Chapter

9 Straight line graphs

What you will learn

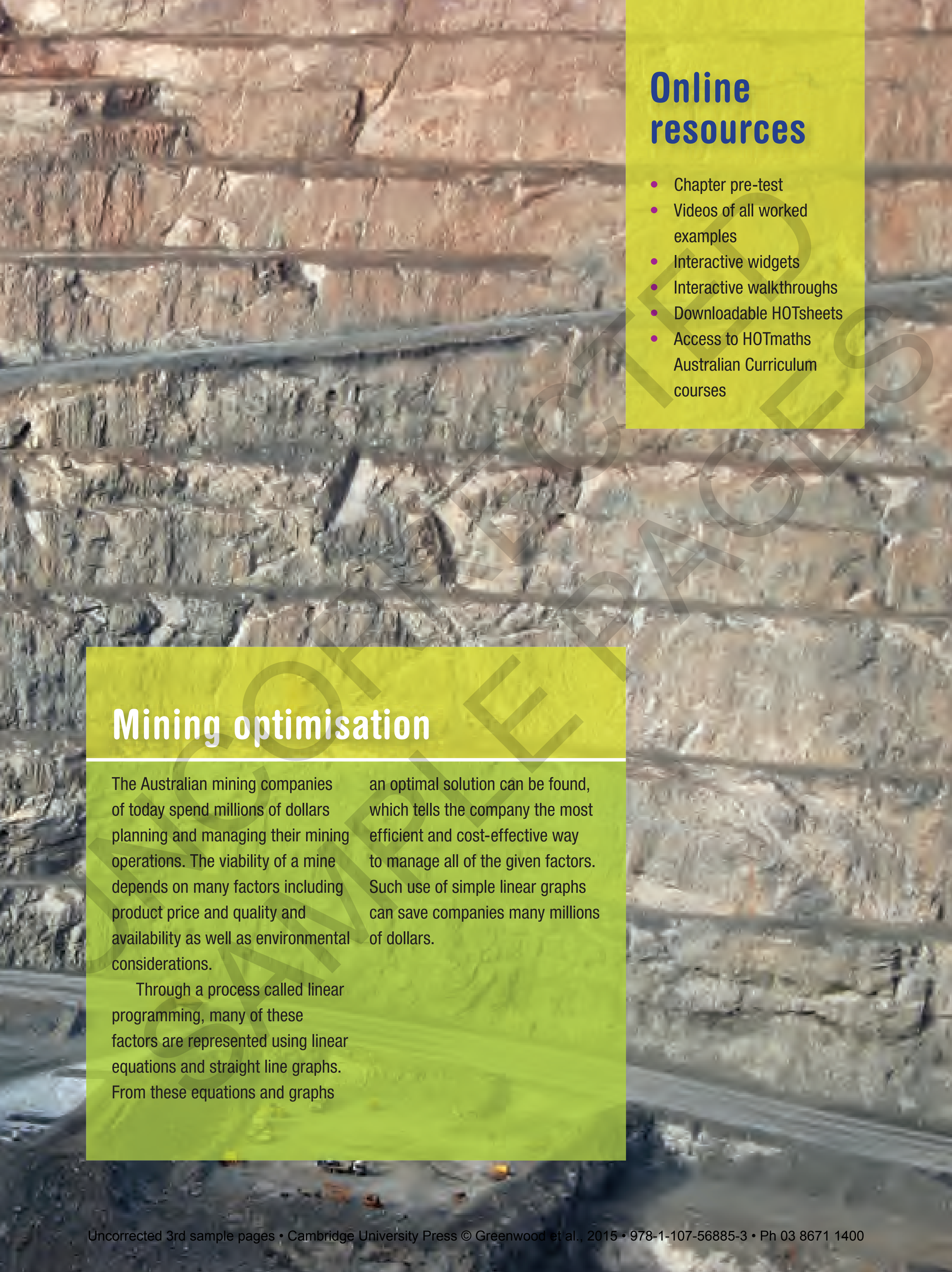
Australian curriculum

NUMBER AND ALGEBRA

Linear and non-linear relationships

Plot linear relationships on the Cartesian plane with and without the use of digital technologies (ACMNA193)

Solve linear equations using algebraic and graphical techniques. Verify solutions by substitution (ACMNA194)



Online resources

- Chapter pre-test
- Videos of all worked examples
- Interactive widgets
- Interactive walkthroughs
- Downloadable HOTsheets
- Access to HOTmaths Australian Curriculum courses

Mining optimisation

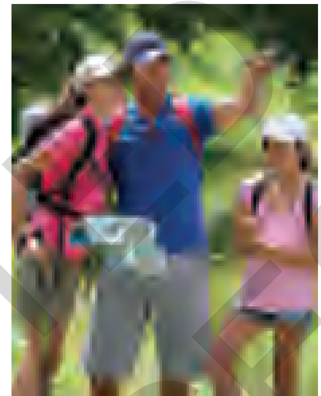
The Australian mining companies of today spend millions of dollars planning and managing their mining operations. The viability of a mine depends on many factors including product price and quality and availability as well as environmental considerations.

Through a process called linear programming, many of these factors are represented using linear equations and straight line graphs. From these equations and graphs an optimal solution can be found, which tells the company the most efficient and cost-effective way to manage all of the given factors. Such use of simple linear graphs can save companies many millions of dollars.

9A The number plane CONSOLIDATING

Interactive
Widgets
HOTsheets
Walkthroughs

On a number plane, a pair of coordinates gives the exact position of a point. The number plane is also called the Cartesian plane after its inventor, Rene Descartes, who lived in France in the 17th century. The number plane extends both a horizontal axis (x) and vertical axis (y) to include negative numbers. The point where these axes cross over is called the origin and it provides a reference point for all other points on the plane.



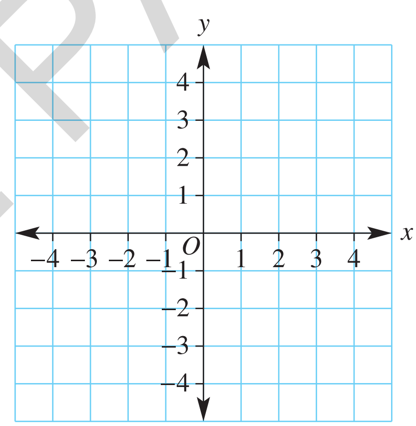
Coordinates on the number plane can be used to pinpoint locations just like you might locate points on a map.

Let's start: Make the shape

In groups or as a class, see if you can remember how to plot points on a number plane. Then decide what type of shape is formed by each set.

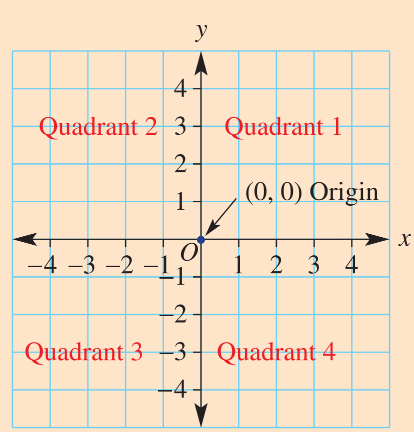
- $A(0, 0)$, $B(3, 1)$, $C(0, 4)$
- $A(-2, 3)$, $B(-2, -1)$, $C(-1, -1)$, $D(-1, 3)$
- $A(-3, -4)$, $B(2, -4)$, $C(0, -1)$, $D(-1, -1)$

Discuss the basic rules for plotting points on a number plane.



Key ideas

- A **number plane** (or **Cartesian plane**) includes a vertical y-axis and a horizontal x-axis intersecting at right angles.
  - There are 4 **quadrants** labelled as shown.
- A point on a number plane has **coordinates** (x, y).
  - The x-coordinate is listed first followed by the y-coordinate.
- The point $(0, 0)$ is called the **origin** (O).
- $(x, y) = \left(\begin{array}{cc} \text{horizontal} & \text{vertical} \\ \text{units from,} & \text{units from} \\ \text{origin} & \text{origin} \end{array}\right)$



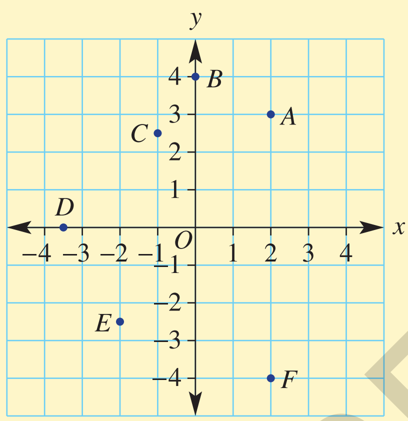
Example 1 Plotting points

Draw a number plane extending from –4 to 4 on both axes then plot and label these points.

a $A(2, 3)$ **b** $B(0, 4)$ **c** $C(-1, 2.5)$

d $D(-3.5, 0)$ **e** $E(-2, -2.5)$ **f** $F(2, -4)$

SOLUTION



EXPLANATION

The *x*-coordinate is listed first followed by the *y*-coordinate.

For each point start at the origin (0, 0) and move left or right or up and down to suit both *x*- and *y*-coordinates. For point $C(-1, 2.5)$, for example, move 1 to the left and 2.5 up.

Exercise 9A

1–3	3	—

UNDERSTANDING

1 Complete these sentences.

a The coordinates of the origin are _____.

b The vertical axis is called the _____-axis.

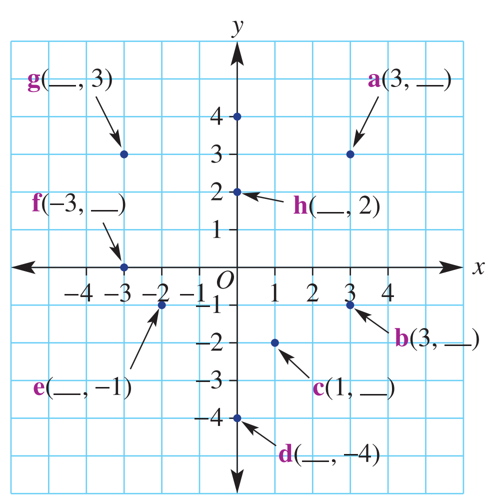
c The quadrant that has positive coordinates for both *x* and *y* is the _____ quadrant.

d The quadrant that has negative coordinates for both *x* and *y* is the _____ quadrant.

e The point (–2, 3) has *x*-coordinate _____.

f The point (1, –5) has *y*-coordinate _____.

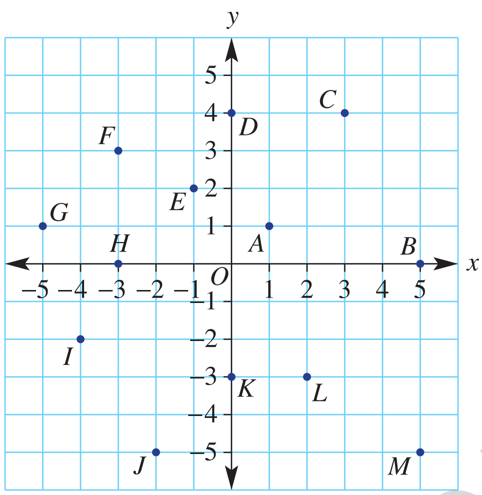
2 Write the missing number for the coordinates of the points **a**–**h**.



9A

UNDERSTANDING

3 Write the coordinates of the points labelled *A* to *M*.



4(½) | 4–5(½) | 4–5(½)

FLUENCY

Example 1

4 Draw a number plane extending from – 4 to 4 on both axes and then the plot and label these points.

- **a** $A(4, 1)$
- **b** $B(2, 3)$
- **c** $C(0, 1)$
- **d** $D(-1, 3)$
- **e** $E(-3, 3)$
- **f** $F(-2, 0)$
- **g** $G(-3, -1)$
- **h** $H(-1, -4)$
- **i** $I(0, -2.5)$
- **j** $J(0, 0)$
- **k** $K(3.5, -1)$
- **l** $L(1.5, -4)$
- **m** $M(-3.5, -3.5)$
- **n** $N(-3.5, 0.5)$
- **o** $O(3.5, 3.5)$
- **p** $P(2.5, -3.5)$

5 Using a scale extending from –5 to 5 on both axes, plot and then join the points for each part. Describe the basic picture formed.

- **a** $(-2, -2), (2, -2), (2, 2), (1, 3), (1, 4), \left(\frac{1}{2}, 4\right), \left(\frac{1}{2}, 3\frac{1}{2}\right), (0, 4), (-2, 2), (-2, -2)$
- **b** $(2, 1), (0, 3), (-1, 3), (-3, 1), (-4, 1), (-5, 2), (-5, -2), (-4, -1), (-3, -1), (-1, -3), (0, -3), (2, -1), (1, 0), (2, 1)$

6–7(½) | 6–7(½) | 7–8(½)

PROBLEM-SOLVING

6 One point in each set is not 'in line' with the other points. Name the point in each case.

- **a** $A(1, 2), B(2, 4), C(3, 4), D(4, 5), E(5, 6)$
- **b** $A(-5, 3), B(-4, 1), C(-3, 0), D(-2, -3), E(-1, -5)$
- **c** $A(-4, -3), B(-2, -2), C(0, -1), D(2, 0)$, E(3, 1)
- **d** $A(6, -4), B(0, -1), C(4, -3), D(3, -2), E(-2, 0)$

PROBLEM-SOLVING

7 Each set of points forms a basic shape. Describe the shape without drawing a graph if you can.

a $A(-2, 4)$, $B(-1, -1)$, $C(3, 0)$
b $A(-3, 1)$, $B(2, 1)$, $C(2, -6)$, $D(-3, -6)$
c $A(-4, 2)$, $B(3, 2)$, $C(4, 0)$, $D(-3, 0)$
d $A(-1, 0)$, $B(1, 3)$, $C(3, 0)$, $D(1, -9)$

8 The midpoint of a line segment (or interval) is the point that cuts the segment in half. Find the midpoint of the line segment joining these pairs of points.

a (1, 3) and (3, 5)
b (−4, 1) and (−6, 3)
c (−2, −3) and (0, −2)
d (3, −5) and (6, −4)

REASONING | 9 | 9, 10 | 10, 11

9 List all the points, using only integer values of x and y that lie on the line segment joining these pairs of points.

a (1, −3) and (1, 2)
b (−2, 0) and (3, 0)
c (−3, 4) and (2, −1)
d (−3, −6) and (3, 12)

10 If (a, b) is a point on a number plane, name the quadrant or quadrants that matches the given description.

a $a > 0$ and $b < 0$
b $a < 0$ and $b > 0$
c $a < 0$
d $b < 0$

11 A set of points has coordinates $(0, y)$ where y is any number. What does this set of points represent?

ENRICHMENT | — | — | 12, 13

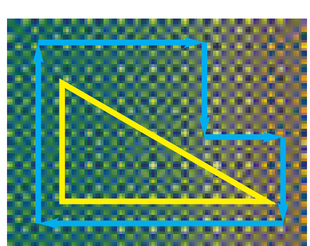
Distances between points

12 Find the distance between these pairs of points.

a (0, 0) and (0, 10)
b (0, 0) and (−4, 0)
c (−2, 0) and (5, 0)
d (0, −4) and (0, 7)
e (−1, 2) and (5, 2)
f (4, −3) and (4, 1)

13 When two points are not aligned vertically or horizontally, Pythagoras' theorem can be used to find the distance between them. Find the distance between these pairs of points.

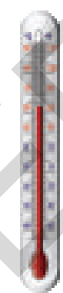
a (0, 0) and (3, 4)
b (0, 0) and (5, 12)
c (−3, −4) and (4, 20)
d (1, 1) and (4, −1)
e (−1, −2) and (2, 7)
f (−3, 4) and (3, −1)



9B Rules, tables and graphs

Interactive
Widgets
HOTsheets
Walkthroughs

From our earlier study of formulas we know that two (or more) variables that have a relationship can be linked by a rule. A rule with two variables can be represented on a graph to illustrate this relationship. The rule can be used to generate a table that shows coordinate pairs (x, y). The coordinates can be plotted to form the graph. Rules that give straight line graphs are described as being **linear**.



The rule for converting Celsius to Fahrenheit gives a straight line graph.

For example, the rule linking degrees Celsius (°C) with degrees Fahrenheit (°F) is given by $°C = \frac{5}{9}(°F - 32)$ and gives a straight line graph.

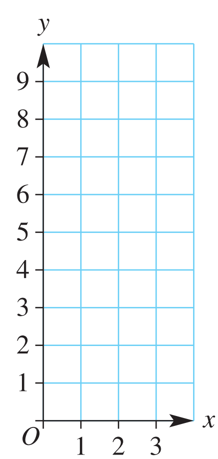
Let's start: They're not all straight

Not all rules give a straight line graph. Here are three rules which can be graphed to give lines or curves.

1 $y = \frac{6}{x}$ **2** $y = x^2$ **3** $y = 2x + 1$

- In groups, discuss which rule(s) might give a straight line graph and which might give curves.
- Use the rules to complete the given table of values.
- Discuss how the table of values can help you decide which rule(s) give a straight line.
- Plot the points to see if you are correct.

x	1	2	3
y_1			
y_2			
y_3			



Key ideas

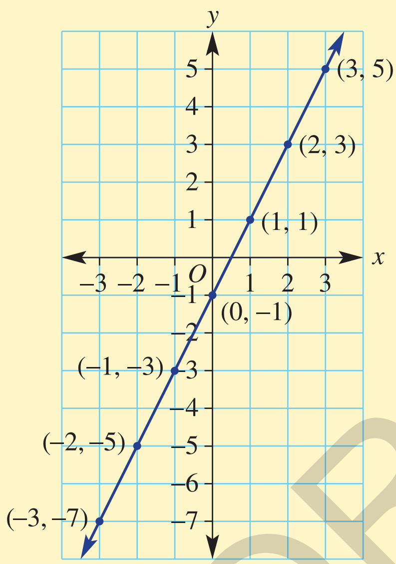
- A **rule** is an equation which describes the relationship between two or more variables.
- A **linear** relationship will result in a straight line graph.
- For two variables, a linear rule is often written with y as the subject. For example, $y = 2x - 3$ or $y = -x + 7$
- One way to graph a linear relationship using a rule is to follow these steps.
  - Construct a table of values finding a y-coordinate for each given x-coordinate by substituting each x-coordinate into the rule.
  - Plot the points given in the table on a set of axes.
  - Draw a line through the points to complete the graph.

Example 2 Plotting a graph from a rule

For the rule $y = 2x - 1$ construct a table and draw a graph.

SOLUTION

x	–3	–2	–1	0	1	2	3
y	–7	–5	–3	–1	1	3	5



EXPLANATION

Substitute each x-coordinate in the table into the rule to find the y-coordinate.

Plot each point (–3, –7), (–2, –5) … and join them to form the straight line graph.

Example 3 Checking if a point lies on a line

Decide if the points (1, 3) and (–2, –4) lie on the graph of $y = 3x$.

SOLUTION

Substitute (1, 3)
$y = 3x$
$3 = 3 \times 1$ (True)
So (1, 3) is on the line.

Substitute (–2, –4)
$y = 3x$
$-4 = 3 \times -2$ (False)
So (–2, –4) is not on the line.

EXPLANATION

Substitute (1, 3) into the rule for the line. The equation is true, so the point is on the line.

Substitute (–2, –4) into the rule for the line. The equation is not true, so the point is not on the line.

Exercise 9B

1–3	3	—

UNDERSTANDING

1 For the rule $y = 2x + 3$ find the y-coordinate for these x-coordinates.

a 1 b 2 c 0 d –1
e –5 f –7 g 11 h –12

9B

UNDERSTANDING

2 Write the missing number in these tables for the given rules.

a $y = 2x$

x	0	1	2	3
y	0		4	6

b $y = x - 3$

x	–1	0	1	2
y	–4	–3		–1

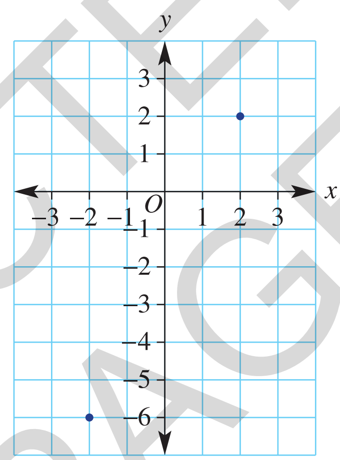
c $y = 5x + 2$

x	–3	–2	–1	0
y		–8	–3	2

3 Complete the graph to form a straight line from the given rule and table. Two points have been plotted for you.

$y = 2x - 2$

x	–2	–1	0	1	2
y	–6	–4	–2	0	2



4–5(½) 4–5(½) 4–5(½)

FLUENCY

Example 2

4 For each rule construct a table like the one shown here and draw a graph.

x	–3	–2	–1	0	1	2	3
y							

a $y = x + 1$
b $y = x - 2$
c $y = 2x - 3$
d $y = 2x + 1$
e $y = -2x + 3$
f $y = -3x - 1$
g $y = -x$
h $y = -x + 4$

Example 3

5 Decide if the given points lie on the graph with the given rule.

a Rule: $y = 2x$ Points: i (2, 4) and ii (3, 5)
b Rule: $y = 3x - 1$ Points: i (1, 1) and ii (2, 5)
c Rule: $y = 5x - 3$ Points: i (–1, 0) and ii (2, 12)
d Rule: $y = -2x + 4$ Points: i (1, 2) and ii (2, 0)
e Rule: $y = 3 - x$ Points: i (1, 2) and ii (4, 0)
f Rule: $y = 10 - 2x$ Points: i (3, 4) and ii (0, 10)
g Rule: $y = -1 - 2x$ Points: i (2, –3) and ii (–1, 1)

6–7(½) 6–7(½) 7, 8

PROBLEM-SOLVING

6 For x-coordinates from –3 to 3, construct a table and draw a graph for these rules. For parts c and d remember that subtracting a negative number is the same as adding its opposite, for example that $3 - (-2) = 3 + 2$.

a $y = \frac{1}{2}x + 1$
b $y = -\frac{1}{2}x - 2$
c $y = 3 - x$
d $y = 1 - 3x$

PROBLEM-SOLVING

7 For the graphs of these rules, state the coordinates of the two points at which the line cuts the x- and y-axes.

a $y = x + 1$ b $y = 2 - x$ c $y = 2x + 4$
d $y = 10 - 5x$ e $y = 2x - 3$ f $y = 7 - 3x$

8 The rules for two lines are $y = x + 2$ and $y = 5 - 2x$. At what point do they intersect?

REASONING

9	9, 10	10, 11

9 a What is the minimum number of points needed to draw a graph of a straight line?
b Draw the graph of these rules by plotting only two points. Use $x = 0$ and $x = 1$.
i $y = \frac{1}{2}x$ ii $y = 2x - 1$

10 a The graphs of $y = x$, $y = 3x$ and $y = -2x$ all pass through the origin (0, 0). Explain why.
b The graphs of $y = x - 1$, $y = 3x - 2$ and $y = 5 - 2x$ do not pass through the origin (0, 0). Explain why.

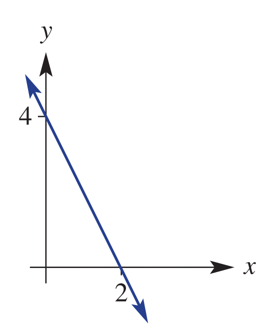
11 The y-coordinates of points on the graphs of the rules in Question 4 parts a to d increase as the x-coordinates increase. Also the y-coordinates of points on the graphs of the rules in Question 4 parts e to h decrease as the x-coordinates increase.
a What do the rules in Question 4 parts a to d have in common?
b What do the rules in Question 4 parts e to h have in common?
c What feature of a rule tells you that a graph increases or decreases as the x-coordinate increases?

Axes-intercepts

ENRICHMENT

—	—	12

12 A sketch of a straight line graph can be achieved by finding only the x-and y-intercepts and labelling these on the graph. The y-intercept is the point where $x = 0$ and the x-intercept is the point where $y = 0$.
For example: $y = 4 - 2x$



y–intercept ($x = 0$)
$$y = 4 - 2 \times 0$$
$$= 4$$

x–intercept ($y = 0$)
$$0 = 4 - 2x$$
$$-4 = -2x$$
$$2 = x$$

Sketch graphs of these rules using the method outlined above.

a $y = x + 4$ b $y = 2x - 4$ c $y = 5 - x$
d $y = -1 + 2x$ e $y = 7x - 14$ f $y = 5 - 3x$
g $y = 3 - 2x$ h $3y - 2x = 6$

9C Finding the rule using tables

Interactive

Widgets

HOTsheets

Walkthroughs

A mathematical rule is an efficient way of describing a relationship between two variables. While a table and a graph are limited by the number of points they show, a rule can be used to find any value of y for any given x value quickly. Finding such a rule from a collection of points on a graph or table is an important step in the development and application of mathematics.

Businesses attempt to find mathematical rules from the data they collect and graph.

Let's start: What's my rule?

Each of the tables here describe a linear relationship between y and x.

x	y
0	4
1	5
2	6
3	7
4	8

x	y
−3	−5
−2	−3
−1	−1
0	1
1	3

x	y
−2	5
−1	4
0	3
1	2
2	1

- For each table write a rule making y the subject.
- Discuss your strategy for finding the three different rules. What patterns did you notice and how did these patterns help determine the rule?

Key ideas

- A **rule** must be true for every pair of coordinates (x, y) in a table or graph.

coefficient of x

$$y = \square \times x + \square$$

constant

- Consider a linear rule of the form $y = \square \times x + \square$.
  - The **coefficient** of x will be the increase in y as x increases by 1. If there is a decrease in y, then the coefficient will be negative.

x	−2	−1	0	1	2
y	−1	1	3	5	7

+2 +2 +2 +2

$y = 2x + 3$

x	−2	−1	0	1	2
y	1	0	−1	−2	−2

−1 −1 −1 −1

$y = -x - 1$

  - The **constant** will be the value of y when $x = 0$.

Key ideas

- If the value of y when $x = 0$ is not given in the table, substitute another pair of coordinates to find the value of the constant.

x	2	3	4	5
y	5	7	9	11

+2 +2 +2

$y = 2x + \square$

$5 = 2 \times 2 + \square$ substituting (2, 5)

So $\square = 1$

Example 4 Finding rules from tables

Find the rule for these tables of values.

a

x	–2	–1	0	1	2
y	–8	–5	–2	1	4

b

x	3	4	5	6	7
y	–5	–7	–9	–11	–13

SOLUTION

a Coefficient of x is 3.
Constant is –2.
$y = 3x - 2$

b Coefficient of x is –2.
$y = -2x + \square$
Substitute (3, –5).
$-5 = -2 \times 3 + \square$
$-5 = -6 + \square$
So $\square = 1$.
$y = -2x + 1$

EXPLANATION

x	–2	–1	0	1	2
y	–8	–5	–2	1	4

+3 +3 +3 +3

$y = 3x - 2$

x	3	4	5	6	7
y	–5	–7	–9	11	–13

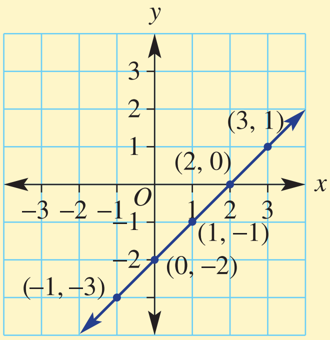
–2 –2 –2 –2

$y = -2x + \square$

To find the constant substitute a point and choose the constant so that the equation is true. This can be done mentally.

Example 5 Finding a rule from a graph

Find the rule for this graph by first constructing a table of (x, y) values.



SOLUTION

x	–1	0	1	2	3
y	–3	–2	–1	0	1

Coefficient of x is 1.

When $x = 0$, $y = -2$

$y = x - 2$

EXPLANATION

Construct a table using the points given on the graph. Change in y is 1 for each increase by 1 in x.

x	–1	0	1	2	3
y	–3	–2	–1	0	1

+1 +1 +1 +1

$y = 1x + (-2)$ or $y = x - 2$

Exercise 9C

1–3 | 3 | —

UNDERSTANDING

1 By how much does y increase for each increase by 1 in x? If y is decreasing give a negative answer.

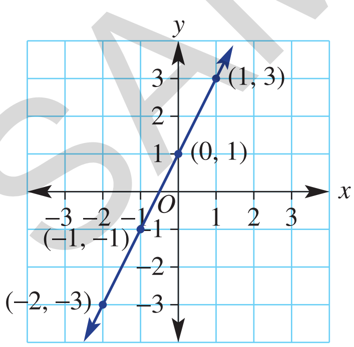
a

x	–2	–1	0	1	2
y	–1	1	3	5	7

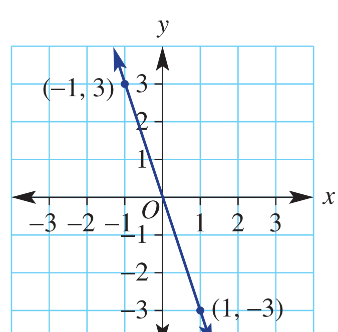
b

x	–3	–2	–1	0	1
y	4	3	2	1	0

c



d



UNDERSTANDING

2 For each of the tables and graphs in Question **1**, state the value of y when $x = 0$.

3 Find the missing number in these equations.

a $4 = 2 + \square$ **b** $3 = 5 + \square$ **c** $-1 = 2 + \square$

d $5 = 3 \times 2 + \square$ **e** $-2 = 2 \times 4 + \square$ **f** $-10 = 2 \times (-4) + \square$

g $-8 = -4 \times 2 + \square$ **h** $-12 = -5 \times 3 + \square$ **i** $16 = -4 \times (-3) + \square$

4–6(½) 4–6(½) 5–6(½)

FLUENCY

Example 4a

4 Find the rule for these tables of values.

a

x	–2	–1	0	1	2
y	0	2	4	6	8

b

x	–2	–1	0	1	2
y	–7	–4	–1	2	5

c

x	–3	–2	–1	0	1
y	4	3	2	1	0

d

x	–1	0	1	2	3
y	8	6	4	2	0

Example 4b

5 Find the rule for these tables of values.

a

x	1	2	3	4	5
y	5	9	13	17	21

b

x	–5	–4	–3	–2	–1
y	–13	–11	–9	–7	–5

c

x	5	6	7	8	9
y	–12	–14	–16	–18	–20

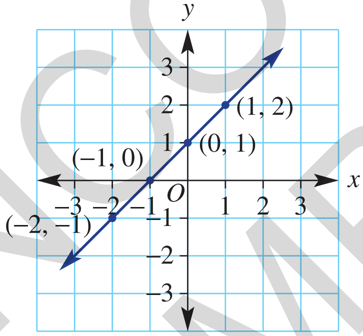
d

x	–6	–5	–4	–3	–2
y	10	9	8	7	6

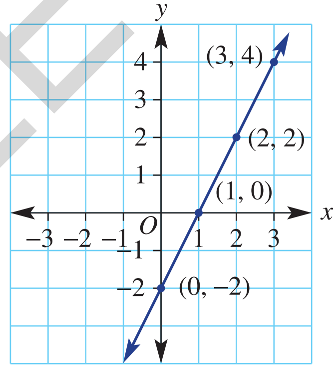
Example 5

6 Find the rule for these graphs by first constructing a table of (x, y) values.

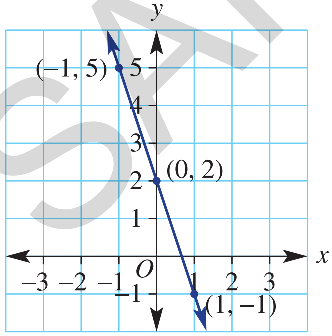
a



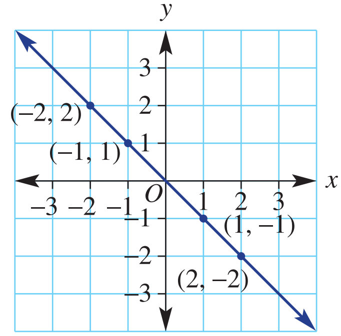
b



c



d



9C

PROBLEM-SOLVING

7, 8	8, 9	8–10

7 Find the rule for these set of points. Try to do it without drawing a graph or table.

a (1, 3), (2, 4), (3, 5), (4, 6)

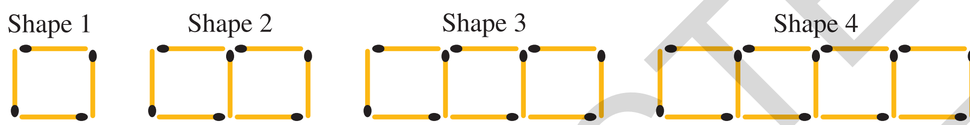
b (–3, –7), (–2, –6), (–1, –5), (0, – 4)

c (–1, –3), (0, –1), (1, 1), (2, 3)

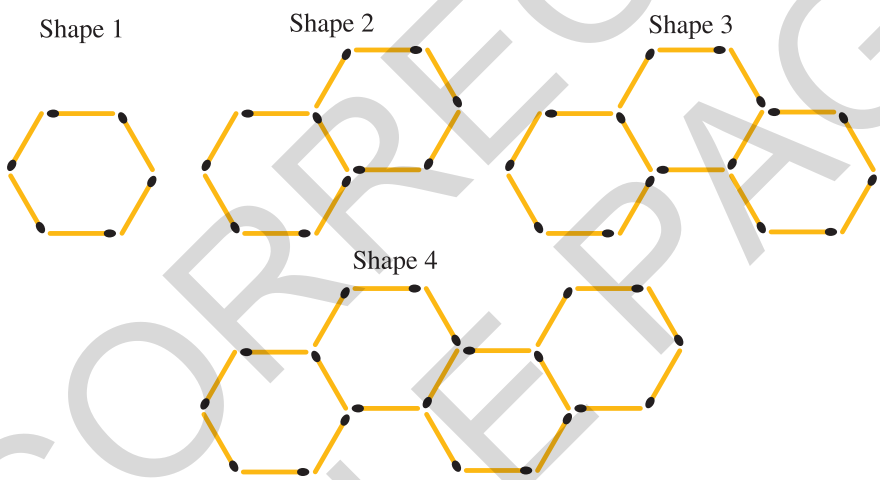
d (–2, 3), (–1, 2), (0, 1), (1, 0)

8 Write a rule for these matchstick patterns.

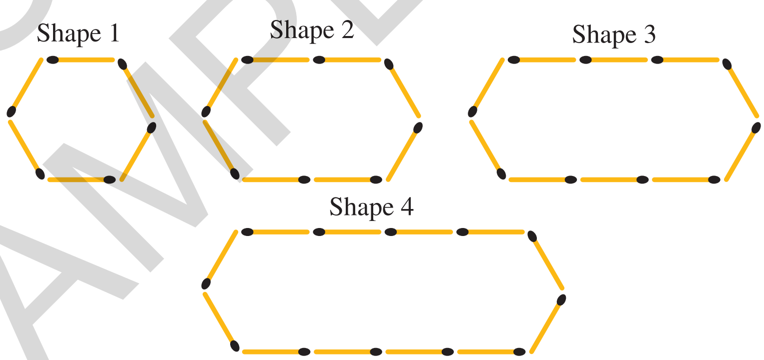
a x = number of squares y = number of matchsticks



b x = number of hexagons y = number of matchsticks



c x = number of matchsticks on top row y = number of matchsticks



9 A straight-line graph passes through the two points (0, –2) and (1, 6). What is the rule of the graph?

10 A straight-line graph passes through the two points (–2, 3) and (4, –3). What is the rule of the graph?

	11	11, 12	12–14

REASONING

11 The rule $y = -2x + 3$ can be written as $y = 3 - 2x$. Write these rules in a similar form.

a $y = -2x + 5$
b $y = -3x + 7$
c $y = -x + 4$
d $y = -4x + 10$

12 A straight line has two points (0, 2) and (1, b).

a Write an expression for the coefficient of x in the rule linking y and x.
b Write the rule for the graph in terms of b.

13 A straight line has two points (0, a) and (1, b).

a Write an expression for the coefficient of x in the rule linking y and x.
b Write the rule for the graph in terms of a and b.

14 In Question 8a you can observe that 3 extra matchsticks are needed for each new shape and 1 matchstick is needed to complete the first square (so the rule is $y = 3x + 1$). In a similar way, describe how many matchsticks are needed for the shapes in:

a Question 8b
b Question 8c

Skipping x values	—	—	15

ENRICHMENT

15 Consider this table of values.

x	–2	0	2	4
y	–4	–2	0	2

a The increase in y for each unit increase in x is not 2. Explain why.
b If the pattern is linear, state the increase in y for each increase by 1 in x.
c Write the rule for the relationship.
d Find the rule for these tables.

i

x	–4	–2	0	2	4
y	–5	–1	3	7	11

ii

x	–3	–1	1	3	5
y	–10	–4	2	8	14

iii

x	–6	–3	0	3	6
y	15	9	3	–3	–9

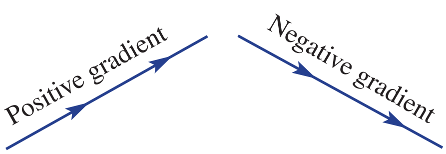
iv

x	–10	–8	–6	–4	–2
y	20	12	4	–4	–12

9D Gradient EXTENDING

Interactive · Widgets · HOTsheets · Walkthroughs

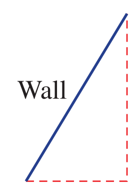
The gradient of a line is a measure of how steep the line is. The steepness or slope of a line depends on how far it rises or falls over a given horizontal distance. This is why the gradient is calculated by dividing the vertical rise by the horizontal run between two points. Lines that rise (from left to right) have a positive gradient and lines that fall (from left to right) have a negative gradient.




It's more difficult for a plane to take off at a steep gradient than a gradual one.

Let's start: Which is the steepest?

At a children's indoor climbing centre there are three types of sloping walls to climb. The blue wall rises 2 metres for each metre across. The red wall rises 3 metres for every 2 metres across and the yellow wall rises 7 metres for every 3 metres across.

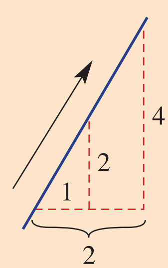


- Draw a diagram showing the slope of each wall.
- Label your diagrams with the information given above.
- Discuss which wall might be the steepest giving reasons.
- Discuss how it might be possible to accurately compare the slope of each wall.

Key ideas

- The **gradient** is a measure of **slope**.
  - It is the increase in y as x increases by 1.
  - It is the ratio of the change in y over the change in x.

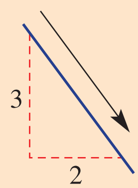
Positive gradient



$$\text{Gradient} = \frac{\text{rise}}{\text{run}} = \frac{4}{2} = \frac{2}{1} = 2$$

  - **Gradient** $= \dfrac{\textbf{rise}}{\textbf{run}}$
  - Rise = change in y
  - Run = change in x

Negative gradient

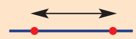


$$\text{Gradient} = \frac{\text{rise}}{\text{run}} = \frac{-3}{2}$$

  - The run is always considered to be positive when moving from left to right on the Cartesian plane.

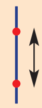
Key ideas

- A gradient is negative if y decreases as x increases. The rise is considered to be negative.
- The gradient of a horizontal line is 0.
- The gradient of a vertical line is undefined.



$\text{Gradient} = \frac{0}{2}$

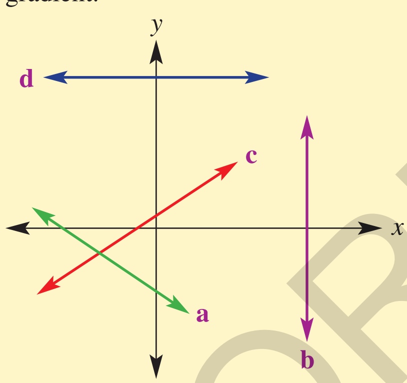
$= 0$



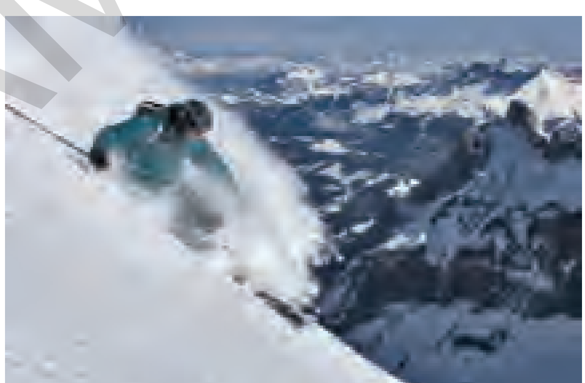
$\text{Gradient} = \frac{2}{0}$ which is undefined

Example 6 Defining a type of gradient

Decide if the lines labelled **a**, **b**, **c** and **d** on this graph have a positive, negative, zero or undefined gradient.



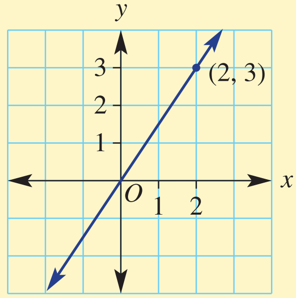
SOLUTION	EXPLANATION
a Negative gradient	As x increases y decreases.
b Undefined gradient	The line is vertical.
c Positive gradient	y increases as x increases.
d Zero gradient	There is no increase or decrease in y as x increases.



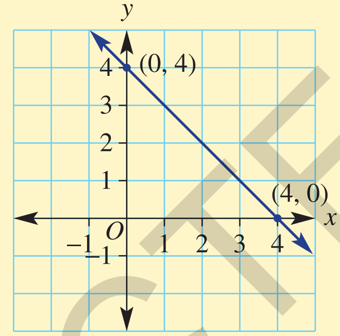
Example 7 Finding the gradient from a graph

Find the gradient of these lines.

a



b

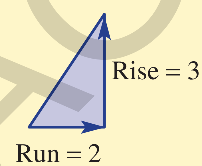


SOLUTION | **EXPLANATION**

a Gradient $= \frac{\text{rise}}{\text{run}}$

$= \frac{3}{2}$ or 1.5

The rise is 3 for every 2 across to the right.

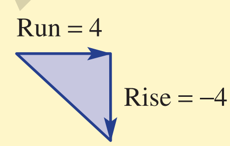


b Gradient $= \frac{\text{rise}}{\text{run}}$

$= \frac{-4}{4}$

$= -1$

The y value falls 4 units while the x value increases by 4.



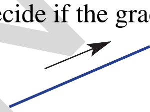
Exercise 9D

1–3(½) | 3(½) | —

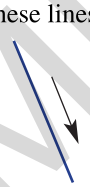
UNDERSTANDING

1 Decide if the gradients of these lines are positive or negative.

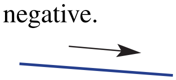
a



b



c



d

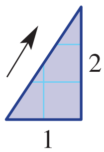
2 Simplify these fractions.

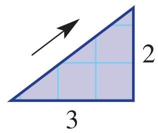
a $\frac{4}{2}$ **b** $\frac{12}{4}$ **c** $\frac{6}{4}$ **d** $\frac{14}{7}$

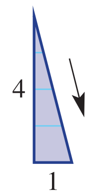
e $-\frac{6}{3}$ **f** $-\frac{20}{4}$ **g** $-\frac{16}{6}$ **h** $-\frac{15}{9}$

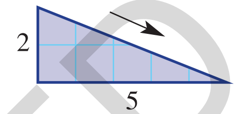
UNDERSTANDING

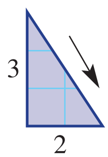
3 Write down the gradient, using $\frac{\text{rise}}{\text{run}}$ for each of these slopes.

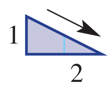
a


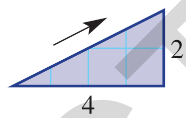
b


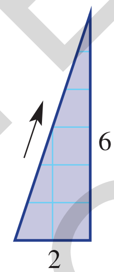
c


d


e


f


g


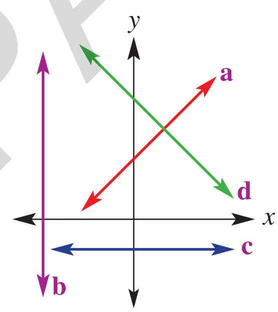
h


4, 5–6(½)	4, 5–6(½)	4, 5–6(½)

FLUENCY

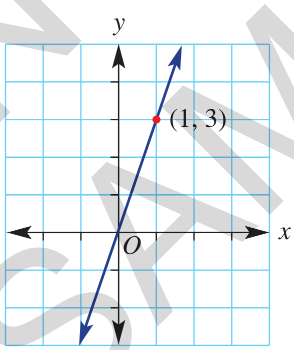
Example 6

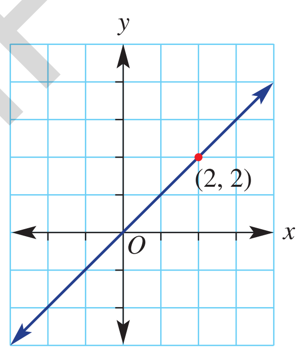
4 Decide if these lines labelled **a**, **b**, **c** and **d** on this graph have a positive, negative, zero or undefined gradient.

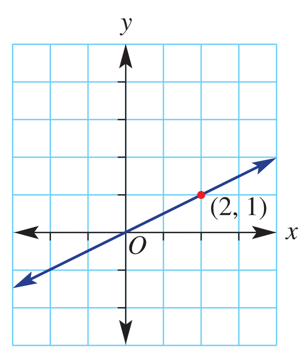


Example 7a

5 Find the gradient of these lines. Use Gradient = $\frac{\text{rise}}{\text{run}}$.

a


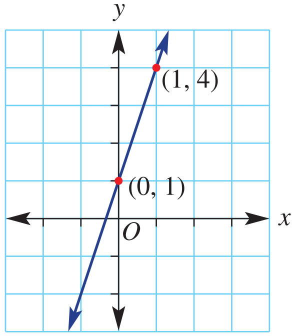
b


c


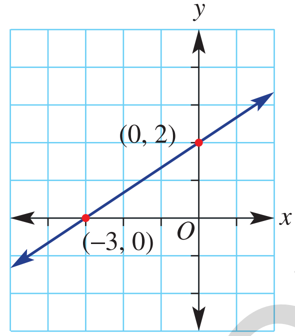
9D

FLUENCY

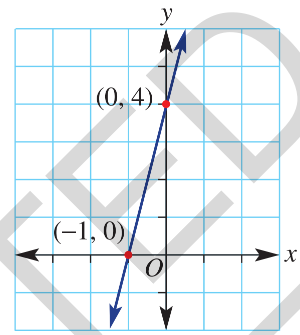
d



e



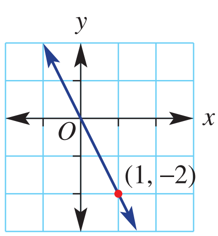
f



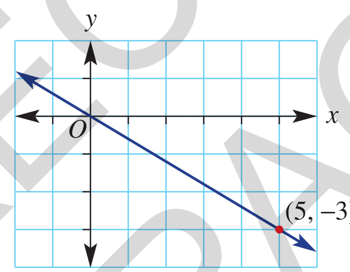
Example 7b

6 Find the gradient of these lines.

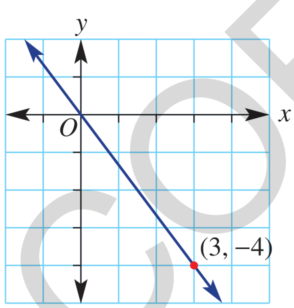
a



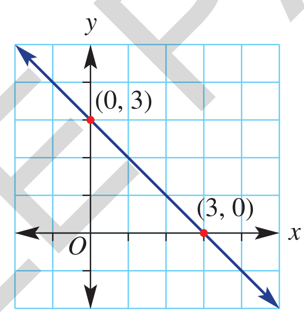
b



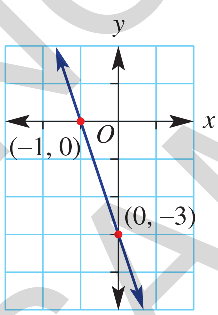
c



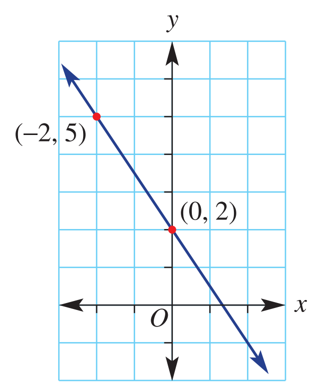
d



e



f

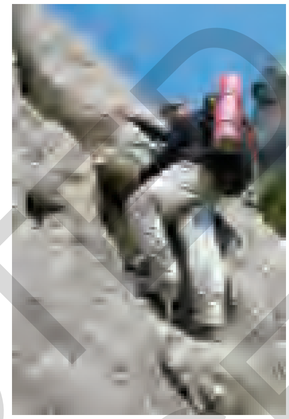


9D

7, 8	8, 9	8–10

PROBLEM-SOLVING

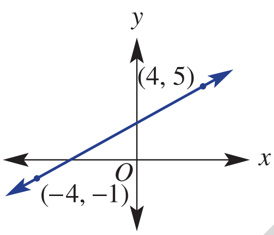
7 Abdullah climbs a rocky slope that rises 12 m for each 6 metres across. His friend Jonathan climbs a nearby grassy slope that rises 25 m for each 12 m across. Which slope is steeper?



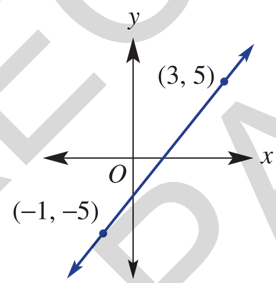
8 A submarine falls 200 m for each 40 m across and a torpedo falls 420 m for each 80 m across in pursuit of the submarine. Which has the steeper gradient, the submarine or torpedo?

9 Find the gradient of these lines. You will need to first calculate the rise and run.

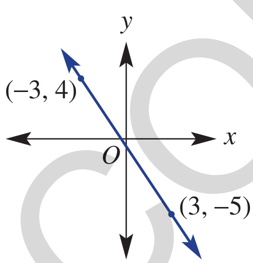
a



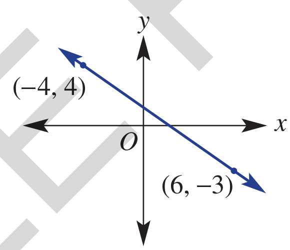
b



c



d



10 Find the gradient of the line joining these pairs of points.

a (0, 2) and (2, 7)
b (0, –1) and (3, 4)
c (–3, 7) and (0, –1)
d (–5, 6) and (1, 2)
e (–2, –5) and (1, 3)
f (–5, 2) and (5, –1)

11	11, 12	12, 13

REASONING

11 A line with gradient 3 joins (0, 0) with another point A.

a Write the coordinates of three different positions for A, using positive integers for both the x- and y-coordinates.

b Write the coordinates of three different positions for A using negative integers for both the x- and y-coordinates.

12 A line joins the point (0, 0) with the point (a, b) with a gradient of 2.

a If $a = 1$ find b.
b If $a = 5$ find b.
c Write an expression for b in terms of a.
d Write an expression for a in terms of b.

9D

REASONING

13 A line joins the point (0, 0) with the point (a, b) with a gradient of $-\frac{1}{2}$.

a If $a = 1$ find b.

b If $a = 3$ find b.

c Write an expression for b in terms of a.

d Write an expression for a in terms of b.

Rise and run as differences

—	—	14

ENRICHMENT

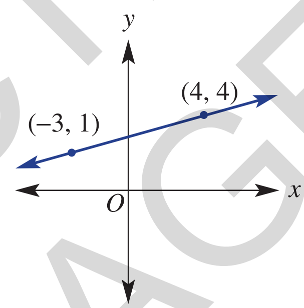
14 The run and rise between two points can be calculated by finding the difference between the pairs of x- and pairs of y-coordinates.

For example:

$$\text{Rise} = 4 - 1 = 3$$

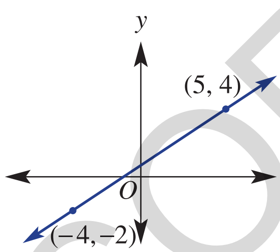
$$\text{Run} = 4 - (-3) = 7$$

$$\text{Gradient} = \frac{3}{7}$$

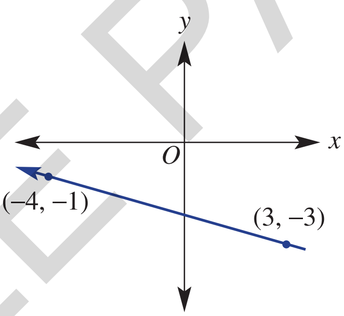


Use this method to find the gradient of the line joining the given points.

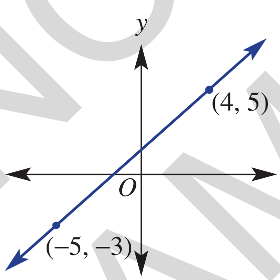
a



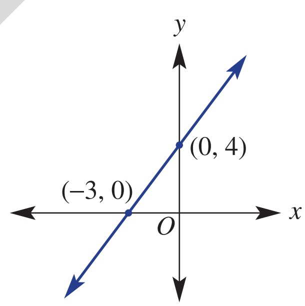
b



c



d



e (–3, 1) and (4, –2)

f (–2, –5) and (1, 7)

g (5, –2) and (–4, 6)

h $\left(-\frac{1}{2}, 2\right)$ and $\left(\frac{9}{2}, -3\right)$

i $\left(-4, \frac{2}{3}\right)$ and $\left(3, -\frac{4}{3}\right)$

j $\left(-\frac{3}{2}, \frac{2}{3}\right)$ and $\left(4, -\frac{2}{3}\right)$

k $\left(-\frac{7}{4}, 2\right)$ and $\left(\frac{1}{3}, \frac{1}{2}\right)$

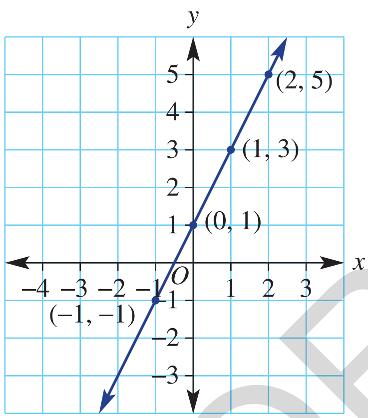
9E Gradient–intercept form EXTENDING

Interactive Widgets HOTsheets Walkthroughs

From previous sections in this chapter, you may have noticed some connections between the numbers that make up a rule for a linear relationship and the numbers that are the gradient and *y*-coordinate with $x = 0$ (the *y*-intercept). This is no coincidence. Once the gradient and *y*-intercept of a graph are known, the rule can be written down without further analysis.

Let's start: What's the connection?

To explore the connection between the rule for a linear relationship and the numbers that are the gradient and the *y*-intercept, complete the missing details for the graph and table below.



x	−1	0	1	2
y	−1	1		

+2

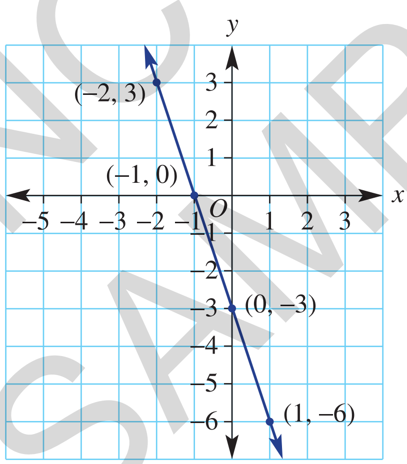
$y = \square \times x + \square$

Gradient = $\frac{\text{rise}}{\text{run}}$

= _______

y-intercept = _______

- What do you notice about the numbers in the rule including the coefficient of *x* and the constant (below the table) and the numbers for the gradient and *y*-intercept?
- Complete the details for this new example below to see if your observations are the same.



x	−2	−1	0	1
y				

$y = \square \times x + \square$

Gradient = $\frac{\text{rise}}{\text{run}}$

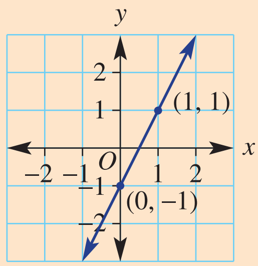
= _______

y-intercept = _______

Key ideas

- The rule for a straight line graph is given by $y = mx + c$ or $y = mx + b$ where:
  - m is the gradient
  - c (or b) is the **y-intercept**

  For example:



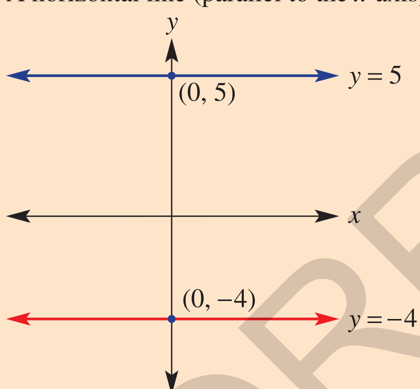
$m = \frac{2}{1} = 2$

$c = -1$

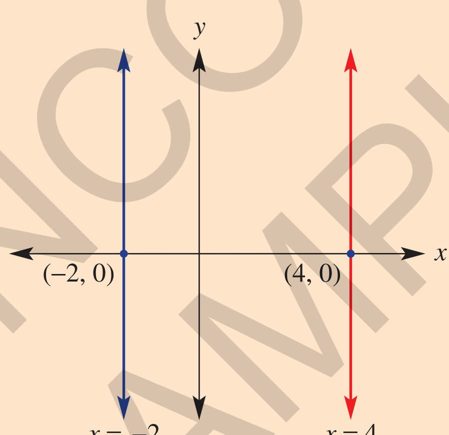
So $y = mx + c$ becomes

$y = 2x - 1$

- A horizontal line (parallel to the x-axis) has the rule $y = c$, since $m = 0$.



- A vertical line (parallel to the y-axis) has the rule $x = k$.



Example 8 Stating the gradient and y-intercept from the rule

Write down the gradient and y-intercept for the graphs of these rules.

a $y = 2x + 3$

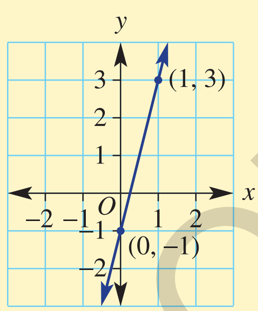
b $y = \frac{1}{3}x - 4$

SOLUTION	EXPLANATION
a $y = 2x + 3$ gradient = 2 y-intercept = 3	The coefficient of x is 2 and this number is the gradient. The y-intercept is the constant.
b $y = \frac{1}{3}x - 4$ gradient = $\frac{1}{3}$ y-intercept = -4	The gradient (m) is the coefficient of x. Remember that $y = \frac{1}{3}x - 4$ is the same as $y = \frac{1}{3}x + (-4)$ so the constant is -4.

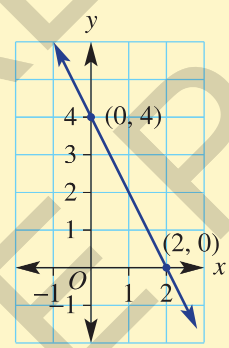
Example 9 Finding a rule from a graph

Find the rule for these graphs by first finding the values of m and the y-intercept.

a



b

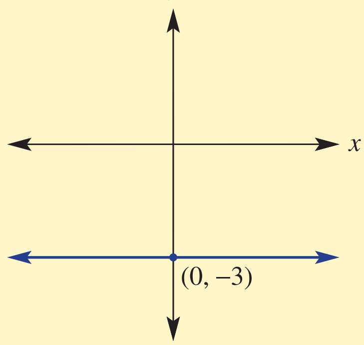


SOLUTION	EXPLANATION
a $m = \frac{\text{rise}}{\text{run}}$ $= \frac{4}{1}$ $= 4$	Between $(0, -1)$ and $(1, 3)$ the rise is 4 and the run is 1.
$c = -1$ $y = 4x - 1$	The line cuts the y-axis at -1. Substitute the value of m and c into $y = mx + c$.
b $m = \frac{\text{rise}}{\text{run}}$ $= \frac{-4}{2}$ $= -2$	Between $(0, 4)$ and $(2, 0)$ y falls by 4 units as x increases by 2.
$c = 4$ $y = -2x + 4$	The line cuts the y-axis at 4.

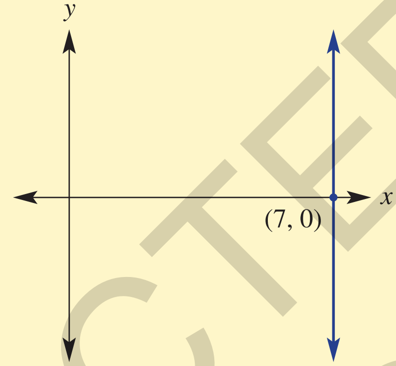
Example 10 Finding rules for horizontal and vertical lines

Write the rule for these horizontal and vertical lines.

a



b



SOLUTION	EXPLANATION
a $y = -3$	All points on the line have a y value of –3 and the gradient is 0.
b $x = 7$	Vertical lines take the form $x = k$. Every point on the line has an x value of 7.

Exercise 9E

1–2(½)	2(½)	—

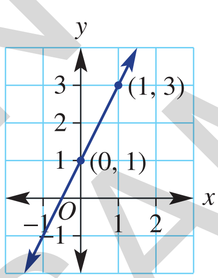
UNDERSTANDING

1 Substitute the given value of m and c into $y = mx + c$ to write a rule.

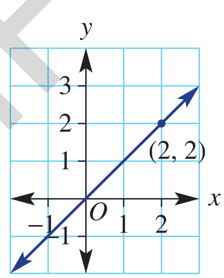
a $m = 2$ and $c = 3$ **b** $m = 4$ and $c = 2$ **c** $m = -3$ and $c = 1$
d $m = -5$ and $c = -3$ **e** $m = 2$ and $c = -5$ **f** $m = -10$ and $c = -11$

2 For these graphs write down the y-intercept and find the gradient using $\frac{\text{rise}}{\text{run}}$.

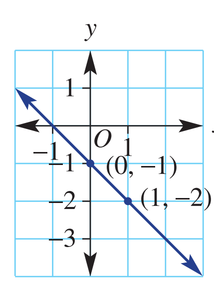
a



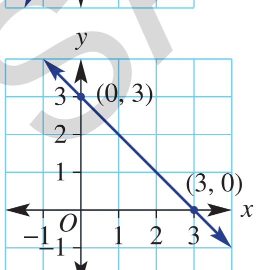
b



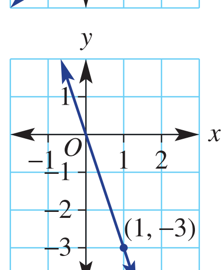
c



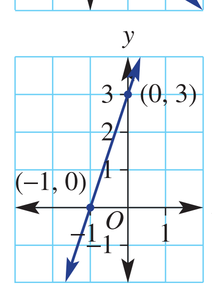
d



e



f



9E

3–6(½)	3–7(½)	3–7(½)

FLUENCY

Example 8

3 Write down the gradient and y-intercept for the graphs of these rules.

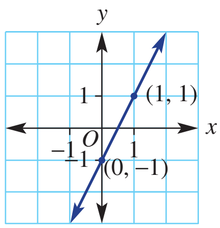
a $y = 4x + 2$ **b** $y = 3x + 7$ **c** $y = \frac{1}{2}x + 1$ **d** $y = \frac{2}{3}x + \frac{1}{2}$

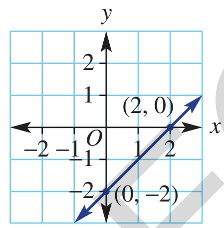
e $y = -2x + 3$ **f** $y = -4x + 4$ **g** $y = -7x - 1$ **h** $y = -3x - 7$

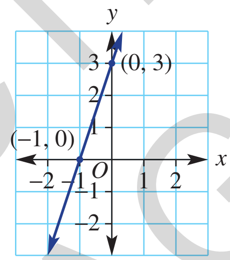
i $y = -x - 6$ **j** $y = -\frac{1}{2}x + 5$ **k** $y = -\frac{2}{3}x - \frac{1}{2}$ **l** $y = \frac{3}{7}x - \frac{4}{5}$

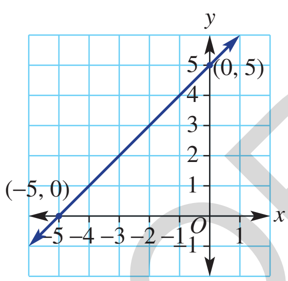
Example 9a

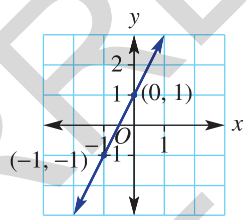
4 Find the rule for these graphs by first finding the values of m and c.

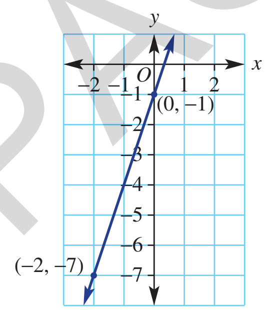
a


b


c


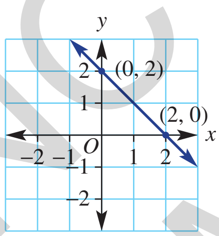
d


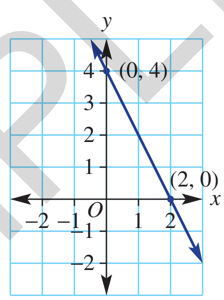
e


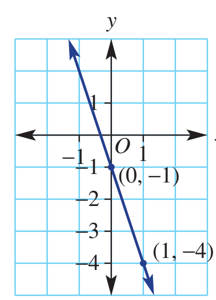
f


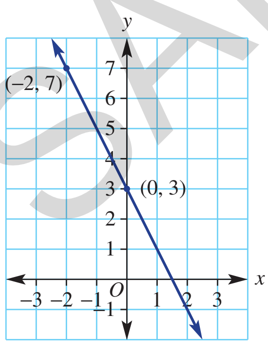
Example 9b

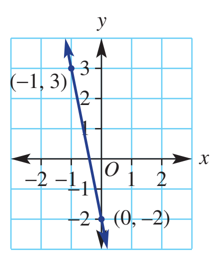
5 Find the rule for these graphs by first finding the values of m and c.

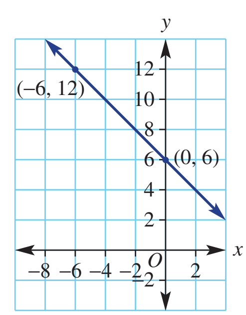
a


b


c


d


e


f


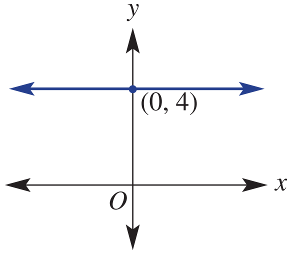
9E

FLUENCY

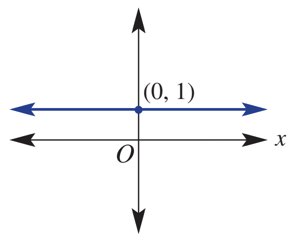
Example 10

6 Write the rule for these horizontal and vertical lines.

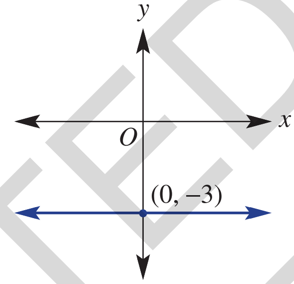
a



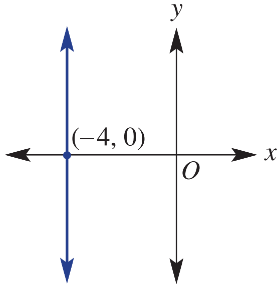
b



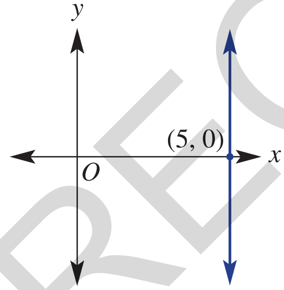
c



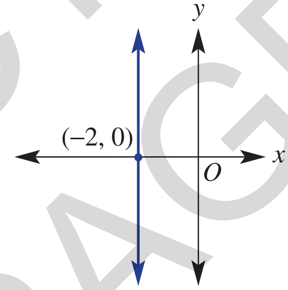
d



e



f



7 Sketch horizontal or vertical graphs for these rules.

a $y = 2$ **b** $y = -1$ **c** $y = -4$ **d** $y = 5$

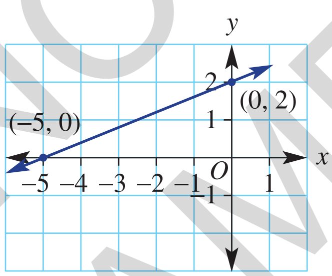
e $x = -3$ **f** $x = 4$ **g** $x = 1$ **h** $x = -1$

8, 9	9–11	9–11

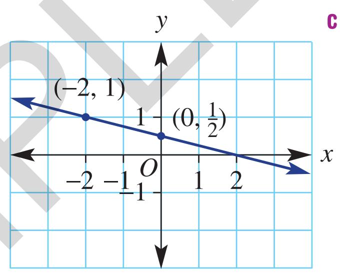
PROBLEM-SOLVING

8 These graphs have rules that involve fractions. Find m and c and write the rule.

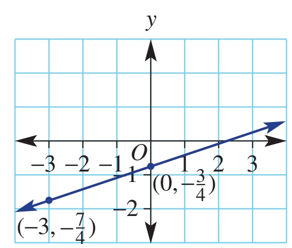
a



b



c



9 Find the rule for the graph of the lines connecting these pairs of points.

a (0, 0) and (2, 6) **b** (–1, 5) and (0, 0) **c** (–2, 5) and (0, 3) **d** (0, –4) and (3, 1)

10 A line passes through the given points. Note that the y-intercept is not given. Find m and c and write the linear rule. A graph may be helpful.

a (–1, 1) and (1, 5) **b** (–2, 6) and (2, 4) **c** (–2, 4) and (3, –1) **d** (–5, 0) and (2, 14)

11 Find the rectangular area enclosed by these sets of lines.

a $x = 4, x = 1, y = 2, y = 7$ **b** $x = 5, x = -3, y = 0, y = 5$

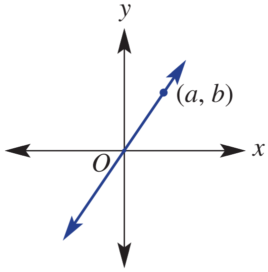
12	12, 13	13, 14

REASONING

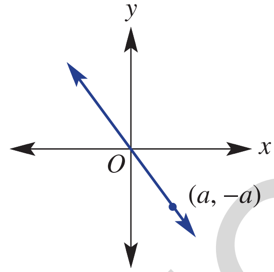
12 a Explain why the rule for the x-axis is given by $y = 0$.
b Explain why the rule for the y-axis is given by $x = 0$.

13 Write the rule for these graphs. Your rule should include the pronumerals a and/or b.

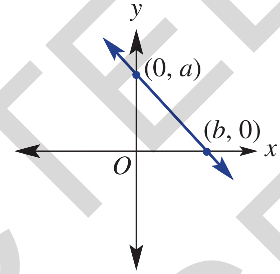
a



b



c



14 Some rules for straight lines may not be written in the form $y = mx + c$. The rule $2y + 4x = 6$, for example, can be rearranged to $2y = -4x + 6$ then to $y = -2x + 3$. So clearly $m = -2$ and $c = 3$. Use this idea to find m and c for these rules.

a $2y + 6x = 10$
b $3y - 6x = 9$
c $2y - 3x = 8$
d $x - 2y = -6$

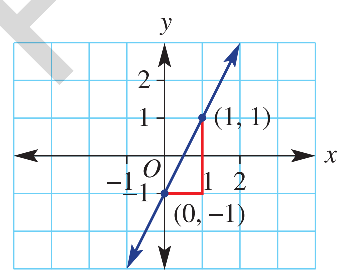
Sketching with m and c

—	—	15

ENRICHMENT

15 The gradient and y-intercept can be used to sketch a graph without the need to plot more than two points.

For example, the graph of the rule $y = 2x - 1$ has $m = 2\left(= \frac{2}{1}\right)$ and $c = -1$. By plotting the point $(0, -1)$ for the y-intercept and moving 1 to the right and 2 up for the gradient, a second point $(1, 1)$ can be found.



Use this idea to sketch the graphs of these rules.

a $y = 3x - 1$
b $y = 2x - 3$
c $y = -x + 2$
d $y = -3x - 1$
e $y = 4x$
f $y = -5x$
g $y = \frac{1}{2}x - 2$
h $y = -\frac{3}{2}x + 1$

9F The x-intercept EXTENDING

Interactive

Widgets

HOTsheets

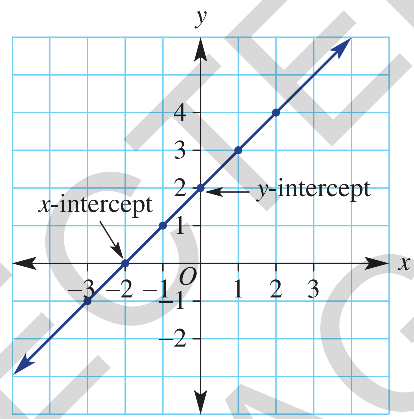
Walkthroughs

We know that the y-intercept marks the point where a line cuts the y-axis. This is also the value of y for the rule where $x = 0$. Similarly the x-intercept marks the point where $y = 0$. This can be viewed in a table of values or found using the algebraic method.

x-intercept where $y = 0$

x	−3	−2	−1	0	1	2
y	−1	0	1	2	3	4

y-intercept where $x = 0$



Let's start: Discover the method

These rules all give graphs that have x-intercepts at which $y = 0$.

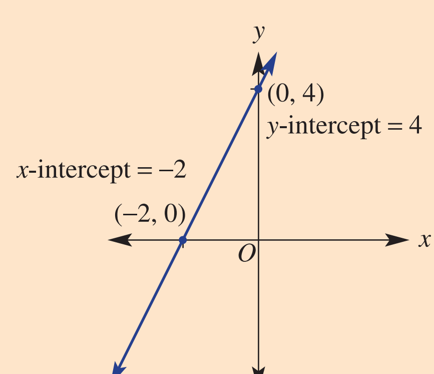
A $y = 2x - 2$ **B** $y = x - 5$ **C** $y = 3x - 9$ **D** $y = 4x + 3$

- First try to guess the x-intercept by a trial and error (guess and check) method. Start by asking what value of x makes $y = 0$.
- Discuss why the rule for **D** is more difficult to work with than the others.
- Can you describe an algebraic method that will give the x-intercept for any rule? How would you show your working for such a method?

Key ideas

- The x **-intercept** is the point on a graph where $y = 0$.
- Find the x-intercept by substituting $y = 0$ into the rule. Solve using algebraic steps. For example:

$$y = 2x + 4$$
$$0 = 2x + 4$$
$$-4 = 2x$$
$$-2 = x$$
$\therefore$ x-intercept is −2



Example 11 Finding the *x*-intercept

For the graphs of these rules, find the *x*-intercept.

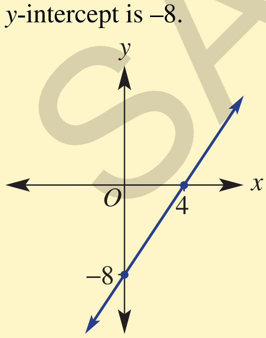
a $y = 3x - 6$

b $y = -2x + 1$

SOLUTION	EXPLANATION
a $y = 3x - 6$	
$0 = 3x - 6$	Substitute $y = 0$ into the rule.
$6 = 3x$	Add 6 to both sides.
$2 = x$	Divide both sides by 3.
$\therefore$ *x*-intercept is 2	
b $y = -2x + 1$	
$0 = -2x + 1$	Substitute $y = 0$ into the rule.
$-1 = -2x$	Subtract 1 from both sides.
$\frac{1}{2} = x$	Divide both sides by -2.
$\therefore$ *x*-intercept is $\frac{1}{2}$	

Example 12 Sketching with intercepts

Find the *x*- and *y*-intercepts and then sketch the graph of the rule $y = 2x - 8$.

SOLUTION	EXPLANATION
$y = 2x - 8$	
$0 = 2x - 8$	Substitute $y = 0$ into the rule.
$8 = 2x$	Add 8 to both sides.
$4 = x$	Divide both sides by 2.
x-intercept is 4.	
y-intercept is -8.	The *y*-intercept is the value of c in $y = mx + c$. Alternatively substitute $x = 0$ to get $y = -8$.
	Sketch by showing the *x*- and *y*-intercepts. There is no need to show a grid.

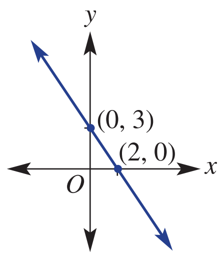
Exercise 9F

1–3(½)	3(½)	—

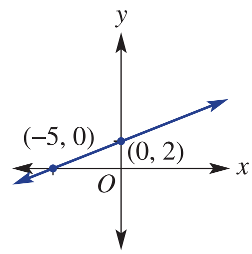
UNDERSTANDING

1 Look as these graphs and write down the x-intercept (the value of x where $y = 0$).

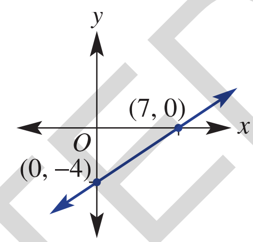
a



b



c



2 For each of these tables state the value of x that gives a y value of 0.

a

x	–2	–1	0	1	2
y	2	0	–2	–4	–6

b

x	1	2	3	4	5
y	3	2	1	0	–1

c

x	–4	–3	–2	–1	0
y	8	4	0	–4	–8

d

x	$-\frac{1}{2}$	0	$\frac{1}{2}$	1	$\frac{3}{2}$
y	–2	0	2	4	6

3 Solve each of these equations for x.

a $0 = x - 4$ **b** $0 = x + 2$ **c** $0 = x + 5$
d $0 = 2x + 10$ **e** $0 = 3x - 12$ **f** $0 = 4x + 28$
g $0 = 3x + 1$ **h** $0 = 5x + 2$ **i** $0 = -2x - 4$
j $0 = -5x - 20$ **k** $0 = -x + 2$ **l** $0 = -x + 45$

4–5(½)	4–5(½)	4–5(½)

FLUENCY

Example 11

4 For the graphs of these rules, find the x-intercept.

a $y = x - 1$ **b** $y = x - 6$ **c** $y = x + 2$
d $y = 2x - 8$ **e** $y = 4x - 12$ **f** $y = 3x + 6$
g $y = 2x + 20$ **h** $y = -2x + 4$ **i** $y = -4x + 8$
j $y = -2x - 4$ **k** $y = -x - 7$ **l** $y = -x + 11$

Example 12

5 Find the x- and y-intercepts and then sketch the graphs of these rules.

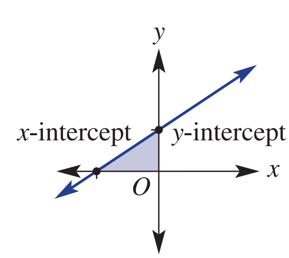
a $y = x + 1$ **b** $y = x - 4$ **c** $y = 2x - 10$
d $y = 3x + 9$ **e** $y = -2x - 4$ **f** $y = -4x + 8$
g $y = -x + 3$ **h** $y = -x - 5$ **i** $y = -3x - 15$

6, 7	6–8	7–9

PROBLEM-SOLVING

6 Find the area of the triangle enclosed by the x-axis, the y-axis and the given line. You will first need to find the x- and y-intercepts.

a $y = 2x + 4$ **b** $y = 3x - 3$
c $y = -x + 5$ **d** $y = -4x - 8$



9F

PROBLEM-SOLVING

7 Write the rule for a graph that matches the given information.
 a y-intercept 4, x-intercept -4
 b y-intercept -1, x-intercept 2
 c x-intercept -2, gradient 3
 d x-intercept 5, gradient -5

8 The height of water (H cm) in a tub is given by $H = -2t + 20$ where t is the time in seconds.
 a Find the height of water initially (i.e. at $t = 0$).
 b How long will it take for the tub to empty?

9 The amount of credit (C cents) on a phone card is given by the rule $C = -t + 200$ where t is time in seconds.
 a How much credit is on the card initially ($t = 0$)?
 b For how long can you use the phone card before the money runs out?

10	10, 11	11, 12

REASONING

10 Some lines have no x-intercept. What type of lines are they? Give two examples.

11 Decide if the x-intercept will be positive or negative for $y = mx + c$ under these conditions.
 a m is positive and c is positive, e.g. $y = 2x + 4$
 b m is positive and c is negative, e.g. $y = 2x - 4$
 c m is negative and c is negative, e.g. $y = -2x - 4$
 d m is negative and c is positive, e.g. $y = -2x + 4$

12 Write the rule for the x-intercept if $y = mx + c$. Your answer will include the pronumerals m and c.

Using $ax + by = d$

—	—	13

ENRICHMENT

13 The x-intercept can be found if the rule for the graph is given in any form. Substituting $y = 0$ starts the process whatever the form of the rule. Find the x-intercept for the graphs of these rules.
 a $x + y = 6$
 b $3x - 2y = 12$
 c $y - 2x = 4$
 d $2y - 3x = -9$
 e $y - 3x = 2$
 f $3y + 4x = 6$
 g $5x - 4y = -10$
 h $2x + 3y = 3$
 i $y - 3x = -1$

Progress quiz

9A **1** For x-coordinates from –2 to 2, construct a table and draw a graph for the rule $y = 2x + 1$.

9B **2** Decide if the given points lie on the graph with the given rule.

a Rule: $y = 3x$ Points: **i** (2, 6) **ii** (4, 7)

b Rule: $y = 4 - x$ Points: **i** (5, 1) **ii** (–2, 6)

9C **3** Find the rule for these tables of values.

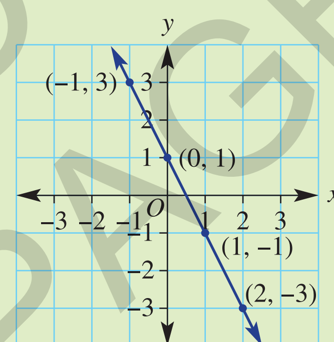
a

x	–2	–1	0	1	2
y	–7	–4	–1	2	5

b

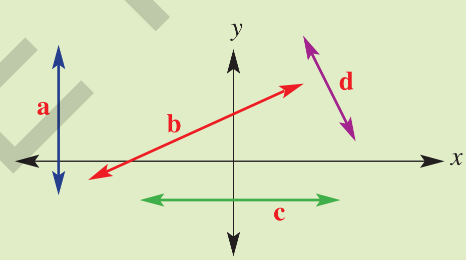
x	2	3	4	5	6
y	–2	–4	–6	–8	–10

9C **4** Find the rule for this graph by first constructing a table of (x, y) values.



9D **5** Decide if the lines labelled **a**, **b**, **c** and **d** on this graph have a positive, negative, zero or undefined gradient.

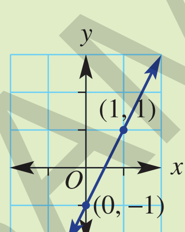
Ext



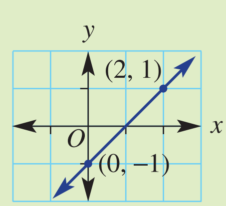
9D **6** Find the gradient of these lines. Use Gradient $= \frac{\text{rise}}{\text{run}}$.

Ext

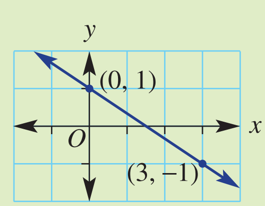
a



b



c



9E Ext

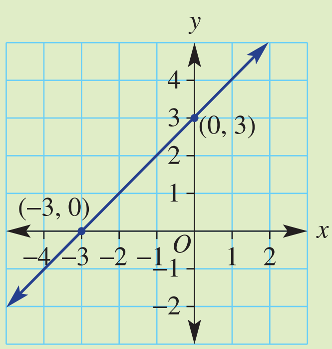
7 Write down the gradient and y-intercept for the graphs of these rules.

a $y = 3x + 2$ b $y = \frac{2}{3}x - 5$ c $y = -3x + 4$ d $y = -x - \frac{4}{5}$

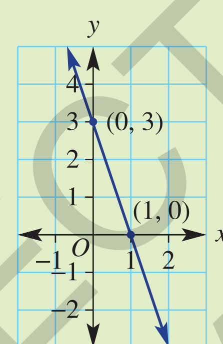
9E Ext

8 Find the rule for these graphs by first finding the values of m and the y-intercept.

a



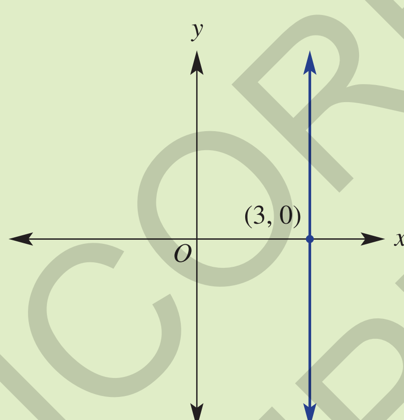
b



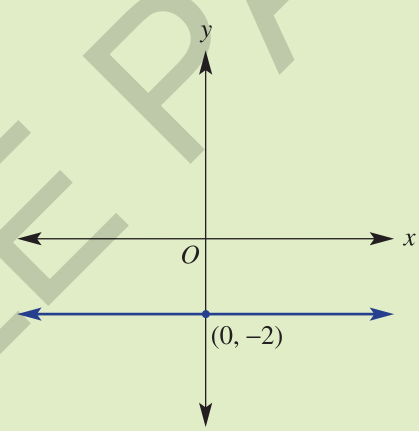
9E Ext

9 Write the rule for these horizontal and vertical lines.

a



b



9F Ext

10 For the graphs of these rules, find the x-intercept.

a $y = 2x - 6$ b $y = -3x - 12$

9F Ext

11 Find the x- and y-intercepts and then sketch the graph of the rule $y = 3x - 6$.

9E Ext

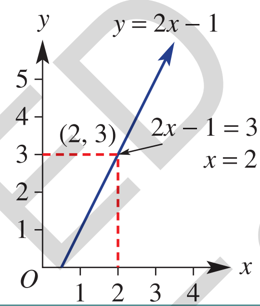
12 Find the rule for the graph of the line connecting this pair of points: $(0, -5)$ and $(2, -9)$.

9G Using graphs to solve linear equations

Interactive Widgets HOTsheets Walkthroughs

The rule for a straight line shows the connection between the x- and y-coordinate of each point on the line. We can substitute a given x-coordinate into the rule to calculate the y-coordinate. When we substitute a y-coordinate into the rule, it makes an equation that can be solved to give the x-coordinate. So, for every point on a straight line, the value of the x-coordinate is the solution to a particular equation.

The point of intersection of two straight lines is the shared point where the lines cross over each other. This is the only point with coordinates that satisfy both equations; that is, makes both equations true (LHS = RHS).



For example, the point (2, 3) on the line $y = 2x - 1$ shows us that when $2x - 1 = 3$ the solution is $x = 2$.

Let's start: Matching equations and solutions

When a value is substituted into an equation and it makes the equation true (LHS = RHS), then that value is a solution to that equation.

- From the lists below, match each equation with a solution. Some equations have more than one solution.

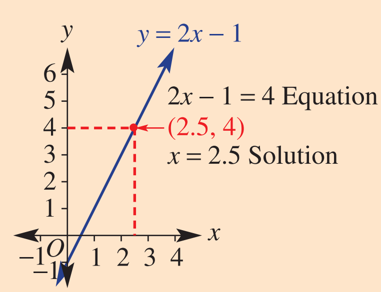
Equations	
$2x - 4 = 8$	$y = x + 4$
$3x + 2 = 11$	$y = 2x - 5$
$y = 10 - 3x$	$5x - 3 = 2$

Possible solutions			
$x = 1$	$(1, 5)$	$x = 2$	$(3, 1)$
$x = -1$	$x = 6$	$(2, -1)$	$(2, 6)$
$(-2, -9)$	$(-2, 16)$	$x = 3$	$(2, 4)$

- Which two equations share the same solution and what is this solution?
- List the equations that have only one solution. What is a common feature of these equations?
- List the equations that have more than one solution. What is a common feature of these equations?

Key ideas

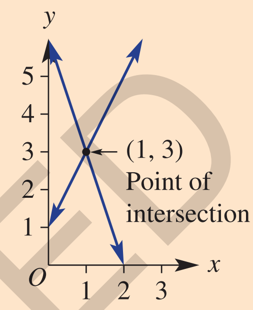
- The x-coordinate of each point on the graph of a straight line is a solution to a particular linear equation.
  - A particular linear equation is formed by substituting a chosen y-coordinate into a linear relationship. For example, if $y = 2x - 1$ and $y = 4$, then the linear equation is $2x - 1 = 4$.
  - The solution to this equation is the x-coordinate of the point with the chosen y-coordinate. For example, the point (2.5, 4) shows that $x = 2.5$ is the solution to $2x - 1 = 4$.



- A point (x, y) is a solution to the equation for a line if its coordinates make the equation true.
  - An equation is true when LHS = RHS after the coordinates are substituted.
  - Every point on a straight line is a solution to the equation for that line.
  - Every point that is not on a straight line is a not a solution to the equation for that line.

Key ideas

- The point of intersection of two straight lines is the only solution that satisfies both equations.
  - The point of intersection is the shared point where two straight lines cross each other.
  - This is the only point with coordinates that make both equations true.
    For example, (1, 3) is the only point that makes both $y = 6 - 3x$ and $y = 2x + 1$ true.
    Substituting (1, 3)



$y = 6 - 3x$	$y = 2x + 1$
$3 = 6 - 3 \times 1$	$3 = 2 \times 1 + 1$
$3 = 3$ (True)	$3 = 3$ (True)

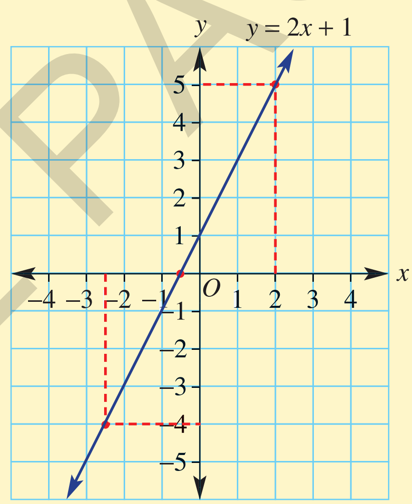
Example 13 Using a linear graph to solve an equation

Use the graph of $y = 2x + 1$ shown here to solve each of the following equations:

a $2x + 1 = 5$

b $2x + 1 = 0$

c $2x + 1 = -4$



SOLUTION	EXPLANATION
a $x = 2$	Locate the point on the line with y-coordinate 5. The x-coordinate of this point is 2 so $x = 2$ is the solution to $2x + 1 = 5$.
b $x = -0.5$	Locate the point on the line with y-coordinate 0. The x-coordinate of this point is -0.5 so $x = -0.5$ is the solution to $2x + 1 = 0$.
c $x = -2.5$	Locate the point on the line with y-coordinate -4. The x-coordinate of this point is 2.5 so $x = -2.5$ is the solution to $2x + 1 = -4$.

Example 14 Using the point of intersection of two lines to solve an equation

Use the graph of $y = 4 - x$ and $y = 2x + 1$, shown here, to answer these questions.

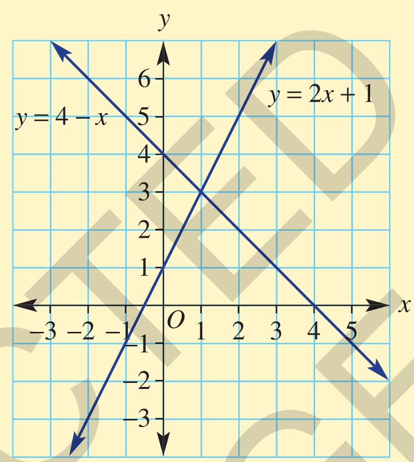
a Write two equations that each have $x = -2$ as a solution.

b Write four solutions (x, y) for the line with equation $y = 4 - x$.

c Write four solutions (x, y) for the line with equation $y = 2x + 1$.

d Write the solution (x, y) that is true for both lines and show that it satisfies both line equations.

e Solve the equation $4 - x = 2x + 1$.



SOLUTION	EXPLANATION
a $4 - x = 6$ $2x + 1 = -3$	$(-2, 6)$ is on the line $y = 4 - x$ so $4 - x = 6$ has solution $x = -2$. $(-2, -3)$ is on the line $y = 2x + 1$ so $2x + 1 = -3$ has solution $x = -2$.
b $(-2, 6)$, $(-1, 5)$, $(1, 3)$, $(4, 0)$	Many correct answers. Each point on the line $y = 4 - x$ is a solution to the equation for that line.
c $(-2, -3)$, $(0, 1)$, $(1, 3)$, $(2, 5)$	Many correct answers. Each point on the line $y = 2x + 1$ is a solution to the equation for that line.
d $(1, 3)$ $(1, 3)$ $y = 4 - x$ $y = 2x + 1$ $3 = 4 - 1$ $3 = 2 \times 1 + 1$ $3 = 3$ True $3 = 3$ True	The point of intersection $(1, 3)$ is the solution that satisfies both equations. Substitute $(1, 3)$ into each equation and show that it makes a true equation (LHS = RHS).
e $x = 1$	The solution to $4 - x = 2x + 1$ is the x-coordinate at the point of intersection. The value of both rules is equal for this x-coordinate.

Exercise 9G

1–4	4	—

UNDERSTANDING

1 Use the given rule to complete this table and then plot and join the points to form a straight line. $y = 2x - 1$.

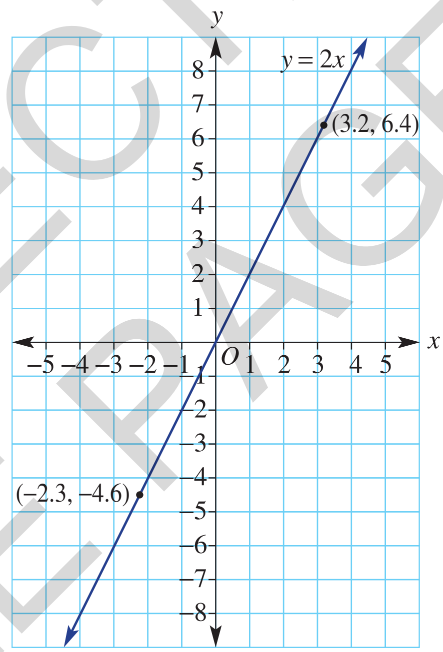
x	-2	-1	0	1	2
y					

2 Substitute each given y-coordinate into the rule $y = 2x - 3$, and then solve the equation algebraically to find the x-coordinate.

a $y = 7$ **b** $y = -5$

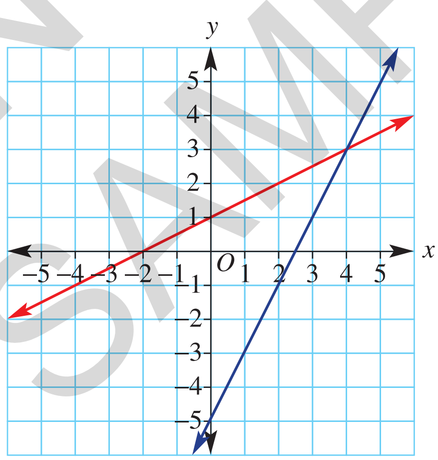
3 State the coordinates (x, y) of the point on this graph of $y = 2x$ where:

a $2x = 4$ (i.e. $y = 4$)
b $2x = 6.4$
c $2x = -4.6$
d $2x = 7$
e $2x = -14$
f $2x = 2000$
g $2x = 62.84$
h $2x = -48.602$
i $2x =$ *any number* (worded answer)

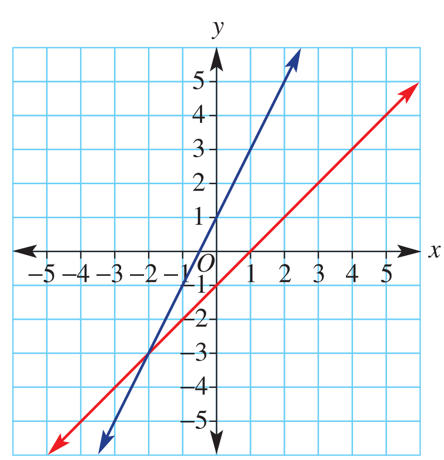


4 For each of these graphs write down the coordinates of the point of intersection (i.e. the point where the lines cross over each other).

a



b



9G

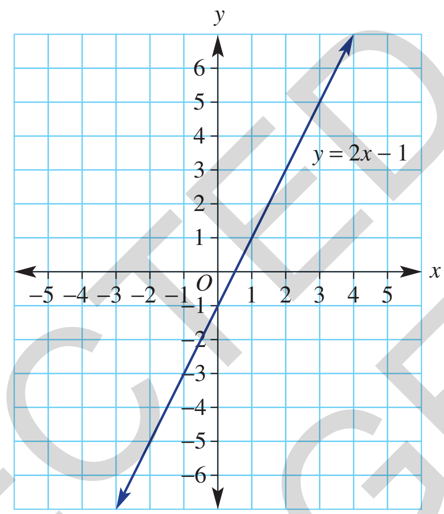
5–6(½), 7,8	5–6(½), 7,8	5–6(½), 7,8

FLUENCY

Example 13

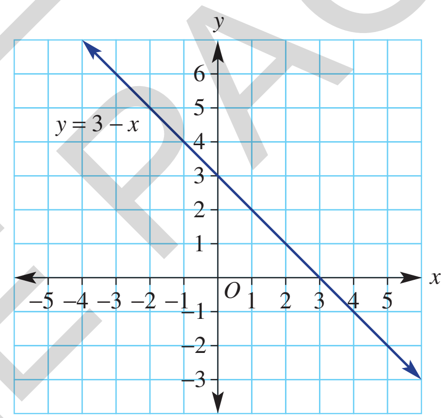
5 Use the graph of $y = 2x - 1$, shown here, to find the solution to each of these equations.

a $2x - 1 = 3$
b $2x - 1 = 0$
c $2x - 1 = 5$
d $2x - 1 = -6$
e $2x - 1 = -4$



6 Use the graph of $y = 3 - x$, shown here, to solve each of the following equations.

a $3 - x = 5.5$
b $3 - x = 0$
c $3 - x = 3.5$
d $3 - x = -1$
e $3 - x = -2$



7 Graph each pair of lines on the same set of axes and read off the point of intersection.

a $y = 2x - 1$

x	–2	–1	0	1	2	3
y						

$y = x + 1$

x	–2	–1	0	1	2	3
y						

b $y = -x$

x	–2	–1	0	1	2	3
y						

$y = x + 2$

x	–2	–1	0	1	2	3
y						

8 Use digital technology to sketch a graph of $y = 1.5x - 2.5$ for x and y values between –7 and 7. Use the graph to solve each of the following equations. Round answers to two decimal places.

a $1.5x - 2.5 = 3$ **b** $1.5x - 2.5 = -4.8$ **c** $1.5x - 2.5 = 5.446$

9 Use digital technology to sketch a graph of each pair of lines and find the coordinates of the points of intersection. Round answers to two decimal places.

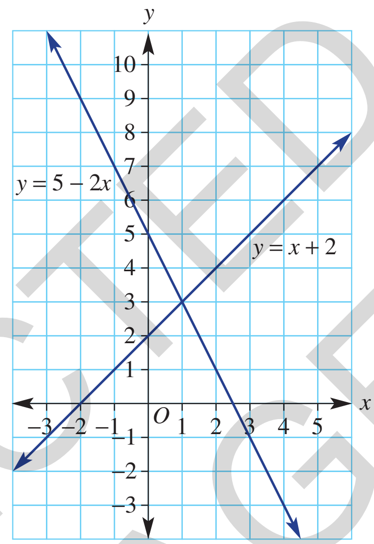
a $y = 0.25x + 0.58$ and $y = 1.5x - 5.4$ **b** $y = 2 - 1.06x$ and $y = 1.2x + 5$

10, 11 | 10–12 | 11–13

PROBLEM-SOLVING

Example 14

10 Use the graph of $y = 5 - 2x$ and $y = x + 2$, shown here, to answer the following questions.

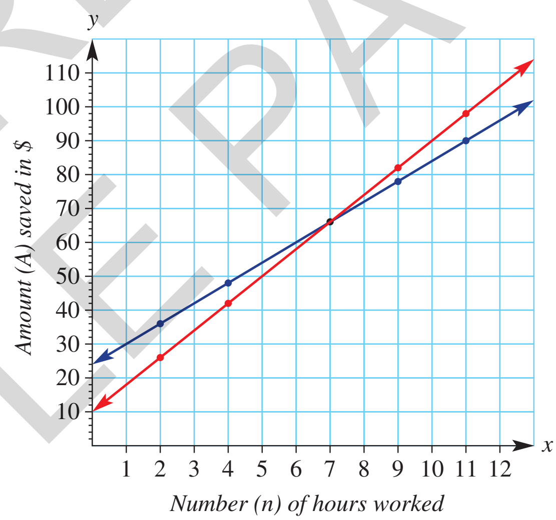


a Write two equations that each have $x = -1$ as a solution.
b Write four solutions (x, y) for the equation $y = 5 - 2x$.
c Write four solutions (x, y) for the equation $y = x + 2$.
d Write the solution (x, y) that is true for both lines and show that it satisfies both line equations.
e Solve the equation $5 - 2x = x + 2$ from the graph.

11 Jayden and Ruby are saving all their money for the school ski trip.

- Jayden has saved \$24 and earns \$6 per hour mowing lawns.
- Ruby has saved \$10 and earns \$8 per hour babysitting.

This graph shows the total Amount (A) in dollars of their savings for the number (n) of hours worked.



a Here are two rules for calculating the Amount (A) saved for working for n hours.
$A = 10 + 8n$ and $A = 24 + 6n$
Which rule applies to Ruby and which to Jayden? Explain why.

b Use the appropriate line on the above graph to find the solution to the following equations.

i $10 + 8n = 42$ **ii** $24 + 6n = 48$ **iii** $10 + 8n = 66$
iv $24 + 6n = 66$ **v** $10 + 8n = 98$ **vi** $24 + 6n = 90$

c From the graph write three solutions (n, A) that satisfy $A = 10 + 8n$.
d From the graph write three solutions (n, A) that satisfy $A = 24 + 6n$.
e Write the solution (n, A) that is true for both Ruby's and Jayden's equations and show that it satisfies both equations.
f From the graph find the solution to the equation: $10 + 8n = 24 + 6n$ (i.e. find the value of n that makes Ruby's and Jayden's savings equal to each other).
g Explain how many hours have been worked and what their savings are at the point of intersection of the two lines.

9G

PROBLEM-SOLVING

12 Jessica and Max have a 10 second running race.

- Max runs at 6 m/second.
- Jessica is given a 10 m head-start and runs at 4 m/second.

a Copy and complete this table showing the distance run by each athlete.

Time (t) in seconds	0	1	2	3	4	5	6	7	8	9	10
Max's distance (d) in metres	0										
Jessica's distance (d) in metres	10										

b Plot these points on a distance-time graph and join to form two straight lines labelling them 'Jessica' and 'Max'.

c Find the rule linking distance d and time t for Max.

d Using the rule for Max's race, write an equation that has the solution:

i $t = 3$ **ii** $t = 5$ **iii** $t = 8$

e Find the rule linking distance d and time t for Jessica.

f Using the rule for Jessica's race, write an equation that has the solution:

i $t = 3$ **ii** $t = 5$ **iii** $t = 8$

g Write the solution (t, d) that is true for both distance equations and show that it satisfies both equations.

h Explain what is happening in the race at the point of intersection and for each athlete state the distance from the starting line and time taken.

13 Some equations can be re-arranged so that a given graph can be used to find a solution. For example $4x - 1 = 2(x - 3)$ can be solved using the graph of $y = 2x - 3$ with this re-arrangement:

$$4x - 1 = 2(x - 3)$$

$$4x - 1 = 2x - 6$$

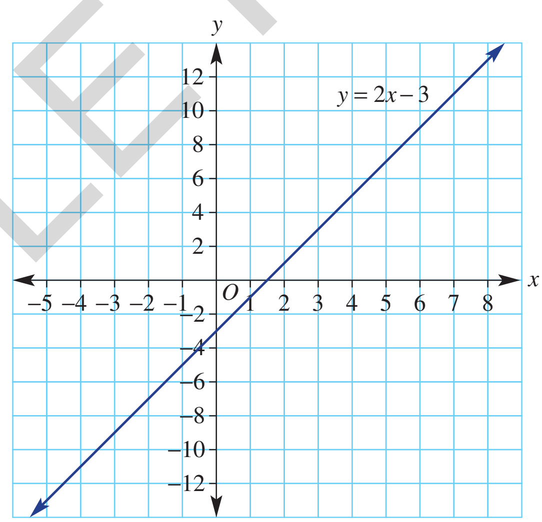
$-2x$ (both sides)

$$2x - 1 = -6$$

-2 (both sides)

$$2x - 3 = -8$$

$y = -8$ on the graph gives the solution $x = -2.5$



Re-arrange the following equations so the left side of each is $2x - 3$ and then use this graph of $y = 2x - 3$ to find each solution.

a $4x - 1 = 2(x - 5)$

b $5x + 7 = 3(x + 4)$

c $3 = 6 - 2(x - 3)$

d $3 + 4x - 1 = x + 14 + x - 4$

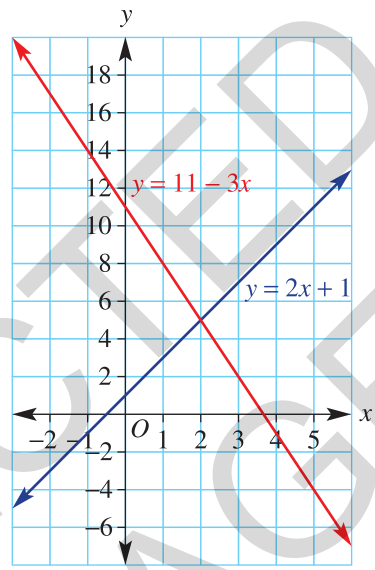
e $3(x - 3) - 2x = 4x + 3 - 5x$

f $4(x - 1) + x = 5x - 7 - 2x$

14	14	15

REASONING

14 This graph shows two lines with equations $y = 11 - 3x$ and $y = 2x + 1$.



a Copy and complete the coordinates of each point that is a solution for the given linear equation.

i $y = 11 - 3x$

(–2, ?), (–1, ?) (0, ?) (1, ?) (2, ?) (3, ?) (4, ?) (5, ?)

ii $y = 2x + 1$

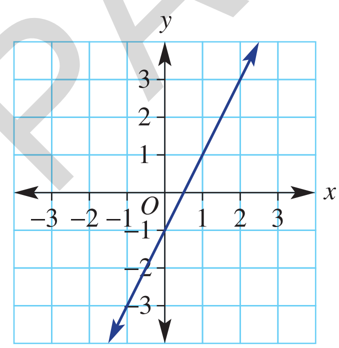
(–2, ?), (–1, ?) (0, ?) (1, ?) (2, ?) (3, ?) (4, ?) (5, ?)

b State the coordinates of the point of intersection and show it is a solution to both equations.

c Explain why the point of intersection is the only solution that satisfies both equations.

15 Here is a table and graph for the line $y = 2x - 1$.

x	–1	–0.5	0	0.5	1	1.5	2
y	–3	–2	–1	0	1	2	3



a Luke says: *"Only seven equations can be solved from points on this line because the y values must be whole numbers."*

i What are three of the equations and their solutions that Luke could be thinking of?

ii Is Luke's statement correct? Explain your conclusion with some examples.

b Chloe says: *"There are sixty equations that can be solved from points on this line because y values can go to one decimal place."*

i What are three extra equations and their solutions that Chloe might have thought of?

ii Is Chloe's statement correct? Explain your conclusion with some examples.

c Jamie says: *"There are an infinite number of equations that can be solved from points on a straight line."*

i Write the equations that can be solved from these two points: (1.52, 2.04) and (1.53, 2.06).

ii Write the coordinates of two more points with x-coordinates between the first two points and write the equations that can be solved from these two points.

iii Is Jamie's statement correct? Explain your reasons.

9G

More than one solution

— — 16

ENRICHMENT

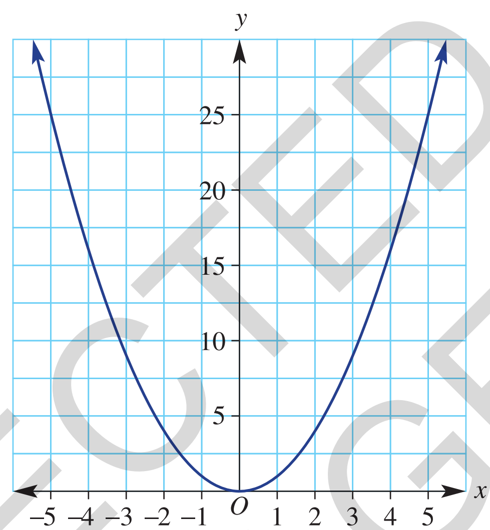
16 a Use this graph of $y = x^2$ to solve the following equations.

i $x^2 = 4$

ii $x^2 = 9$

iii $x^2 = 16$

iv $x^2 = 25$



b Explain why there are two solutions to each of the equations in question **a** above.

c Use digital technology to graph $y = x^2$ and graphically solve the following equations, rounding answers to two decimal places.

i $x^2 = 5$

ii $x^2 = 6.8$

iii $x^2 = 0.49$

iv $x^2 = 12.75$

v $x^2 = 18.795$

d Give one reason why the graph of $y = x^2$ does *not* give a solution to the equation $x^2 = -9$.

e List three more equations of the form x^2 = 'a number' that *can't* be solved from the graph of $y = x^2$.

f List the categories of numbers that *will* give a solution to the equation: x^2 = 'a number'.

g Graph $y = x + 2$ and $y = x^2$ on the same screen and graphically solve $x^2 = x + 2$ by finding the x values of the points of intersection.

h Use digital technology to solve the following equations using graphical techniques. Round answers to two decimal places.

i $x^2 = 3x + 16$

ii $x^2 = 27 - 5x$

iii $x^2 = 2x - 10$

iv $x^2 = 6x - 9$

9H Applying linear graphs EXTENDING

Interactive Widgets HOTsheets Walkthroughs

Rules and graphs can be used to help analyse many situations in which there is a relationship between two variables. If the rate of change of one variable with respect to another is constant, then the relationship will be linear and a graph will give a straight line. For example, if a pile of dirt being emptied out of a pit increases at a rate of 12 tonnes per hour, then the graph of the mass of dirt over time would be a straight line. For every hour, the mass of dirt increases by 12 tonnes.

A constant rate of excavation creates a linear relationship and a straight line graph.

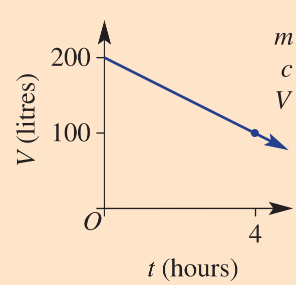
Let's start: Water storage

The volume of water in a tank starts at 1000 litres and with constant rainfall the volume of water increases by 2000 litres per hour for 5 hours.

- Describe the two related variables in this situation.
- Discuss whether or not the relationship between the two variables is **linear**.
- Use a table and a graph to illustrate the relationship.
- Find a rule that links the two variables and discuss how your rule might be used to find the volume of water in the tank at a given time.

Key ideas

- If the rate of change of one variable with respect to another is constant, then the relationship between the two variables is **linear**.
- When applying straight line graphs, choose letters to replace x and y to suit the variables. For example, V for volume and t for time.
- $y = mx + c$ can be used to help find the linear rule linking two variables.



$m = -25$
$c = 200$
$V = -25t + 200$

Example 15 Linking distance with time

A hiker walks at a constant rate of 4 kilometres per hour for 4 hours.

a Draw a table of values using t for time in hours and d for distance in kilometres. Use t between 0 and 4.

b Draw a graph by plotting the points given in the table in part a.

c Write a rule linking d with t.

d Use your rule to find the distance travelled for 2.5 hours of walking.

e Use your rule to find the time taken to travel 8 km.

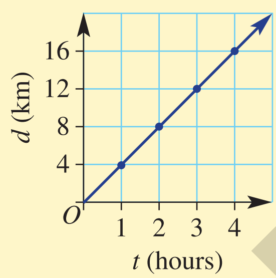
SOLUTION / **EXPLANATION**

a

t	0	1	2	3	4
d	0	4	8	12	16

d increases by 4 for every increase in t by 1.

b



Plot the points on a graph using a scale that matches the numbers in the table.

c

$m = \frac{4}{1} = 4$

$c = 0$

$y = mx + c$

$d = 4t + 0$

So $d = 4t$

Using $y = mx + c$ find the gradient (m) and the y-intercept (c). Replace y with d and x with t.

d

$d = 4t$

$= 4 \times 2.5$

$= 10$

The distance is 10 km after 2.5 hours of walking.

Substitute $t = 2.5$ into your rule and find the value for d.

e

$d = 4t$

$8 = 4t$

$2 = t$

It takes 2 hours to travel 8 km.

Substitute $d = 8$ into your rule then divide both sides by 4.

Example 16 Applying graphs when the rate is negative

The initial volume of water in a dish in the sun is 300 mL. The water evaporates and the volume decreases by 50 mL per hour for 6 hours.

a Draw a table of values using t for time in hours and V for volume in millilitres.

b Draw a graph by plotting the points given in the table in part **a**.

c Write a rule linking V with t.

d Use your rule to find the volume of water in the dish after 4.2 hours in the sun.

e Use your rule to find the time taken for the volume to reach 75 mL.

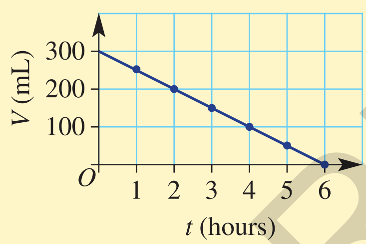
SOLUTION

a

t	0	1	2	3	4	5	6
V	300	250	200	150	100	50	0

The volume starts at 300 millilitres and decreases by 50 millilitres every hour.

b



Use numbers from 0 to 300 on the V-axis and 0 to 6 on the t-axis to accommodate all the numbers in the table.

c $m = \frac{-50}{1} = -50$

$c = 300$

$y = mx + c$

$V = -50t + 300$

The gradient m in $y = mx + c$ is given by $\frac{\text{rise}}{\text{run}} = \frac{-50}{1}$

c is the y-intercept = 300

d $V = -50t + 300$

$= -50 \times 4.2 + 300$

$= 90$

The volume of water in the dish is 90 millilitres after 4.2 hours.

Substitute $t = 4.2$ into your rule to find V.

e $V = -50t + 300$

$75 = -50t + 300$

$-225 = -50t$

$4.5 = t$

It takes 4.5 hours for the volume to reach 75 mL.

Substitute $V = 75$ into your rule.
Subtract 300 from both sides.
Divide both sides by –50.

Exercise 9H

1–4(½)	4	—

UNDERSTANDING

1 A rule linking distance d and time t is given by $d = 10t + 5$. Use this rule to find the value of d for the given values of t.

a $t = 1$ b $t = 4$ c $t = 0$ d $t = 12$

2 Write down the gradient (m) and the y-intercept (c) for these rules.

a $y = 3x + 2$ b $y = -2x - 1$ c $y = 4x + 2$

d $y = -10x - 3$ e $y = -20x + 100$ f $y = 5x - 2$

3 The height (in cm) of fluid in a flask increases at a rate of 30 cm every minute starting at 0 cm. Find the height of fluid in the flask at these times.

a 2 minutes b 5 minutes c 11 minutes

4 The volume of gas in a tank decreases from 30 L by 2 L every second. Find the volume of gas in the tank at these times.

a 1 second b 3 seconds c 10 seconds

5–7	5–8	6–8

FLUENCY

Example 15

5 A jogger runs at a constant rate of 6 kilometres per hour for 3 hours.

a Draw a table of values using t for time in hours and d for distance in kilometres. Use t between 0 and 3.

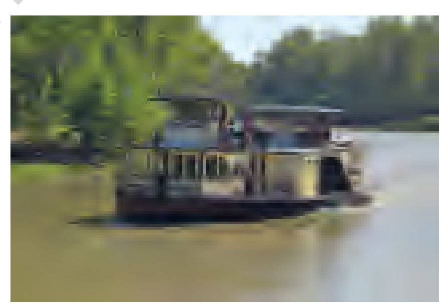
b Draw a graph by plotting the points given in the table in part a.

c Write a rule linking d with t.

d Use your rule to find the distance travelled for 1.5 hours of jogging.

e Use your rule to find how long it takes to travel 12 km.

6 A paddle steamer moves up the Murray River at a constant rate of 5 kilometres per hour for 8 hours.



a Draw a table of values using t for time in hours and d for distance in kilometres. Use t between 0 and 8.

b Draw a graph by plotting the points given in the table in part a.

c Write a rule linking d with t.

d Use your rule to find the distance travelled after 4.5 hours.

e Use your rule to find how long it takes to travel 20 km.

FLUENCY

Example 16

7 The volume of water in a sink is 20 L. The plug is pulled out and the volume decreases by 4 L per second for 5 seconds.

a Draw a table of values using t for time in seconds and V for volume in litres.
b Draw a graph by plotting the points given in the table in part a.
c Write a rule linking V with t.
d Use your rule to find the volume of water in the sink 2.2 seconds after the plug is pulled.
e Use your rule to find how long it takes for the volume to fall to 8 L.

8 A weather balloon at a height of 500 m starts to descend at a rate of 125 m per minute for 4 minutes.

a Draw a table of values using t for time in minutes and h for height in metres.
b Draw a graph by plotting the points given in the table in part a.
c Write a rule linking h with t.
d Use your rule to find the height of the balloon after 1.8 minutes.
e Use your rule to find how long it takes for the balloon to fall to a height of 125 m.

9, 10	9, 10	10, 11

PROBLEM-SOLVING

9 A BBQ gas bottle starts with 3.5 kg of gas. Gas is used at a rate of 0.5 kg per hour for a long lunch.

a Write a rule for the mass of gas M in terms of time t.
b How long will it take for the gas bottle to empty?
c How long will it take for the mass of the gas in the bottle to reduce to 1.25 kg?

10 A cyclist races 50 km at an average speed of 15 km per hour.

a Write a rule for the distance travelled d in terms of time t.
b How long will it take the cyclist to travel 45 km?
c How long will the cyclist take to complete the 50-km race? Give your answer in hours and minutes.

11 An oil well starts to leak and the area of an oil slick increases by 8 km^2 per day. How long will it take the slick to increase to 21 km^2? Give your answer in days and hours.

12	12	12, 13

REASONING

12 The volume of water in a tank (in litres) is given by $V = 2000 - 300t$ where t is in hours.

a What is the initial volume?
b Is the volume of water in the tank increasing or decreasing? Explain your answer.
c At what rate is the volume of water changing?

9H

REASONING

13 The cost of a phone call is 10 cents plus 0.5 cents per second.

a Explain why the cost c cents of the phone call for t seconds is given by $c = 0.5t + 10$.

b Explain why the cost C dollars of the phone call for t seconds is given by $C = 0.005t + 0.1$.

Danger zone

—	—	14

ENRICHMENT

14 Two small planes take off and land at the same airfield. One plane takes off from the runway and gains altitude at a rate of 15 metres per second. At the same time, the second plane flies near the runway and reduces its altitude from 100 metres at rate of 10 metres per second.

a Draw a table of values using t between 0 and 10 seconds and h for height in metres of both planes.

t (s)	0	1	2	3	4	5	6	7	8	9	10
h_1 (m)											
h_2 (m)											

b On the one set of axes draw a graph of the height of each plane during the 10-second period.

c How long does it take for the second plane to touch the ground?

d Write a rule for the height of each plane.

e At what time are the planes at the same height?

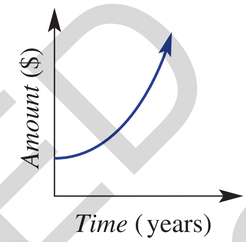
f At what time is the first plane at a height of 37.5 m?

g At what time is the second plane at a height of 65 m?

h At the same time, a third plane at an altitude of 150 m descends at a rate of 25 m per second. Will all three planes ever be at the same height at the same time? What are the heights of the three planes at the 4-second mark?

9I Non-linear graphs EXTENDING

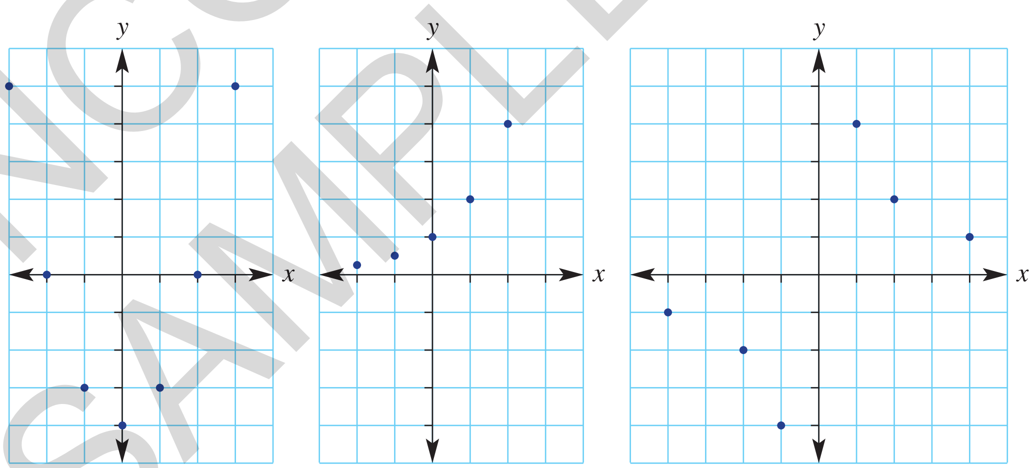
Not all relationships between two variables are linear. The amount of money invested in a compound interest account, for example, will not increase at a constant rate. Over time, the account balance will increase more rapidly, meaning that the graph of the relationship between *Amount* and *Time* will be a curve and not a straight line.



Compound interest means your account balance increases at a more rapid pace over time.

Let's start: Drawing curves by hand

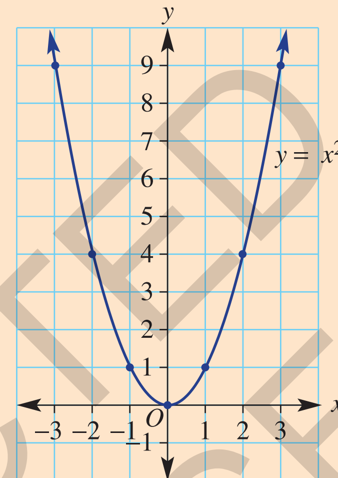
Here are three common non-linear patterns shown as a set of points on a graph.



- Test your drawing skill by copying these graphs and drawing a smooth curve through the points on each graph. For the third graph, you will need to draw two separate curves.
- Discuss why it might not be correct to join each point with a straight line segment.
- The three graphs above are $y = 2^x$, $y = \frac{4}{x}$ and $y = x^2 - 4$. Can you match each rule with its graph? Explain how you can check your answers.

Key ideas

- To plot **non-linear** curves given their rule follow these steps.
  - Construct a table of values using the rule.
  - Plot the points on a set of axes.
  - Join the plotted points to form a smooth curve.
- The graph of $y = x^2$ is an example of a non-linear graph called a **parabola**.

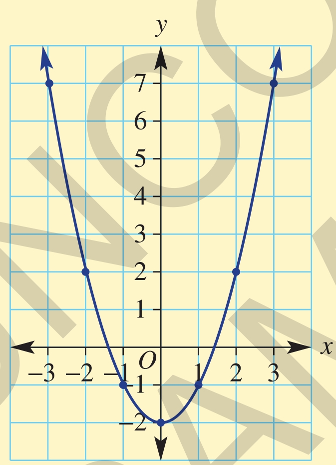


Example 17 Plotting a non-linear relationship

Plot points to draw the graph of $y = x^2 - 2$ using a table.

SOLUTION

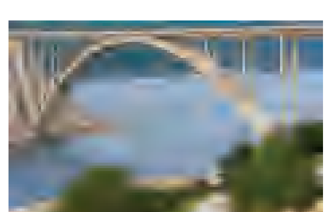
x	–3	–2	–1	0	–1	2	3
y	7	2	–1	–2	–1	2	7



EXPLANATION

Find the value of y by substituting each value of x into the rule.

Plot the points and join with a smooth curve. The curve is called a parabola.



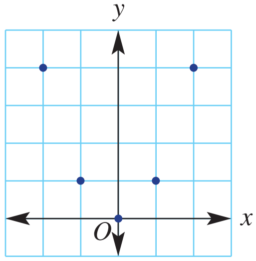
This bridge support arch is an inverted parabola.

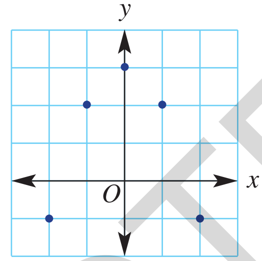
Exercise 9I

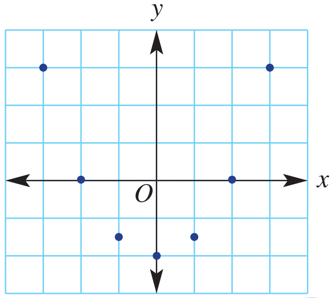
1–3	1–3	—

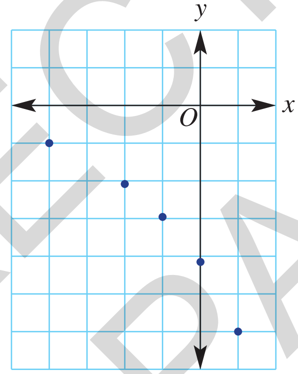
UNDERSTANDING

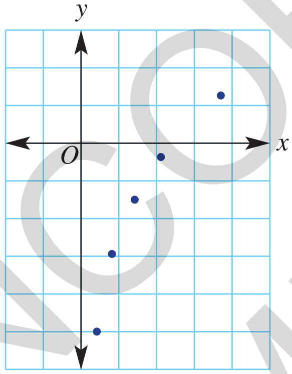
1 Copy these graphs then join the points to form smooth curves.

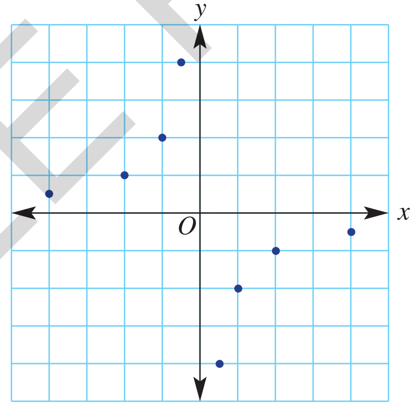
a


b


c


d


e


f


2 If $y = x^2 - 1$, find the value of y for these x values.

a $x = 0$
b $x = 3$
c $x = 2$
d $x = -4$

3 If $y = 2^x$, find the value of y for these x values.

a $x = 1$
b $x = 2$
c $x = 3$
d $x = 4$

9I

	4	4	4(½)

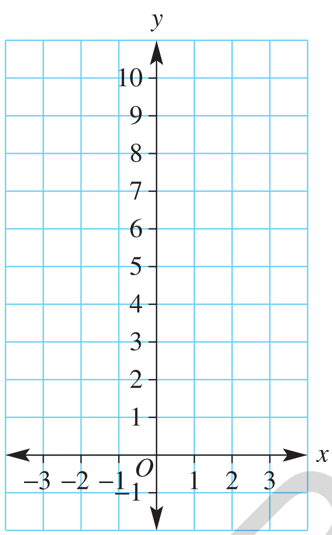
FLUENCY

Example 17

4 Plot points to draw the graph of each of the given rules. Use the table and set of axes as a guide.

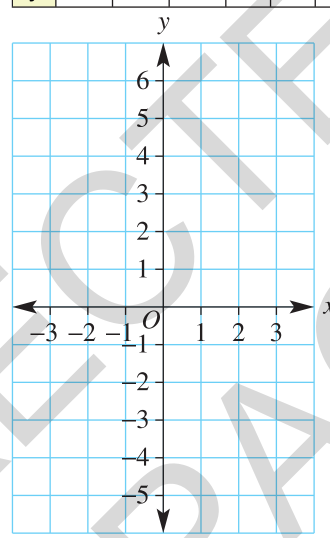
a $y = x^2$

x	−3	−2	−1	0	1	2	3
y	9						



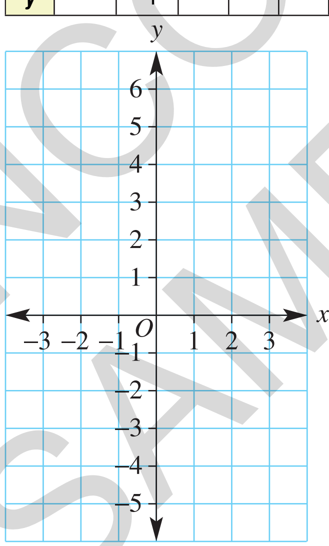
b $y = x^2 - 4$

x	−3	−2	−1	0	1	2	3
y	5						



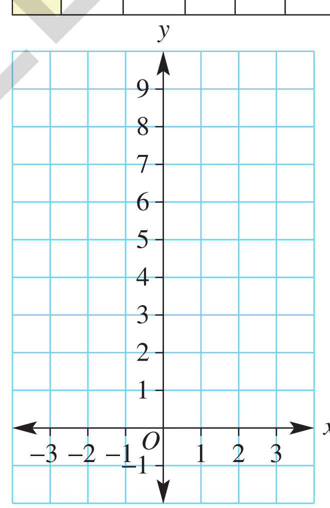
c $y = 5 - x^2$

x	−3	−2	−	0	1	2	3
y		1					



d $y = 2^x$

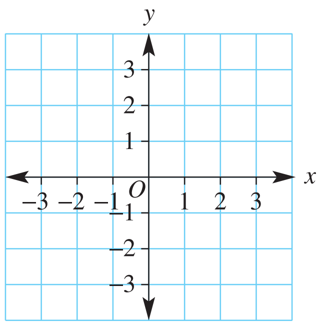
x	−2	−1	0	1	2	3
y	$\frac{1}{4}$	$\frac{1}{2}$	1			



FLUENCY

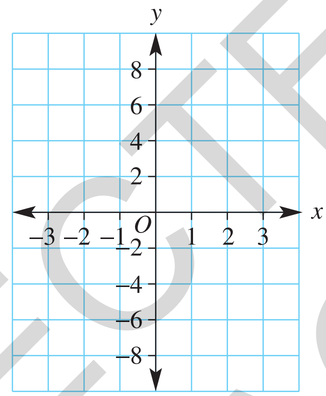
e $y = \frac{1}{x}$

x	−2	−1	$-\frac{1}{2}$	$\frac{1}{2}$	1	2
y			−2	2		



f $y = x^3$

x	−2	−1	0	1	2
y					

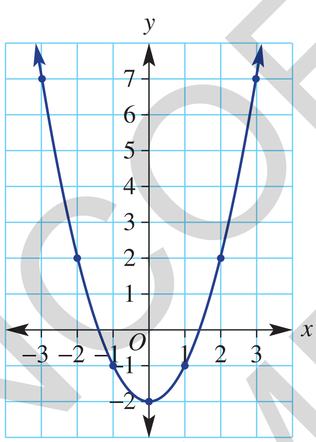


5	5, 6	5, 6

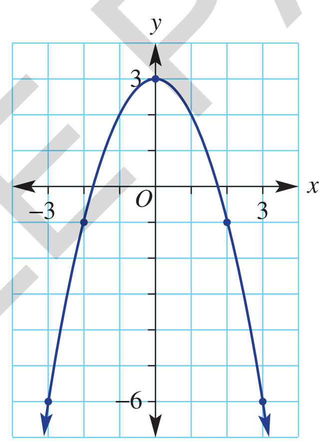
PROBLEM-SOLVING

5 Find a rule for these non-linear graphs.

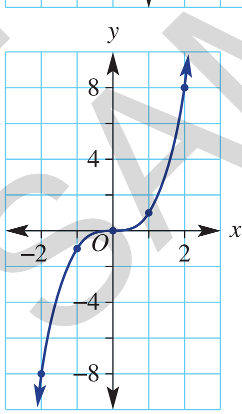
a



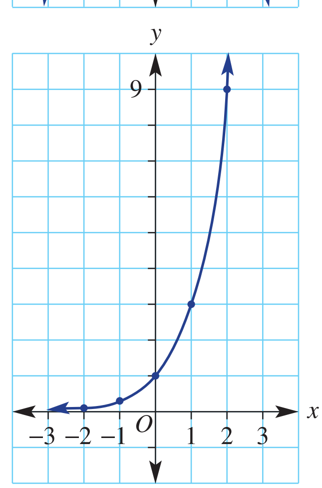
b



c

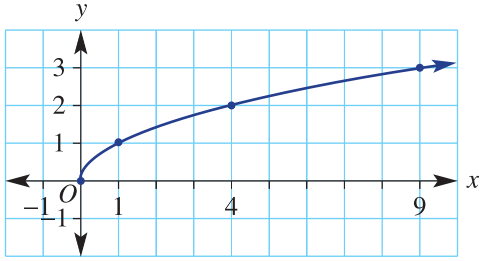


d

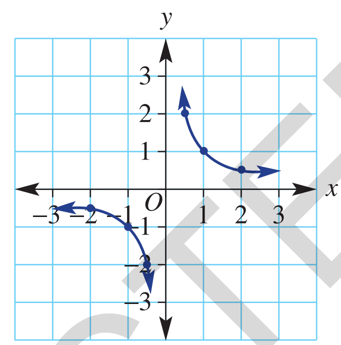


9I

e



f



PROBLEM-SOLVING

6 What are the smallest and largest values of y on a graph of these rules if x ranges from –3 to 3?

a $y = x^2$ b $y = x^2 - 10$ c $y = 6 - x^2$

d $y = x^3$ e $y = -x^3 - 1$ f $y = (x - 1)^3$

7 7 7

REASONING

7 Here are some rules classified by the name of their graph.

Line	Parabola	Hyperbola	Exponential
$y = 2x - 3$	$y = x^2$	$y = \frac{1}{x}$	$y = 3^x$
$y = -5x + 1$	$y = 3 - x^2$	$y = -\frac{5}{x}$	$y = 4^x - 3$

Name the type of graph that is produced by each of these rules.

a $y = x^2 + 7$ b $y = -2x + 4$ c $y = \frac{2}{x}$ d $y = 7^x$

e $y = \frac{-3}{x} + 1$ f $y = 4 - x^2$ g $y = 1 - x$ h $y = (0.5)^x$

Families of parabolas

— — 8

ENRICHMENT

8 For each family of parabolas, plot graphs by hand or use technology to draw each set on the same set of axes. Then describe the features of each family. Describe the effect on the graph when the number a changes.

a Family 1: $y = ax^2$ Use $a = \frac{1}{2}$, $a = 1$, $a = 2$ and $a = 3$.

b Family 2: $y = -ax^2$ Use $a = \frac{1}{2}$, $a = 1$, $a = 2$ and $a = 3$.

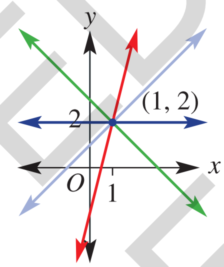
c Family 3: $y = x^2 + a$ Use $a = -3$, $a = -1$, $a = 0$, $a = 2$ and $a = 5$.

d Family 4: $y = (x - a)^2$ Use $a = -2$, $a = 0$, $a = 1$ and $a = 4$.

Investigation

Families of straight lines

A set of lines are said to be in the same family if the lines have something in common. For example, four lines that pass through the point (1, 2).



The parallel family

1 Complete the table for these rules.
   a $y_1 = 2x - 5$
   b $y_2 = 2x - 2$
   c $y_3 = 2x$
   d $y_4 = 2x + 3$

x	–3	–2	–1	0	1	2	3
y_1							
y_2							
y_3							
y_4							

2 Plot the points given in your table to draw graphs of the four rules in part 1 on the one set of axes. Label each line with its rule.

3 What do you notice about the graphs of the four rules? Describe how the numbers in the rule relate to its graph.

4 How would the graphs for the rules $y = 2x + 10$ and $y = 2x - 7$ compare with the graphs you have drawn above? Explain.

The point family

1 Complete the table for these rules.
   a $y_1 = x + 1$
   b $y_2 = 2x + 1$
   c $y_3 = 1$
   d $y_4 = -x + 1$
   e $y = -\frac{1}{2}x + 1$

x	–3	–2	–1	0	1	2	3
y_1							
y_2							
y_3							
y_4							
y_5							

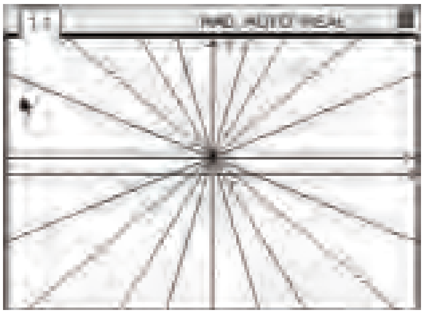
2 Plot the points given in your table to draw graphs of the five rules in part 1 on one set of axes. Label each line with its rule.

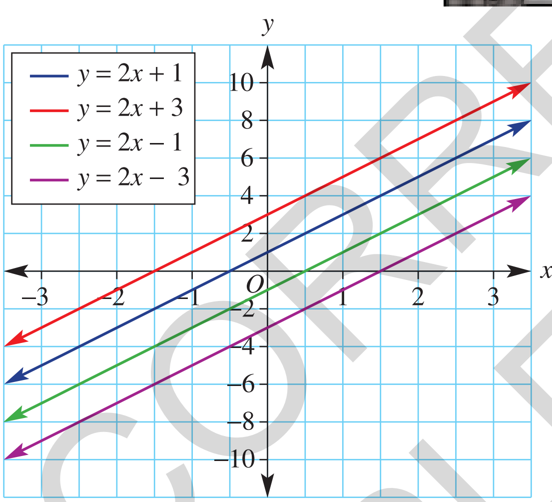
3 What do you notice about the graphs of the five rules? Describe how the numbers in the rule relate to its graph.

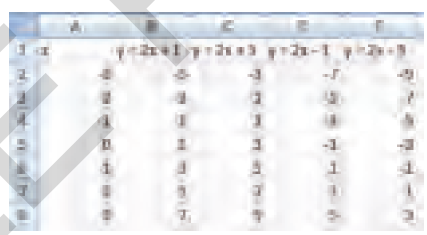
4 How would the graphs for the rules $y = 3x + 1$ and $y = -\frac{1}{3}x + 1$ compare with the graphs you have drawn above? Explain.

Exploring families with technology

Graphics or CAS calculators and spreadsheets are useful tools to explore families of straight lines. Here are some screenshots showing the use of technology.



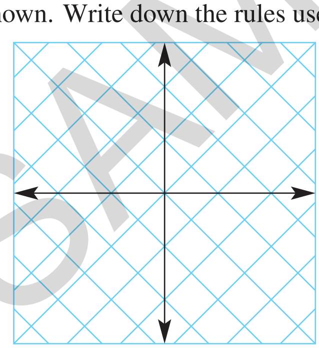




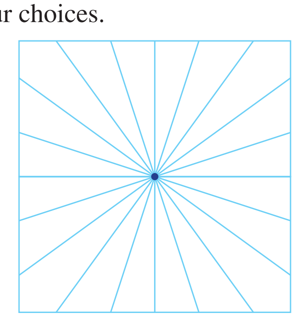
1 Choose one type of technology and sketch the graphs for the two families of straight lines shown in the previous two sections: the 'parallel family' and the 'point family'.

2 Use your chosen technology to help design a family of graphs that produces the patterns shown. Write down the rules used and explain your choices.

a

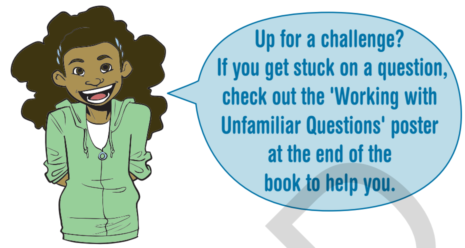


b



3 Make up your own design then use technology to produce it. Explain how your design is built and give the rules that make up the design.

Problems and challenges



1 A trekker hikes down a track at 3 km per hour. Two hours later, a second trekker sets off on the same track at 5 km per hour. How long is it before the second trekker to catches up with the first?

2 Find the rules for the non-linear relations with these tables.

a

x	–2	–1	0	1	2
y	1	–2	–3	–2	1

b

x	–2	–1	0	1	2
y	6	9	10	9	6

c

x	0	1	4	9	16
y	1	2	3	4	5

d

x	–3	–2	–1	0	1
y	–30	–11	– 4	–3	–2

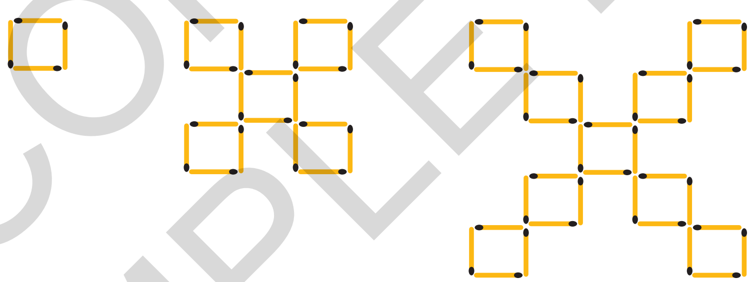
3 A line with a gradient of 3 intersects another line at right angles. Find the gradient of the other line.

4 Two cars travel toward each other on a 100 km stretch of road. One car travels at 80 km per hour and the other at 70 km per hour. If they set off at the same time, how long will it be before the cars meet?

5 Find the y-intercept of a line joining the two points $(–1, 5)$ and $(2, 4)$.

6 Find the rule of a line that passes through the two points $(–3, –1)$ and $(5, 3)$.

7 Find the number of matchsticks needed in the 100th diagram in the pattern given below. The first three diagrams in the pattern are given.



8 At a luxury car hire shop, a Ferrari costs \$300 plus \$40 per hour. A Porsche costs \$205 plus \$60 per hour. What hire time makes both cars the same cost? Give the answer in hours and minutes.

9 Find the area of the figure enclosed by the four lines: $x = 6, y = 4, y = –2$ and $y = x + 5$.

10 A rectangle $ABCD$ is rotated about A by 90° clockwise.

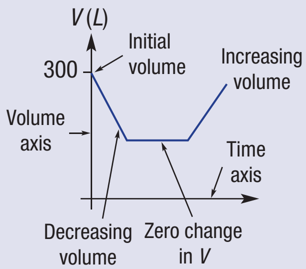
a In two different ways, explain why the diagonal AC is perpendicular to its image $A'C'$.

b If $AB = p$ and $BC = q$, find the simplified product of the gradients of AC and $A'C'$.

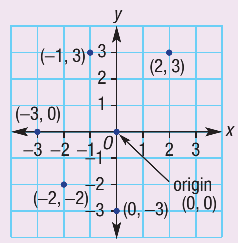
Chapter summary

Straight line graphs

Graphs



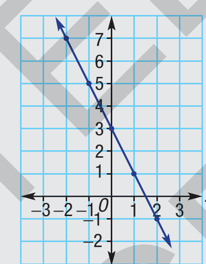
Number/Cartesian plane



Rules, tables and graphs

$y = -2x + 3$

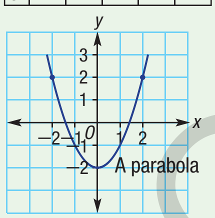
x	–2	–1	0	1	2
y	7	5	3	1	–1



Non-linear graphs (Ext)

$y = x^2 - 2$

x	–2	–1	0	1	2
y	2	–1	–2	–1	2



Finding the rule from tables

x	–2	–1	0	1	2
y	–8	–5	–2	1	4

+3 +3 +3 +3

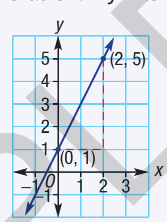
$y = \square \times x + \square$

$= 3x + (-2)$

$= 3x - 2$

$y = mx + c$ (Ext)

Gradient (m) — y-intercept (c)



$m = \frac{4}{2} = 2$

$c = 1$

$y = 2x + 1$

Applications (Ext)

- Distance increases by 20 km per hour.

t	0	1	2	3
d	0	20	40	60

$d = 20t$

- Volume decreases from 1000 L by 200 L per minute.

t	0	1	2	3	4	5
V	1000	800	600	400	200	0

$V = -200t + 1000$

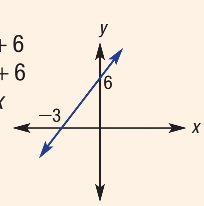
x-intercept (at $y = 0$) (Ext)

$y = 2x + 6$

$0 = 2x + 6$

$-6 = 2x$

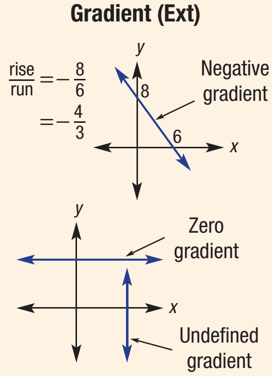
$-3 = x$



Gradient (Ext)

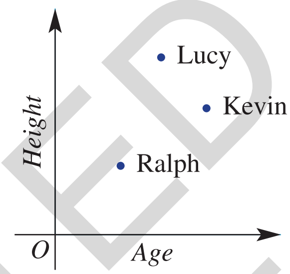
$\frac{\text{rise}}{\text{run}} = -\frac{8}{6}$

$= -\frac{4}{3}$



Multiple-choice questions

9A **1** This graph shows the relationship between the height and age of 3 people. Who is the tallest person?



A Ralph
B Lucy
C Kevin
D Lucy and Ralph together
E Kevin and Lucy together

9A **2** The name of the point (0, 0) on a number (Cartesian) plane is:

A y-intercept **B** gradient **C** origin **D** axis **E** x-intercept

9A **3** Which point is not in line with the other points? $A(-2, 3)$, $B(-1, 2)$, $C(0, 0)$, $D(1, 0)$, $E(2, -1)$

A A **B** B **C** C **D** D **E** E

9B **4** Which of the points A(1, 2), $B(2, -1)$ or $C(3, -4)$ lie on the line $y = -x + 1$?

A C **B** A and C **C** A
D B **E** None

9E Ext **5** A rule gives this table of values.

x	−2	−1	0	1	2
y	3	2	1	0	−1

The y-intercept for the graph of the rule would be:

A 3 **B** 1 **C** 0
D −2 **E** −3

9D Ext **6** The gradient of a line joining the two points (0, 0) and (1, −6) is:

A 3 **B** 6 **C** 1
D −6 **E** −3

9E Ext **7** The gradient of a line is −1 and its y-intercepts is −3. The rule for the line is:

A $y = -x - 3$ **B** $y = x - 3$ **C** $y = -x + 3$
D $y = x + 3$ **E** $-(x - 1)$

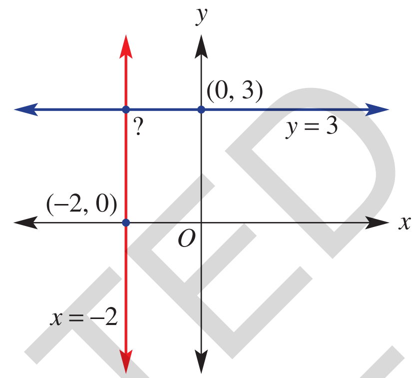
9E Ext **8** The rule for a horizontal line passing through (0, 6) is:

A $y = 6x + 1$ **B** $y = 6x$ **C** $y = x + 6$
D $y = x - 6$ **E** $y = 6$

9E Ext

9 The coordinates of the point of intersection of the graphs of $y = 3$ and $x = -2$ are:

A (0, 3) **B** (2, –3)
C (–2, –3) **D** (–2, 3)
E (2, 3)



9H Ext

10 The water level (h centimetres) in a dam starts at 300 cm deep and decreases by 5 cm every day
for 10 days. The gradient of the graph of this relationship is:

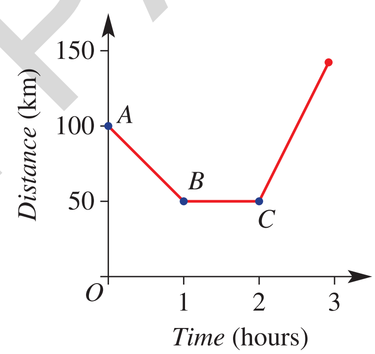
A 5 **B** 300 **C** –300
D –60 **E** –5

Short-answer questions

9A

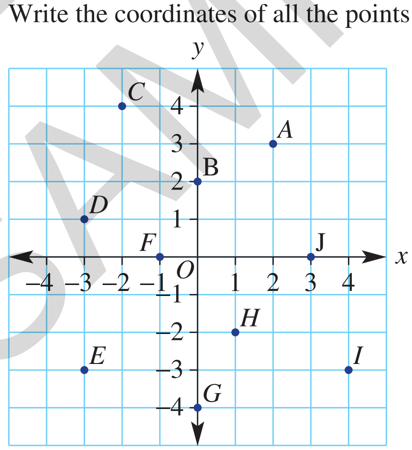
1 The graph shows how far a bird is from its nest.

a How far was the bird from its nest initially (at the start)?
b For how long did the bird rest?
c How far did the bird travel in:
i section A? **ii** section C?
d During which section did the bird fly the fastest?



9A

2 Write the coordinates of all the points $A - J$ in the graph below.



9B **3** For each rule create a table using x values from –3 to 3 and plot to draw a straight line graph.

a $y = 2x$ **b** $y = 3x - 1$ **c** $y = 2x + 2$

d $y = -x + 1$ **e** $y = -2x + 3$ **f** $y = 3 - x$

9C **4** Write the rule for these tables of values.

a

x	–2	–1	0	1	2
y	–3	–1	1	3	5

b

x	–2	–1	0	1	2
y	–4	–1	2	5	8

c

x	3	4	5	6	7
y	6	7	8	9	10

d

x	–3	–2	–1	0	1
y	4	3	2	1	0

e

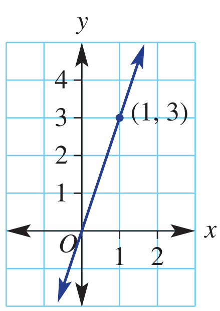
x	–1	0	1	2	3
y	3	–1	–5	–9	–13

f

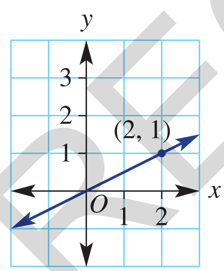
x	–6	–5	–4	–3	–2
y	8	7	6	5	4

9D Ext **5** Find the gradient of each of these lines.

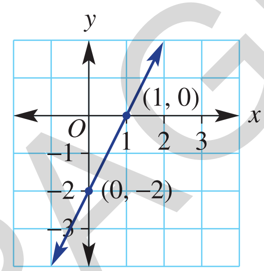
a



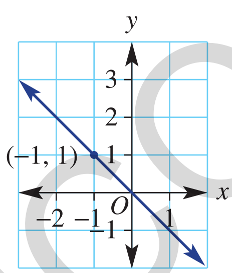
b



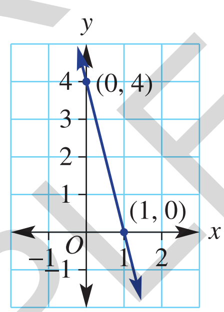
c



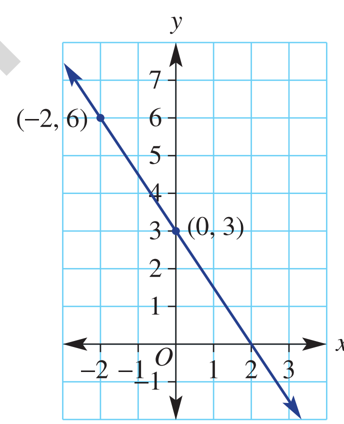
d



e



f



9D Ext **6** Find the gradient of the line joining these pairs of points.

a (0, 0) and (3, –12) **b** (–4, 2) and (0, 0)

c (1, 1) and (4, 4) **d** (–5, 3) and (1, –9)

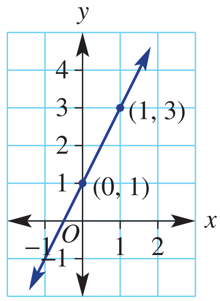
9E Ext **7** Write the gradient (m) and y-intercept (c) for the graphs of these rules.

a $y = 5x + 2$ **b** $y = 2x - 4$ **c** $y = -3x + 7$ **d** $y = -x - \frac{1}{2}$

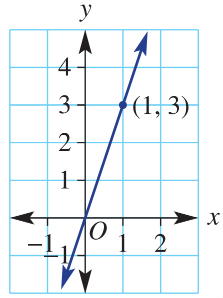
9E
Ext

8 Write the rule for these graphs by first finding the values of m and c.

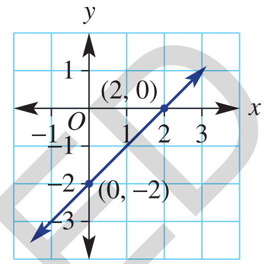
a



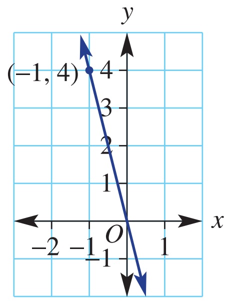
b



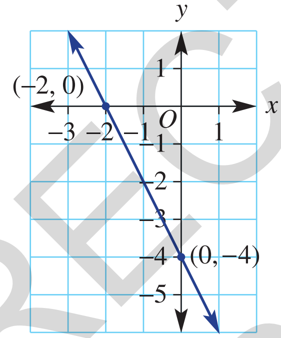
c



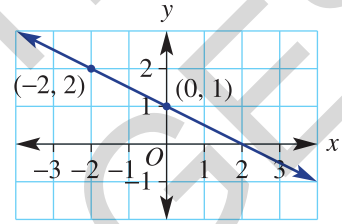
d



e



f

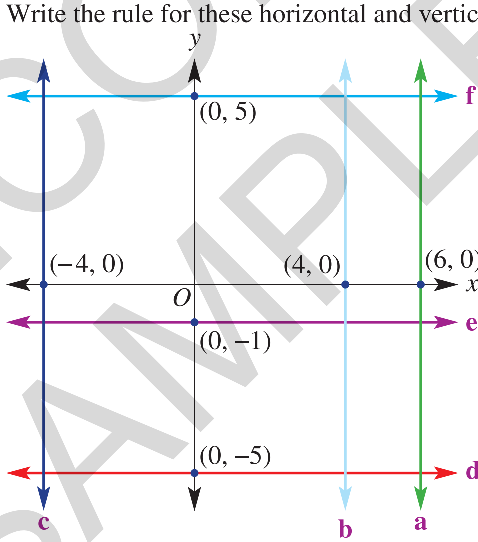


9E
Ext

9 Find the rule for the graphs of the lines connecting these points by first finding the value of m and c.

a (0, 0) and (1, 6)
b (–1, 4) and (0, 0)
c (–2, 3) and (0, 1)
d (0, –2) and (6, 1)

10 Write the rule for these horizontal and vertical lines.



9F
Ext

11 Find the x-intercept for the graphs of the following rules.

a $y = 2x - 12$
b $y = 3x + 9$
c $y = -x - 4$
d $y = -4x + 8$

9F
Ext

12 Find the x and y-intercepts for the graphs of these rules and then sketch a graph.

a $y = x + 3$
b $y = 2x - 10$
c $y = -4x + 8$

9I Ext

13 Using a table with x values between –2 and 2 draw a smooth curve for the non-linear graphs of these rules.

a $y = x^2 - 2$

b $y = x^3$

c $y = \frac{2}{x}$

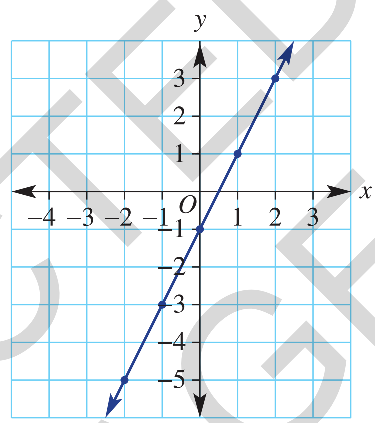
9G

14 Use this graph of $y = 2x - 1$ shown here to solve each of the following equations.

a $2x - 1 = 3$

b $2x - 1 = -5$

c $2x - 1 = 0$



Extended-response questions

1 A seed sprouts and the plant grows 3 millimetres per day in height for 6 days.

a Construct a table of values using t for time in days and h for height in millimetres.

b Draw a graph using the points from your table. Use t on the horizontal axis.

c Find a rule linking h with t.

d Use your rule to find the height of the plant after 3.5 days.

e If the linear pattern continued, what would be the height of the plant after 10 days?

f How long will it be before the plant grows to 15 mm in height?

2 A speed boat at sea is initially 12 km from a distant buoy. The boat travels towards the buoy at a rate of 2 km per minute. The distance between the boat and the buoy will therefore decrease over time.

a Construct a table showing t for time in minutes and d for distance to the buoy in kilometres.

b Draw a graph using the points from your table. Use t on the horizontal axis.

c How long does it take the speed boat to reach the buoy?

d What is the gradient of the line drawn in part **b**?

e Find a rule linking d with t.

f Use your rule to find the distance from the buoy at the 2.5 minute mark.

g How long does it take for the distance to reduce to 3.5 km?