{1}{5}\%$  is equivalent to which fraction in simplest form?  
 A  $\frac{1}{5}$       B  $12\frac{1}{5}$       C  $\frac{61}{500}$       D  $\frac{61}{5}$
- Which percentage is closest in value to  $\frac{9}{11}$ ?  
 A 80%      B 81%      C 82%      D 83%
- Jaymee has saved \$112 towards an iPod that retails for \$248. The amount she has saved as a percentage of the amount she requires is closest to which value?  
 A 45%      B 55%      C 221%      D \$136
- The calculation of  $17\frac{3}{4}\%$  of \$190.70 can be performed by completing which one of these?  
 A  $17.75 \times \$190.70$       B  $17.34 \times \$190.70$   
 C  $0.1775 \times \$190.70$   
 D  $17.75 \div 190.70 \times \$100$
- During a sale, a store offers computer accessories at a discount of 22.5%. An external hard drive is originally priced at \$145.95. Which of these shows the calculation for the after-discount sale price?  
 A 22.5% of \$145.95      B 122.5% of \$145.95  
 C 77.5% of \$145.95  
 D  $\$145.95 - 77.5\% \text{ of } \$145.95$

Questions 7 and 8 refer to this information.  
 A men's clothing store applies a mark-up of 115.25% to the wholesale price of their clothing to determine the selling price to the public.

- A pair of jeans has a wholesale price of \$85. What is the mark-up amount on these jeans to the nearest dollar?
- Which shows the correct method to calculate the selling price of the jeans?  
 A  $\$85 + 15.25\% \text{ of } \$85$   
 B  $\$85 + 215.25\% \text{ of } \$85$   
 C  $1.1525 \times \$85$       D  $215.25\% \text{ of } \$85$
- Vanessa sells her jewellery at the market for \$25 per piece. This price includes her profit margin of 150%, applied to her cost price. What is the cost price to Vanessa of each jewellery piece?

- \$10 000 is invested at 6.5% p.a. simple interest for 18 months. Which set of values should be substituted into the simple interest formula?  
 A  $P = 10\ 000, R = 6.5, T = 1.5$   
 B  $P = 10\ 000, R = 0.065, T = 1.5$   
 C  $P = 10\ 000, R = 6.5, T = 18$   
 D  $P = 10\ 000, R = 0.065, T = 18$

Questions 11 and 12 refer to this information.  
 A loan of \$15 750 is charged a simple interest rate of 9.8% p.a. for a period of 2 years.

- How much interest is charged to the loan?
- What is the total amount to be repaid?
- Bryce invests \$1845 in a bank that offers him a simple interest rate of 4.5% p.a. He hopes for this investment to grow to at least \$2000. From the simple interest formula, which variable do you not know the value of?  
 A  $I$       B  $P$       C  $R$       D  $T$
- How much needs to be invested at an interest rate of 5.25% p.a. for 3 years to earn \$1500 in simple interest?

Questions 15–18 refer to this information.  
 An investment of \$25 000 is made with a bank that offers 4.75% interest compounded 6-monthly. The investment is made for 4 years.

- How many compounding periods are there for the term of the investment?
- Which represents the value used for  $R$  in the compound interest formula?  
 A 4.75      B 0.0475  
 C 0.023 75      D 0.011 875
- Which calculation gives the final value of the investment at the end of the term?  
 A  $A = \$25\ 000(1.023\ 75)^4$   
 B  $A = \$25\ 000(1.023\ 75)^8$   
 C  $A = \$25\ 000(1.0475)^4$   
 D  $A = \$25\ 000(1.0475)^8$
- How much interest is compounded over the term of the investment?

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**19** Missy borrows \$4500 from a bank who charges her interest at a rate of 6.6% p.a. with interest compounded monthly. The term of the loan is 2 years. Which statement is *not* correct?

- A** There are 24 compounding periods for the loan
- B** The rate per compounding period is 0.55%
- C** The amount of interest charged on the loan is \$594
- D** The value at the end of the investment is close to \$5130

Questions **20** and **21** refer to this information.

An investment of \$5850 is made for 4 years with a bank that offers an interest rate of 6.45% p.a. with interest compounded annually.

- 20** What is the value of the investment after the second year?
- 21** How much interest is earned through the entire term of the loan?
- 22** Consider an investment of \$16 000 that grows to \$20 000 over a 3-year period with interest compounded annually. From the compound interest formula, which variable do you not know the value of?

- A**  $A$       **B**  $P$       **C**  $R$       **D**  $n$

## ANALYSIS

Susan made an investment of \$12 000 for 4 years with her bank with interest compounded quarterly throughout the term. The interest rate earned on the investment varied at different stages.

In the first year, the interest rate was  $5\frac{1}{2}\%$  p.a., for the next 2 years the interest rate remained fixed at 5.7% p.a., and the rate increased to 6.2% p.a. for the entire final year.

- a** How many compounding periods were there for the term of the investment?
- b** Consider the annual interest rates stated. What value for  $R$  would be used in the compound interest formula in the:
  - i** first year    **ii** second 2 years    **iii** final year?

**23** How much must be invested at an interest rate of 4.4% p.a. in order for the value to grow to \$8000 over 3 years, with interest compounded quarterly?

- A** 3      **B** 12      **C** 15      **D** 36

Questions **24** and **25** refer to this information.

Over a 5-year period, Ingrid's parents invested \$9000 for her into a savings account where interest was calculated 6-monthly. Over that time, the investment grew to \$12 000.

**24** Which correctly displays the calculation after substituting the values into the compound interest formula?

- A**  $\$12\ 000 = \$9000(1 + R)^{10}$
- B**  $\$12\ 000 = \$9000(1 + R)^5$
- C**  $\$9000 = \$12\ 000(1 + R)^{10}$
- D**  $\$12\ 000 = \$9000(1 + R)^{30}$

**25** What was the annual interest rate applied to the investment?

- c** Use the compound interest formula to determine the value of the investment at the end of the first year.
- d** State the values for  $P$ ,  $R$  and  $n$  you would use to calculate the growth in the investment from the start of the second year to the end of the third year.
- e** What was the value of the investment at the end of the third year?
- f** What was the value of the investment at the end of the term?
- g** How much interest did Susan earn on the investment?

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# CONNECT

## Repayment options

Every day, people borrow money in one form or another to make large purchases. Such purchases may be from the money as obtained through the use of a credit card, store purchase plans or by a loan with a bank, to list some examples. Regardless of how the money is obtained, the common feature in all these cases is that the money is expected to be repaid, generally with interest.

Consider a loan of \$5000 at  $12\frac{1}{2}\%$  p.a. over 2 years. How much is to be repaid if interest is calculated:

- as simple interest based on the amount borrowed?
- as compound interest based on the amount borrowed?
- on the amount owing at the time of calculation?

Your task is to perform calculations to determine which method offers the best option for repaying the money.

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### Your task

You are to:

- work out the amount to be repaid if interest is charged as simple interest based on the amount borrowed
- work out the amount to be repaid if interest is charged as compound interest based on the amount borrowed
- explore how the amount to be repaid changes if interest is calculated based on the amount owing at the time of calculation
- compare all options and determine which method best suits the given situation
- investigate the repayment options and methods of interest calculations offered by stores through their purchase plans and the methods various banks apply to their loans.

Include all necessary working to justify your answers.





You may like to present your findings as a report.  
Your report could be in the form of:

- an advertising brochure displaying tables comparing values
- a PowerPoint presentation
- a technology demonstration
- other (check with your teacher).