



Month	J	F	M	A	M	J	J	A	S	O	N	D
Av. sunlight (hours)	8	9	7	6	5	4	4	5	6	7	8	7

Range =  $9 - 4 = 5$

Mean =  $6.4$

Median =  $6.5$

Mode =  $8$

# 8

Pr (rainy day) =  $0.3$   
 Pr (sunny day) =  $1 - 0.3 = 0.7$

Area of panel  
 $1.5\text{m} \times 2\text{m} = 3\text{m}^2$

# ESSENTIAL MATHEMATICS

FOR THE AUSTRALIAN CURRICULUM

SECOND EDITION

UNCORRECTED SAMPLE



DAVID GREENWOOD | BRYN HUMBERSTONE  
 JUSTIN ROBINSON | JENNY GOODMAN  
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477 Williamstown Road, Port Melbourne, VIC 3207, Australia

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[www.cambridge.edu.au](http://www.cambridge.edu.au)

Information on this title: [www.cambridge.org/9781107568839](http://www.cambridge.org/9781107568839)

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Jennifer Vaughan 2015

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First published 2011

Reprinted 2011, 2012 (twice), 2013 (twice), 2014 (twice), 2015

Second edition 2015

Cover designed by Sardine Design

Typeset by Diacritech

Printed in China by C & C Offset Printing Co. Ltd.

*A Cataloguing-in-Publication entry is available from the catalogue of the National Library of Australia at [www.nla.gov.au](http://www.nla.gov.au)*

ISBN 978-1-107-56883-9 Paperback

Additional resources for this publication at [www.cambridge.edu.au/GO](http://www.cambridge.edu.au/GO)

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# Table of contents

<i>About the authors</i>	ix
<i>Introduction and how to use this series</i>	x

<b>1</b>	<b>Integers</b>	<b>2</b>	<b>Number and Algebra</b>
			<b>Number and place value</b>
1A	Whole number addition and subtraction <b>CONSOLIDATING</b>	4	
1B	Whole number multiplication and division <b>CONSOLIDATING</b>	9	
1C	Number properties <b>CONSOLIDATING</b>	14	
1D	Divisibility and prime factorisation <b>CONSOLIDATING</b>	20	
1E	Negative numbers <b>CONSOLIDATING</b>	26	
1F	Addition and subtraction of negative integers <b>CONSOLIDATING</b>	30	
	Progress quiz	34	
1G	Multiplication and division of integers	35	
1H	Order of operations and substitution	39	
	Investigation	43	
	Problems and challenges	45	
	Review: Chapter summary	46	
	Multiple-choice questions	47	
	Short-answer questions	48	
	Extended-response questions	49	
<b>2</b>	<b>Lines, shapes and solids</b>	<b>50</b>	<b>Measurement and Geometry</b>
2A	Angles at a point <b>CONSOLIDATING</b>	52	<b>Geometric reasoning</b>
2B	Parallel lines <b>CONSOLIDATING</b>	58	
2C	Triangles <b>CONSOLIDATING</b>	64	

2D	Quadrilaterals	71
2E	Polygons <b>EXTENDING</b>	77
	Progress quiz	82
2F	Solids and Euler's rule <b>EXTENDING</b>	84
	Investigation	90
	Problems and challenges	92
	Review: Chapter summary	93
	Multiple-choice questions	94
	Short-answer questions	95
	Extended-response questions	97

### 3 Fractions, decimals and percentages 98

3A	Equivalent fractions <b>CONSOLIDATING</b>	100
3B	Operations with fractions <b>CONSOLIDATING</b>	106
3C	Operations with negative fractions	114
3D	Understanding decimals <b>CONSOLIDATING</b>	120
3E	Operations with decimals <b>CONSOLIDATING</b>	126
3F	Terminating, recurring and rounding decimals	133
	Progress quiz	138
3G	Converting fractions, decimals and percentages <b>CONSOLIDATING</b>	139
3H	Finding a percentage and expressing as a percentage	146
3I	Decreasing and increasing by a percentage	152
3J	Calculating percentage change	157
3K	Percentages and the unitary method <b>EXTENDING</b>	162
	Investigation	166
	Problems and challenges	168
	Review: Chapter summary	169
	Multiple-choice questions	171
	Short-answer questions	171
	Extended-response questions	173

### Number and Algebra

Real Numbers

Money and financial mathematics

### 4 Measurement and introduction to Pythagoras' theorem 174

4A	Length and perimeter <b>CONSOLIDATING</b>	176
4B	Circumference of a circle	183
4C	Area	188
4D	Area of special quadrilaterals	195
4E	Area of a circle	200
4F	Sectors and composite shapes <b>EXTENDING</b>	206

### Measurement and Geometry

Using units of measurement

Number and Algebra

Real numbers

4G	Surface area of a prism	EXTENDING	212
4H	Volume and capacity		217
4I	Volume of prisms and cylinders		222
	Progress quiz		227
4J	Time		229
4K	Introduction to Pythagoras' theorem		
	EXTENDING		238
4L	Using Pythagoras' theorem	EXTENDING	243
4M	Finding the length of a shorter side		
	EXTENDING		248
	Investigation		253
	Problems and challenges		255
	Review: Chapter summary		256
	Multiple-choice questions		257
	Short-answer questions		258
	Extended-response questions		260

## 5 Algebra 262

### Number and Algebra

5A	The language of algebra	CONSOLIDATING	264
5B	Substitution and equivalence		
	CONSOLIDATING		269
5C	Adding and subtracting terms		273
5D	Multiplying and dividing terms		278
5E	Adding and subtracting algebraic fractions	EXTENDING	282
5F	Multiplying and dividing algebraic fractions	EXTENDING	287
5G	Expanding brackets		292
	Progress quiz		297
5H	Factorising expressions		298
5I	Applying algebra		302
5J	Index laws: multiplying and dividing powers		306
5K	Index laws: raising powers		311
	Investigation		315
	Problems and challenges		317
	Review: Chapter summary		318
	Multiple-choice questions		319
	Short-answer questions		319
	Extended-response questions		321

#### Number and place value

#### Patterns and algebra

## Semester Review 1 322

## 6 Ratios and rates 332

### Number and Algebra

#### Real numbers

6A	Introducing ratios	334
6B	Simplifying ratios	339
6C	Dividing a quantity in a given ratio	
	EXTENDING	344
6D	Scale drawings	350
6E	Introducing rates	357
	Progress quiz	362
6F	Ratios and rates and the unitary method	363
6G	Solving rate problems EXTENDING	368
6H	Speed EXTENDING	373
	Investigation	380
	Problems and challenges	382
	Review: Chapter summary	383
	Multiple-choice questions	384
	Short-answer questions	385
	Extended-response questions	387

## 7 Equations and inequalities 388

### Number and Algebra

#### Linear and non-linear relationships

7A	Equations review CONSOLIDATING	390
7B	Equivalent equations CONSOLIDATING	395
7C	Equations with fractions	401
7D	Equations with pronumerals on both sides	406
7E	Equations with brackets	411
7F	Formulas and relationships	415
	Progress quiz	419
7G	Applications	420
7H	Inequalities EXTENDING	425
7I	Solving inequalities EXTENDING	429
	Investigation	434
	Problems and challenges	436
	Review: Chapter summary	437
	Multiple-choice questions	438
	Short-answer questions	439
	Extended-response questions	441

## 8 Probability and statistics 442

## Statistics and Probability

8A	Interpreting graphs and tables	
	CONSOLIDATING	444
8B	Frequency tables and tallies	
	CONSOLIDATING	451
8C	Histograms	457
8D	Measures of centre	464
8E	Measures of spread EXTENDING	471
8F	Surveying and sampling	476
	Progress quiz	482
8G	Probability	484
8H	Two-step experiments EXTENDING	490
8I	Tree diagrams EXTENDING	495
8J	Venn diagrams and two-way tables	500
8K	Experimental probability	507
	Investigation	513
	Problems and challenges	515
	Review: Chapter summary	516
	Multiple-choice questions	517
	Short-answer questions	518
	Extended-response questions	521

### Chance

### Data representation and interpretation

## 9 Straight line graphs 522

## Number and Algebra

9A	The number plane CONSOLIDATING	524
9B	Rules, tables and graphs	528
9C	Finding the rule using tables	532
9D	Gradient EXTENDING	538
9E	Gradient-intercept form EXTENDING	545
9F	The x-intercept EXTENDING	552
	Progress quiz	556
9G	Using graphs to solve linear equations	558
9H	Applying linear graphs EXTENDING	567
9I	Non-linear graphs EXTENDING	573
	Investigation	579
	Problems and challenges	581
	Review: Chapter summary	582
	Multiple-choice questions	583
	Short-answer questions	584
	Extended-response questions	587

### Linear and non-linear relationships

10A	Reflection	590
10B	Translation	597
10C	Rotation	601
10D	Congruent figures	607
10E	Congruent triangles	613
	Progress quiz	620
10F	Congruence and quadrilaterals	622
10G	Similar figures <b>EXTENDING</b>	627
10H	Similar triangles <b>EXTENDING</b>	633
	Investigation	641
	Problems and challenges	642
	Review: Chapter summary	643
	Multiple-choice questions	644
	Short-answer questions	645
	Extended-response questions	648

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**Geometric reasoning**
**Semester Review 2****650***Answers**658*

# About the Authors

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**Stuart Palmer**

Introduction and How to Use This Series  
(Yet to Come)

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## Online Components and Associated Websites (Yet to Come)

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## Working with unfamiliar problems: Part 1

Choose one or more strategies to solve these problems. Clearly communicate your solution and final answer.

- 1 Find the sum of the first 100 counting numbers.
- 2 If you start with 10 and then add half of it and then keep adding half of the previous number, what total do you achieve?
- 3 Find the total of  $9 + 99 + 999 + 999 + 9999 + \dots + 999999999$



For questions 1-3, try starting with smaller numbers and look for a pattern.

- 4 In the Melbourne Cup there are 24 horses that enter the race. If you select 3 horses that you think will win, then a Box Trifecta allows your selections to finish first, second and third in any order. How many mystery Box Trifectas are possible in the Melbourne Cup?



For questions 4 and 5, try making a list or table.

- 5 Arrange all the digits 1, 2, 3, 4 and 5 into the form  $\square\square\square \times \square\square$  so that the three digit number multiplied by the two digit number gives the smallest possible answer.
- 6 Arrange the first 16 counting numbers in a four by four magic square with a magic number of 34. What do the four numbers in the middle of the magic square add to?
- 7 A rectangle is divided up into 3 rows of 8 small squares (so 24 identical squares in total). If the rectangle has perimeter 55 cm, find the perimeter and area of each small square.



For questions 6 and 7, draw a labelled diagram to help you visualise the problem.

- 8 What is the best integer estimate for  $6756.893 \div \frac{4}{3}$ ?



For question 8, try estimating by rounding the values in the question.

- 9 How many people in your school have used the canteen in the last week?



For question 9, try working with a smaller sample first.

- 10 A square has a diagonal of 46 cm, what is the area of this square?

- 11 Find the size of the largest angle of a triangle if it is 10 degrees more than the second largest angle, which in turn is 10 degrees more than the smallest angle?



For questions 10 and 11, try using a formula or rule to find a shortcut to the answer.

- 12 The sum of three consecutive even integers is 390. What is the product of the largest and smallest of the integers?



For question 12, try using algebra as a tool: define the pronumerals, form an equation and then solve it.

- 13 What are the dimensions of the rectangle with the largest area if the perimeter is 96 cm?

- 14 Each student in a round robin table tennis competition must play every other student once. How many (singles) matches are played if there are 12 students in the competition?



For questions 13 and 14, try using concrete, everyday materials to represent the problem.

- 15 Ken can wash a car in 10 minutes. Sean takes 15 minutes to wash the same size car. How long should it take them to wash 10 same size cars, if they work as a team of two, washing each car together?



For question 15, try applying one or more mathematical procedures, such as a rule for adding fractions.

## Working with unfamiliar problems: Part 2

Choose one or more strategies to solve these problems, which are in no particular order. Clearly communicate your solution and final answer.

- 1 The number of bacteria in a petri dish double every 30 seconds. If 5 bacteria are present at 1 p.m., how many will be present after 10 minutes?
- 2 If each letter represents a different digit (0–9), find the value of each letter so that the statement  $\text{FOUR} + \text{FIVE} = \text{NINE}$  is true.
- 3 A red and a blue four-sided die are each numbered  $-4, -2, 0, 3$ . If these dice are rolled and the uppermost results are multiplied together, what is the probability that the result is positive?
- 4 A four-digit number has the following properties: the second and fourth digits form a number that is a multiple of 11; the first two digits form a number divisible by 3 and less than 40; the first and third digits form the square of a whole number. Find all such four digit numbers that are divisible by:
  - i 4
  - ii 5
  - iii 6
  - iv 9
- 5 In a school science competition there were 20 boys and 30 girls who entered. Prizes were awarded to 40% of the boys and 30% of the girls. Find the total percentage of entrants who received a prize.



