Chapter 7

Equations and inequalities

What you will learn

Australian curriculum

NUMBER AND ALGEBRA

Linear and non-linear relationships

Solve linear equations using algebraic and graphical techniques. Verify solutions by substitution (ACMNA194)

Online resources

- Chapter pre-test
- Videos of all worked examples
- Interactive widgets
- Interactive walkthroughs
- Downloadable HOTsheets
- Access to HOTmaths Australian Curriculum courses

Protecting sea turtles

Loggerhead sea turtles are magnificent marine predators that feed on shellfish, crabs, sea urchins and jellyfish. Sadly, Australian loggerhead turtles have lost more than 50% of their nesting females in 10 years. These diminishing numbers mean that loggerhead turtles are now an endangered species. They are under threat for a number of reasons, including human activity on the beaches where the females lay eggs and accidental death in fishing nets. In order to best work out how to save the loggerhead sea turtle, scientists need to work out what effect various actions, such as closing beaches, will have on the loggerhead turtle population.

To do this scientists use mathematics! With a combination of data and equations, they use computer models to predict population numbers of future generations of the loggerhead turtle.

For example, this simple equation describes the number of turtles that there will be next year:

$$F = C(1 + B - D)$$

The variables used in this equation are:

- F = future population
- C = current population
- B = birth rate
- D = death rate

By mathematically predicting the future of the loggerhead turtle population, environmental scientists can advise governments of the best decisions to help save the loggerhead sea turtle from extinction.

7A Equations review CONSOLIDATING

Interactive

Widgets

HOTsheets

Walkthroughs

An equation is a statement that two things are equal, such as:

$$2 + 2 = 4$$

$$7 \times 5 = 30 + 5$$

$$4 + x = 10 + y.$$

It consists of two expressions separated by the equals sign (=), and it is considered true if the left-hand side and right-hand side are equal. True equations include $7 + 10 = 20 - 3$ and $8 = 4 + 4$; examples of false equations are $2 + 2 = 7$ and $10 \times 5 = 13$.

If an equation has a pronumeral in it, such as $3 + x = 7$, then a **solution** to the equation is a value to substitute for the pronumeral to form a true equation. In this case a solution is $x = 4$ because $3 + 4 = 7$ is a true equation.

Let's start: Solving the equations

- Find a number that would make the equation $25 = b \times (10 - b)$ true.
- How can you prove that this value is a solution?
- Try to find a solution to the equation $11 \times b = 11 + b$.

Key ideas

- An **equation** is a mathematical statement that two expressions are equal, such as $4 + x = 32$. It could be true (e.g. $4 + 28 = 32$) or false (e.g. $4 + 29 = 32$).
- A false equation can be made into a true statement by using the ≠ sign. For instance, $4 + 29 \neq 32$ is a true statement.
- An equation has a left-hand side (LHS) and a right-hand side (RHS).
- A **solution** to an equation is a value that makes an equation true. The process of finding a solution is called **solving**. In an equation with a pronumeral, the pronumeral is also called an **unknown**.
- An equation could have no solutions or it could have one or more solutions.

Example 1 Classifying equations as true or false

For each of the following equations, state whether they are true or false.

a $3 + 8 = 15 - 4$ **b** $7 \times 3 = 20 + 5$ **c** $x + 20 = 3 \times x$, if $x = 10$

SOLUTION	EXPLANATION
a True	Left-hand side (LHS) is $3 + 8$, which is 11. Right-hand side (RHS) is $15 - 4$, which is also 11. Since LHS equals RHS, the equation is true.
b False	LHS $= 7 \times 3 = 21$ RHS $= 20 + 5 = 25$ Since LHS and RHS are different, the equation is false.
c True	If $x = 10$ then LHS $= 10 + 20 = 30$. If $x = 10$ then RHS $= 3 \times 10 = 30$. LHS equals RHS, so the equation is true.

Example 2 Stating a solution to an equation

State a solution to each of the following equations.

a $4 + x = 25$ **b** $5y = 45$ **c** $26 = 3z + 5$

SOLUTION	EXPLANATION
a $x = 21$	We need to find a value of x that makes the equation true. Since $4 + 21 = 25$ is a true equation, $x = 21$ is a solution.
b $y = 9$	If $y = 9$ then $5y = 5 \times 9 = 45$, so the equation is true.
c $z = 7$	If $z = 7$ then $3z + 5 = 3 \times 7 + 5$ $= 21 + 5$ $= 26$ Note: The fact that z is on the right-hand side of the equation does not change the procedure.

Example 3 Writing equations from a description

Write equations for the following scenarios.

a The number k is doubled, then three is added and the result is 52.

b Akira works n hours, earning \$12 per hour. The total she earned was \$156.

SOLUTION	EXPLANATION
a $2k + 3 = 52$	The number k is doubled, giving $k \times 2$. This is the same as $2k$. Since 3 is added, the left-hand side is $2k + 3$, which must be equal to 52 according to the description.
b $12n = 156$	If Akira works n hours at \$12 per hour, the total amount earned is $12 \times n$, or $12n$.

Exercise 7A

UNDERSTANDING	1–4	3, 4	—

Example 1a, 1b

1 Classify these equations as true or false.

a $5 \times 3 = 15$ **b** $7 + 2 = 12 + 3$ **c** $5 + 3 = 16 \div 2$

d $8 - 6 = 6$ **e** $4 \times 3 = 12 \times 1$ **f** $2 = 8 - 3 - 3$

2 If the value of x is 3, what is the value of the following?

a $10 + x$ **b** $3x$ **c** $5 - x$ **d** $6 \div x$

3 State the value of the missing number to make the following equations true.

a $5 + \Box = 12$ **b** $10 \times \Box = 90$ **c** $\Box - 3 = 12$ **d** $3 + 5 = \Box$

4 Consider the equation $15 + 2x = x \times x$.

a If $x = 5$, find the value of $15 + 2x$.

b If $x = 5$, find the value of $x \times x$.

c Is $x = 5$ a solution to the equation $15 + 2x = x \times x$?

d Give an example of another equation with $x = 5$ as a solution.

FLUENCY	5–6(½), 8(½)	5–8(½)	5–8(½)

Example 1c

5 If $x = 2$, state whether the following equations are true or false.

a $7x = 8 + 3x$ **b** $10 - x = 4x$ **c** $3x = 5 - x$

d $x + 4 = 5x$ **e** $10x = 40 \div x$ **f** $12x + 2 = 15x$.

6 If $a = 3$, state whether the following equations are true or false.

a $7 + a = 10$ **b** $2a + 4 = 12$ **c** $8 - a = 5$

d $4a - 3 = 9$ **e** $7a + 2 = 8a$ **f** $a = 6 - a$

FLUENCY

7 Someone has attempted to solve the following equations. State whether the solution is correct (C) or incorrect (I).

- **a** $5 + 2x = 4x - 1$, proposed solution: $x = 3$
- **b** $4 + q = 3 + 2q$, proposed solution: $q = 10$
- **c** $13 - 2a = a + 1$, proposed solution: $a = 4$
- **d** $b \times (b + 3) = 4$, proposed solution, $b = -4$

Example 2

8 State a solution to each of the following equations.

- **a** $5 + x = 12$
- **b** $3 = x - 10$
- **c** $4v + 2 = 14$
- **d** $17 = p - 2$
- **e** $10x = 20$
- **f** $16 - x = x$
- **g** $4u + 1 = 29$
- **h** $7k = 77$
- **i** $3 + a = 2a$

9, 10(½)	9–10(½),11, 12	11–13

PROBLEM-SOLVING

Example 3

9 Write equations for each of the following problems. You do not need to solve the equations.

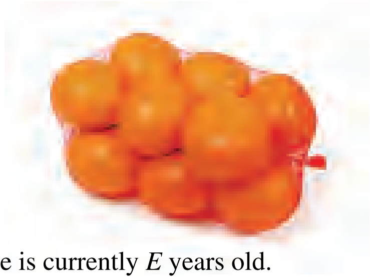
- **a** A number x is doubled and then 7 is added. The result is 10.
- **b** The sum of x and half of x is 12.
- **c** Aston's age is a. His father, who is 25 years older, is twice as old as Aston.
- **d** Fel's height is h cm and her brother Pat is 30 cm taller. Pat's height is 147 cm.
- **e** Coffee costs \$$c$ per cup and tea costs \$$t$. Four cups of coffee and three cups of tea cost a total of \$21.
- **f** Chairs cost \$$c$ each. To purchase 8 chairs and a \$2000 table costs a total of \$3600.

10 Find the value of the unknown number for each of the following.

- **a** A number is tripled to obtain the result 21.
- **b** Half of a number is 21.
- **c** Six less than a number is 7.
- **d** A number is doubled and the result is –16.
- **e** Three-quarters of a number is 30.
- **f** Six more than a number is –7.

11 Berkeley buys x kg of oranges at \$3.20 per kg. He spends a total of \$9.60.

- **a** Write an equation involving x to describe this situation.
- **b** State a solution to this equation.



12 Emily's age in 10 years' time will be triple her current age. She is currently E years old.

- **a** Write an equation involving E to describe this situation.
- **b** Find a solution to this equation.
- **c** How old is Emily now?
- **d** How many years will she have to wait until she is four times her current age?

13 Find two possible values of t that make the equation $t(10 - t) = 21$ true.

14 | 14, 15 | 14, 16

REASONING

14 **a** Why is $x = 3$ a solution to $x^2 = 9$?
b Why is $x = -3$ a solution to $x^2 = 9$?
c Find the two solutions to $x^2 = 64$ (Hint: one is negative).
d Explain why $x^2 = 0$ has only one solution, but $x^2 = 1$ has two.
e Explain why $x^2 = -9$ has no solutions. (Hint: consider multiplying positive and negative numbers.)

15 **a** Explain why the equation $x + 3 = x$ has no solutions.
b Explain why the equation $x + 2 = 2 + x$ is true, regardless of the value of x.
c Show that the equation $x + 3 = 10$ is sometimes true and sometimes false.
d Classify the following equations as always true (A), sometimes true (S) or never true (N).

- **i** $x + 2 = 10$
- **ii** $5 - q = q$
- **iii** $5 + y = y$
- **iv** $10 + b = 10$
- **v** $2 \times b = b + b$
- **vi** $3 - c = 10$
- **vii** $3 + 2z = 2z + 1$
- **viii** $10p = p$
- **ix** $2 + b + b = (b + 1) \times 2$

e Give a new example of another equation that is always true.

16 **a** The equation $p \times (p + 2) = 3$ has two solutions. State the two solutions.
Hint: One of them is negative.
b How many solutions are there for the equation $p + (p + 2) = 3$?
c Try to find an equation that has three solutions.

More than one unknown

— | — | 17, 18

ENRICHMENT

17 **a** There are six equations in the square below. Find the values of a, b, c, d and e to make all six equations true.

a	$+$	12	$=$	22
$\times$		$\div$		$-$
2	$\times$	b	$=$	c
$=$		$=$		$=$
d	$\div$	e	$=$	10

b Find the value of f that makes the equation $a \times b \times e = c \times d \times f$ true.

18 For each of the following pairs of equations, find values of c and d that make both equations true. More than one answer may be possible.

a $c + d = 10$ and $cd = 24$.
b $c - d = 8$ and $c + d = 14$
c $c \div d = 4$ and $c + d = 30$
d $cd = 0$ and $c - d = 7$

7B Equivalent equations CONSOLIDATING

If we have an equation, we can obtain an **equivalent** equation by performing the same operation to both sides. For example, if we have $2x + 4 = 20$, we can add 3 to both sides to obtain $2x + 7 = 23$.

The new equation will be true for exactly the same values of x as the old equation. This observations helps us to solve equations algebraically. For example, $2x + 4 = 20$ is equivalent to $2x = 16$ (subtract 4 from both sides), and this is equivalent to $x = 8$ (divide both sides by 2). The bottom equation is only true if x has the value 8, so this means the solution to the equation $2x + 4 = 20$ is $x = 8$. We write this as:

$$\begin{tikzcd} 2x + 4 = 20 \arrow[d, bend right, "-4"'] \arrow[d, bend left, "-4"] \\ 2x = 16 \arrow[d, bend right, "\div 2"'] \arrow[d, bend left, "\div 2"] \\ x = 8 \end{tikzcd}$$

Let's start: Attempted solutions

Below are three attempts at solving the equation $4x - 8 = 40$. Each has a problem.

Attempt 1

$$\begin{tikzcd} 4x - 8 = 40 \arrow[d, bend right, "+8"'] \arrow[d, bend left, "+8"] \\ 4x = 48 \arrow[d, bend right, "-4"'] \arrow[d, bend left, "-4"] \\ x = 44 \end{tikzcd}$$

Attempt 2

$$\begin{tikzcd} 4x - 8 = 40 \arrow[d, bend right, "+8"'] \arrow[d, bend left, "-8"] \\ 4x = 32 \arrow[d, bend right, "\div 4"'] \arrow[d, bend left, "\div 4"] \\ x = 8 \end{tikzcd}$$

Attempt 3

$$\begin{tikzcd} 4x - 8 = 40 \arrow[d, bend right, "\div 4"'] \arrow[d, bend left, "\div 4"] \\ x - 8 = 10 \arrow[d, bend right, "+8"'] \arrow[d, bend left, "+8"] \\ x = 18 \end{tikzcd}$$

- Can you prove that these results are not the correct solutions to the equation above?
- For each one, find the mistake that was made.
- Can you solve $4x - 8 = 40$ algebraically?

Key ideas

- Two equations are **equivalent** if you can get from one to the other by repeatedly:
  - adding a number to both sides
  - subtracting a number from both sides
  - multiplying both sides by a number other than zero
  - dividing both sides by a number other than zero
  - swapping the left-hand side and right-hand sides of the equation.
- To solve an equation **algebraically**, repeatedly find an equivalent equation that is simpler.
  For example:

$$\begin{tikzcd}
5x + 2 = 32 \arrow[d, bend right, "-2"'] \arrow[d, bend left, "-2"] \\
5x = 30 \arrow[d, bend right, "\div 5"'] \arrow[d, bend left, "\div 5"] \\
x = 6
\end{tikzcd}$$

- To check a solution, substitude the unknown's value in to both sides to see if the equation is true, e.g. LHS = 5(6) + 2 = 32 and RHS = 32.

Example 4 Finding equivalent equations

Show the result of applying the given operation to both sides of these equations.

a $8y = 40$ [$\div 8$] **b** $10 + 2x = 36$ [-10] **c** $5a - 3 = 12$ [$+3$]

SOLUTION	EXPLANATION
a $\begin{tikzcd} 8y = 40 \arrow[d, bend right, "\div 8"'] \arrow[d, bend left, "\div 8"] \\ y = 5 \end{tikzcd}$	Write the equation out and then divide both sides by 8. $40 \div 8$ is 5 and $8y \div 8$ is y.
b $\begin{tikzcd} 10 + 2x = 36 \arrow[d, bend right, "-10"'] \arrow[d, bend left, "-10"] \\ 2x = 26 \end{tikzcd}$	Write the equation out and then subtract 10 from both sides. $36 - 10$ is 26 $10 + 2x - 10$ is $2x$
c $\begin{tikzcd} 5a - 3 = 12 \arrow[d, bend right, "+3"'] \arrow[d, bend left, "+3"] \\ 5a = 15 \end{tikzcd}$	Write the equation out and then add 3 to both sides. $12 + 3$ is 15 $5a - 3 + 3$ is $5a$

Example 5 Solving equations algebraically

Solve the following equations algebraically and check the solution by substituting.

a $x - 4 = 16$ **b** $2u + 7 = 17$ **c** $40 - 3x = 22$

SOLUTION	EXPLANATION
a $$\begin{tikzcd} x-4=16 \arrow[d, bend right, "+4"'] \arrow[d, bend left, "+4"] \\ x=20 \end{tikzcd}$$	By adding 4 to both sides of the equation, we get an equivalent equation.
So the solution is $x = 20$.	Check: $20-4=16$ ✓
b $$\begin{tikzcd} 2u+7=17 \arrow[d, bend right, "-7"'] \arrow[d, bend left, "-7"] \\ 2u=10 \arrow[d, bend right, "\div 2"'] \arrow[d, bend left, "\div 2"] \\ u=5 \end{tikzcd}$$	To get rid of the +7, we subtract 7 from both sides. Finally we divide by 2 to reverse the $2u$. Remember that $2u$ means $2\times u$
So the solution is $u = 5$.	Check: $2(5)+7=10+7=17$ ✓
c $$\begin{tikzcd} 40-3x=22 \arrow[d, bend right, "-40"'] \arrow[d, bend left, "-40"] \\ -3x=-18 \arrow[d, bend right, "\div -3"'] \arrow[d, bend left, "\div -3"] \\ x=6 \end{tikzcd}$$	We subtract 40 from both sides to get rid of the 40 at the start of the LHS. Since $-3\times x=-18$, we divide by -3 to get the final solution.
So the solution is $x = 6$.	Check: $40-3(6)=40-18=22$ ✓

Exercise 7B

1–4	2–4	—

UNDERSTANDING

1 For each of the following equations add 4 to both sides to obtain an equivalent equation.

a $3x = 10$ **b** $7 + k = 14$ **c** $5 = 2x$

2 For each equation fill in the blank to get an equivalent equation.

a
$$\begin{tikzcd} 5x=10 \arrow[d, bend right, "+2"'] \arrow[d, bend left, "+2"] \\ 5x+2=\underline{\quad} \end{tikzcd}$$

b
$$\begin{tikzcd} 10-2x=20 \arrow[d, bend right, "+5"'] \arrow[d, bend left, "+5"] \\ 15-2x=\underline{\quad} \end{tikzcd}$$

c
$$\begin{tikzcd} 3q+4=\underline{\quad} \arrow[d, bend right, "-4"'] \arrow[d, bend left, "-4"] \\ 3q=12 \end{tikzcd}$$

d
$$\begin{tikzcd} 7z+12=4z+10 \arrow[d, bend right, "-10"'] \arrow[d, bend left, "-10"] \\ 7z+2=\underline{\quad} \end{tikzcd}$$

3 Consider the equation $4x = 32$.

a Copy and complete the following working.

$$\begin{tikzcd} 4x=32 \arrow[d, bend right, "\div 4"'] \arrow[d, bend left, "\div 4"] \\ x=\underline{\quad} \end{tikzcd}$$

b What is the solution to the equation $4x = 32$?

7B

UNDERSTANDING

4 To solve the equation $10x + 5 = 45$, which of the following operations would you first apply to both sides?

A Divide by 5 **B** Subtract 5 **C** Divide by 10 **D** Subtract 45

5–6, 7–9(½) 5–10(½) 6–10(½)

FLUENCY

Example 4

5 For each equation, show the result of applying the given operation to both sides.

a $10 + 2x = 30\ [-10]$
b $4 + q = 12\ [-2]$
c $13 = 12 - q\ [+5]$
d $4x = 8\ [\times 3]$
e $7p = 2p + 4\ [+6]$
f $3q + 1 = 2q + 1\ [-1]$

6 Copy and complete the following to solve the given equation algebraically.

a
$$\begin{tikzcd} 10x = 30 \arrow[d, bend right, "\div 10"'] \arrow[d, bend left, "\div 10"] \\ x = \_\_ \end{tikzcd}$$

b
$$\begin{tikzcd} q + 5 = 2 \arrow[d, bend right, "-5"'] \arrow[d, bend left, "-5"] \\ \_\_ = \_\_ \end{tikzcd}$$

c
$$\begin{tikzcd} 4x + 2 = 22 \arrow[d, bend right, "-2"'] \arrow[d, bend left, "-2"] \\ 4x = \_\_ \arrow[d, bend right, "\div 4"'] \arrow[d, bend left, "\div 4"] \\ \_\_ = \_\_ \end{tikzcd}$$

d
$$\begin{tikzcd} 30 = 4p + 2 \arrow[d, bend right, "-2"'] \arrow[d, bend left, "-2"] \\ \_\_ = \_\_ \arrow[d, bend right, "\square"'] \arrow[d, bend left, "\square"] \\ \_\_ = \_\_ \end{tikzcd}$$

e
$$\begin{tikzcd} 20 - 4x = 8 \arrow[d, bend right, "-20"'] \arrow[d, bend left, "-20"] \\ -4x = -12 \arrow[d, bend right, "\div -4"'] \arrow[d, bend left, "\div -4"] \\ x = \_\_ \end{tikzcd}$$

f
$$\begin{tikzcd} p \div 3 + 6 = 8 \arrow[d, bend right, "-6"'] \arrow[d, bend left, "-6"] \\ p \div 3 = 2 \arrow[d, bend right, "\square"'] \arrow[d, bend left, "\square"] \\ \_\_ = \_\_ \end{tikzcd}$$

Example 5a

7 Solve the following equations algebraically.

a $a + 5 = 8$
b $t \times 2 = 14$
c $7 = q - 2$
d $11 = k + 2$
e $19 = x + 9$
f $-30 = 3h$
g $-36 = 9l$
h $g \div 3 = -3$

Example 5b

8 Solve the following equations algebraically. Check your solutions using substitution.

a $5 + 9h = 32$
b $9u - 6 = 30$
c $13 = 5s - 2$
d $-18 = 6 - 3w$
e $-12 = 5x + 8$
f $-44 = 10w + 6$
g $8 = -8 + 8a$
h $4y - 8 = -40$

Example 5c

9 Solve the following equations algebraically and check your solutions.

a $20 - 4d = 8$
b $34 = 4 - 5j$
c $21 - 7a = 7$
d $6 = 12 - 3y$
e $13 - 8k = 45$
f $44 = 23 - 3n$
g $13 = -3b + 4$
h $-22 = 14 - 9b$

10 The following equations do not all have whole number solutions. Solve the following equations algebraically, giving each solution as a fraction.

a $2x + 3 = 10$
b $5 + 3q = 6$
c $12 = 10b + 7$
d $15 = 10 + 2x$
e $15 = 10 - 2x$
f $13 + 2p = -10$
g $22 = 9 + 5y$
h $12 - 2y = 15$

11 For each of the following, write an equation and solve it algebraically.

a The sum of p and 8 is 15.

b The product of q and –3 is 12.

c 4 is subtracted from double the value of k and the result is 18.

d When r is tripled and 4 is added the result is 34.

e When x is subtracted from 10 the result is 6.

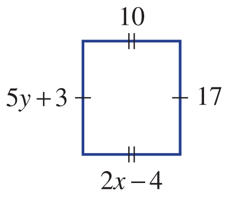
f When triple y is subtracted from 10 the result is 16.

12 Solve the following equations algebraically. More than two steps are involved.

a $14 \times (4x + 2) = 140$ **b** $8 = (10x - 4) \div 2$ **c** $-12 = (3 - x) \times 4$

13 The following shapes are rectangles. By solving equations algebraically, find the value of the variables. Some of the answers will be fractions.

a



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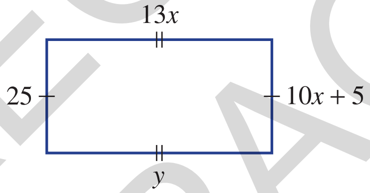

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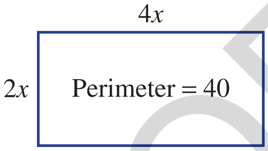
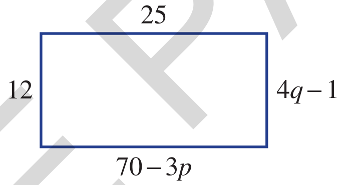
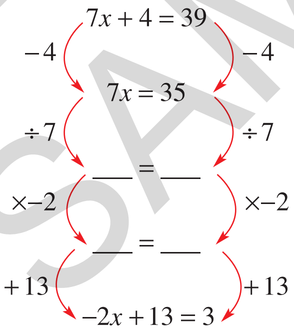


d



14 Sidney works for 10 hours at the normal rate of pay (\$$x$ per hour) and then the next three hours at double that rate. If he earns a total of \$194.88, write an equation and solve it to find his normal hourly rate.

15 **a** Prove that $7x + 4 = 39$ and $-2x + 13 = 3$ are equivalent by filling in the missing steps.



b Prove that $10k + 4 = 24$ and $3k - 1 = 5$ are equivalent.

7B

REASONING

16 **a** Prove that $4x + 3 = 11$ and $2x = 4$ are equivalent. Try to use just two steps to get from one equation to the other.

b Are the equations $5x + 2 = 17$ and $x = 5$ equivalent?

c Prove that $10 - 2x = 13$ and $14x + 7 = 20$ are not equivalent, no matter how many steps are used.

17 A student has taken the equation $x = 5$ and performed some operations to both sides.

$$\begin{tikzcd}
x = 5 \arrow[d, bend right, "\times 4"'] \arrow[d, bend left, "\times 4"] \\
4x = 20 \arrow[d, bend right, "+3"'] \arrow[d, bend left, "+3"] \\
4x + 3 = 23 \arrow[d, bend right, "\times 2"'] \arrow[d, bend left, "\times 2"] \\
(4x + 3) \times 2 = 46
\end{tikzcd}$$

a Solve $(4x + 3) \times 2 = 46$ algebraically.

b Describe how the steps you used in your solution compare with the steps the student used.

c Give an example of another equation that has $x = 5$ as its solution.

d Explain why there are infinitely many different equations with the solution $x = 5$.

Dividing whole expressions — — 18

ENRICHMENT

18 It is possible to solve $2x + 4 = 20$ by first dividing both sides by 2, as long as every term is divided by 2. So you could solve it in either of these fashions.

$$\begin{tikzcd}
2x + 4 = 20 \arrow[d, bend right, "-4"'] \arrow[d, bend left, "-4"] \\
2x = 16 \arrow[d, bend right, "\div 2"'] \arrow[d, bend left, "\div 2"] \\
x = 8
\end{tikzcd}
\qquad
\begin{tikzcd}
2x + 4 = 20 \arrow[d, bend right, "\div 2"'] \arrow[d, bend left, "\div 2"] \\
x + 2 = 10 \arrow[d, bend right, "-2"'] \arrow[d, bend left, "-2"] \\
x = 8
\end{tikzcd}$$

Note that $2x + 4$ divided by 2 is $x + 2$, not $x + 4$. Use this method of dividing first to solve the following equations and then check that you get the same answer as if you subtracted first.

a $2x + 6 = 12$
b $4x + 12 = 16$
c $10x + 30 = 50$
d $2x + 5 = 13$
e $5x + 4 = 19$
f $3 + 2x = 5$
g $7 = 2x + 4$
h $10 = 4x + 10$
i $12 = 8 + 4x$

7C Equations with fractions

Recall from algebra that a fraction such as $\frac{x}{3}$ represents $x \div 3$. This means that to solve an equation with $\frac{x}{3}$ on one side, we should first multiply both sides by 3. For example:

$$\begin{tikzcd} \frac{x}{3}=10 \arrow[d, bend right, "\times 3"'] \arrow[d, bend left, "\times 3"] \\ x=30 \end{tikzcd}$$

Let's start: Practising with fractions

- If x is a number greater than 1, evaluate these expressions and put them into ascending order (smallest to largest).

$$\frac{2x+1}{2} \qquad 2\left(\frac{x}{2}+1\right) \qquad \frac{2}{x+1} \qquad \frac{2+2x}{2} \qquad 2\left(x+\frac{1}{2}\right)$$

- Investigate how the order might change if x could be a number less than 1.

Key ideas

- $\frac{a}{b}$ means $a \div b$.
- To solve an equation with a fraction on one side, multiply both sides by the denominator.
  For example:

$$\begin{tikzcd} \frac{q}{4}=12 \arrow[d, bend right, "\times 4"'] \arrow[d, bend left, "\times 4"] \\ q=48 \end{tikzcd}$$

Example 6 Solving equations with fractions

Solve the following equations algebraically.

a $\frac{4x}{3} = 8$

b $\frac{4y+15}{9} = 3$

c $4 + \frac{5x}{2} = 29$

d $7 - \frac{2x}{3} = 5$

SOLUTION	EXPLANATION
a $\begin{tikzcd} \frac{4x}{3}=8 \arrow[d, bend right, "\times 3"'] \arrow[d, bend left, "\times 3"] \\ 4x=24 \arrow[d, bend right, "\div 4"'] \arrow[d, bend left, "\div 4"] \\ x=6 \end{tikzcd}$	Multiplying both sides by 3 removes the denominator of 3. Both sides are divided by 4 to solve the equation.

b

$$\begin{tikzcd}
\frac{4y+15}{9} = 3 \arrow[d, bend right, "\times 9"'] \arrow[d, bend left, "\times 9"] \\
4y + 15 = 27 \arrow[d, bend right, "-15"'] \arrow[d, bend left, "-15"] \\
4y = 12 \arrow[d, bend right, "\div 4"'] \arrow[d, bend left, "\div 4"] \\
y = 3
\end{tikzcd}$$

Multiplying both sides by 9 removes the denominator of 9.

The equation $4y + 15 = 27$ is solved in the usual fashion (subtract 15, divide by 4).

c

$$\begin{tikzcd}
4 + \frac{5x}{2} = 29 \arrow[d, bend right, "-4"'] \arrow[d, bend left, "-4"] \\
\frac{5x}{2} = 25 \arrow[d, bend right, "\times 2"'] \arrow[d, bend left, "\times 2"] \\
5x = 50 \arrow[d, bend right, "\div 5"'] \arrow[d, bend left, "\div 5"] \\
x = 10
\end{tikzcd}$$

We must subtract 4 first because we do not have a fraction by itself on the left-hand side. Once there is a fraction by itself, multiply by the denominator (2).

d

$$\begin{tikzcd}
7 - \frac{2x}{3} = 5 \arrow[d, bend right, "-7"'] \arrow[d, bend left, "-7"] \\
-\frac{2x}{3} = -2 \arrow[d, bend right, "\times -1"'] \arrow[d, bend left, "\times -1"] \\
\frac{2x}{3} = 2 \arrow[d, bend right, "\times 3"'] \arrow[d, bend left, "\times 3"] \\
2x = 6 \arrow[d, bend right, "\div 2"'] \arrow[d, bend left, "\div 2"] \\
x = 3
\end{tikzcd}$$

Subtract 7 first to get a fraction. When both sides are negative, multiplying (or dividing) by –1 makes them both positive.

Exercise 7C

1–3	2, 3	—

UNDERSTANDING

1 **a** If $x = 4$ find the value of $\frac{x}{2} + 6$.

b If $x = 4$ find the value of $\frac{x+6}{2}$.

c Are $\frac{x}{2} + 6$ and $\frac{x+6}{2}$ equivalent expressions?

2 Fill in the missing steps to solve these equations.

a

$$\begin{tikzcd}
\frac{x}{3} = 10 \arrow[d, bend right, "\times 3"'] \arrow[d, bend left, "\times 3"] \\
x = \_\_
\end{tikzcd}$$

b

$$\begin{tikzcd}
\frac{m}{5} = 2 \arrow[d, bend right, "\times 5"'] \arrow[d, bend left, "\times 5"] \\
m = \_\_
\end{tikzcd}$$

c

$$\begin{tikzcd}
11 = \frac{q}{2} \arrow[d, bend right, "\square"'] \arrow[d, bend left, "\square"] \\
\_\_ = q
\end{tikzcd}$$

d

$$\begin{tikzcd}
\frac{p}{10} = 7 \arrow[d, bend right, "\square"'] \arrow[d, bend left, "\square"] \\
p = \_\_
\end{tikzcd}$$

UNDERSTANDING

3 Match each of these equations with the correct first step to solve it.

a $\frac{x}{4} = 7$ **b** $\frac{x-4}{2} = 5$ **c** $\frac{x}{2} - 4 = 7$ **d** $\frac{x}{4} + 4 = 3$

A Multiply both sides by 2.
B Add 4 to both sides.
C Multiply both sides by 4.
D Subtract 4 from both sides.

4–5(½) 4–5(½) 4–5(½)

FLUENCY

Example 6a

4 Solve the following equations algebraically.

a $\frac{b}{5} = 4$ **b** $\frac{g}{10} = 2$ **c** $\frac{a}{5} = 3$ **d** $\frac{k}{6} = 3$

e $\frac{2l}{5} = 8$ **f** $\frac{7w}{10} = -7$ **g** $\frac{3s}{2} = -9$ **h** $\frac{5v}{4} = 15$

i $\frac{3m}{7} = 6$ **j** $\frac{2n}{7} = 4$ **k** $\frac{-7j}{5} = 7$ **l** $\frac{-6f}{5} = -24$

Example 6b, 6c, 6d

5 Solve the following equations algebraically. Check your solutions by substituting.

a $\frac{t-8}{2} = -10$ **b** $\frac{h+10}{3} = 4$ **c** $\frac{a+12}{5} = 2$ **d** $\frac{c-7}{2} = -5$

e $-1 = \frac{s-2}{8}$ **f** $\frac{5j+6}{8} = 2$ **g** $3 = \frac{7v}{12} + 10$ **h** $\frac{4n}{9} - 6 = -2$

i $\frac{7q+12}{5} = -6$ **j** $-4 = \frac{f-15}{3}$ **k** $15 = \frac{3-12l}{5}$ **l** $9 - \frac{4r}{7} = 5$

m $-6 = \frac{5x-8}{-7}$ **n** $\frac{5u-7}{-4} = -2$ **o** $\frac{5k+4}{-8} = -3$ **p** $20 = \frac{3+13b}{-7}$

q $\frac{7m}{12} - 12 = -5$ **r** $4 + \frac{-7y}{8} = -3$ **s** $4 = \frac{p-15}{-3}$ **t** $\frac{g-3}{5} = -1$

6 6(½), 7 6(½) 7, 8

PROBLEM-SOLVING

6 For the following puzzles, write an equation and solve it to find the unknown number.
a A number x is divided by 5 and the result is 7.
b Half of y is –12.
c A number p is doubled and then divided by 7. The result is 4.
d Four is added to x. This is halved to get a result of 10.
e x is halved and then 4 is added to get a result of 10.
f A number k is doubled and then 6 is added. This result is halved to obtain –10.

7 The average of two numbers can be found by adding them and then dividing the result by 2.

a If the average of x and 5 is 12, what is x? Solve the equation $\frac{x+5}{2} = 12$ to find out.
b The average of 7 and p is –3. Find p by writing and solving an equation.
c The average of a number and double that number is 18. What is that number?
d The average of $4x$ and 6 is 19. What is the average of $6x$ and 4? (Hint: Find x first.)

8 A restaurant bill of \$100 is to be paid. Blake puts in one-third of the amount in his wallet, leaving \$60 to be paid by the other people at the table.

a Write an equation to describe this situation, if b represents the amount in Blake's wallet before he pays.

b Solve the equation algebraically, and hence state how much money Blake has in his wallet.

9	9, 10	9, 10(½)

9 In solving $\frac{2x}{3} = 10$ we have first been multiplying by the denominator, but we could have written $2\left(\frac{x}{3}\right) = 10$ and divided both sides by 2.

a Solve $2\left(\frac{x}{3}\right) = 10$.

b Is the solution the same as the solution for $\frac{2x}{3} = 10$ if both sides are first multiplied by 3?

c Solve $\frac{147q}{13} = 1470$ by first:

i multiplying both sides by 13 **ii** dividing both sides by 147

d What is one advantage in dividing first rather than multiplying?

e Solve the following equations.

i $\frac{20p}{14} = 40$ **ii** $\frac{13q}{27} = -39$ **iii** $\frac{-4p}{77} = 4$ **iv** $\frac{123r}{17} = 246$

10 To solve an equation with a pronumeral on the denominator we can first multiply both sides by that pronumeral.

$$\begin{tikzcd}
\frac{30}{x} = 10 \arrow[d, bend right, "\times x"'] \arrow[d, bend left, "\times x"] \\
30 = 10x \arrow[d, bend right, "\div 10"'] \arrow[d, bend left, "\div 10"] \\
3 = x
\end{tikzcd}$$

Use this method to solve the follows equations.

a $\frac{12}{x} = 2$ **b** $\frac{-15}{x} = -5$ **c** $\frac{1}{x} + 3 = 4$

d $4 + \frac{20}{x} = 14$ **e** $\frac{16}{x} + 1 = 3$ **f** $5 = \frac{-10}{x} + 3$

Fractional solutions

— — 11, 12

11 Solve the following equations. Note that the solutions should be given as fractions.

a $\frac{4x+3}{5} = 12$

b $\frac{8+3x}{5} = 6$

c $7 = \frac{x}{4} + \frac{1}{3}$

d $2 = \frac{10-3x}{4}$

12 Recall from Section 5E (Adding and subtracting algebraic fractions) that algebraic fractions can be combined by finding a common denominator, for example:

$$\begin{aligned}\frac{2x}{3} + \frac{5x}{4} &= \frac{8x}{12} + \frac{15x}{12}\\ &= \frac{23x}{12}\end{aligned}$$

Use this simplification to solve the following equations.

a $\frac{2x}{3} + \frac{5x}{4} = 46$

b $\frac{x}{5} + \frac{x}{6} = 22$

c $10 = \frac{x}{2} + \frac{x}{3}$

d $4 = \frac{x}{2} - \frac{x}{3}$

e $\frac{6x}{5} + \frac{2x}{3} = 28$

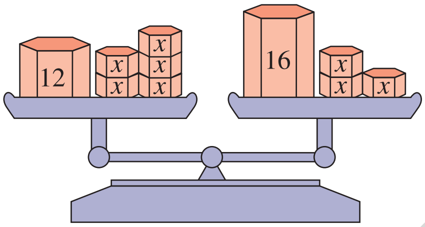
f $4 = \frac{3x}{7} - \frac{x}{3}$

7D Equations with pronumerals on both sides

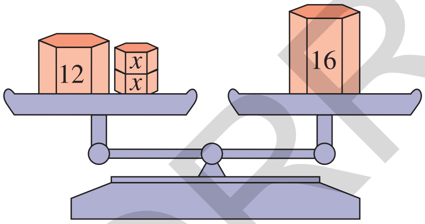
Interactive
Widgets
HOTsheets
Walkthroughs

So far all the equations we have considered involved a pronumeral either on the left-hand side, for example, $2x + 3 = 11$ or on the right side, for example, $15 = 10 - 2x$. But how can you solve an equation with pronumerals on both sides, for example, $12 + 5x = 16 + 3x$? The idea is to look for an equivalent equation with pronumerals on just one side.

The equation $12 + 5x = 16 + 3x$ can be thought of as balancing scales.



Then $3x$ can be removed from both sides of this equation to get



The equation $12 + 2x = 16$ is straightforward to solve.

Let's start: Moving pronumerals

You are given the equation $11 + 5x = 7 + 3x$.

- Can you find an equivalent equation with x just on the left-hand side?
- Can you find an equivalent equation with x just on the right-hand side?
- Try to find an equivalent equation with $9x$ on the left-hand side.
- Do all of these equations have the same solution? Try to find it.

Key ideas

- If both sides of an equation have a term added or subtracted, the new equation will be **equivalent** to the original equation.
- If pronumerals are on both sides of an equation, add or subtract terms so that the pronumeral appears only one side.

  For example:

$$\begin{tikzcd} 10 + 5a = 13 + 2a \arrow[d, bend right, "-2a"'] \arrow[d, bend left, "-2a"] \\ 10 + 3a = 13 \end{tikzcd} \qquad \begin{tikzcd} 4b + 12 = 89 - 3b \arrow[d, bend right, "+3b"'] \arrow[d, bend left, "+3b"] \\ 7b + 12 = 89 \end{tikzcd}$$

Example 7 Solving equations with pronumerals on both sides

Solve the following equations and check your solutions using substitution.

a $7t + 4 = 5t + 10$ **b** $6x + 4 = 22 - 3x$ **c** $2u = 7u - 20$

SOLUTION

EXPLANATION

a

$$\begin{tikzcd} 7t + 4 = 5t + 10 \arrow[d, bend right, "-5t"'] \arrow[d, bend left, "-5t"] \\ 2t + 4 = 10 \arrow[d, bend right, "-4"'] \arrow[d, bend left, "-4"] \\ 2t = 6 \arrow[d, bend right, "\div 2"'] \arrow[d, bend left, "\div 2"] \\ t = 3 \end{tikzcd}$$

Pronumerals are on both sides of the equation, so subtract $5t$ from both sides.

Once $5t$ is subtracted, the usual procedure is applied for solving equations.

$\text{LHS} = 7(3) + 4$ $\quad$ $\text{RHS} = 5(3) + 10$

$= 25$ $\quad$ $= 25$ ✓

b

$$\begin{tikzcd} 6x + 4 = 22 - 3x \arrow[d, bend right, "+3x"'] \arrow[d, bend left, "+3x"] \\ 9x + 4 = 22 \arrow[d, bend right, "-4"'] \arrow[d, bend left, "-4"] \\ 9x = 18 \arrow[d, bend right, "\div 9"'] \arrow[d, bend left, "\div 9"] \\ x = 2 \end{tikzcd}$$

Pronumerals are on both sides. To get rid of $3x$, we add $3x$ to both sides of the equation. Alternatively, $6x$ could have been subtracted from both sides of the equation to get $4 = 22 - 9x$.

$\text{LHS} = 6(2) + 4$ $\quad$ $\text{RHS} = 22 - 3(2)$

$= 16$ $\quad$ $= 16$ ✓

c

$$\begin{tikzcd} 2u = 7u - 20 \arrow[d, bend right, "-2u"'] \arrow[d, bend left, "-2u"] \\ 0 = 5u - 20 \arrow[d, bend right, "+20"'] \arrow[d, bend left, "+20"] \\ 20 = 5u \arrow[d, bend right, "\div 5"'] \arrow[d, bend left, "\div 5"] \\ 4 = u \end{tikzcd}$$

$$\therefore\ u = 4$$

Choose to get rid of $2u$ by subtracting it.

Note that $2u - 2u$ is equal to 0, so the LHS of the new equation is 0.

$\text{LHS} = 2(4)$ $\quad$ $\text{RHS} = 7(4) - 20$

$= 8$ $\quad$ $= 8$ ✓

Exercise 7D

1–3	2, 3	—

UNDERSTANDING

1 If $x = 3$ are the following equations true or false?

a $5 + 2x = 4x - 1$
b $7x = 6x + 5$
c $2 + 8x = 12x$
d $9x - 7 = 3x + 11$

2 Fill in the blanks for these equivalent equations.

a

$$\begin{tikzcd} 5x+3 \arrow[d, bend right, "-2x"'] & = & 2x+8 \arrow[d, bend left, "-2x"] \\ \_\_\_ & = & 8 \end{tikzcd}$$

b

$$\begin{tikzcd} 9q+5 \arrow[d, bend right, "-9q"'] & = & 12q+21 \arrow[d, bend left, "-9q"] \\ \_\_\_ & = & 3p+21 \end{tikzcd}$$

c

$$\begin{tikzcd} 3p+9 \arrow[d, bend right, "+2p"'] & = & 5-2p \arrow[d, bend left, "+2p"] \\ \_\_\_ & = & \_\_\_ \end{tikzcd}$$

d

$$\begin{tikzcd} 15k+12 \arrow[d, bend right, "+7k"'] & = & 13-7k \arrow[d, bend left, "+7k"] \\ \_\_\_ & = & \_\_\_ \end{tikzcd}$$

3 To solve the equation $12x + 2 = 8x + 16$, which one of the following first steps will ensure that x is only on one side of the equation?

A Subtract 2
B Subtract $8x$
C Add $12x$
D Subtract 16
E Add $20x$

4–5(½)	4–6(½)	4–7(½)

FLUENCY

Example 7a

4 Solve the following equations algebraically. Check your solution using substitution.

a $10f + 3 = 23 + 6f$
b $10y + 5 = 26 + 3y$
c $7s + 7 = 19 + 3s$
d $9j + 4 = 4j + 14$
e $2t + 8 = 8t + 20$
f $4 + 3n = 10n + 39$
g $4 + 8y = 10y + 14$
h $5 + 3t = 6t + 17$
i $7+5q = 19 + 9q$

Example 7b

5 Solve the following equations algebraically.

a $9 + 4t = 7t + 15$
b $2c - 2 = 4c - 6$
c $6t - 3 = 7t - 8$
d $7z - 1 = 8z - 4$
e $8t - 24 = 2t - 6$
f $2q - 5 = 3q - 3$
g $5x + 8 = 6x - 1$
h $8w - 15 = 6w + 3$
i $6j + 4 = 5j - 1$

Example 7c

6 Solve the following equations algebraically.

a $1 - 4a = 7 - 6a$
b $6 - 7g = 2 - 5g$
c $12 - 8n = 8 - 10n$
d $2 + 8u = 37 + 3u$
e $21 - 3h = 6 - 6h$
f $37 - 4j = 7 - 10j$
g $13 - 7c = 8c - 2$
h $10 + 4n = 4 - 2n$
i $10a + 32 = 2a$
j $10v + 14 = 8v$
k $18 + 8c = 2c$
l $2t + 7 = 22 - 3t$
m $6n - 47 = 9 - 8n$
n $3n = 15 + 8n$
o $38 - 10l = 10 + 4l$

7 Solve the following equation, giving your solutions as improper fractions where necessary.

a $3x + 5 = x + 6$
b $5k - 2 = 2k$
c $3 + m = 6 + 3m$
d $9j + 4 = 5j + 14$
e $3 - j = 4 + j$
f $2z + 3 = 4z - 8$

8 8–10 9–11

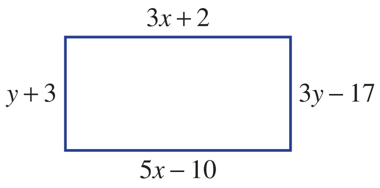
PROBLEM-SOLVING

8 Write an equation and solve it algebraically to find the unknown number in these problems.

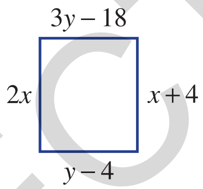
- **a** Doubling x and adding 3 is the same as tripling x and adding 1.
- **b** If z is increased by 9, this is the same as doubling the value of z.
- **c** The product of 7 and y is the same as the sum of y and 12.
- **d** When a number is increased by 10 this has the same effect as tripling the number and subtracting 6.

9 Find the value of x and y in the following rectangles.

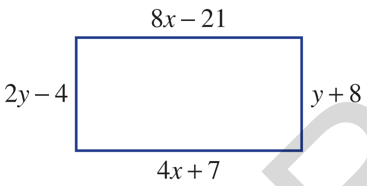
a



b



10 Find the area and the perimeter of this rectangle.



11 At a newsagency, Preeta bought 4 pens and a \$1.50 newspaper, while her husband Levy bought 2 pens and a \$4.90 magazine. To their surprise the cost was the same.

- **a** Write an equation to describe this, using p for the cost of a single pen in dollars.
- **b** Solve the equation to find the cost of pens.
- **c** If Fred has a \$20 note, what is the maximum number of pens that he can purchase?

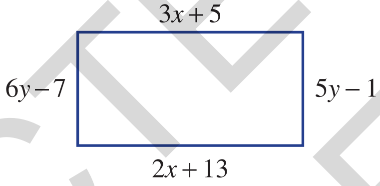
	12	12, 13	13, 14

REASONING

12 To solve the equation $12 + 3x = 5x + 2$ you can first subtract $3x$ or subtract $5x$.

- **a** Solve the equation above by first subtracting $3x$.
- **b** Solve the equation above by first subtracting $5x$.
- **c** What is the difference between the two methods?

13 Prove that the rectangular shape shown to the right must be a square. (Hint: First find the values of x and y.)



14 **a** Try to solve the equation $4x + 3 = 10 + 4x$.

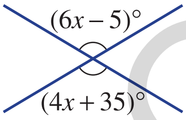
- **b** This tells you that the equation you are trying to solve has no solutions (because $10 = 3$ is never true). Prove that $2x + 3 = 7 + 2x$ has no solutions.
- **c** Give an example of another equation that has no solutions.

Geometric equations	—	—	15

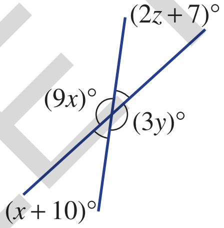
ENRICHMENT

15 Find the values of the unknown variables in the following geometric diagrams.

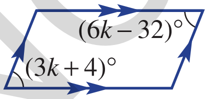
a



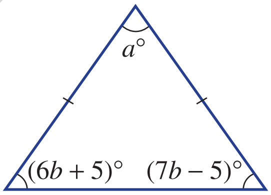
b



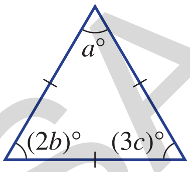
c



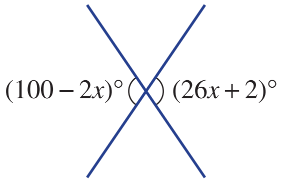
d



e



f



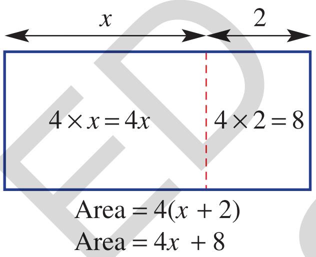
7E Equations with brackets

Interactive

Widgets

HOTsheets

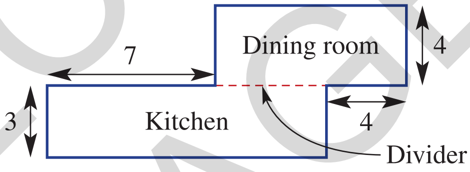
Walkthroughs



In Chapter 5 it was noted that expressions with brackets could be expanded by considering rectangle areas.

So $4(x + 2)$ and $4x + 8$ are equivalent. This becomes quite helpful when solving an equation like $4(x + 2) = 5x + 1$. We just solve $4x + 8 = 5x + 1$ using the techniques from the previous section.

Let's start: Architect's dilemma



In the following house plans, the kitchen and dining room are separated by a dividing door.

- If the width of the divider is x, what is the area of the kitchen? What is the area of the dining room?
- Try to find the width of the divider if the areas of the two rooms are equal.
- Is it easier to solve $3(7 + x) = 4(x + 4)$ or $21 + 3x = 4x + 16$? Which of these did you solve when trying to find the width of the divider?

Key ideas

- To expand brackets, use the **distributive law**, which states that
  - $a(b + c) = ab + ac$. For example, $3(x + 4) = 3x + 12$.
  - $a(b - c) = ab - ac$. For example, $4(b - 2) = 4b - 8$.
- **Like terms** are terms that contain exactly the same variables and can be collected to simplify expressions. For example, $5x + 10 + 7x$ can be simplified to $12x + 10$.
- Equations involving brackets can be solved by first expanding brackets and collecting like terms.

Example 8 Solving equations with brackets

Solve the following equations by first expanding any brackets.

a $3(p + 4) = 18$

b $-12(3q + 5) = -132$

c $4(2x - 5) + 3x = 57$

d $2(3k + 1) = 5(2k - 6)$

SOLUTION	EXPLANATION
a $3(p + 4) = 18$	Use the distributive law to expand the brackets.
$-12 \quad 3p + 12 = 18 \quad -12$ $\div 3 \quad 3p = 6 \quad \div 3$ $p = 2$	Solve the equation by performing the same operations to both sides.

b

$$\begin{aligned} -12(3q+5) &= -132 \\ -36q+(-60) &= -132 \\ +60 \quad -36q-60 &= -132 \quad +60 \\ \div -36 \quad -36q &= -72 \quad \div -36 \\ q &= 2 \end{aligned}$$

Use the distributive law to expand the brackets. Simplify $-36q + (-60)$ to $-36q - 60$.

Solve the equation by performing the same operations to both sides.

c

$$\begin{aligned} 4(2x-5)+3x &= 57 \\ 8x-20+3x &= 57 \\ +20 \quad 11x-20 &= 57 \quad +20 \\ \div 11 \quad 11x &= 77 \quad \div 11 \\ x &= 7 \end{aligned}$$

Use the distributive law to expand the brackets. Combine the like terms: $8x + 3x = 11x$.

Solve the equation by performing the same operations to both sides.

d

$$\begin{aligned} 2(3k+1) &= 5(2k-6) \\ -6k \quad 6k+2 &= 10k-30 \quad -6k \\ +30 \quad 2 &= 4k-30 \quad +30 \\ \div 4 \quad 32 &= 4k \quad \div 4 \\ 8 &= k \\ \therefore\ k &= 8 \end{aligned}$$

Use the distributive law on both sides to expand the brackets.

Solve the equation by performing the same operations to both sides.

Exercise 7E

1–4 | 1, 4 | —

UNDERSTANDING

1 Fill in the missing numbers.

a $4(y+3) = 4y + \square$

b $7(2p-5) = \square p - 35$

c $2(4x+5) = \square x + \square$

d $10(5+3q) = \square + \square q$

2 Match each expression **a**–**d** with its expanded form **A**–**D**.

a	$2(x+4)$	**A**	$4x+8$
b	$4(x+2)$	**B**	$2x+4$
c	$2(2x+1)$	**C**	$2x+8$
d	$2(x+2)$	**D**	$4x+2$

3 Rolf is unsure whether $4(x+3)$ is equivalent to $4x+12$ or $4x+3$.

a Fill out the table below.

x	0	1	2
$4(x+3)$			
$4x+12$			
$4x+3$			

b What is the correct expansion of $4(x+3)$?

4 Simplify the following expressions by collecting like terms.

a $4x+3x$

b $7p+2p+3$

c $8x-2x+4$

d $2k+4+5k$

e $3x+6-2x$

f $7k+21-2k$

5, 6–7(½)	5, 6–8(½)	5–9(½)

FLUENCY

Example 8a

5 Solve the following equations by first expanding the brackets.

a $2(4u + 2) = 52$ **b** $3(3j - 4) = 15$ **c** $5(2p - 4) = 40$

d $15 = 5(2m - 5)$ **e** $2(5n + 5) = 60$ **f** $26 = 2(3a + 4)$

Example 8b

6 Solve the following equations involving negative numbers.

a $-6(4p + 4) = 24$ **b** $-2(4u - 5) = 34$ **c** $-2(3v - 4) = 38$

d $28 = -4(3r + 5)$ **e** $-3(2b - 2) = 48$ **f** $-6 = -3(2d - 4)$

Example 8c

7 Solve the following equations by expanding and combining like terms.

a $4(3y + 2) + 2y = 50$ **b** $5(2l - 5) + 3l = 1$ **c** $4(5 + 3w) + 5 = 49$

d $49 = 5(3c + 5) - 3c$ **e** $28 = 4(3d + 3) - 4d$ **f** $58 = 4(2w - 5) + 5w$

g $23 = 4(2p - 3) + 3$ **h** $44 = 5(3k + 2) + 2k$ **i** $49 = 3(2c - 5) + 4$

Example 8d

8 Solve the following equations by expanding brackets on both sides.

a $5(4x - 4) = 5(3x + 3)$ **b** $6(4 + 2r) = 3(5r + 3)$ **c** $5(5f - 2) = 5(3f + 4)$

d $4(4p - 3) = 2(4 + 3p)$ **e** $2(5h + 4) = 3(4 + 3h)$ **f** $4(4r - 5) = 2(5 + 5r)$

g $4(3r - 2) = 4(2r + 3)$ **h** $2(2p + 4) = 2(3p - 2)$ **i** $3(2a + 1) = 11(a - 2)$

9 Solve the following equations algebraically.

a $2(3 + 5r) + 6 = 4(2r + 5) + 6$ **b** $3(2l + 2) + 18 = 4(4l + 3) - 8$

c $2(3x - 5) + 16 = 3 + 5(2x - 5)$ **d** $3(4s + 3) - 3 = 3(3s + 5) + 15$

e $4(4y + 5) - 4 = 6(3y - 3) + 20$ **f** $3(4h + 5) + 2 = 14 + 3(5h - 2)$

10, 11	11, 12	11–13

PROBLEM-SOLVING

10 Desmond notes that in 4 years' time his age when doubled will give the number 50.

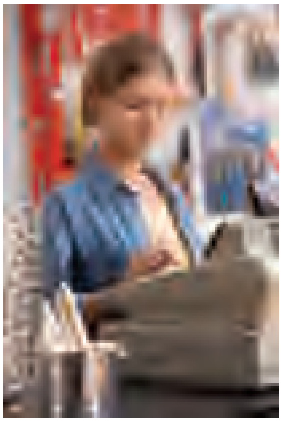
Desmond's current age is d years old.

a Write an expression for Desmond's age in 4 years' time.

b Write an expression for double his age in 4 years' time.

c Write an equation to describe the situation described above.

d Solve the equation to find his current age.



11 Rahda's usual hourly wage is $\$w$. She works for 5 hours at this wage and then 3 more hours at an increased wage of $\$(w + 4)$.

a Write an expression for the total amount Rahda earns for the 8 hours.

b Rahda earns \$104 for the 8 hours. Write and solve an equation to find her usual hourly wage.

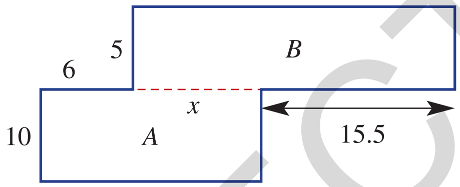
PROBLEM-SOLVING

12 Kate's age 5 years ago, when doubled, is equal to triple her age 10 years ago.

- **a** Write an equation to describe this, using k for Kate's current age.
- **b** Solve the equation to find Kate's current age.

13 Rectangles A and B have the same area.

- **a** What is the value of x?
- **b** State the perimeter of the shape shown at right.



14	14,15	15,16

REASONING

14 Abraham is asked how many people are in the room next door. He answers that if three more people walked in and then the room's population was doubled, this would have the same effect as quadrupling the population and then 11 people leaving. Prove that what Abraham said cannot be true.

15 Ajith claims that three times his age 5 years ago is the same as nine times how old he will be next year. Prove that what Ajith is saying cannot be true.

16 A common mistake when expanding is to write $2(n + 3)$ as $2n + 3$. These are not equivalent, since, for example, $2(5 + 3) = 16$ and $2 \times 5 + 3 = 13$.

- **a** Prove that they are never equal by trying to solve $2(n + 3) = 2n + 3$.
- **b** Prove that $4(2x + 3)$ is never equal to $8x + 3$ but it is sometimes equal to $4x + 12$.

Challenging expansions

—	—	17

ENRICHMENT

17 Solve the following equations. Note that, in general, your answers will not be integers.

- **a** $2(3x + 4) + 5(6x + 7) = 64x + 1$
- **b** $-5(3p + 2) + 5(2p + 3) = -31$
- **c** $-10(n + 1) + 20(2n + 13) = 7$
- **d** $4(2q + 1) - 5(3q + 1) = 11q - 1$
- **e** $x + 2(x + 1) + 3(x + 2) = 11x$
- **f** $m - 2(m + 1) - 3(m - 1) = 2(1 - 4m)$

7F Formulas and relationships

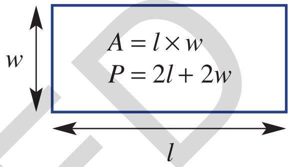
Interactive

Widgets

HOTsheets

Walkthroughs

Some equations involve two or more variables that are related. For example, you know from measurement that the area of a rectangle is related to its length and width, given by the formula $A = l \times w$ and its perimeter is given by $P = 2l + 2w$. Although these are often used as a definition for the area and a definition of perimeter, they are also just equations – two expressions written on either side of an equals sign.



Let's start: Rectangular dimensions

You know that the area and perimeter of a rectangle are given by $A = l \times w$ and $P = 2l + 2w$.

- If $l = 10$ and $w = 7$, find the perimeter and the area.
- If $l = 2$ and $w = 8$, find the perimeter and the area.
- Notice that sometimes the area is bigger than the perimeter and sometimes the area is less than the perimeter. If $l = 10$, is it possible to make the area and the perimeter equal?
- If $l = 2$, can you make the area and the perimeter equal? Discuss.

Key ideas

- The **subject** of an equation is a pronumeral (or variable) that occurs by itself on the left-hand side.
  For example, V is the subject of $V = 3x + 2y$.
- A **formula** or **rule** is an equation containing two or more variables, one of which is the subject of the equation.
- To use a formula, substitute all known values and then solve the equation to find the unknown value.

Example 9 Applying a formula

Apply the formula for a rectangle's perimeter $P = 2l + 2w$ to find:

a P when $l = 4$ and $w = 7$

b l when $P = 40$ and $w = 3$

SOLUTION	EXPLANATION
a $P = 2l + 2w$	Write the formula.
$P = 2(4) + 2(7)$	Substitute in the values for l and w.
$P = 22$	Simplify the result.
b $P = 2l + 2w$	Write the formula.
$40 = 2l + 2(3)$	Substitute in the values for P and w to obtain an equation.
$$\begin{tikzcd} 40 = 2l + 6 \arrow[d, "-6"', "-6"] \\ 34 = 2l \arrow[d, "\div 2"', "\div 2"] \\ 17 = l \end{tikzcd}$$ $\therefore\ l = 17$	Solve the equation to obtain the value of l.

Exercise 7F

1–3	1, 3	—

UNDERSTANDING

1 **a** Substitute $x = 4$ into the expression $x + 7$.
  b Substitute $a = 2$ into the expression $3a$.
  c Substitute $p = 5$ into the expression $2p - 3$.
  d Substitute $r = -4$ into the expression $7r$.

2 If you substitute $P = 10$ and $x = 2$ into the formula $P = 3m + x$, which of the following equations would you get?

A $10 = 6 + x$
B $10 = 3m + 2$
C $2 = 3m + 10$
D $P = 30 + 2$

3 If you substitute $k = 10$ and $L = 12$ into the formula $L = 4k + Q$, which of the following equations would you get?

A $12 = 40 + Q$
B $L = 40 + 12$
C $12 = 410 + Q$
D $10 = 48 + Q$

4–7	4–9	5–9

FLUENCY

Example 9a

4 Consider the rule $A = 4p + 7$.

a Find A if $p = 3$.
b Find A if $p = 11$.
c Find A if $p = -2$.
d Find A if $p = \frac{13}{2}$.

Example 9b

5 Consider the rule $U = 8a + 4$.

a Find a if $U = 44$. Set up and solve an equation.
b Find a if $U = 92$. Set up and solve an equation.
c If $U = -12$, find the value of a.

6 Consider the relationship $y = 2x + 4$.

a Find y if $x = 3$.
b By solving an appropriate equation, find the value of x that makes $y = 16$.
c Find the value of x if $y = 0$.

7 Use the formula $P = mv$ to find the value of m when $P = 22$ and $v = 4$.

8 Assume that x and y are related by the equation $4x + 3y = 24$.

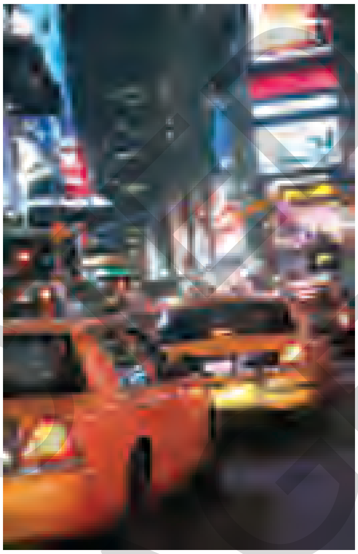
a If $x = 3$, find y by solving an equation.
b If $x = 0$, find the value of y.
c If $y = 2$, find x by solving an equation.
d If $y = 0$, find the value of x.

9 Consider the formula $G = k(2a + p) + a$.

a If $k = 3$, $a = 7$ and $p = -2$, find the value of G.
b If $G = 78$, $k = 3$ and $p = 5$, find the value of a.

10 | 10, 11 | 11, 12

PROBLEM-SOLVING



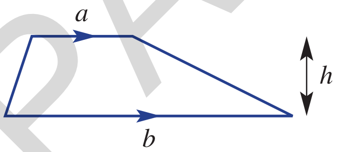
10 The cost \$C to hire a taxi for a trip of length d km is $C = 3 + 2d$.

- **a** Find the cost of a 10 km trip (i.e. for $d = 10$).
- **b** A trip has a total cost of \$161.
  - **i** Set up an equation by substituting $C = 161$.
  - **ii** Solve the equation algebraically.
  - **iii** How far did the taxi travel? (Give your answer in km.)

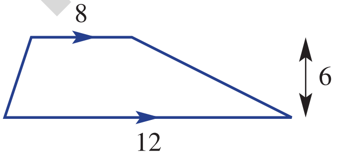
11 To convert temperature between Celsius and Fahrenheit, the rule is $F = 1.8C + 32$.

- **a** Find F if $C = 10$.
- **b** Find C if $F = 95$.
- **c** Vinod's body temperature is 100° Fahrenheit. What temperature is this in degrees Celsius? Answer correct to one decimal place.

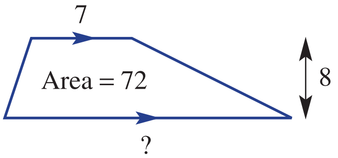
12 The formula for the area of a trapezium is $A = \frac{1}{2}h(a + b)$.



- **a** Find the area of the trapezium shown at right.



- **b** Find the value of h if $A = 20$, $a = 3$ and $b = 7$.
- **c** Find the missing value in the trapezium at right.



13 | 13, 14 | 15, 16

REASONING

13 Katy is a scientist who tries to work out the relationship between the volume of a gas, V mL, and its temperature T°C. She makes a few measurements.

V	10	20
T	10	15

- **a** What is a possible rule between V and T?
- **b** Use your rule to find the volume at a temperature of 27°C.
- **c** Prove that the rule $T = \frac{(V - 10)^2}{20} + 10$ would also work for Katy's results.

14 Consider the rule $G = 120 - 4p$.

- **a** If p is between 7 and 11, what is the largest value of G?
- **b** If p and G are equal, what value do they have?

15 Marie is a scientist who is trying to discover the relationship between the volume of a gas V, its temperature T and its transparency A. She makes a few measurements.

	Test 1	Test 2
V	10	20
A	2	5
T	15	12

Which one or more of the following rules are consistent with the experiment's results?

A $T = \frac{3V}{A}$ **B** $T = V + 2A$ **C** $T = 17 - A$

16 Temperatures in degrees Fahrenheit and Celsius are related by the rule $F = 1.8C + 32$.

a By substituting $F = x$ and $C = x$, find a value such that the temperature in Fahrenheit and the temperature in Celsius are equal.

b By substituting $F = 2x$ and $C = x$, find a temperature in Celsius that doubles to give the temperature in Fahrenheit.

c Prove that there are no Celsius temperatures that can be multiplied by 1.8 to give the temperature in Fahrenheit.

Mobile phone plans

—	—	17

17 Two companies have mobile phone plans that factor in the number of minutes spent talking each month (t) and the total number of calls made (c).

Company A's cost in cents: $A = 20t + 15c + 300$
Company B's cost in cents: $B = 30t + 10c$

a In one month 12 calls were made, totalling 50 minutes on the phone. Find the cost in dollars that company A and company B would have charged.

b In another month, a company A user was charged \$15 (1500 cents) for making 20 calls. How long were these calls in total?

c In another month, a company B user talked for 60 minutes in total and was charged \$21. What was the average length of these calls?

d Briony notices one month that for her values of t and c, the two companies cost exactly the same. Find a possible value of t and c that would make this happen.

e Briony reveals that she made exactly 20 calls for the month in which the two companies' charges would be the same. How much time did she spend talking?

Progress quiz

7A **1** For each of the following equations, state whether they are true (T) or false (F).
a $5 + 11 = 8 \times 2$ **b** $x + 9 = 18 - x$, if $x = 4$ **c** $a \times (a - 4) = 2a$, if $a = 6$

7A **2** State a solution to each of the following equations (no working required).
a $k + 5 = 21$ **b** $7 = c - 6$ **c** $4m = 32$ **d** $10 + t = 3t$

7A **3** Write equations for the following scenarios. You do not need to solve the equations.
a A number n is doubled and then 5 is added. The result is 17.
b Archie's age is a. Archie's mother, who is 26 years older than Archie, is triple Archie's age.

7B **4** Solve the following equations algebraically.
a $a + 8 = 15$ **b** $12 = 9 - k$ **c** $-42 = 6h$
d $5 + 3y = 29$ **e** $4u - 8 = 40$ **f** $52 = 8j - 4$
g $68 - 12d = 8$ **h** $59 = -13 - 9m$

7C **5** Solve the following equations algebraically.
a $\frac{5u}{2} = 100$ **b** $\frac{-3h}{7} = -6$ **c** $3 + \frac{4x}{3} = 15$ **d** $\frac{2w + 7}{3} = 5$

7D **6** Solve the following equations algebraically.
a $4n + 3 = 2n + 17$ **b** $9w - 7 = 4w - 17$ **c** $e + 8 = -28 - 3e$

7E **7** Solve the following equations by first expanding any brackets.
a $6(a + 2) = 42$ **b** $3(4w - 6) = 114$ **c** $5(2q - 1) - 3q = 30$ **d** $-8(2 - p) = 3(2p - 8)$

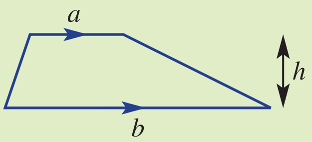
7F **8** **a** Apply the formula for a rectangle's perimeter $P = 2l + 2w$ to find the length l, when $P = 32$ cm and $w = 6$ cm.
b The rule for the area of a triangle is $A = \frac{bh}{2}$. By solving an appropriate equation, find the base length b, for a triangle of height $h = 3$ cm and area $A = 24$ cm^2.

7B/C **9** For each of the following, write an equation and solve it algebraically to find the unknown number.
a The product of q and -6 is 30.
b Two thirds of a number m gives a result of 12.
c A number k is tripled and then 4 is added. This result is halved to obtain -13.
d The average of $3x$ and 10 is 14.

7E **10** Maddie's age 8 years ago when multiplied by 5, is the same as triple Maddie's age in 2 years' time. Write and solve an equations to find Maddie's current age.

7F **11** The formula for the area of a trapezium is $A = \frac{h}{2}(a + b)$.

Find the value of the height h, in a trapezium with $A = 162$ cm^2, $a = 12$ cm, $b = 15$ cm.



7G Applications

Interactive

Widgets

HOTsheets

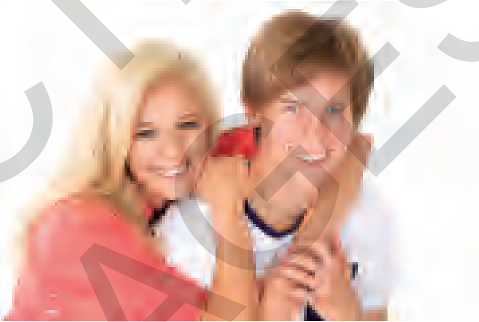
Walkthroughs

Although knowing how to solve equations is useful, it is important to be able to recognise when real-world situations can be thought of as equations. This is the case whenever it is known that two values are equal. In this case, an equation can be constructed and solved. It is important to translate this solution into a meaningful answer within the real-world context.

Let's start: Sibling sum

John and his elder sister are 4 years apart in their ages.

- If the sum of their ages is 26, describe how you could work out how old they are.
- Could you write an equation to describe the situation above, if x is used for John's age?
- How would the equation change if x is used for John's sister's age instead?



The difference in two people's ages can be expressed as an equation.

Key ideas

- An equation can be used to describe any situation in which two values are equal.
- To solve a problem follow these steps.
  1. Define pronumerals to stand for unknown numbers (e.g. let j = John's current age).
  2. Write an equation to describe the problem.
  3. Solve the equation algebraically if possible, or by inspection.
  4. Ensure you answer the original question, including the correct units (e.g. dollars, years, cm).
- Your final solution should be checked to see if it solves the problem correctly.

Example 10 Solving a problem using equations

The weight of 6 identical books is 1.2 kg. What is the weight of one book?

SOLUTION	EXPLANATION
Let b = weight of one book in kg.	Define a pronumeral to stand for the unknown number.
$6b = 1.2$	Write an equation to describe the situation.
$\begin{tikzcd} 6b \arrow[d, bend right, "\div 6"'] & = & 1.2 \arrow[d, bend left, "\div 6"] \\ b & = & 0.2 \end{tikzcd}$	Solve the equation.
The books weigh 0.2 kg each, or 200 g each.	Answer the original question. It is not enough to give a final answer as 0.2; this is not the weight of a book, it is just a number.

Example 11 Solving a harder problem using equations

Purchasing 5 apples and a $2.40 mango costs the same as purchasing 7 apples and a mandarin that costs 60 cents. What is the cost of each apple?

SOLUTION	EXPLANATION
Let c = cost of one apple in dollars.	Define a pronumeral to stand for the unknown number.
$5c + 2.4 = 7c + 0.6$	Write an equation to describe the situation. Note that 60 cents must be converted to $0.6 to keep the units the same throughout the equation.
$\begin{tikzcd} 5c + 2.4 = 7c + 0.6 \arrow[d, bend right, "-5c"'] \arrow[d, bend left, "-5c"] \\ 2.4 = 2c + 0.6 \arrow[d, bend right, "-0.6"'] \arrow[d, bend left, "-0.6"] \\ 1.8 = 2c \arrow[d, bend right, "\div 2"'] \arrow[d, bend left, "\div 2"] \\ 0.9 = c \end{tikzcd}$	Solve the equation.
Apples cost 90 cents each.	Answer the original question. It is not enough to give a final answer as 0.9; this is not the cost of an apple, it is just a number.

Example 12 Solving problems with two related unknowns

Jane and Luke have a combined age of 60. Given that Jane is twice as old as Luke, find the ages of Luke and Jane.

SOLUTION	EXPLANATION
Let l = Luke's age	Define a pronumeral for the unknown value. Once Luke's age is found, we can double it to find Jane's age.
$l + 2l = 60$	Write an equation to describe the situation. Note that Jane's age is $2l$ because she is twice as old as Luke.
$\begin{tikzcd} 3l = 60 \arrow[d, bend right, "\div 3"'] \arrow[d, bend left, "\div 3"] \\ l = 20 \end{tikzcd}$	Solve the equation by first combining like terms.
Luke is 20 years old and Jane is 40 years old.	Answer the original question.

Exercise 7G

1–3	2	—

UNDERSTANDING

1 Match each of these worded descriptions with an appropriate expression.

a The sum of x and 3	**A** $2x$
b The cost of 2 apples if they cost \$$x$ each	**B** $x + 1$
c The cost of x oranges if they cost \$1.50 each	**C** $3x$
d Triple the value of x	**D** $x + 3$
e One more than x	**E** $1.5x$

2 For the following problems choose the equation to describe them.

a The sum of x and 5 is 11.

A $5x = 11$ **B** $x + 5 = 11$ **C** $x - 5 = 11$ **D** $11 - 5$

b The cost of 4 pens is \$12. Each pen costs \$$p$.

A $4 = p$ **B** $12p$ **C** $4p = 12$ **D** $12p = 4$

c Josh's age next year is 10. His current age is j.

A $j + 1 = 10$ **B** $j = 10$ **C** 9 **D** $j - 1 = 10$

d The cost of n pencils is \$10. Each pencil costs \$2.

A $n \div 10 = 2$ **B** 5 **C** $10n = 2$ **D** $2n = 10.$

3 Solve the following equations.

a $5p = 30$ **b** $5 + 2x = 23$ **c** $12k - 7 = 41$ **d** $10 = 3a + 1$

4–6	4–7	5–7

FLUENCY

Example 10

4 Jerry buys 4 cups of coffee for \$13.20.

a Choose a pronumeral to stand for the cost of one cup of coffee.

b Write an equation to describe the problem.

c Solve the equation algebraically.

d Hence state the cost of one cup of coffee.

5 A combination of 6 chairs and a table costs \$3000. The table alone costs \$1740.

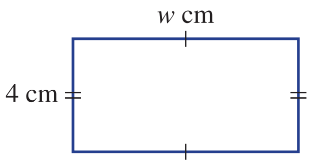
a Define a pronumeral for the cost of one chair.

b Write an equation to describe the problem.

c Solve the equation algebraically.

d Hence state the cost of one chair.

6 The perimeter of this rectangle is 72 cm.



a Write an equation to describe the problem, using w for the width.

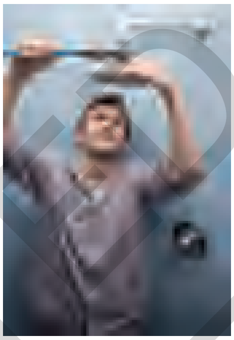
b Solve the equation algebraically.

c Hence state the width of the rectangle.

FLUENCY

7 A plumber charges a $70 call-out fee and $52 per hour. The total cost of a particular visit was $252.

- **a** Define a variable to stand for the length of the visit in hours.
- **b** Write an equation to describe the problem.
- **c** Solve the equation algebraically.
- **d** State the length of the plumber's visit, giving your answer in minutes.



9	8–11	11–13

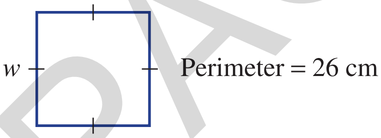
PROBLEM-SOLVING

Example 11

8 A number is tripled, then 2 is added. This gives the same result as if the number were quadrupled. Set up and solve an equation to find the original number.

9 A square has a perimeter of 26 cm.

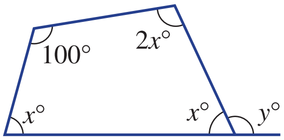
- **a** Solve an equation to find its width.
- **b** Hence state the area of the square.



Example 12

10 Alison and Flynn's combined age is 40. Given that Flynn is 4 years older than Alison, write an equation and use it to find Alison's age.

11 Recall that in a quadrilateral the sum of all angles is 360°. Find the values of x and y in the diagram shown.



12 The sum of three consecutive numbers is 357.

- **a** Use an equation to find the smallest of the three numbers.
- **b** What is the average of these three numbers?
- **c** If the sum of three consecutive numbers is 38 064, what is their average?

13 The width of a rectangular pool is 5 metres longer than the length. The perimeter of the pool is 58 metres.

- **a** Draw a diagram of this situation.
- **b** Use an equation to find the pool's length.
- **c** Hence state the area of the pool.

14	14	14, 15

REASONING

14 The average of two numbers can be found by adding them and dividing by 2.

a If the average of x and 10 is 30, what is the value of x?
b If the average of x and 10 is 2, what is the value of x?
c If the average of x and 10 is some number R, create a formula for the value of x.

15 Sometimes you are given an equation to solve a puzzle, but the solution of the equation is not actually possible for the situation. Consider these five equations.

A $10x = 50$ **B** $8 + x = 10$ **C** $10 + x = 8$ **D** $10x = 8$ **E** $3x + 5 = x + 5$

a You are told that the number of people in a room can be determined by solving an equation. Which of these equations could be used to give a reasonable answer?
b If the length of an insect is given by the variable x cm, which of the equations could be solved to give a reasonable value of x?
c Explain why equation **D** could not be used to find the number of people in a room but could be used to find the length of an insect.
d Give an example of a puzzle that would make equation **C** reasonable.

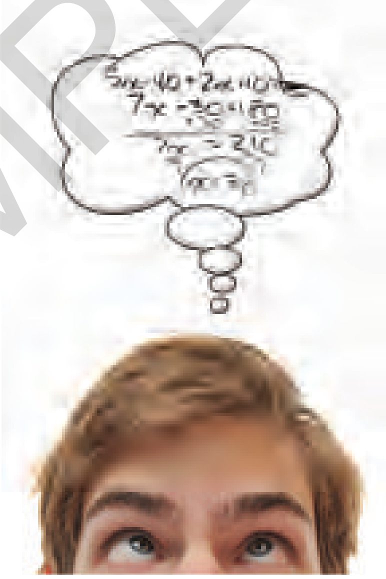
Unknown numbers

—	—	16

ENRICHMENT

16 Find the unknown number using equations. The answers might not be whole numbers.

a The average of a number and double the number is 25.5.
b Adding 3 to twice a number is the same as subtracting 9 from half the number.
c The average of a number and double the number gives the same result as adding one to the original number and then multiplying by one-third.
d The product of 5 and a number is the same as the sum of four and twice the original number.
e The average of 5 numbers is 7. When one more number is added the average becomes 10. What number was added?



7H Inequalities EXTENDING

Interactive

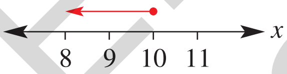
Widgets

HOTsheets

Walkthroughs

An **inequality** is like an equation but, instead of indicating that two expressions are equal, it indicates which of the two has a greater value. For example, $2 + 4 < 7$, $3 \times 5 \geq 15$ and $x \leq 10$ are all inequalities. The first two are true, and the last one could be true or false depending on the value of x. For instance, the numbers 9.8, 8.45, 7 and –120 all make this inequality true.

We could represent all the values of x that make $x \leq 10$ a true statement.



Let's start: Small sums

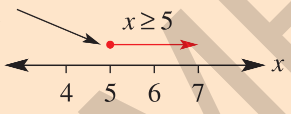
Two positive whole numbers are chosen: x and y. You are told that $x + y \leq 5$.

- How many possible pairs of numbers make this true? For example, $x = 2$ and $y = 1$ is one pair and it is different from $x = 1$ and $y = 2$.
- If $x + y \leq 10$, how many pairs are possible? Try to find a pattern rather than listing them all.
- If all you know about x and y is that $x + y > 10$, how many pairs of numbers could there be?

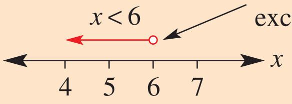
Key ideas

- An **inequality** is a statement of the form:
  - LHS > RHS (greater than). For example, $5 > 2$
  - LHS ≥ RHS (greater than or equal). For example, $7 \geq 7$ or $10 \geq 7$
  - LHS < RHS (less than). For example, $2 < 10$
  - LHS ≤ RHS (less than or equal). For example, $5 \leq 5$ or $2 \leq 5$
- Inequalities can be reversed: $3 < x$ and $x > 3$ are equivalent.
- Inequalities can be represented on a number line, using closed circles at the end points if the value is included, or open circles if it is excluded.

Closed circle indicates 5 is included



Open circle indicates 6 is excluded



- A range can be represented as a segment on the number line using appropriate closed and open end points.

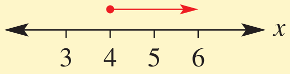
Example 13 Representing inequalities on a number line

Represent the following inequalities on a number line.

a $x \geq 4$ **b** $x < 6$ **c** $1 < x \leq 5$

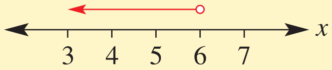
SOLUTION | **EXPLANATION**

a



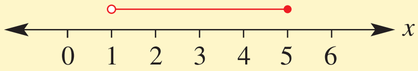
A circle is placed at 4 and then the arrow points to the right, towards all numbers greater than 4. The circle is filled (closed) because 4 is included in the set.

b



A circle is placed at 6 and then the arrow points to the left, towards all numbers less than 6. The circle is hollow (open) because 6 is not included in the set.

c



Circles are placed at 1 and 5, and a line goes between them to indicate that all numbers in between are included. The circle at 1 is open because the inequality is < not ≤ .

Example 14 Using inequalities to describe real-life situations

Describe the following situations as an inequality, using x to stand for the unknown quantity.

a Fred is shorter than 160 cm.

b John is at least as old as Maria, who is 10.

c Rose's test score is between 40 and 50 inclusive.

SOLUTION	EXPLANATION
a $x < 160$	Using x to stand for Fred's height in cm, x must be less than 160.
b $x \geq 10$	John is at least 10, so his age is greater than or equal to 10.
c $40 \leq x \leq 50$	x is between 40 and 50. The word 'inclusive' tells us that 40 and 50 are both included, so ≤ is used (rather than < if the word 'exclusive' is used).

Exercise 7H

1–4	4	—

UNDERSTANDING

1 Classify the following statements as true or false.

a $5 > 3$ **b** $7 < 5$ **c** $2 < 12$

d $13 < 13$ **e** $13 \leq 13$ **f** $4 \geq 2$

2 Match each of these inequalities with the appropriate description.

a $x > 5$ **b** $x < 5$ **c** $x \geq 3$ **d** $x \leq 3$

A The number x is less than 5.

B The number x is greater than or equal to 3.

C The number x is less than or equal to 3.

D The number x is greater than 5.

UNDERSTANDING

3 For each of the following, state whether they make the inequality $x > 4$ true or false.

a $x = 5$ **b** $x = -2$ **c** $x = 4$ **d** $x = 27$

4 If $x = 12$, classify the following inequalities as true or false.

a $x > 2$ **b** $x < 11$ **c** $x \geq 13$ **d** $x \leq 12$

5(½), 6	5(½), 6, 7(½)	5(½), 6, 7(½)

FLUENCY

Example 13a, 13b

5 Represent the following inequalities on separate number lines.

a $x > 3$ **b** $x < 10$ **c** $x \geq 2$ **d** $x < 1$

e $x \geq 1$ **f** $x \geq 4$ **g** $x < -5$ **h** $x < -9$

i $x \leq 2$ **j** $x < -6$ **k** $x \geq -3$ **l** $x \leq 5$

m $10 > x$ **n** $2 < x$ **o** $5 \geq x$ **p** $-3 \leq x$

Example 13c

6 **a** List which of the following numbers make the inequality $2 \leq x < 7$ true.

8, 1, 3, 4, 6, 4.5, 5, 2.1, 7, 6.8, 2

b Represent the inequality $2 \leq x < 7$ on a number line.

7 Represent the following inequalities on separate number lines.

a $1 \leq x \leq 6$ **b** $4 \leq x < 11$ **c** $-2 < x \leq 6$ **d** $-8 \leq x \leq 3$

e $2 < x \leq 5$ **f** $-8 < x < -1$ **g** $7 < x \leq 8$ **h** $0 < x < 1$

8	8, 10	9, 10

PROBLEM-SOLVING

Example 14

8 For each of the following descriptions, choose an appropriate inequality from **A**–**H** below.

a John is more than 12 years old.

b Marika is shorter than 150 cm.

c Matthew is at least 5 years old but he is younger than 10.

d The temperature outside is between –12°C and 10°C inclusive.

A $x < 150$ **B** $x < 12$ **C** $x > 12$ **D** $x \leq 150$

E $10 \leq x \leq -12$ **F** $-12 \leq x \leq 10$ **G** $5 \leq x < 10$ **H** $5 < x \leq 10$

9 It is known that Tim's age is between 20 and 25 inclusive, and Nick's age is between 23 and 27 inclusive.

a If t = Tim's age and n = Nick's age, write two inequalities to represent these facts.

b Represent both inequalities on the same number line.

c Nick and Tim are twins. What is the possible range of their ages? Represent this on a number line.

PROBLEM-SOLVING

10 At a certain school the following grades are awarded for different scores.

Score	$x \geq 80$	$60 \leq x < 80$	$40 \leq x < 60$	$20 \leq x < 40$	$x < 20$
Grade	A	B	C	D	E

a Convert the following scores into grades.

i 15 **ii** 79 **iii** 80 **iv** 60 **v** 30

b Emma got a B on one test, but her sister Rebecca got an A with just 7 more marks. What is the possible range for Emma's score?

c Hugh's mark earned him a C. If he had scored half this mark, what grade would he have earned?

d Alfred and Reuben earned a D and a C respectively. If their scores were added together, what grade or grades could they earn?

e Michael earned a D and was told that if he doubled his mark he would have a B. What grade or grades could he earn if he got an extra 10 marks?

11a, b	11	11, 12(½)

REASONING

11 Sometimes multiple inequalities can be combined to a simpler inequality.

a Explain why the combination $x \geq 5, x \geq 7$ is equivalent to the inequality $x \geq 7$.

b Simplify the following pairs of inequalities to a single inequality.

i $x > 5, x \geq 2$ **ii** $x < 7, x < 3$ **iii** $x \geq 1, x > 1$

iv $x \leq 10, x < 10$ **v** $x > 3, x < 10$ **vi** $x > 7, x \leq 10$

c Simplify the following pairs of inequalities to a single inequality.

i $3 < x < 5, 2 < x < 7$ **ii** $-2 \leq x < 4, -2 < x \leq 4$

iii $7 < x \leq 10, 2 \leq x < 8$ **iv** $5 \leq x < 10, 9 \leq x \leq 11$

12 Some inequalities, when combined, have no solutions; some have one solution and some have infinitely many solutions. Label each of the following pairs using 0, 1 or ∞ (infinity) to say how many solutions they have.

i $x \geq 5$ and $x \leq 5$ **ii** $x > 3$ and $x < 10$

iii $x \geq 3$ and $x < 4$ **iv** $x > 3$ and $x < 2$

v $-2 < x < 10$ and $10 < x < 12$ **vi** $-3 \leq x \leq 10$ and $10 \leq x \leq 12$

vii $x > 2.5$ and $x \leq 3$ **viii** $x \geq -5$ and $x \leq -7$

Working within boundaries

—	—	13, 14

ENRICHMENT

13 If it is known that $0 \leq x \leq 10$ and $0 \leq y \leq 10$, which of the following inequalities must be true? Justify your answers.

a $x + y \leq 30$ **b** $2x \leq 20$ **c** $10 \leq 2y \leq 20$

d $x \times y \leq 100$ **e** $0 \leq x - y \leq 10$ **f** $x + 5y \leq 100$

14 If it is known that $0 \leq a \leq 10, 0 \leq b \leq 10$ and $0 \leq c \leq 10$, what is the largest value that the following expressions could have?

a $a + b + c$ **b** $ab + c$ **c** $a(b + c)$ **d** $a \times b \times c$

e $a - b - c$ **f** $a - (b - c)$ **g** $3a + 4$ **h** $a - bc$

7I Solving inequalities EXTENDING

Interactive
Widgets
HOTsheets
Walkthroughs

Sometimes a problem arises in which an inequality is more complicated than something such as $x > 5$ or $y \leq 40$. For instance, you could have the inequality $2x + 4 > 100$. To **solve** an inequality means to find all the values that make it true. For the inequality above, $x = 50$, $x = 90$ and $x = 10\,000$ are all part of the solution, but the solution is best described as $x > 48$, because any number greater than 48 will make this inequality true and any other number makes it false.

The rules for solving inequalities are very similar to those for equations: perform the same operation to both sides. The one exception occurs when multiplying or dividing by a negative number. We can do this, but we must flip the sign because of the following observation.

$$\begin{tikzcd} 5 > 2 \arrow[d, bend right, "\times -1"'] \arrow[d, bend left, "\times -1"] \\ -5 > -2 \end{tikzcd}$$

Incorrect method

$$\begin{tikzcd} 5 > 2 \arrow[d, bend right, "\times -1"'] \arrow[d, bend left, "\times -1"] \\ -5 < -2 \end{tikzcd}$$

Correct method

Let's start: Limousine costing

A limousine is hired for a wedding. The charge is a $50 hiring fee plus $200 per hour.

- If the total hire time was more than 3 hours, what can you say about the total cost?
- If the total cost is less than $850 but more than $450, what can you say about the total time the limousine was hired?

Key ideas

- Given an inequality, an **equivalent** inequality can be obtained by:
  - adding or subtracting an expression from both sides
  - multiplying or dividing both sides by any positive number
  - multiplying or dividing both sides by a negative number and reversing the inequality
  - Switching sides and reversing the inequality.
    For example:

$$\begin{tikzcd} 2x + 4 < 10 \arrow[d, bend right, "-4"'] \arrow[d, bend left, "-4"] \\ 2x < 6 \arrow[d, bend right, "\div 2"'] \arrow[d, bend left, "\div 2"] \\ x < 3 \end{tikzcd}$$

$$\begin{tikzcd} -4x + 2 < 6 \arrow[d, bend right, "-2"'] \arrow[d, bend left, "-2"] \\ -4x < 4 \arrow[d, bend right, "\div -4"'] \arrow[d, bend left, "\div -4"] \\ x > -1 \end{tikzcd}$$

Sign is reversed

$$\begin{tikzcd} 2 > x + 1 \\ 1 > x \arrow[d] \\ x < 1 \end{tikzcd}$$

Sign is reversed

Example 15 Solving inequalities

Solve the following inequalities.

a $5x + 2 < 47$ **b** $\dfrac{3 + 4x}{9} \geq 3$ **c** $15 - 2x > 1$

SOLUTION

EXPLANATION

a

$$\begin{tikzcd} 5x + 2 < 47 \arrow[d, bend right, "-2"'] \arrow[d, bend left, "-2"] \\ 5x < 45 \arrow[d, bend right, "\div 5"'] \arrow[d, bend left, "\div 5"] \\ x < 9 \end{tikzcd}$$

The inequality is solved in the same way as an equation is solved: 2 is subtracted from each side and then both sides are divided by 5. The sign does not change throughout.

b

$$\begin{tikzcd} \dfrac{3 + 4x}{9} \geq 3 \arrow[d, bend right, "\times 9"'] \arrow[d, bend left, "\times 9"] \\ 3 + 4x \geq 27 \arrow[d, bend right, "-3"'] \arrow[d, bend left, "-3"] \\ 4x \geq 24 \arrow[d, bend right, "\div 4"'] \arrow[d, bend left, "\div 4"] \\ x \geq 6 \end{tikzcd}$$

The inequality is solved in the same way as an equation is solved. Both sides are multiplied by 9 first to eliminate 9 from the denominator.

c

$$\begin{tikzcd} 15 - 2x > 1 \arrow[d, bend right, "-15"'] \arrow[d, bend left, "-15"] \\ -2x > -14 \arrow[d, bend right, "\div -2"'] \arrow[d, bend left, "\div -2"] \\ x < 7 \end{tikzcd}$$

15 is subtracted from each side.

Both sides are divided by –2. Because this is a negative number, the inequality is reversed from > to <.

Exercise 7I

1–4	4	—

UNDERSTANDING

1 If $x = 3$, classify the following inequalities as true or false.

a $x + 4 > 2$ **b** $5x \geq 10$ **c** $10 - x < 5$ **d** $5x + 1 < 16$

2 State whether the following choices of x make the inequality $2x + 4 \geq 10$ true or false.

a $x = 5$ **b** $x = 1$ **c** $x = -5$ **d** $x = 3$

3 **a** Copy and complete the following.

$$\begin{tikzcd} 2x < 8 \arrow[d, bend right, "\div 2"'] \arrow[d, bend left, "\div 2"] \\ x < \underline{\quad} \end{tikzcd}$$

b What is the solution to the inequality $2x < 8$?

UNDERSTANDING

4 **a** Copy and complete the following.

$$\begin{tikzcd}[row sep=large]
2x + 4 \geq 10 \arrow[d, "-4"', "-4"] \\
2x \geq \_\_ \arrow[d, "\div 2"', "\div 2"] \\
x \geq \_\_
\end{tikzcd}$$

b What is the solution to the inequality $2x + 4 \geq 10$?

c If $x = 7.1328$, is $2x + 4 \geq 10$ true or false?

5–6(½) 5–7(½) 5–7(½), 8

FLUENCY

Example 15a

5 Solve the following inequalities.

a $x + 9 > 12$ **b** $4l + 9 \geq 21$ **c** $8g - 3 > 37$ **d** $2r - 8 \leq 6$

e $9k + 3 > 21$ **f** $8s - 8 < 32$ **g** $8a - 9 > 23$ **h** $2 + n \geq 7$

i $9 + 2d \geq 23$ **j** $8 + 6h < 38$ **k** $10 + 7r \leq 24$ **l** $6 + 5y < 26$

Example 15b

6 Solve the following inequalities involving fractions.

a $\frac{d-9}{2} > 10$ **b** $\frac{y+4}{2} \leq 7$ **c** $\frac{x-3}{4} > 2$ **d** $\frac{q+4}{2} \leq 11$

e $\frac{2x+4}{3} > 6$ **f** $\frac{7+3h}{2} < 5$ **g** $\frac{4+6p}{4} \geq 4$ **h** $\frac{8j+2}{7} < 6$

Example 15c

7 Solve the following inequalities involving negative numbers. Remember to reverse the inequality when multiplying or dividing by a negative number.

a $6 - 2x < 4$ **b** $24 - 6s \geq 12$ **c** $43 - 4n > 23$

d $34 - 2j < 14$ **e** $2 - 9v \leq 20$ **f** $2 - 7j \leq 37$

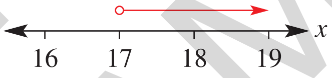
g $48 - 8c \geq 32$ **h** $42 - 8h \leq 42$ **i** $7 - 8s > 31$

j $6 - 8v > 22$ **k** $10 - 4v \geq 18$ **l** $4 - 5v < 29$

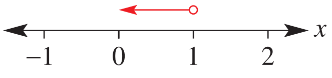
8 Match the following inequalities with their solutions depicted on a number line.

a $5x + 2 \geq 17$ **b** $\frac{x+1}{6} > 3$ **c** $9(x + 4) < 45$ **d** $5 - 2x < 3$

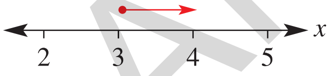
A



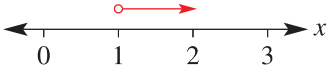
B



C



D



9, 10 9, 10 10, 11

PROBLEM-SOLVING

9 Kartik buys 4 cartons of milk and a \$20 phone card. The total cost of his shopping was greater than \$25.

a If c is the cost of a carton of milk, write an inequality to describe the situation above.

b Solve the inequality to find the possible values of c

c If the milk's cost is a multiple of 5 cents, what is the minimum price it could be?

PROBLEM-SOLVING

10 In AFL football the score is given by $6g + b$ where g is the number of goals and b is the number of behinds. A team scored 4 behinds and their score was less than or equal to 36.

a Write an inequality to describe this situation.

b Solve the inequality.

c Given that the number of goals must be a whole number, what is the maximum number of goals that they could have scored?

11 Recall that to convert degrees Celsius to Fahrenheit the rule is $F = 1.8C + 32$. Pippa informs you that the temperature is between 59° and 68° Fahrenheit inclusive.

a Solve $1.8C + 32 \geq 59$.

b Solve $1.8C + 32 \leq 68$.

c Hence state the solution to $59 \leq 1.8C + 32 \leq 68$, giving your answer as a single inequality.

d Pippa later realised that the temperatures she gave you should have been doubled – the range was actually 118° to 136° Fahrenheit. State the range of temperatures in Celsius, giving your answer as an inequality.

12a, b	12, 14	13–15

REASONING

12 To say that a number x is positive is to say that $x > 0$.

a If $10x - 40$ is positive, find all the possible values of x. That is, solve $10x - 40 > 0$.

b Find all k values that make $2k - 6$ positive.

c If $3a + 6$ is negative and $10 - 2a$ is positive, what are the possible values of a?

d If $5a + 10$ is negative and $10a + 30$ is positive, what are the possible values of a?

13 **a** Prove that if $5x - 2$ is positive then x is positive.

b Prove that if $2x + 6$ is positive then $x + 5$ is positive.

c Is it possible that $10 - x$ is positive and $10 - 2x$ is positive but $10 - 3x$ is negative? Explain.

d Is it possible that $10 - x$ is positive and $10 - 3x$ is positive but $10 - 2x$ is negative? Explain.

REASONING

14 A puzzle is given below with four clues.

Clue A: $3x > 12$
Clue B: $5 - x \leq 4$
Clue C: $4x + 2 \leq 42$
Clue D: $3x + 5 < 36$

a Two of the clues are unnecessary. State which two clues are not needed.

b Given that x is a whole number divisible by 4, what is the solution to the puzzle?

15 Multiplying or dividing by a negative number can be avoided by adding the variable to the other side of the equation. For example:

$$\begin{tikzcd}
-4x+2<6 \arrow[d, bend right, "+4x"'] \arrow[d, bend left, "+4x"] \\
2<6+4x \arrow[d, bend right, "-6"'] \arrow[d, bend left, "-6"] \\
-4<4x \arrow[d, bend right, "\div 4"'] \arrow[d, bend left, "\div 4"] \\
-1<x
\end{tikzcd}$$

This can be rearranged to $x > -1$, which is the same as the answer obtained using the method shown in the **Key ideas**. Use this method to solve the following inequalities.

a $-5x + 20 < 10$ **b** $12 - 2a \geq 16$ **c** $10 - 5b > 25$ **d** $12 < -3c$

Pronumerals on both sides

—	—	16

ENRICHMENT

16 This method for solving inequalities allows both sides to have any expression subtracted from them. This allows us to solve inequalities with pronumerals on both sides. For example:

$$\begin{tikzcd}
12x+5 \leq 10x+11 \arrow[d, bend right, "-10x"'] \arrow[d, bend left, "-10x"] \\
2x+5 \leq 11
\end{tikzcd}$$

which can then be solved as usual. If we end up with a pronumeral on the right-hand side, such as $5 < x$, the solution is rewritten as $x > 5$.

Solve the following inequalities.

a $12x + 5 \leq 10x + 11$ **b** $7a + 3 > 6a$ **c** $5 - 2b \geq 3b - 35$

d $7c - 5 < 10c - 11$ **e** $14k > 200 + 4k$ **f** $9g + 40 < g$

g $4(2a + 1) > 7a + 12$ **h** $2(3k - 5) \leq 5k - 1$ **i** $2(3p + 1) > 4(p + 2) + 3$

Investigation

Tiling a pool edge

The Sunny Swimming Pool Company constructs rectangular pools each 4 m wide with various lengths. There are non-slip square tiles, 50 cm by 50 cm, that can be used for the external edging around the pool perimeter where swimmers walk.

1 Draw a neat diagram illustrating the pool edge with one row of flat tiles bordering the perimeter of a rectangular pool 4 m wide and 5 m long.

2 Develop a table showing the dimensions of rectangular pools each of width 4 m and ranging in length from 5 m to 10 m. Add a column for the total number of tiles required for each pool when one row of flat tiles borders the outside edge of the pool.

3 Develop an algebraic rule for the total number of tiles, T, required for a bordering the perimeter of rectangular pools that are 4 m wide and x m long.

4 **a** Use your algebraic rule to form equations for each of the following total number of tiles when a single row of flat tiles is used for pool edging.

i 64 tiles **ii** 72 tiles **iii** 80 tiles **iv** 200 tiles

b By manually solving each equation, determine the lengths of the various pools that use each of the above numbers of tiles.

5 Develop an algebraic rule for the total number of tiles, T, required for two rows of flat tiles bordering rectangular pools that are 4 m wide and x m long.

6 **a** Use your algebraic rule to form equations for each of the following total numbers of tiles when two rows of flat tiles are used for pool edging.

i 96 tiles **ii** 120 tiles **iii** 136 tiles **iv** 248 tiles

b By manually solving each equation, determine the lengths of the pools that use these numbers of tiles.

7 Determine an algebraic rule for the total number of tiles, T, required for n rows of flat tiles bordering rectangular pools that are 4 m wide and x m in length.

8 Use this algebraic rule to form equations for each of the following pools, and then manually solve each equation to determine the length of each pool.

Pool	Width of pool 4 m	Length of pool x m	Number of layers n	Total number of tiles, T
A	4		3	228
B	4		4	288
C	4		5	500

Hiring a tuk-tuk taxi

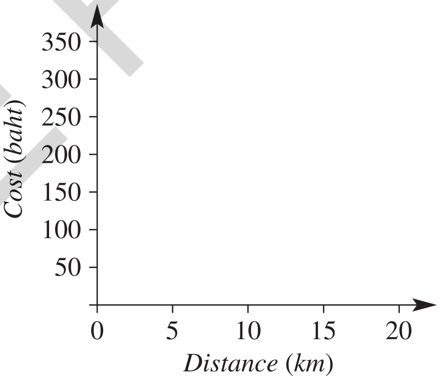
The cost of hiring a tuk-tuk taxi in Thailand is 50 baht hiring cost (flag fall) plus 15 baht every 2 km.

1 Using a table, show the costs of hiring a tuk-tuk for 2, 4, 6, 8, 10 and 20 km.

2 On graph paper, draw a graph of the hiring cost (vertical axis) versus the distance (horizontal axis) travelled. Use the grid at right as a guide.

3 On your graph rule lines to show the distance travelled for hiring costs of 125 baht and 170 baht.

4 Write an algebraic formula for the hiring cost C of an x km ride in a tuk-tuk taxi.

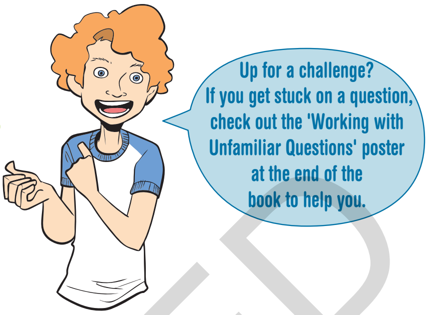


5 a Use your algebraic formula to form equations for each of the following trip costs:
   i 320 baht ii 800 baht iii 1070 baht

   b Use your equation-solving procedures to calculate the distance travelled for each trip.

6 A tourist was charged A\$100 for the 116 km trip from Pattaya Beach to the Bangkok International airport. If 1 baht can be bought for A\$0.03318, do you think that this was a fair price? Justify your answer with calculations.

Problems and challenges



1 Find the unknown value in the following problems.

- **a** A number is increased by 2, then doubled, then increased by 3 and then tripled. The result is 99.
- **b** A number is doubled and then one third of the number is subtracted. The result is 5 larger than the original number.
- **c** In five years' time Alf will be twice as old as he was two years ago. How old is Alf now?
- **d** The price of a shirt is increased by 10% for GST and then decreased by 10% on a sale. The new price is \$44. What was the original price?
- **e** One-third of a number is subtracted from 10 and then the result is tripled, giving the original number back again.
- **f** The sides of a quadrilateral are four consecutive integers. If the longest side is 26% of the perimeter, find the perimeter.

2 Consider the following 'proof' that $0 = 1$.

- **a** Which step caused the problem in this proof? (Hint: Consider the actual solution to the equation.)
- **b** Prove that $0 = 1$ is equivalent to the equation $22 = 50$ by adding, subtracting, multiplying and dividing both sides.

$$\begin{tikzcd}
2x + 5 = 3x + 5 \arrow[d, "-5"', "-5"] \\
2x = 3x \arrow[d, "\div x"', "\div x"] \\
2 = 3 \arrow[d, "-2"', "-2"] \\
0 = 1
\end{tikzcd}$$

3 Find all the values of x that would make both these inequalities false.
$19 - 2x < 5$ and $20 + x > 4x + 2$

4 The following six expressions could appear on either side of an equation. Using just two of the expressions, create an equation that has no solution.
$2x \quad 3x + 1 \quad 7x + 4 \quad 4(x + 7) \quad 2 + 3(x + 1) \quad 2(3 + x) - 1$

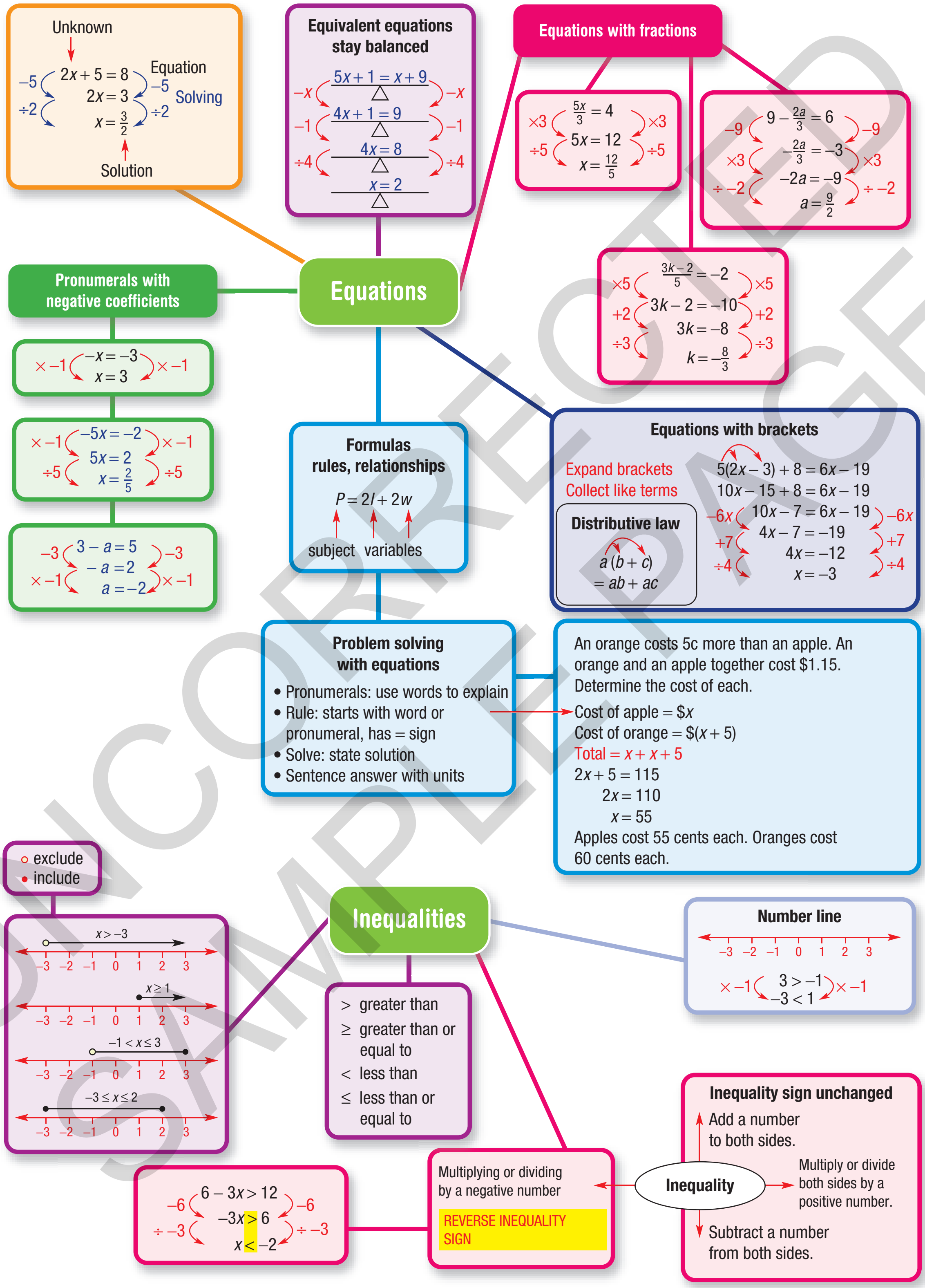
5 A certain pair of scales only registers weights between 100 kg and 150 kg, but it allows more than one person to get on at a time.

- **a** If three people weigh themselves in pairs and the first pair weighs 117 kg, the second pair weighs 120 kg and the third pair weighs 127 kg, what are their individual weights?
- **b** If another three people weigh themselves in pairs and get weights of 108 kg, 118 kg and 130 kg, what are their individual weights?
- **c** A group of four children who all weigh less than 50 kg, weigh themselves in groups of three, getting the weights 122 kg, 128 kg, 125 kg and 135 kg. How much do they each weigh?

6 Each link in a chain has an outer length of 44 mm and is made of wire 4 mm thick.

- **a** Find the greatest length (in mm) of a chain with 5 links.
- **b** Determine an expression for the greatest length L (in mm) of a chain with n links.
- **c** Find the smallest number of links required to make a chain with length greater than 7 metres.

Chapter summary



Equations

Unknown

$-5 \quad 2x+5=8 \quad$ Equation

$\div 2 \quad 2x=3 \quad -5$ Solving

$x=\frac{3}{2} \quad \div 2$

Solution

Equivalent equations stay balanced

$-x \quad 5x+1=x+9 \quad -x$

$-1 \quad 4x+1=9 \quad -1$

$\div 4 \quad 4x=8 \quad \div 4$

$x=2$

Equations with fractions

$\times 3 \quad \frac{5x}{3}=4 \quad \times 3$

$\div 5 \quad 5x=12 \quad \div 5$

$x=\frac{12}{5}$

$-9 \quad 9-\frac{2a}{3}=6 \quad -9$

$\times 3 \quad -\frac{2a}{3}=-3 \quad \times 3$

$\div -2 \quad -2a=-9 \quad \div -2$

$a=\frac{9}{2}$

$\times 5 \quad \frac{3k-2}{5}=-2 \quad \times 5$

$+2 \quad 3k-2=-10 \quad +2$

$\div 3 \quad 3k=-8 \quad \div 3$

$k=-\frac{8}{3}$

Pronumerals with negative coefficients

$\times -1 \quad -x=-3 \quad \times -1$

$x=3$

$\times -1 \quad -5x=-2 \quad \times -1$

$\div 5 \quad 5x=2 \quad \div 5$

$x=\frac{2}{5}$

$-3 \quad 3-a=5 \quad -3$

$\times -1 \quad -a=2 \quad \times -1$

$a=-2$

Formulas rules, relationships

$P=2l+2w$

subject variables

Equations with brackets

Expand brackets $\quad 5(2x-3)+8=6x-19$

Collect like terms $\quad 10x-15+8=6x-19$

$-6x \quad 10x-7=6x-19 \quad -6x$

$+7 \quad 4x-7=-19 \quad +7$

$\div 4 \quad 4x=-12 \quad \div 4$

$x=-3$

Distributive law

$a(b+c)$

$=ab+ac$

Problem solving with equations

- Pronumerals: use words to explain
- Rule: starts with word or pronumeral, has = sign
- Solve: state solution
- Sentence answer with units

An orange costs 5c more than an apple. An orange and an apple together cost $1.15. Determine the cost of each.

Cost of apple = $\$x$

Cost of orange = $\$(x+5)$

Total $= x+x+5$

$2x+5=115$

$2x=110$

$x=55$

Apples cost 55 cents each. Oranges cost 60 cents each.

Inequalities

○ exclude

● include

$x>-3$

$x\geq 1$

$-1<x\leq 3$

$-3\leq x\leq 2$

−3 −2 −1 0 1 2 3

$>$ greater than

$\geq$ greater than or equal to

$<$ less than

$\leq$ less than or equal to

Number line

−3 −2 −1 0 1 2 3

$\times -1 \quad 3>-1 \quad \times -1$

$-3<1$

$-6 \quad 6-3x>12 \quad -6$

$\div -3 \quad -3x>6 \quad \div -3$

$x<-2$

Multiplying or dividing by a negative number

REVERSE INEQUALITY SIGN

Inequality sign unchanged

Add a number to both sides.

Inequality

Multiply or divide both sides by a positive number.

Subtract a number from both sides.

Multiple-choice questions

7A **1** Which one of the following equations is true?

A $20 \div (2 \times 5) = 20 \div 10$ **B** $12 - 8 = 2$ **C** $5 \div 5 \times 5 = \frac{1}{5}$

D $15 + 5 \times 4 = 20 + 4$ **E** $10 - 2 \times 4 = 4 \times 2 - 5$

7A **2** Which one of the following equations does not have the solution $x = 9$?

A $4x = 36$ **B** $x + 7 = 16$ **C** $\frac{x}{3} = 3$

D $x + 9 = 0$ **E** $14 - x = 5$

7B **3** The solution to the equation $3a + 8 = 29$ is:

A $a = 21$ **B** $a = 12\frac{1}{3}$ **C** $a = 7$ **D** $a = 18$ **E** $a = 3$

7G **4** ‘Three less than half a number is the same as four more than the number’ can be expressed as an equation by:

A $\frac{x}{2} - 3 = 4x$ **B** $\frac{(x-3)}{2} = x + 4$ **C** $\frac{x}{2} - 3 = x + 4$

D $\frac{x}{2} + 3 = x + 4$ **E** $\frac{x}{2} - 3 + 4$

7E **5** The solution to the equation $-3(m + 4) = 9$ is:

A $m = 7$ **B** $m = -7$ **C** $m = -1$ **D** $m = 1$ **E** $m = -3$

7D **6** If $12 + 2x = 4x - 6$ then x equals:

A 8 **B** 9 **C** 12 **D** 15 **E** 23

7H **7** If ☐ < -4, then ☐ could have the value:

Ext

A 0 **B** $-\frac{1}{4}$ **C** -3 **D** -5 **E** 3

7A **8** Which one of the following equations has the solution $n = 10$?

A $4 - n = 6$ **B** $2n + 4 = 3n + 5$ **C** $50 - 4n = 90$

D $2(n + 5) = 3(n + 1)$ **E** $70 - 6n = n$

7B **9** Malcolm solves an equation as follows:

$$\begin{aligned} 5 - 2x + 4 &= 11 && \text{line 1} \\ 1 - 2x &= 11 && \text{line 2} \\ -2x &= 10 && \text{line 3} \\ x &= -5 && \text{line 4} \end{aligned}$$

Choose the correct statement.

A The only mistake was made in moving from line 1 to line 2.

B The only mistake was made in moving from line 2 to line 3.

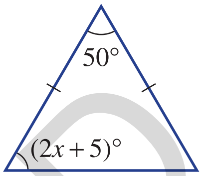
C The only mistake was made in moving from line 3 to line 4.

D A mistake was made in moving from line 1 to line 2 and from line 3 to line 4.

E No mistakes were made.

7G **10** The value of x in this isosceles triangle is:

A 30 **B** 45 **C** 62.5
D 65 **E** 130



Short-answer questions

7A **1** Classify each of the following as true or false.

- **a** If $3x = 6$ then $x = 3$.
- **b** If $a = 21$ then $a + a - a = a$.
- **c** $5 \times 4 = 10 + 10$

7B **2** Find the solutions to these equations.

- **a** $4m = 16$
- **b** $\frac{m}{3} = -4$
- **c** $9 - a = 10$
- **d** $10m = 2$
- **e** $2m + 6 = 36$
- **f** $a + a = 12$

7A **3** Write an equation to represent each of the following statements. You do not need to solve the equations.

- **a** Double m plus 3 equals three times m
- **b** The sum of n and four is multiplied by five; the answer is 20.
- **c** The sum of two consecutive even numbers, the first being x, is 74.

7B **4** For each equation below, state the first operation you would apply to both sides.

- **a** $15 + 2x = 45$
- **b** $\frac{x}{2} - 5 = 6$
- **c** $3a + 3 = 2a + 10$

7B **5** Solve the following equations.

- **a** $a + 8 = 12$
- **b** $6 - y = 15$
- **c** $2x - 1 = -9$
- **d** $5 + 3x = 17$
- **e** $20 - 4x = 12$
- **f** $8a - 8 = 0$

7C **6** Solve the following equations algebraically.

- **a** $\frac{m}{3} = -2$
- **b** $\frac{5x}{2} = 20$
- **c** $\frac{-2y}{3} = 12$
- **d** $\frac{k+3}{11} = -5$
- **e** $\frac{8-2w}{3} = 4$
- **f** $\frac{2a-8}{6} = 13$

7C **7** Solve these equations including fractions.

- **a** $2 = \frac{x+2}{8}$
- **b** $\frac{x}{6} + 2 = 3$
- **c** $\frac{2x-10}{7} = 10$

7D **8** Solve these equations.

- **a** $8a - 3 = 7a + 5$
- **b** $2 - 3m = m$
- **c** $3x + 5 = 7x - 11$
- **d** $10 - 4a = a$
- **e** $2x + 8 = x$
- **f** $4x + 9 = 5x$

7E

9 Solve the following equations by first expanding the brackets.

a $2(x + 5) = 16$
b $3(x + 1) = -9$
c $18 = -2(2x - 1)$
d $3(2a + 1) = 27$
e $5(a + 4) = 3(a + 2)$
f $2(3m + 5) = 16 + 3(m + 2)$
g $8(3 - a) + 16 = 64$
h $2x + 10 = 4(x - 6)$
i $4(2x - 1) - 3(x - 2) = 10x - 3$

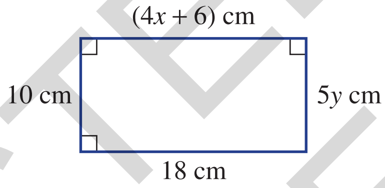
7F

10 **a** If $P = 2(I + b)$, find I when $P = 48$ and $b = 3$.

b If $M = \frac{f}{f - d}$, find M when $f = 12$ and $d = 8$.

c If $F = 2.5c + 20$, find c when $F = 30$.

d Find the value of x and y for this rectangle.



7G

11 **a** The sum of three consecutive numbers is 39. First write an equation and then find the value of the smallest number.

b Four times a number less 5 is the same as double the number plus 3. Write an equation to find the number.

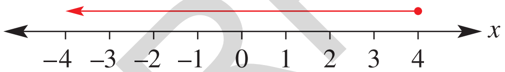
c The difference between a number and three times that number is 17. What is the number?

7H

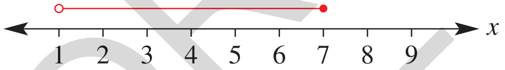
Ext

12 Write the inequality represented by each number line below.

a



b



7H

Ext

13 Represent the following inequalities on separate number lines.

a $x \leq -1$
b $x > 2$
c $x < -1$
d $x \geq -2$
e $1 > x$
f $-3 \leq x$
g $x < 7$
h $-2 < x \leq 4$
i $-1 \leq x \leq 1$

7H

Ext

14 Write an inequality to represent these situations.

a The profit of a company is at least \$100 000.

b The cost of a new car cannot exceed \$6700.

c To ride on the roller-coaster, a person's height must be between 1.54 m and 1.9 m inclusive.

7I

Ext

15 Solve the following inequalities.

a $x + 3 > 5$
b $x - 2 < 6$
c $x - 2 < -6$
d $6x \geq 12$
e $4x < -8$
f $\frac{x}{4} \geq 2$
g $\frac{2x}{3} < -8$
h $\frac{2x + 1}{3} > 9$
i $\frac{6 - 5x}{4} \leq -1$

Extended-response questions

1 To upload an advertisement to the www.searches.com.au website costs \$20 and then 12 cents whenever someone clicks on it.

a Write a formula relating the total cost (\$$S$) and the number of clicks (n) on the advertisement.

b If the total cost is \$23.60, write and solve an equation to find out how many times the advertisement has been clicked on.

c To upload to the www.yousearch.com.au website costs \$15 initially and then 20 cents for every click. Write a formula for the total cost \$$Y$ when the advertisement has been clicked n times.

d If a person has at most \$20 to spend, what is the maximum number of clicks they can afford on their advertisement at yousearch.com.au?

e Set up and solve an equation to find the minimum number of clicks for which the total cost of posting an advertisement to searches.com.au is less than the cost of posting to yousearch.com.au.

2 Mahni plans to spend the next 12 weeks saving some of her income. She will save \$$x$ a week for the first 6 weeks and then \$$(2x - 30)$ a week for the following 6 weeks.

a Write an expression for the total amount saved over the 12 weeks.

b If she managed to save \$213 in the first six weeks, how much did she save:

i in the first week? **ii** in the 7th week?

iii in total over the 12 weeks?

c If Mahni wants to save a total of \$270, write and solve an equation to find out how much she would have to save in the first week.

d If Mahni wants to save the same amount in the first 6 weeks as in the last 6 weeks, how much would she save each week?

e In the end Mahni decides that she does not mind exactly how much she saves but wants it to be between \$360 and \$450. State the range of x values that would achieve this goal, giving your answer as an inequality.