Chapter 6 Ratios and rates

What you will learn

Australian curriculum

NUMBER AND ALGEBRA

Real numbers

Solve a range of problems involving rates and ratios, with and without digital technologies (ACMNA188) AC

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Online resources

- Chapter pre-test
- Videos of all worked examples
- Interactive widgets
- Interactive walkthroughs
- Downloadable HOTsheets
- Access to HOTmaths Australian Curriculum courses

Bicycle gear ratios

The bicycle is renowned as the most energy-efficient means of human transport. A bicycle with a range of gears is an extremely versatile and enjoyable machine. The 'gear ratio' of a bicycle is the ratio of pedal rotations compared to the rate at which the wheels rotate. Manufacturers apply the mathematics of gear ratios to provide cyclists with the range of gears that is most useful for the style of bike.

Have you ever tried to ride up a steep hill in too high a gear or reached the bottom of a hill while still in a very low gear? A bicycle rider has a comfortable range of pressures and rotation rates that they can apply to the pedals. Gears make it easier to stay in this comfortable range over a wide range of riding conditions such as BMX downhill racing, a fast time trial for racing bikes on flat ground or climbing steep, rough trails on a mountain bike.

Unicycles have only one wheel and so have a ratio of 1:1 for pedal turns to wheel turns. Unicycles can't reach high speeds because it is very difficult to rotate pedals more than about 100 rpm; however, riding a unicycle is fun and it requires strength and agility to remain stable, especially when playing unicycle hockey!

6A Introducing ratios

Interactive

Widgets

HOTsheets

Walkthroughs

Ratios are regularly used in everyday life. They are used to show the relationship between two (or more) related quantities.

Here are five common uses of ratios:

- Ingredients – the ratio of different ingredients in a recipe (cooking, industrial)
- Maps – most maps include a scale which is written as a ratio
- Sporting success – showing a team's win to loss ratio or the ratio of kicking goals to points
- Comparing size – the ratio of length, area or volume of different shapes
- Legal requirements – minimum standards of supervision, staff to student ratios

The ratio of eggs to butter to sugar will affect the flavour and texture of a cake.

When dealing with ratios, the order in which the ratio is written is very important. For example, a team's win : loss ratio of 5 : 2 is very different to a team's win : loss ratio of 2 : 5.

Ratios compare quantities of the same type and given in the same unit. Therefore a ratio is not generally written with a unit of measurement.

Let's start: Estimating ratios

Estimate the following ratios.

- Ratio of boys to girls in your school
- Ratio of Year 8 students to Year 7 students in your school
- Ratio of your teacher's age to your age!
- Ratio of hours you spend outside to hours you spend inside
- Ratio of hours you are awake to hours you are asleep
- Ratio of parents to children in an average Australian family
- Ratio of the length to the width of an A4 sheet of paper

Discuss your answers as a class.

Key ideas

- Ratios show the relationship between two (or more) related quantities.
  For example, a drink was made with the ratio of cordial to water of 1 : 3.
- The **colon :** is the mathematical symbol used to represent ratios.
  The written ratio of **a : b** is read as the ratio of '**a to b**' or '**a is to b**'.

Key ideas

- The order in which the quantities are written in a ratio is important.
- Before ratios can be written and simplified, the quantities must be expressed in the same unit.
- If each value in a ratio is multiplied or divided by the same number an **equivalent ratio** is formed.
  For example, 1 : 3 and 2 : 6 and 10 : 30 are equivalent ratios.

Example 1 Writing ratios

A sample of mixed nuts contains 5 cashews, 12 peanuts and 2 macadamia nuts.
Write down:

a the ratio of cashews to peanuts to macadamias

b the ratio of cashews to the total number of nuts

c the ratio of peanuts to other nuts

SOLUTION	EXPLANATION
a 5 : 12 : 2	cashews : peanuts : macadamias
b 5 : 19	5 cashews, total nuts 19
c 12 : 7	12 peanuts, other nuts 7

Example 2 Producing equivalent ratios

Complete each pair of equivalent ratios.

a 4 : 9 = 16 : ☐ **b** 30 : 15 = ☐ : 5

SOLUTION	EXPLANATION
a $\times 4$ (4 : 9 → 16 : 36) $\times 4$	Both numbers are multiplied by 4.
b $\div 3$ (30 : 15 → 10 : 5) $\div 3$	Both numbers are divided by 3.

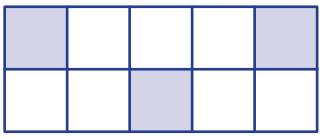
Exercise 6A

1–3 | 3 | —

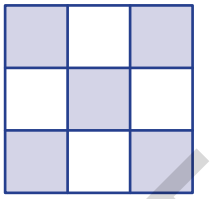
UNDERSTANDING

1 Write down the ratio of shaded parts to unshaded parts for each grid.

a



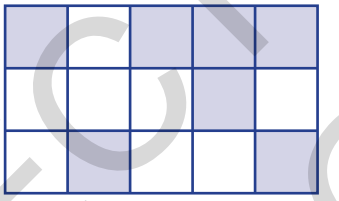
b



2 Write down the ratio of shaded parts to total parts for each grid.

a

b



3 Express each of the following pairs of quantities as a ratio.

a 9 goals to 4 goals

b 52 litres to 17 litres

c 7 potatoes to 12 carrots

d 3 blue marbles to 5 green marbles

4–6, 7–8(½) | 4–6, 7(½), 8–9 | 4–9(½)

FLUENCY

Example 1

4 A bag contains 5 green marbles, 7 blue marbles and 3 yellow marbles Write down the ratio of:

a green marbles to yellow marbles

b blue marbles to total marbles

c yellow to blue to green marbles

d green to yellow to blue marbles

5 Over the past fortnight, it has rained on eight days and it has snowed on three days. Write down the ratio of:

a rainy days to snowy days

b snowy days to total days

c fine days to rainy and snowy days

d rainy days to non-rainy days

6 In a box of 40 flavoured icy poles there were 13 green, 9 lemonade, 11 raspberry and 7 orange icy poles. Write down the ratio of:

a green icy poles to orange icy poles

b raspberry icy poles to lemonade icy poles

c the four different flavours of icy poles

d green and orange icy poles to raspberry and lemonade icy poles

6A

FLUENCY

Example 2

7 Complete each pair of equivalent ratios.

a 1 : 3 = 4 : ☐ **b** 1 : 7 = 2 : ☐ **c** 2 : 5 = ☐ : 10

d 3 : 7 = ☐ : 21 **e** 5 : 10 = 1 : ☐ **f** 12 : 16 = 3 : ☐

g 12 : 18 = ☐ : 3 **h** 20 : 50 = ☐ : 25 **i** 4 : 7 = 44 : ☐

j 14 : 17 = 12 : ☐ **k** 27 : 6 = ☐ : 2 **l** 121 : 66 = ☐ : 6

8 Complete each pair of equivalent ratios.

a 2 : 3 : 5 = 4 : ☐ : ☐ **b** 4 : 12 : 16 = ☐ : 6 : ☐

c 1 : 7 : 9 = ☐ : ☐ : 63 **d** 22 : 110 : 66 = 11 : ☐ : ☐

9 Write three equivalent ratios for each of the following ratios.

a 1 : 2 **b** 2 : 5 **c** 8 : 6 **d** 9 : 3

PROBLEM-SOLVING

10, 11 | 11–13 | 12–14

10 Sort the following ratios into three pairs of equivalent ratios.

2 : 5, 6 : 12, 7 : 4, 1 : 2, 4 : 10, 70 : 40

11 During a recent dry spell, it rained on only 3 days during the month of September.

a What was the ratio of wet days to total days for the month of September?

b Write equivalent ratios for a total of 10 days and 100 days.

12 On their way to work, Andrew passes 15 sets of traffic lights and Pauline passes 10 sets of traffic lights. One morning Andrew was stopped by 12 red traffic lights. How many green traffic lights would Pauline need to pass through to have the equivalent ratio of red to green traffic lights as Andrew?

13 Write the ratio of vowels to consonants for each of the following place names.

a Queensland **b** Canberra **c** Wagga Wagga **d** Australia

14 Name any Australian states that have a vowel to consonant ratio of 1 : 1.

REASONING

15 | 15, 16 | 16–18

15 Can 10 people be divided into two groups with a ratio of 1 : 2? Explain.

16 Bertrand wins one-third of his games of tennis this season. Write down his win to loss ratio.

17 **a** What is the ratio of black keys to white keys in one octave of the piano?

b What is the ratio of black keys to white keys for the entire 88 keys of the piano?

c Are these equivalent ratios?

18 Write the missing expression.

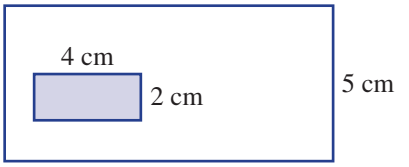
a 2 : x = 6 : ☐ **b** 2 : x = 5 : ☐ **c** y : 2x = ☐ : 8x **d** 12xy : 6y = ☐ : 1

6A Area ratios — — 19–21

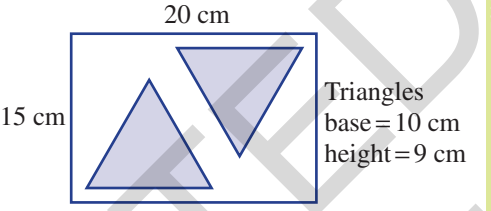
ENRICHMENT

19 Using the dimensions provided, find the ratio of the shaded area to the unshaded area for each of the following diagrams.

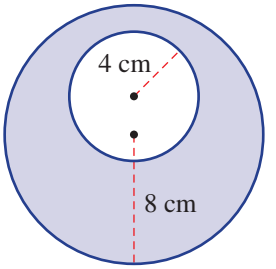
a



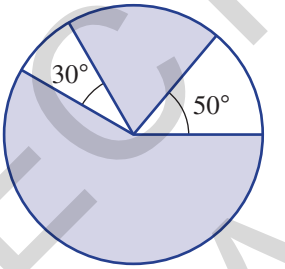
b



c



d



20 Estimate the ratio of the shaded floor to the unshaded floor in this photo.

21 Design your own diagram for which the ratio of shaded area to unshaded area is:

a 1 : 3 **b** 1 : 7

6B Simplifying ratios

Interactive

Widgets

HOTsheets

Walkthroughs

In a similar way to fractions, ratios are simplified by dividing each term by a common factor. A ratio is said to be in its simplest form when it contains whole numbers only and the highest common factor (HCF) between the terms in the ratio is 1.

If a ratio contains fractions or decimals, it can be simplified by multiplying rather than dividing or cancelling.

Let's start: Class ratios

Look around your classroom and write down the following ratios:

- **a** Ratio of girls to boys
- **b** Ratio of teachers to students
- **c** Ratio of wearing a watch to not wearing a watch
- **d** Ratio of white socks to black socks
- **e** Ratio of textbooks open to textbooks closed
- **f** Ratio of not having a pencil case to having a pencil case
- **g** Ratio of blonde hair to brown hair to black hair
- **h** Ratio of blue eyes to brown eyes to other colour eyes

Design your own ratio question for your class or classroom.

Can any of your ratio answers be simplified?

Key ideas

- **Simplifying ratios**
  A ratio is simplified by dividing both numbers in the ratio by their highest common factor (HCF).
  For example, the ratio 15 : 25 can be simplified to 3 : 5.

$$15 : 25 \xrightarrow{\div 5} 3 : 5 \quad \text{Simplest form}$$

- It is convention to express ratios in their simplest form.
- Ratios in simplest form use whole numbers only.
- If a ratio is expressed with fractions, it is simplified by converting the quantities to whole numbers. This is generally done by multiplying by the lowest common denominator (LCD).
- Before ratios are simplified the quantities must be expressed in the same unit.

Example 3 Simplifying ratios

Simplify the following ratios.

a 7 : 21 **b** 450 : 200

SOLUTION

a $7 : 21 \xrightarrow{\div 7} 1 : 3$

b $450 : 200 \xrightarrow{\div 50} 9 : 4$

EXPLANATION

HCF of 7 and 21 is 7.
Divide both numbers by 7.

HCF of 450 and 200 is 50.
Divide both numbers by 50.

Example 4 Simplifying ratios involving fractions

Simplify the following ratios.

a $\frac{3}{5} : \frac{1}{2}$ **b** $2\frac{1}{3} : 1\frac{1}{4}$

SOLUTION

a $\frac{3}{5} : \frac{1}{2} \xrightarrow{\times 10} 6 : 5$

b $\frac{7}{3} : \frac{5}{4} \xrightarrow{\times 12} 28 : 15$

EXPLANATION

LCD of 5 and 2 is 10, so multiply by 10.
Alternatively, $\frac{3}{5} : \frac{1}{2} = \frac{6}{10} : \frac{5}{10} = 6 : 5$

Convert mixed numerals to improper fractions.
LCD of 3 and 4 is 12.
Multiply both numbers by 12.

Example 5 Changing quantities to the same units

First change the quantities to the same unit, and then express each pair of quantities as a ratio in simplest form.

a 4 mm to 2 cm **b** 25 minutes to 2 hours

SOLUTION

a 4 mm to 2 cm = 4 mm to 20 mm
= 4 : 20
= 1 : 5

b 25 minutes to 2 hours
= 25 minutes to 120 minutes
= 25 : 120
= 5 : 24

EXPLANATION

2 cm = 20 mm
Once in same unit, write as a ratio.
Simplify ratio by dividing by the HCF of 4.

2 hours = 120 minutes
Once in same unit, write as a ratio.
Simplify ratio by dividing by the HCF of 5.

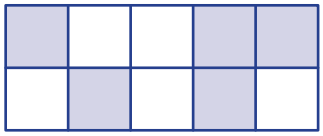
Exercise 6B

1–4 | 3, 4 | —

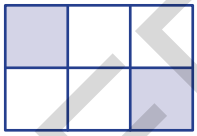
UNDERSTANDING

1 Write down the ratio of shaded parts to unshaded parts for each of the following in simplest form.

a



b



2 To express the ratio 4 : 16 in simplest form, you would:

A multiply both quantities by 2 **B** subtract 4 from both quantities
C divide both quantities by 2 **D** divide both quantities by 4

3 Decide which one of the following ratios is not written in simplest form.

A 1 : 5 **B** 3 : 9 **C** 2 : 5 **D** 11 : 17

4 Decide which one of the following ratios is written in simplest form.

A 2 : 28 **B** 15 : 75 **C** 14 : 45 **D** 13 : 39

5–7(½), 9(½) | 5–9(½) | 5–9(½)

FLUENCY

Example 3

5 Simplify the following ratios.

a 2 : 8 **b** 10 : 50 **c** 4 : 24 **d** 6 : 18
e 8 : 10 **f** 25 : 40 **g** 21 : 28 **h** 24 : 80
i 18 : 14 **j** 26 : 13 **k** 45 : 35 **l** 81 : 27
m 51 : 17 **n** 20 : 180 **o** 300 : 550 **p** 150 : 75
q 1200 : 100 **r** 70 : 420 **s** 200 : 125 **t** 90 : 75

6 Simplify the following ratios.

a 2 : 4 : 6 **b** 12 : 21 : 33 **c** 42 : 60 : 12 **d** 85 : 35 : 15
e 12 : 24 : 36 **f** 100 : 300 : 250 **g** 270 : 420 : 60 **h** 24 : 48 : 84

Example 4a

7 Simplify the following ratios.

a $\frac{1}{3} : \frac{1}{2}$ **b** $\frac{1}{4} : \frac{1}{5}$ **c** $\frac{2}{5} : \frac{3}{4}$ **d** $\frac{2}{7} : \frac{1}{5}$
e $\frac{3}{8} : \frac{1}{4}$ **f** $\frac{7}{10} : \frac{4}{5}$ **g** $\frac{11}{10} : \frac{2}{15}$ **h** $\frac{9}{8} : \frac{7}{12}$

Example 4b

8 Simplify the following ratios.

a $1\frac{1}{2} : \frac{3}{4}$ **b** $2\frac{1}{5} : \frac{3}{5}$ **c** $3\frac{1}{3} : 1\frac{2}{5}$ **d** $4\frac{1}{3} : 3\frac{3}{4}$

Example 5

9 First change the quantities to the same unit, and then express each pair of quantities as a ratio in simplest form.

a 12 mm to 3 cm **b** 7 cm to 5 mm **c** 120 m to 1 km
d 60 mm to 2.1 m **e** 3 kg to 450 g **f** 200 g to 2.5 kg
g 2 tonnes to 440 kg **h** 1.25 L to 250 mL **i** 400 mL to 1 L
j 20 minutes to 2 hours **k** 3 hours to 15 minutes **l** 3 days to 8 hours
m 180 minutes to 2 days **n** 8 months to 3 years **o** 4 days to 4 weeks
p 8 weeks to 12 days **q** 50 cents to $4 **r** $7.50 to 25 cents

6B

10, 11 | 11–13 | 11–13

PROBLEM-SOLVING

10 Which of these ratios is the simplest form of the ratio $\frac{1}{2} : 2$?

A $2 : \frac{1}{2}$ **B** $1 : 4$ **C** $\frac{1}{4} : 1$ **D** $1 : 1$

11 Kwok was absent from school on 8 days in Term 1. There were 44 school days in Term 1. Write the following ratios in simplest form.

a Ratio of days absent to total days

b Ratio of days present to total days

c Ratio of days absent to days present

12 Over the past four weeks, the Paske family had eaten takeaway food for dinner on eight occasions. They had hamburgers twice, fish and chips three times and pizza three times. Every other night they had home-cooked dinners. Write the following ratios in simplest form.

a Ratio of hamburgers to fish and chips to pizza

b Ratio of fish and chips to pizza

c Ratio of takeaway dinners to home-cooked dinners

d Ratio of home-cooked dinners to total dinners

13 When Lisa makes fruit salad for her family, she uses 5 bananas, 5 apples, 2 passionfruit, 4 oranges, 3 pears, 1 lemon (for juice) and 20 strawberries.

a Write the ratio of the fruits in Lisa's fruit salad.

b Lisa wanted to make four times the amount of fruit salad to take to a party. Write an equivalent ratio that shows how many of each fruit Lisa would need.

c Write these ratios in simplest form:

i bananas to strawberries **ii** strawberries to other fruits

14 | 14, 15 | 15–17

REASONING

14 Andrew incorrectly simplified 12 cm to 3 mm as a ratio of 4 : 1. What was Andrew's mistake and what is the correct simplified ratio?

15 Mariah has $4 and Rogan has 50 cents. To write a ratio in simplest form, values must be written in the same unit.

a First convert both units to dollars, and then express the ratio of Mariah's money to Rogan's money in simplest form.

b As an alternative, convert both units to cents and express the ratio in simplest form. Do you arrive at the same simplified ratio?

16 a Write two quantities of time, in different units, which have a ratio of 2 : 5.

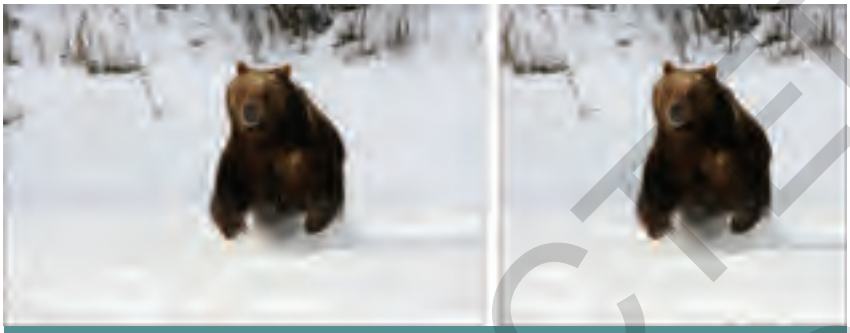
b Write two quantities of distance, in different units, which have a ratio of 4 : 3.

17 Simplify the following ratios.

a $2a : 4b$ **b** $15x : 3y$ **c** $a : a^2$

d $5f : 24f$ **e** $hk : 3k$ **f** $24xyz : 60yz$

18 Aspect ratio is the relationship between the width and height of the image as displayed on a screen.



A high-definition television screen (left) compared to the same image on an analogue television screen (right).

- **a** An old analogue television has an aspect ratio of $1.\dot{3} : 1$. Write this ratio in simplest form.
- **b** A high definition digital television has an aspect ratio of $1.\dot{7} : 1$. Write this ratio in simplest form.
- **c** Although these two ratios are clearly not equivalent, there is a connection between the two. What is it?
- **d** The digital television aspect ratio of $1.\dot{7} : 1$ was chosen as the best compromise to show widescreen movies on television. Many major films are presented in Panavision, which has an aspect ratio of $2.35 : 1$. Write this ratio in simplest form.
- **e** Investigate the history of aspect ratio in films and television.
- **f** Research how formats of unequal ratios are converted through the techniques of cropping, zooming, letterboxing, pillarboxing or distorting.
- **g** Investigate aspect ratios of other common media.
  - **i** Find the aspect ratio for several different newspapers.
  - **ii** Find the aspect ratio for several common sizes of photographs.
  - **iii** Find the aspect r

6C Dividing a quantity in a given ratio EXTENDING

Ratios are closely connected to fractions. To help solve problems involving ratios, we can think of each quantity in terms of a 'number of parts'.

A drink made in the ratio of cordial to water of 1 : 4 means that there is '1 part' of cordial and '4 parts' of water. There is a total of '5 parts' in the drink. In terms of ratios, it does not matter whether the drink is a 250 mL glass, a 2 L bottle or a 50 L urn. Changing the size of the drink will not change the ratio of 'parts'; it will simply change the size of each part.

The fraction of cordial in the drink is $\frac{\text{number of cordial parts}}{\text{total number of parts}} = \frac{1}{5}$.

The fraction of water in the drink is $\frac{\text{number of water parts}}{\text{total number of parts}} = \frac{4}{5}$.

Thinking about ratios in terms of 'parts' helps us to divide quantities into particular ratios.

Let's start: Sharing $120

For each situation below you have $120 to share out in the given ratios.

With a partner, work out how much each person receives.

Situation 1: Two people, Jack and Pippa, ratio of 1 : 1

Situation 2: Two people, Andrew and Alex, ratio of 2 : 2

Situation 3: Two people, Jess and Teresa, ratio of 2 : 3

Situation 4: Two people, Kyle and Kara, ratio of 3 : 5

Situation 5: Three people, Matt, Leos and Djarrin, ratio of 1 : 1 : 1

Situation 6: Three people, Christine, Prue and Carolyn, ratio of 3 : 5 : 7

Discuss the strategy that you used to share the $120. Does your strategy work for every situation?

Key ideas

- Think of a ratio in terms of '**parts**'.
  A ratio of 2 : 3 has 2 parts of one quantity for every 3 parts of another quantity and a total of 5 parts.
- Using the **unitary method** to divide a quantity in a given ratio:
  - Find the total number of parts in the ratio.
  - Find the value of one part.
  - Find the value of the number of parts required in the ratio.

  For example: Share $20 in ratio of 2 : 3.
  Think of sharing $20 into 2 parts and 3 parts.
  Total number of parts = 2 + 3 = 5.
  Value of one part = $20 ÷ 5 = $4.
  Therefore 2 parts = $8, and 3 parts = $12.

$$\begin{matrix} \div 5 \\ \times 2 \end{matrix} \begin{matrix} \$20 = 5 \text{ parts} \\ \$4 = 1 \text{ part} \\ \$8 = 2 \text{ parts} \end{matrix} \begin{matrix} \div 5 \\ \times 2 \end{matrix}$$

Key ideas

- Using fractions to divide a quantity in a given ratio:
  - Fraction of the amount required = $\frac{\text{number in ratio}}{\text{total number of parts}}$
  - Calculate the fraction of the amount for each share of the ratio.

  For example, share \$20 in ratio of 2 : 3.

  Fractions of the amount required are $\frac{2}{5}$ and $\frac{3}{5}$.

  Therefore $\frac{2}{5}$ of \$20 = \$8 and $\frac{3}{5}$ of \$20 = \$12.
- To find a total quantity from a given ratio:
  - Use the concept of 'parts' and the unitary method to find the value of one part and therefore the value of the total parts can be calculated.

  Or
  - Use equivalent ratios to find the value of each quantity in the ratio and then add the numbers together to find the total.

Example 6 Dividing a quantity in a particular ratio

Divide 54 m in a ratio of 4 : 5.

SOLUTION

Unitary method

Total number of parts = 9

÷9 (9 parts = 54 m) ÷9
×4 (1 part = 6 m) ×4
(4 parts = 24 m)

×5 (1 part = 6 m) ×5
(5 parts = 30 m)

54 m divided in a ratio of 4 : 5 is 24 m and 30 m.

Fractions method

$\frac{4}{9}$ of $54 = \frac{4}{9} \times \frac{54}{1} = 24$

$\frac{5}{9}$ of $54 = \frac{5}{9} \times \frac{54}{1} = 30$

54 m divided in a ratio of 4 : 5 is 24 m and 30 m.

EXPLANATION

Total number of parts = 4 + 5 = 9

Value of 1 part = 54 m ÷ 9 = 6 m

Check numbers add to total: 24 + 30 = 54

Fraction = $\frac{\text{number in ratio}}{\text{total number of parts}}$

Check numbers add to total:
24 + 30 = 54

Example 7 Dividing a quantity in a ratio with three terms

Divide \$300 in the ratio of 2 : 1 : 3.

SOLUTION

EXPLANATION

Unitary method

Total number of parts = 6

$\div 6$ (6 parts = \$300) $\div 6$
$\times 2$ (1 part = \$50) $\times 2$
2 parts = \$100

$\times 3$ (1 part = \$50) $\times 3$
3 parts = \$150

\$300 divided in a ratio of 2 : 1 : 3 is

\$100, \$50 and \$150.

Total number of parts = 2 + 1 + 3 = 6

Value of 1 part = \$300 ÷ 6 = \$50

Check numbers add to total:

\$100 + \$50 + \$150 = \$300

Fractions method

$\frac{2}{6}$ of $300 = \frac{2}{6} \times \frac{300}{1} = 100$

$\frac{1}{6}$ of $300 = \frac{1}{6} \times \frac{300}{1} = 50$

$\frac{3}{6}$ of $300 = \frac{3}{6} \times \frac{300}{1} = 150$

\$300 divided in a ratio of 2 : 1 : 3 is

\$100, \$50 and \$150.

Fraction = $\frac{\text{number in ratio}}{\text{total number of parts}}$

Check numbers add to total:

\$100 + \$50 + \$150 = \$300

An example of dividing a quantity (a length of steel) in a particular ratio.

Example 8 Finding a total quantity from a given ratio

The ratio of boys to girls at Birdsville College is 2 : 3. If there are 246 boys at the school, how many students attend Birdsville College?

SOLUTION

Unitary method

$\div 2$ (2 parts = 246) $\div 2$
$\times 5$ (1 part = 123) $\times 5$
(5 parts = 615)

615 students attend Birdsville College.

EXPLANATION

Ratio of boys : girls is 2 : 3

Boys have '2 parts' = 246

Value of 1 part = $246 \div 2 = 123$

Total parts = $2 + 3 = 5$ parts

5 parts = $5 \times 123 = 615$

Equivalent ratios method

$\times 123$ (boys : girls = 2 : 3 = 246 : 369) $\times 123$

615 students attend Birdsville College.

Use equivalent ratios.

Multiply each quantity by 123.

Total number of students

= 246 boys + 369 girls = 615

Exercise 6C

1–4 | 3, 4 | —

UNDERSTANDING

1 Find the total number of parts in the following ratios.

a 3 : 7 **b** 1 : 5 **c** 11 : 3 **d** 2 : 3 : 4

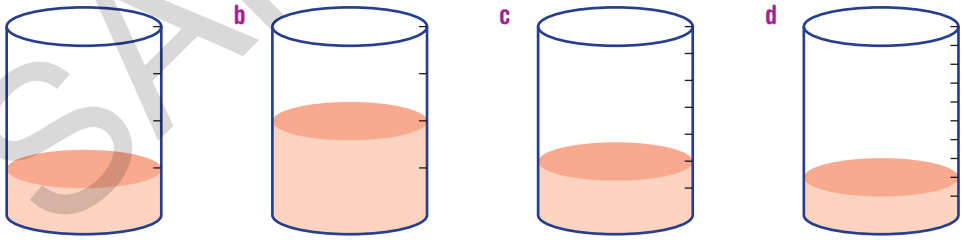
2 The ratio of girls to boys in a class is 3 : 5.

a What fraction of the class is girls?

b What fraction of the class is boys?

3 The diagram shows four glasses that contain different amounts of cordial. Water is then added to fill each glass. For each drink shown, what is the ratio of cordial to water?

a **b** **c** **d**



4 What fraction of each of the drinks above is cordial?

6C

5(½), 6, 7(½) | 5–8(½) | 5–8(½)

FLUENCY

Example 6

5 Divide:

a $60 in the ratio of 2 : 3 **b** $110 in the ratio of 7 : 4 **c** $1000 in the ratio of 3 : 17

d 48 kg in the ratio of 1 : 5 **e** 14 kg in the ratio of 4 : 3 **f** 360 kg in the ratio of 5 : 7

g 72 m in the ratio of 1 : 2 **h** 40 m in the ratio of 3 : 5 **i** 155 m in the ratio of 4 : 1

6 Share $400 in the ratio:

a 1 : 3 **b** 2 : 3 **c** 3 : 5 **d** 9 : 11

Example 7

7 Divide:

a $200 in the ratio of 1 : 2 : 2 **b** $400 in the ratio of 1 : 3 : 4

c 12 kg in the ratio of 1 : 2 : 3 **d** 88 kg in the ratio of 2 : 1 : 5

e 320 kg in the ratio of 12 : 13 : 15 **f** $50 000 in the ratio of 1 : 2 : 3 : 4

8 Share 600 lollies in the ratio:

a 1 : 9 **b** 2 : 1 : 3 **c** 2 : 5 : 5 **d** 12 : 7 : 8 : 3

9–11 | 10–13 | 12–15

PROBLEM-SOLVING

9 Evergreen Fertiliser is made up of the three vital nutrients nitrogen, potassium and phosphorus in a mass ratio of 4 : 5 : 3. How much of each nutrient is in a 1.5 kg bag?

10 The angles of a triangle are in the ratio of 2 : 3 : 4. Find the size of each angle.

11 Three friends, Cam, Molly and Seb, share a prize of $750 in a ratio of 3 : 4 : 8. How much more money does Seb receive than Cam?

12 Trudy and Bella made 80 biscuits. If Trudy made 3 biscuits in the time that Bella made 2 biscuits, how many biscuits did Trudy make?

Example 8

13 In Year 8, the ratio of boys to girls is 5 : 7. If there are 140 girls in Year 8, what is the total number of students in Year 8?

14 A light rye bread requires a ratio of wholemeal flour to plain flour of 4 : 3. A baker making a large quantity of loaves uses 126 cups of plain flour. What is the total amount of flour used by the baker?

15 A textbook contains three chapters and the ratio of pages in these chapters is 3 : 2 : 5. If there are 24 pages in the smallest chapter, how many pages are in the textbook?

16 | 16, 17 | 16, 17 | 6C

REASONING

16 The ratio of the cost of a shirt to the cost of a jacket is 2 : 5. If the jacket cost $240 more than the shirt, find the cost of the shirt and the cost of the jacket.

17 In a class of 24 students the ratio of girls to boys is 1 : 2.

- **a** How many more girls are needed to make the ratio 1 : 1?
- **b** If 4 more girls and 4 more boys joined the class, what would be the new ratio?
- **c** On one day, the ratio was 3 : 7. How many boys and how many girls were absent?

A fair share — — 18

ENRICHMENT

18 Three students Ramshid, Tony and Maria entered a group Geography competition.

- Ramshid spent 5 hours preparing the PowerPoint presentation.
- Tony spent 3 hours researching the topic.
- Maria spent $2\frac{1}{2}$ hours designing the poster.

Their group won second prize in the competition and received a prize of $250.
Ramshid, Tony and Maria decide to share the prize in a ratio of 3 : 2 : 1.

- **a** How much money did each student receive? Round the answer to nearest cent.
- **b** If the prize was divided up according to the time spent on the project, what would be the new ratio? Write the ratio in whole numbers.
- **c** How much money did each student receive with the new ratio? Round the answer to the nearest cent.
- **d** Although she spent the least time on the project, Maria would prefer to divide the prize according to time spent. How much more money did Maria receive with the new ratio?
- **e** Tony preferred that the original ratio remained. How much was Tony better off using the original ratio?
- **f** Which ratio would Ramshid prefer and why?
- **g** The group ended up going with a ratio based on time spent but then rounded amounts to the nearest $10. How much prize money did each student receive?

6D Scale drawings

Interactive

Widgets

HOTsheets

Walkthroughs

Scale drawings are a special application of ratios. The purpose of a scale drawing is to provide accurate information about an object which has dimensions that are either too large or too small to be shown on a page.

If a scale drawing has a ratio of 1 : 1, then the drawing would be exactly the same size as the actual (real-life) object. For example, it is not practical to have a map that is the same size as the actual distance you need to travel, so a much smaller map (a scaled drawing) is used. The map shows a scale to indicate the relationship of the map distance to the actual distance. The scale is displayed as a ratio.

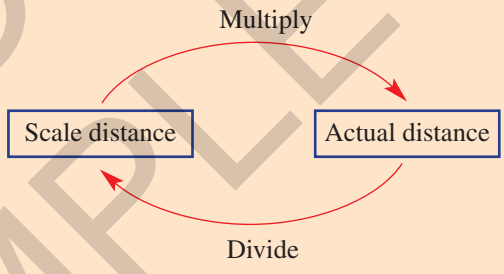
Three common travel maps with their scales are shown below.

Let's start: Scaling down and scaling up

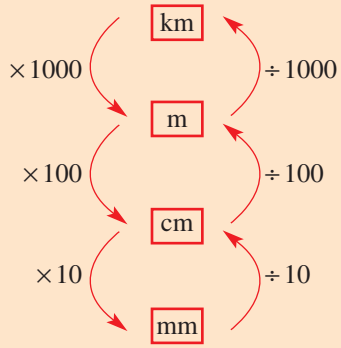
- Brainstorm specific examples where you have come across scale drawings. Think of examples where a scale drawing has been used to show very large objects or distances, and also think of examples where a scale drawing has been used to show very small objects.
- Share your list of examples with your partner.
- As a class, make a list on the board of specific examples of scale drawings.
- What are some examples of common scales that are used?

Key ideas

- A **scale drawing** has exactly the same shape as the original object, but a different size. All scale drawings should indicate the scale ratio.
- The scale on a drawing is written as a scale ratio:
  Drawing length : Actual length
  Drawing length represents the length on the diagram, map or model.
  Actual length represents the real length of the object.
- A **scale ratio** of 1 : 100 means the actual or real lengths are 100 times greater than the lengths measured on the diagram.
- A scale ratio of 20 : 1 means the scaled lengths on the model are 20 times greater than the actual or real lengths.
- It is helpful if scales begin with a 1. Then the second number in the ratio can be referred to as the **scale factor**. Scale ratios that do not start with a 1 can be converted using equivalent ratios.
- To convert a scaled distance to an actual distance you multiply by the scale factor.
- To convert an actual (real) distance to a scaled distance you divide by the scale factor.



- **Conversion of units**. It is important to remember how to correctly convert measurements of length when dealing with scales.
  1 km = 1000 m
  1 m = 100 cm
  1 cm = 10 mm



Example 9 Converting scale distance to actual distance

A map has a scale of 1 : 20 000.
Find the actual distance for each scaled distance.

a 2 cm **b** 5 mm **c** 14.3 cm

SOLUTION	EXPLANATION
a Actual distance = 2 cm × 20 000	Scale factor = 20 000
= 40 000 cm	Multiply scaled distance by scale factor.
= 400 m (or 0.4 km)	Convert answer into sensible units.
b Actual distance = 5 mm × 20 000	Multiply scaled distance by scale factor.
= 100 000 mm	
= 100 m	Convert answer into sensible units.
c Actual distance = 14.3 cm × 20 000	Multiply scaled distance by scale factor.
= 286 000 cm	Shortcut: × 2, then × 10 000
= 2.86 km	Convert answer into sensible units.

Example 10 Converting actual distance to scaled distance

A model boat has a scale of 1 : 500.
Find the scaled length for each of these actual lengths.

a 50 m **b** 75 cm **c** 4550 mm

SOLUTION	EXPLANATION
a Scaled distance = 50 m ÷ 500	Scale factor = 500
= 5000 cm ÷ 500	Convert metres to centimetres.
= 10 cm (0.1 m)	Divide actual distance by scale factor. Convert answer into sensible units.
b Scaled distance = 75 cm ÷ 500	Convert centimetres to millimetres.
= 750 mm ÷ 500	Divide actual distance by scale factor.
= 1.5 mm	
c Scaled distance = 4550 mm ÷ 500	Divide actual distance by scale factor.
= 45.5 mm ÷ 5	Shortcut: ÷ 100, then ÷ 5 (or vice versa)
= 9.1 mm	

Example 11 Determining the scale factor

State the scale factor in the following situations.

a A length of 4 mm on a scale drawing represents an actual distance of 50 cm.

b An actual length of 0.1 mm is represented by 3 cm on a scaled drawing.

SOLUTION	EXPLANATION
a Scale ratio = 4 mm : 50 cm	Write the ratio drawing length : actual length.
= 4 mm : 500 mm	Convert to 'like' units.
Scale ratio = 4 : 500	Write the scale ratio without units.
= 1 : 125	Divide both numbers by 4.
Scale factor = 125	Ratio is now in the form 1 : scale factor. The actual size is 125 times larger than the scaled drawing.
b Scale ratio = 3 cm : 0.1 mm	Write the ratio drawing length : actual length.
= 30 mm : 0.1 mm	Convert to 'like' units.
Scale ratio = 30 : 0.1	Write the scale ratio without units.
= 300 : 1	Multiply both numbers by 10.
$= 1 : \frac{1}{300}$	Divide both numbers by 300.
Scale factor = $\frac{1}{300}$	Ratio is now in the form 1 : scale factor. The actual size is 300 times smaller than the scaled drawing.

Exercise 6D

1–4 | 3, 4 | —

UNDERSTANDING

1 A map has a scale of 1 : 50 000. How many centimetres in real life does 1 cm on the map equal?

2 A map has a scale of 1 : 100. How many metres on the map does 300 m in real life equal?

3 Convert 10 000 cm to:

a mm **b** m **c** km

4 Convert 560 m to:

a km **b** cm **c** mm

6D

5–6(½), 7 | 5–6(½), 7, 8(½) | 5–8(½)

FLUENCY

Example 9

5 Find the actual distance for each of the following scaled distances. Give your answer in an appropriate unit.

a	Scale 1 : 10 000	**i** 2 cm	**ii** 4 mm	**iii** 7.3 cm
b	Scale 1 : 20 000	**i** 80 cm	**ii** 18.5 mm	**iii** 1.25 m
c	Scale 1 : 400	**i** 16 mm	**ii** 72 cm	**iii** 0.03 m
d	Scale 1 : 300	**i** 5 mm	**ii** 8.2 cm	**iii** 7.1 m
e	Scale 1 : 2	**i** 44 m	**ii** 310 cm	**iii** 2.5 mm
f	Scale 1 : 5	**i** 4 m	**ii** 24 cm	**iii** 155 mm
g	Scale 1 : 0.5	**i** 12 cm	**ii** 3.2 mm	**iii** 400 m
h	Scale 100 : 1	**i** 3 cm	**ii** 11.5 km	**iii** 81.5 cm

Example 10

6 Find the scaled length for each of these actual lengths.

a	Scale 1 : 200	**i** 200 m	**ii** 4 km	**iii** 60 cm
b	Scale 1 : 500	**i** 10 000 m	**ii** 1 km	**iii** 75 cm
c	Scale 1 : 10 000	**i** 1350 m	**ii** 45 km	**iii** 736.5 m
d	Scale 1 : 20 000	**i** 12 km	**ii** 1800 m	**iii** 400 mm
e	Scale 1 : 250 000	**i** 5000 km	**ii** 750 000 m	**iii** 1250 m
f	Scale 1 : 10	**i** 23 m	**ii** 165 cm	**iii** 6.15 m
g	Scale 1 : 0.1	**i** 30 cm	**ii** 5 mm	**iii** 0.2 mm
h	Scale 200 : 1	**i** 1 mm	**ii** 7.5 cm	**iii** 8.2 mm

Example 11

7 Determine the scale ratio for each of the following.

a A length of 2 mm on a scale drawing represents an actual distance of 50 cm.
b A length of 4 cm on a scale drawing represents an actual distance of 2 km.
c A length of 1.2 cm on a scale drawing represents an actual distance of 0.6 km.
d A length of 5 cm on a scale drawing represents an actual distance of 900 m.
e An actual length of 7 mm is represented by 4.9 cm on a scaled drawing.
f An actual length of 0.2 mm is represented by 12 cm on a scaled drawing.

8 Convert the two measurements provided in each scale into the same unit and then write the scale as a ratio of two numbers in simplest form.

a	2 cm : 200 m	**b**	5 mm : 500 cm
c	12 mm : 360 cm	**d**	4 mm : 600 m
e	4 cm : 5 m	**f**	1 cm : 2 km
g	28 mm : 2800 m	**h**	3 cm : 0.6 mm
i	1.1 m : 0.11 mm		

9–11 | 10–13 | 11–14

6D

PROBLEM-SOLVING

9 A model city has a scale ratio of 1 : 1000.

- **a** Find the actual height of a skyscraper that has a scaled height of 8 cm.
- **b** Find the scaled length of a train platform that is 45 m long in real life.

10 Blackbottle and Toowoola are 17 cm apart on a map with a scale of 1 : 50 000. How far apart are the towns in real life?

11 Using the house plans shown below, state the actual dimensions of the following rooms.

- **a** Bedroom 1
- **b** Family room
- **c** Patio

12 From the scaled drawing below, calculate the actual length and height of the truck, giving your answer to the nearest metre.

13 The photograph on the right shows Jackson enjoying a walk with his dog. Jackson is 1.8 m tall in real life.
Find a scale for the photograph and use this to estimate the height of Jackson's dog.

14 A scale length of $5\frac{1}{2}$ cm is equal to an actual distance of 44 km.

- **a** How many kilometres does a scale length of 3 cm equal?
- **b** How many kilometres does a scale length of 20 cm equal?
- **c** How many centimetres does an actual distance of 100 km equal?

6D

15 | 15–17 | 17–19

REASONING

15 Group the six ratios below into pairs of equivalent ratios.
1 : 0.01 25 : 1 20 : 1 1 : 0.05 100 : 1 50 : 2

16 Which of the following scales would be most appropriate if you wanted to make a scale model of:
a a car **b** your school grounds **c** Mt Kosciuszko
A 1 : 10 000 **B** 1 : 1000 **C** 1 : 100 **D** 1 : 10

17 A map maker wants to make a map of a mountainous area that is 64 km^2 (side length of 8 km). She decides to use a scale of 1 : 1000. Describe the problem she is going to have.

18 A scientist says that the image of a 1 mm long insect is magnified by a scale of 1 : 1000 through his magnifying glass. Explain what might be wrong with the scientist's ratio.

19 Obtain a map of Australia.
a What is the scale of your map?
b Which two capital cities in Australia are the furthest apart? State the distance.
c Which two capital cities in Australia are closest together? State the distance.
d Which capital city is furthest away from Sydney? State the distance.
e Which capital city is closest to Adelaide? State the distance.
f Check the accuracy of your answers on the internet.

Scaled drawing

— | — | 20

ENRICHMENT

20 Design a scaled drawing of one of the following:
- Your classroom and the associated furniture
- Your bedroom and the associated furniture
- A room in your house (bathroom, family room, garage ...)

Your scaled drawing should fit comfortably onto an A4 page.
a Measure the dimensions of your chosen room and the dimensions of an A4 page, and determine the most appropriate scale for your diagram.
b Show size and location of doors and windows where appropriate.
c Produce a second scale diagram of your room, but show a different arrangement of the furniture.

Can you produce the scale diagram on a computer software package?

6E Introducing rates

If you were to monitor what you said each day, you might well find that you speak about rates many times per day!

A **ratio** shows the relationship between the same type of quantities with the same units, but a **rate** shows the relationship between two different types of quantities with different units.

The following are all examples of rates:

- Cost of petrol was $1.45 per litre.
- Rump steak was on special for $18/kg.
- Dad drove to school at an average speed of 52 km/h.
- After the match, your heart rate was 140 beats/minute.

The top readout on this heart monitor shows heart rate in beats per minute.

Unlike ratios, a **rate** compares different types of quantities, and so units must be shown.

For example:

- The ratio of boys to girls in a school was 4 : 5.
- The average rate of growth of an adolescent boy is 12 cm/year.

Let's start: State the rate

For each of the following statements, write down a corresponding rate.

- The Lodges travelled 400 km in 5 hours.
- Gary was paid $98 for a 4-hour shift at work.
- Felicity spent $600 on a two-day shopping spree.
- Max had grown 9 cm in the last three months.
- Vuong paid $37 for half a cubic metre of crushed rock.
- Paul cycled a total distance of 350 km for the week.

What was the rate (in questions/minute) at which you answered these questions?

Key ideas

- **Rates** compare quantities measured in different units.
- All rates must include units for each quantity.
- The two different units are separated by a slash '/', which is the mathematical symbol for 'per'.
  For example: 20 km/h = 20 km per hour = 20 km for each hour
- It is convention to write rates in their simplest form. This involves writing the rate for only one unit of the second quantity.
  For example: spent \$45 in 3 hours = \$45 in 3 hours ← Non simplified rate
  ÷3 ↓ ↓ ÷3
  = \$15 in 1 hour ← Simplified rate (\$15/h)
- The **average rate** is calculated by dividing the total change in one quantity by the total change in the second quantity.
  For example: reading a 400 page book in 4 days
  Average reading rate = 400 pages in 4 days
  ÷4 ↓ ↓ ÷4
  = 100 pages in 1 day
  Therefore average reading rate = 100 pages/day.

Example 12 Writing simplified rates

Express each of the following as a simplified rate.

a 12 students for two teachers

b \$28 for 4 kilograms

SOLUTION

a 6 students/teacher

b \$7/kg

EXPLANATION

12 students for 2 teachers
÷2 ↓ ↓ ÷2
6 students for 1 teacher

\$28 for 4 kg
÷4 ↓ ↓ ÷4
\$7 for 1 kg

Example 13 Finding average rates

Tom was 120 cm tall when he turned 10 years old. He was 185 cm tall when he turned 20 years old. Find Tom's average rate of growth per year between 10 and 20 years of age.

SOLUTION

÷10 ↓ 65 cm in 10 years ↓ ÷10
6.5 cm in 1 year

Average rate of growth = 6.5 cm/year.

EXPLANATION

Growth = 185 – 120 = 65 cm

Divide both numbers by 10.

Exercise 6E

1–3 | 3 | —

UNDERSTANDING

1 Which of the following are examples of rates?

A \$5.50 **B** 180 mL/min **C** \$60/h **D** $\frac{5}{23}$

E 4.2 runs/over **F** 0.6 g/L **G** 200 cm^2 **H** 84c/L

2 Match each rate in the first column with its most likely rate in the second column.

Employee's wage	90 people/day
Speed of a car	\$2100/m^2
Cost of building a new home	68 km/h
Population growth	64 beats/min
Resting heart rate	\$15/h

3 Write typical units for each of the following rates.

a Price of sausages **b** Petrol costs

c Typing speed **d** Goal conversion rate

e Energy nutrition information **f** Water usage in the shower

g Pain relief medication **h** Cricket team's run rate

4, 5 | 4(½), 5 | 4–5(½)

FLUENCY

Example 12

4 Write each of the following as a simplified rate.

a 12 days in 4 years **b** 15 goals in 3 games

c \$180 in 6 hours **d** \$17.50 for 5 kilograms

e \$126 000 to purchase 9 acres **f** 36 000 cans in 8 hours

g 12 000 revolutions in 10 minutes **h** 80 mm rainfall in 5 days

i 60 minutes to run 15 kilometres **j** 15 kilometres run in 60 minutes

Example 13

5 Find the average rate of change for each situation.

a Relma drove 6000 kilometres in 20 days. **b** Holly saved \$420 over three years.

c A cricket team scored 78 runs in 12 overs. **d** Saskia grew 120 centimetres in 16 years.

e Russell gained 6 kilograms in 4 years. **f** The temperature dropped 5°C in 2 hours.

6–9 | 8–10 | 9–11

PROBLEM-SOLVING

6 A dripping tap filled a 9 litre bucket in 3 hours.

a What was the dripping rate of the tap in litres/hour?

b How long would it take the tap to fill a 21 litre bucket?

7 Martine grew at an average rate of 6 cm/year for the first 18 years of her life. If Martine was 50 cm long when she was born, how tall was Martine when she turned 18?

8 If 30 salad rolls were bought to feed 20 people at a picnic, and the total cost was \$120, find the following rates.

a Salad rolls/person **b** Cost/person **c** Cost/roll

6E

PROBLEM-SOLVING

9 The number of hours of sunshine was recorded each day for one week in April. The results are listed.
Monday 6 hours, Tuesday 8 hours, Wednesday 3 hours, Thursday 5 hours
Friday 7 hours, Saturday 6 hours, Sunday 7 hours

a Find the average number of hours of sunshine:
- **i** per weekday
- **ii** per weekend day
- **iii** per week
- **iv** per day

b Given the above rates, how many hours of sunshine would you expect in April?

10 Harvey finished a 10 kilometre race in 37 minutes and 30 seconds. Jacques finished a 16 kilometre race in 53 minutes and 20 seconds. Calculate the running rate of each runner. Which runner had a faster running pace?

11 The Tungamah Football Club had 12 000 members. After five successful years and two premierships, they now have 18 000 members.

a What has been the average rate of membership growth per year for the past 5 years?

b If this membership growth rate continues, how many more years will it take for the club to have 32 000 members?

12 | 12, 13 | 12, 13

REASONING

12 **a** A car uses 24 L of petrol to travel 216 km. Express these quantities as a simplified rate in:
i km/L **ii** L/km

b How can you convert km/L to L/km?

13 The Teleconnect telecommunications company has a variable call charge rate for phone calls of up to 30 minutes. The charges are 50c/min for first 10 minutes, 75c/min for the second 10 minutes and $1/min for the third 10 minutes.

a Find the cost of phone calls of these given lengths.
i 8 minutes **ii** 13 minutes **iii** 24 minutes **iv** 30 minutes

b What is the average charge rate per minute for a 30 minute call?

Connectplus, a rival telecommunications company, charges a constant call rate of 60c/minute.

c If you normally made calls that were 15 minutes long, which company has the better deal for you?

d If you normally made calls that were 25 minutes long, which company has the better deal for you?

e What is the length of phone call for which both companies would charge the same amount?

Target 155 — — 14 6E

ENRICHMENT

14 In Victoria, due to drought conditions, the state government in 2008 urged all residents to save water. The goal was set for each Victorian to use no more than 155 litres of water per day.

- **a** How many people live in your household?
- **b** According to the Victorian government, how many litres of water can your household use per day?
- **c** Perform some experiments and calculate the following rates of water flow.
  - **i** Shower rate (L/min)
  - **ii** Washing machine (L/load)
  - **iii** Hose (L/min)
  - **iv** Toilet (L/flush, L/half flush)
  - **v** Running tap (L/min)
  - **vi** Drinking water (L/day)
  - **vii** Dishwasher (L/wash)
  - **viii** Water for food preparation (L/day)
- **d** Estimate the average daily rate of water usage for your household.
- **e** Ask your parents for a recent water bill and find out what your family household water usage rate was for the past three months.

Before the initiative, Victorians were using an average of 164 litres/day/person. Twelve months after the initiative, Victorians were using 151 litres/day/person.

- **f** How much water per year for the state of Victoria does this saving of 13 litres/day/person represent?
- **g** What is the rate at which your family is charged for its water?

Progress quiz

6A **1** An excursion to Sea World has 6 teachers, 144 students, 69 boys the rest girls. Write down, in simplest form, the ratio of:

a teachers to students
b boys to girls
c girls to the total of people on the excursion

6A **2** Complete each pair of equivalent ratios.

a 2 : 5 = ____ : 15
b ____ : 3 = 12 : 9
c 24 : 8 = 12 : ____
d 7 : 8 : 3 = 21 : ____ : ____

6B **3** Simplify the following ratios.

a 24 : 36
b 24 : 36 : 40
c 3/4 : 9
d 0.9 : 12
e 40 cents to $1.20

6C **4** Divide:

a $800 in the ratio of 5 : 3
b $7500 in the ratio of 8 : 3 : 1
c 1 km in the ratio of 2 : 3

6C **5** Tony and Vi share their company's profits in the ratio of 15 : 7. Tony's share totals $1200.

a How much profit did the company make?
b What is Vi's share?

6D **6** A map uses a scale of 1 : 40 000. Find the actual distance, in metres, if the distance on the map measures 8 mm.

6D **7** A model car has a scale of 1 : 100. Find the scale length, in centimetres, of a Lexus that is 4.32 metres long.

6D **8** Determine the scale ratio for 4mm on a drawing representing an actual distance of 8 km.

6E **9** Express each of the following as a simplified rate.

a 180 students on 3 buses
b $5.60 for 4 kg
c 186 km in $2\frac{1}{2}$ hours

6E **10** Find the average rate for each situation.

a Thelma drove 8000 km in 50 days
b Callum saved $1250 in 6 months
c Ainslie grew 20 cm in $2\frac{1}{2}$ years

6E **11** Who earns the most, and by how much, if Kelly is paid $96 570 a year and Todd earns $7985 each month?

6E **12** Which is faster 70 km/h or 21 m/s ?

6F Ratios and rates and the unitary method

Interactive

Widgets

HOTsheets

Walkthroughs

The concept of solving problems using the unitary method was introduced in Chapter 3. The unitary method involves finding the value of 'one unit'. This is done by dividing the amount by the given quantity. Once the value of 'one unit' is known, multiplying will find the value of the number of units required.

Let's start: Finding the value of '1 unit'?

For each of the following, find the value of 1 unit or 1 item.

- 8 basketballs cost $200.
- 4 cricket bats cost $316.
- 5 kg of watermelon cost $7.50.

For each of the following, find the rate per 1 unit.

- Car travelled 140 km in 2 hours.
- 1000 L of water leaked out of the tank in 8 hours.
- $51 was the price paid for 3 kg of John Dory fish.

For each of the following, find the value of 1 'part'.

- Ratio of books to magazines read was 2 : 5. Milli had read 14 books.
- Ratio of pink to red flowers is 7 : 11. A total of 330 red flowers are in bloom.
- Ratio of girls to boys is 8 : 5. There are 40 girls in a group.

Key ideas

- The **unitary method** involves finding the value of 'one unit' and then using this information to solve the problem.
- When dealing with **ratios**, find the value of 1 'part' of the ratio.
  For example:
  Ratio of cars to motorbikes is 35 : 2 and there are 6 motorbikes.

  2 parts = 6 motorbikes
  ÷ 2 ↓ ↓ ÷ 2
  1 part = 3 motorbikes

- When dealing with **rates**, find the value of the rate per 1 'unit'.
  For example: Pedro earned $64 for a 4 hour shift at work.
  Therefore, wage rate = $64 per 4 hours = $16 per hour = $16/h.
- Once the value of one 'part' or the rate per one 'unit' is known, the value of any number of parts or units can be found by multiplying.
- The technique of dividing and/or multiplying values in successive one-step calculations can be applied to the concept of converting rates from a set of given units to a different set of units.

Example 14 Reviewing the unitary method

Andy travels 105 km in 7 identical car trips from home to school. How far would she travel in 11 such car trips?

SOLUTION

÷7 ↓ 7 car trips = 105 km ↓ ÷7
×11 ↓ 1 car trip = 15 km ↓ ×11
11 car trips = 165 km

Andy travels 165 km.

EXPLANATION

Find the value of 1 unit by dividing both quantities by 7.

Solve the problem by multiplying both quantities by 11.

Example 15 Solving ratio problems using the unitary method

The ratio of apples to oranges is 3 : 5. If there are 18 apples, how many oranges are there?

SOLUTION

÷3 ↓ 3 parts = 18 apples ↓ ÷3
×5 ↓ 1 part = 6 pieces ↓ ×5
5 parts = 30 oranges

There are 30 oranges.

EXPLANATION

Apples = 3 'parts', Oranges = 5 'parts'

Need to find the value of 1 'part'.

To find 5 'parts' multiply the value of 1 'part' by 5.

Example 16 Solving rate problems using the unitary method

A truck uses 4 L of petrol to travel 36 km. How far will it travel if it uses 70 L of petrol?

SOLUTION

Rate of petrol consumption

÷4 ↓ 36 km for 4 L ↓ ÷4
×70 ↓ 9 km for 1 L ↓ ×70
630 km for 70 L

Truck will travel 630 km on 70 L.

EXPLANATION

Find the petrol consumption rate of 1 unit by dividing both quantities by 4.

Solve the problem by multiplying both quantities by 70.

Example 17 Converting units using the unitary method

Melissa works at the local supermarket and earns $57.60 for a 4 hour shift.
How much does she earn in c/min?

SOLUTION

Wage = $57.60 for 4 hours

÷4 ↓ $14.40 for 1 hour ↓ ÷4

÷60 ↓ 1440c for 60 minutes ↓ ÷60

24c for 1 minute

Melissa earns 24c/min.

EXPLANATION

Write down Melissa's wage rate.

Find the rate of $ per 1 hour.

Convert $ to cents and hours to minutes.

Divide rate by 60 to find rate of cents per minute.

Exercise 6F

1–4 | 4 | —

UNDERSTANDING

Example 14

1 If 8 kg of chicken fillets cost $72, how much would 3 kg of chicken fillets cost?

2 If one dozen tennis balls cost $9.60, how much would 22 tennis balls cost?

3 If three pairs of socks cost $12.99, how much would 10 pairs of socks cost?

4 If 500 g of mince meat costs $4.50, how much would 4 kg of mince meat cost?

5, 6, 8(½) | 5–7, 8(½) | 5–6(½), 7, 8(½)

FLUENCY

Example 15

5 Solve the following ratio problems.

a The required staff to student ratio for an excursion is 2 : 15. If 10 teachers attend the excursion, what is the maximum number of students who can attend?

b The ratio of commercials to actual show time for a particular TV channel is 2 : 3. How many minutes of actual show were there in 1 hour?

c A rectangle has length and width dimensions in a ratio of 3 : 1. If a particular rectangle has a length of 21 m, what is its width?

d Walter and William have a height ratio of 7 : 8. If William has a height of 152 cm, how tall is Walter?

Example 16

6 Solve the following rate problems.

a A tap is dripping at a rate of 200 mL every 5 minutes. How much water drips in 13 minutes?

b A professional footballer scores an average of 3 goals every 6 games. How many goals is he likely to score in a full season of 22 games?

c A snail travelling at a constant speed travels 400 mm in 8 minutes. How far does it travel in 7 minutes?

d A computer processor can process 500 000 kilobytes of information in 4 seconds. How much information can it process in 15 seconds?

7 Leonie, Spencer and Mackenzie have just won a prize. They decide to share it in the ratio of 4 : 3 : 2. If Spencer receives $450, how much do Leonie and Mackenzie receive, and what was the total value of the prize?

6F

Example 17

8 Convert the following rates into the units given in the brackets.

a $15/h (c/min)
b $144/h (c/s)
c 3.5 L/min (L/h)
d 20 mL/min (L/h)
e 0.5 kg/month (kg/year)
f 120 g/day (kg/week)
g 60 g/c (kg/$)
h $38/m (c/mm)
i 108 km/h (m/s)
j 14 m/s (km/h)

FLUENCY

9, 10 | 10–12 | 10–12

PROBLEM-SOLVING

9 The Mighty Oats breakfast cereal is sold in boxes of three different sizes: small (400 g) for $5.00, medium (600 g) for $7.20, large (750 g) for $8.25

a Find the value of each box in $/100 g.
b What is the cheapest way to buy a minimum of 4 kg of the cereal?

10 Gemma runs 70 metres in 8.4 seconds. If she maintains the same average speed, in what time will she run 100 metres?

11 Zana's hair grew 6 cm in 5 months.

a Find Zana's average rate of hair growth in cm/month and in m/year.
b How long would it take for Zana's hair to grow 30 cm?

12 Maria can paint 15 m^2 in 20 minutes.

a What is the rate at which Maria paints in m^2/h?
b What area can Maria paint in 20 hours?
c Maria must paint 1000 m^2 in 20 hours. Find the rate at which she will need to paint in m^2/min.

14a | 13, 14 | 13–15

REASONING

13 If x donuts cost $$y$$:

a how much would 1 donut cost?
b how much would one dozen donuts cost?
c how much would z donuts cost?

14 **a** A triangle has side lengths in a ratio of 19 : 22 : 17. If the shortest side is 17 cm, find the lengths of the other two sides and the perimeter of the triangle.

b A triangle has side lengths in a ratio of 3 : 5 : 4. If the longest side is 35 cm, find the lengths of the other two sides and the perimeter of the triangle.

15 In a faraway galaxy, a thriving alien colony uses the following units:
For money they have puks and paks: 1 puk (pu) = 1000 pak (pa)
For length they have doits and minidoits: 1 doit (D) = 80 minidoits (mD)
Polynaute rope is priced at 4 pu/D. Find the cost of the rope in terms of pa/mD.

Where will we meet? — — 16

16 Phil lives in Perth and his friend Werner lives in Sydney. The distance, by road, between their two houses is 4200 km (rounded to the nearest 100 km).

Phil decides to drive to Sydney and Werner decides to drive to Perth. They leave home at the same time and travel the same route, but in opposite directions.

Phil drives at a constant speed of 75 km/h and Werner drives at a constant speed of 105 km/h.

a Will they meet on the road at a spot closer to Sydney or closer to Perth?
b How long will it take Phil to travel to Sydney?
c How long will it take Werner to travel to Perth?
d State the location of each friend after they have been driving for 15 hours.
e At what location (distance from Sydney and/or Perth) will they meet?

When they meet, Phil decides to change into Werner's car and they drive back to Werner's home at an average speed of 105 km/h.

f How long did it take Phil to travel to Sydney?
g Design a similar problem for two friends travelling at different constant speeds between two different capital cities in Australia.

6G Solving rate problems EXTENDING

'One thing that is certain in life is that change is inevitable.'

We are constantly interested in rates of change and how things change over a period of time. We are also regularly faced with problems involving specific rates. Strong arithmetic skills and knowing whether to multiply or divide by the given rate allows many rate problems to be solved quickly and accurately.

Over the next few days, keep a record of any rates you observe or experience. Look out for the slash '/' sign and listen for the 'per' word.

Let's start: Estimate the rate

For each of the following statements, estimate a corresponding rate.

- Commercial rate: The number of commercials on TV per hour
- Typing rate: Your typing speed in words per minute
- Laughing rate: The number of times a teenager laughs per hour
- Growth rate: The average growth rate of a child from 0 to 15 years of age
- Running rate: Your running pace in metres per second
- Homework rate: The number of subjects with homework per night
- Clapping rate: The standard rate of audience clapping in claps per minute
- Thankyou rate: The number of opportunities to say thank you per day

Compare your rates. Which rate is the 'highest'? Which rate is the 'lowest'? Discuss your answers.

Key ideas

- When a rate is provided, a change in one quantity implies that an equivalent change must occur in the other quantity.
  For example: Patrick earns $20/hour. How much will he earn in 6 hours?

$$\begin{array}{ccc} & \$20 \text{ for } 1 \text{ hour} & \\ \times 6 \swarrow & & \searrow \times 6 \\ & \$120 \text{ for } 6 \text{ hours} & \end{array}$$

  For example: Patrick earns $20/hour. How long will it take him to earn $70?

$$\begin{array}{ccc} & \$20 \text{ for } 1 \text{ hour} & \\ \times 3\frac{1}{2} \swarrow & & \searrow \times 3\frac{1}{2} \\ & \$70 \text{ for } 3\frac{1}{2} \text{ hours} & \end{array}$$

- Carefully consider the units involved in each question and answer.

Example 18 Solving rate problems

a Rachael can touch type at 74 words/minute. How many words can she type in 15 minutes?

b Leanne works in a donut van and sells on average 60 donuts every 15 minutes. How long is it likely to take her to sell 800 donuts?

SOLUTION

a

$\times 15$ (74 words in 1 minute → 1110 words in 15 minutes) $\times 15$

Rachael can type 1110 words in 15 minutes.

b

$\div 15$ (60 donuts in 15 minutes → 4 donuts in 1 minute) $\div 15$

$\times 200$ (4 donuts in 1 minute → 800 donuts in 200 minutes) $\times 200$

Leanne is likely to take 3 hours and 20 minutes to sell 800 donuts.

EXPLANATION

$$\begin{array}{r} 74 \\ \times\ 15 \\ \hline 370 \\ 740 \\ \hline 1110 \end{array}$$

Selling rate = 60 donuts/15 minutes

Divide both quantities by HCF of 15.

Multiply both quantities by 200.

Convert answer to hours and minutes.

Example 19 Solving combination rate problems

Three water hoses from three different taps are used to fill a large swimming pool. The first hose alone takes 200 hours to fill the pool. The second hose alone takes 120 hours to fill the pool and the third hose alone takes only 50 hours to fill the pool. How long would it take to fill the pool if all three hoses were used?

SOLUTION

In 600 hours:

hose 1 would fill 3 pools

hose 2 would fill 5 pools

hose 3 would fill 12 pools

Therefore in 600 hours the three hoses together would fill 20 pools.

Filling rate = 600 h/20 pools

$\div 20$ (→) $\div 20$

30 h/pool

It would take 30 hours to fill the pool if all three hoses were used.

EXPLANATION

LCM of 200, 120 and 50 is 600,

Hose 1 = 600 h ÷ 200 h/pool = 3 pools

Hose 2 = 600 h ÷ 120 h/pool = 5 pools

Hose 3 = 600 h ÷ 50 h/pool = 12 pools

Together = 3 + 5 + 12 = 20 pools filled.

Simplify rate by dividing by HCF.

Exercise 6G

1, 2 | 2 | —

UNDERSTANDING

1 Fill in the gaps.

a ×3 (60 km in 1 hour) ×3
180 km in ___

b ×5 ($25 in 1 hour) ___
$125 in ___

c ___ (7 questions in 3 minutes) ___
70 questions in ___

d ___ (120 litres in 1 minute) ___
___ in 6 minutes

2 Fill in the gaps.

a ÷3 ($36 for 3 hours) ÷3
×5 (___ for 1 hour) ___
___ for 5 hours

b ___ (150 rotations in 5 minutes) ___
___ (_____ in 1 minute) ___
___ (_____ in 7 minutes) ___

3–7 | 3–8 | 4–8

FLUENCY

Example 18a

3 A factory produces 40 plastic bottles/minute.
- **a** How many bottles can the factory produce in 60 minutes?
- **b** How many bottles can the factory produce in an 8 hour day of operation?

Example 18b

4 Mario is a professional home painter. When painting a new home he uses an average of 2.5 litres of paint per hour. How many litres of paint would Mario use in a week if he paints for 40 hours?

5 A truck travels 7 km per litre of fuel. How many litres are needed for the truck to travel 280 km?

6 Daniel practises his guitar for 40 minutes every day. How many days will it take him to log up 100 hours of practice?

7 A flywheel rotates at a rate of 1500 revolutions per minute.
- **a** How many revolutions does the flywheel make in 15 minutes?
- **b** How many revolutions does the flywheel make in 15 seconds?
- **c** How long does it take for the flywheel to complete 15 000 revolutions?
- **d** How long does it take for the flywheel to complete 150 revolutions?

8 Putra is an elite rower. When training, he has a steady working heart rate of 125 beats per minute (bpm). Putra's resting heart rate is 46 bpm.
- **a** How many times does Putra's heart beat during a 30 minute workout?
- **b** How many times does Putra's heart beat during 30 minutes of 'rest'?
- **c** If his coach says that he can stop his workout once his heart has beaten 10 000 times, for how long would Putra need to train?

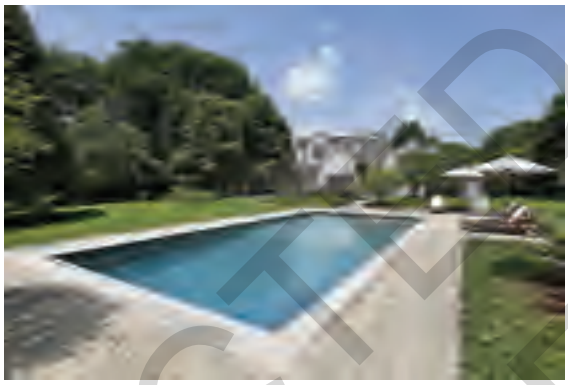
6G

9, 10 | 10–12 | 10–12

PROBLEM-SOLVING

9 What is the cost of paving a driveway that is 18 m long and 4 m wide, if the paving costs $35 per square metre?

10 A saltwater swimming pool requires 2 kg of salt to be added for every 10 000 litres of water. Joan's swimming pool is 1.5 metres deep, 5 metres wide and 15 metres long. How much salt will she need to add to her pool?



11 The Bionic Woman gives Batman a 12 second start in a 2 kilometre race. If the Bionic Woman runs at 5 km/min, and Batman runs at 3 km/min, who will win the race and by how many seconds?

12 At a school camp there is enough food for 150 students for 5 days.

a How long would the food last if there were only 100 students?

b If the food ran out after only 4 days, how many students attended the camp?

13 | 13–15 | 14–17

REASONING

Example 19

13 Michelle can complete a landscaping job in 6 days and Danielle can complete the same job in 4 days. Working together, in how many days could they complete the job?

14 Three bricklayers Maric, Hugh and Ethan are cladding a new home. If Maric were to work alone, the job would take him 8 days to complete. If Hugh were to work alone, the job would take him 6 days to complete, and if Ethan were to work by himself, the job would take him 12 days to complete.

a If the three men work together, how long will it take them to complete the job?

b What fraction of the house will each bricklayer complete?

6G

REASONING

15 Four cans of dog food will feed 3 dogs for 1 day.

- **a** How many cans are needed to feed 10 dogs for 6 days?
- **b** How many dogs can be fed for 9 days from 60 cans?
- **c** For how many days will 40 cans feed 2 dogs?

16 If it takes 4 workers, 4 hours to dig 4 holes, how long would it take 2 workers to dig 6 holes?

17 State the units required for the answer to each of the following rate calculations.

- **a** \$205/kg × 48 kg
- **b** 62 s × 12 m/s
- **c** 500 beats ÷ 65 beats/min (bpm)
- **d** 4000 revolutions ÷ 120 revs/min

ENRICHMENT

Value for money — — 18

18 Soft drink can be purchased from supermarkets in a variety of sizes.
Below are the costs for four different sizes of a certain brand of soft drink.

600 mL 'buddies'	1.25 L bottles	2 L bottles	10 × 375 mL cans
\$2.70 each	\$1.60 each	\$2.20 each	\$6.00 per pack

- **a** Find the economy rate (in \$/L) for each size of soft drink.
- **b** Find and compare the cost of 30 litres of soft drink purchased entirely in each of the four different sizes.
- **c** If you only have \$60 to spend on soft drink for a party, what is the difference between the greatest amount and the least amount of soft drink you could buy? Assume you have less than \$1.60 left.

Most supermarkets now include the economy rate of each item underneath the price tag to allow customers to compare value for money.

- **d** Carry out some research at your local supermarket on the economy rates of a particular food item with a range of available sizes (such as drinks, breakfast cereals, sugar, flour). Write a report on your findings.

6H Speed EXTENDING

Interactive

Widgets

HOTsheets

Walkthroughs

A rate that we come across almost every day is speed. **Speed** is the rate of distance travelled per unit of time.

On most occasions, speed is not constant and therefore we are interested in the average speed of an object. Average speed is calculated by dividing the distance travelled by the time taken.

$$\text{Average speed} = \frac{\text{Distance travelled}}{\text{Time taken}}$$

Given the average speed formula, we can tell that all units of speed must have a unit of length in the numerator, followed by a unit of time in the denominator. Therefore 'mm/h' is a unit of speed and could be an appropriate unit for the speed of a snail!

The most common units of speed are m/s and km/h.

This snail's average speed won't be high, but it can still be calculated.

Let's start: Which is faster?

With a partner, determine the faster of the two listed alternatives.

a	Car A travelling at 10 m/s	Car B travelling at 40 km/h
b	Walker C travelling at 4 km/h	Walker D travelling at 100 m/min
c	Jogger E running at 1450 m/h	Jogger F running at 3 m/s
d	Plane G flying at 700 km/h	Plane H flying at 11 km/min

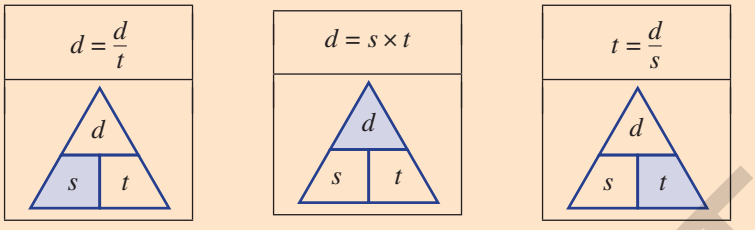
Key ideas

- **Speed** is a measure of how fast an object is travelling.
- If the speed of an object does not change over time, the object is travelling at a **constant speed**. 'Cruise control' helps a car travel at a constant speed.
- When speed is not constant, due to acceleration or deceleration, we are often interested to know the **average speed** of the object.
- Average speed is calculated by the formula:

$$\text{Average speed} = \frac{\text{Distance travelled}}{\text{Time taken}} \quad \text{or} \quad s = \frac{d}{t}$$

Key ideas

- Depending on the unknown value, the above formula can be rearranged to make d or t the subject. The three formulas involving s, d, and t are:



- Care must be taken with units for speed, and on occasions units will need to be converted. The most common units of speed are m/s and km/h.

Example 20 Finding average speed

Find the average speed in km/h of:

a a cyclist who travels 140 km in 5 hours

b a runner who travels 3 km in 15 minutes

SOLUTION	EXPLANATION
a $s = \frac{d}{t}$	The unknown value is speed.
	Write the formula with s as the subject.
$= \frac{140 \text{ km}}{5 \text{ h}}$	Distance travelled = 140 km
	Time taken = 5 h
$= 28$ km/h	Speed unit is km/h.
Alternative unitary method	
$\div 5$ (140 km in 5 hours ; 28 km in 1 hour) $\div 5$	Write down the rate provided in the question.
	Divide both quantities by 5.
Average speed = 28 km/h	
b $s = \frac{d}{t}$	
$= \frac{3 \text{ km}}{15 \text{ min}}$	Distance travelled = 3 km
	Time taken = 15 min or $\frac{1}{4}$ h
$= 3 \div \frac{1}{4}$	Dividing by $\frac{1}{4}$, is the same as multiplying by $\frac{4}{1}$.
$= 3 \times 4 = 12$ km/h	
Alternative unitary method	Write down the rate provided in the question.
$\times 4$ (3 km in 15 minutes ; 12 km in 60 minutes) $\times 4$	Multiply both quantities by 4.
Average speed = 12 km/h	

Example 21 Finding the distance travelled

Find the distance travelled by a truck travelling for 15 hours at an average speed of 95 km/h.

SOLUTION	EXPLANATION
$d = s \times t$	The unknown value is distance.
$= 95 \text{ km/h} \times 15 \text{ h}$	Write the formula with d as the subject.
$= 1425 \text{ km}$	Distance unit is km.
Alternative unitary method	
$\times 15$ (95 km in 1 hour → 1425 km in 15 hours) $\times 15$	Write the rate provided in the question. Multiply both quantities by 15.
Truck travels 1425 km in 15 hours.	

Example 22 Finding the time taken

Find the time taken for a hiker walking at 4 km/h to travel 15 km.

SOLUTION	EXPLANATION
$t = \frac{d}{s}$	The unknown value is time.
$= \frac{15 \text{ km}}{4 \text{ km/h}}$	Write the formula with t as the subject.
$= 3.75 \text{ h}$	The time unit is h.
$= 3 \text{ h } 45 \text{ min}$	Leave answer as a decimal or convert to hours and minutes.
	$0.75 \text{ h} = 0.75 \times 60 = 45 \text{ min}$
Alternative unitary method	
$\div 4$ (4 km in 1 hour → 1 km in $\frac{1}{4}$ hour) $\div 4$ $\times 15$ (15 km in $\frac{15}{4}$ hours) $\times 15$	Express the rate as provided in the question. Divide both quantities by 4. Multiply both quantities by 15.
It takes 3 h 45 min to travel 15 km.	

Example 23 Converting units of speed

a Convert 72 km/h to m/s. **b** Convert 8 m/s to km/h.

SOLUTION	EXPLANATION
a 72 km in 1 hour	Express rate in kilometres per hour.
÷60 ↓ 72 000 m in 60 minutes ↓ ÷60 1200 m in 1 minute	Convert km to m and hour to minutes. Divide both quantities by 60.
÷60 ↓ 1200 m in 60 seconds ↓ ÷60 20 m in 1 second	Convert 1 minute to 60 seconds. Divide both quantities by 60.
∴ 72 km/h = 20 m/s	Shortcut for converting km/h → m/s ÷ 3.6.
b 8 m in 1 second	Express rate in metres per second.
×60 ↓ 8 m in 1 second ↓ ×60 480 m in 1 minute	Multiply by 60 to find distance in 1 minute.
×60 ↓ 480 m in 1 minute ↓ ×60 28 800 m in 1 hour	Multiply by 60 to find distance in 1 hour.
∴ 28.8 km in 1 hour	Convert metres to kilometres. Shortcut: m/s × 3.6 → km/h. 8 m/s × 3.6 = 28.8km/h

Exercise 6H

1–4 | 3, 4 | —

UNDERSTANDING

1 Which of the following is not a unit of speed?
A m/s **B** km/h **C** cm/h **D** L/kg **E** m/min

2 If Average speed = $\frac{\text{Distance travelled}}{\text{Time taken}}$ then the Distance travelled must equal:
A Average speed × Time taken **B** $\frac{\text{Average speed}}{\text{Time taken}}$
C $\frac{\text{Time taken}}{\text{Average speed}}$

3 If Average speed = $\frac{\text{Distance travelled}}{\text{Time taken}}$ then Time taken must equal:
A Distance travelled × Average speed **B** $\frac{\text{Distance travelled}}{\text{Average speed}}$
C $\frac{\text{Average speed}}{\text{Distance travelled}}$

4 If an object travels 800 metres in 10 seconds, the average speed of the object is:
A 8000 m/s **B** 800 km/h **C** 80 km/h **D** 80 m/s

Example 20

5 Calculate the average speed of:

- **a** a sprinter running 200 m in 20 seconds
- **b** a skateboarder travelling 840 m in 120 seconds
- **c** a car travelling 180 km in 3 hours
- **d** a truck travelling 400 km in 8 hours
- **e** a train travelling 60 km in 30 minutes
- **f** a tram travelling 15 km in 20 minutes

Example 21

6 Calculate the distance travelled by:

- **a** a cyclist travelling at 12 m/s for 90 seconds
- **b** an ant travelling at 2.5 m/s for 3 minutes
- **c** a bushwalker who has walked for 8 hours at an average speed of 4.5 km/h
- **d** a tractor ploughing fields for 2.5 hours at an average speed of 20 km/h

Example 22

7 Calculate the time taken by:

- **a** a sports car to travel 1200 km at an average speed of 150 km/h
- **b** a bus to travel 14 km at an average speed of 28 km/h
- **c** a plane to fly 6900 km at a constant speed of 600 km/h
- **d** a ball moving through the air at a speed of 12 m/s to travel 84 m

Example 23a

8 Convert the following speeds to m/s.

a 36 km/h **b** 180 km/h **c** 660 m/min **d** 4 km/s

Example 23b

9 Convert the following speeds to km/h.

a 15 m/s **b** 2 m/s **c** 12 m/min **d** 1 km/s

10 Complete the following table.

	Speed s	Distance d	Time t
a		50 km	2 h
b	30 m/s	1200 m	
c	5 km/h		12 h
d		600 m	5 min
e	210 km/h		20 min
f	100 m/s	10 km	

6H

11, 12 | 12–15 | 13–16

PROBLEM-SOLVING

11 A plane is flying at a cruising speed of 900 km/h. How far will the plane travel from 11:15 a.m. to 1:30 p.m. on the same day?

12 The wheels on Charlie's bike have a circumference of 1.5 m. When Charlie is riding fastest, the wheels rotate at a speed of five turns per second.

- **a** What is the fastest speed Charlie can ride his bike, in km/h?
- **b** How far would Charlie travel in 5 minutes at his fastest speed?

13 The back end of a 160-metre-long train disappears into a 700-metre-long tunnel. Twenty seconds later the front of the train emerges from the tunnel. Determine the speed of the train in m/s.

14 Anna rode her bike to school one morning, a distance of 15 km, at an average speed of 20 km/h. It was raining in the afternoon, so Anna decided to take the bus home. The bus trip home took 30 minutes. What was Anna's average speed for the return journey to and from school?

15 In Berlin 2009, Jamaican sprinter Usain Bolt set a new 100 m world record time of 9.58 seconds. Calculate Usain Bolt's average speed in m/s and km/h for this world record. (Round the answers correct to one decimal place.)

16 The Ghan train is an Australian icon. You can board The Ghan in Adelaide and 2979 km later, after travelling via Alice Springs, you arrive in Darwin. (Round the answers correct to one decimal place.)

- **a** If you board The Ghan in Adelaide on Sunday at 2:20 p.m. and arrive in Darwin on Tuesday at 5:30 p.m., what is the average speed of the train journey?
- **b** There are two major rest breaks. The train stops for $4\frac{1}{4}$ hours at Alice Springs and 4 hours at Katherine. Taking these breaks into account, what is the average speed of the train when it is moving?

6H

17	17, 18	18, 19

REASONING

17 Write two speeds in km/h that are between 40 m/s and 45 m/s.

18 Nina, Shanti and Belle run a 1000 m race at a constant speed. When Nina crossed the finish line first, she was 200 m ahead of Shanti and 400 m ahead of Belle. When Shanti crossed the finish line, how far ahead of Belle was she?

19 Julie and Jeanette enjoy finishing their 6 km morning run together. Julie runs at an average speed of 10 km/h and Jeanette runs at an average speed of 3 m/s. If Julie leaves at 8 a.m., at what time should Jeanette leave if they are to finish their run at the same time?

Speed research

—	—	20

ENRICHMENT

20 Carry out research to find answers to the following questions.

Light and sound

a What is the speed of sound in m/s?
b What is the speed of light in m/s?
c How long would it take sound to travel 100 m?
d How long would it take light to travel 100 km?
e How many times quicker is the speed of light than the speed of sound?
f What is a Mach number?

Spacecraft

g What is the escape velocity needed by a spacecraft to 'break free' of Earth's gravitational pull? Give this answer in km/h and also km/s.
h What is the orbital speed of planet Earth? Give your answer in km/h and km/s.
i What is the average speed of a space shuttle on a journey from Earth to the International Space Station?

Knots

Wind speed and boat speed are often given in terms of knots (kt).

j What does a knot stand for?
k What is the link between nautical miles and a system of locating positions on Earth?
l How do you convert a speed in knots to a speed in km/h?

Investigation

Fun run investigation

Three maths teachers, Mrs M, Mr P and Mr A, trained very hard to compete in a Brisbane Fun Run. The 10.44 km route passed through the botanical gardens, along the Brisbane River to New Farm Park, and back again. Their times and average stride lengths were:

- Mrs M: 41 minutes, 50 seconds: 8 strides per 9 m
- Mr P: 45 minutes, 21 seconds: 7 strides per 9 m
- Mr A: 47 minutes, 6 seconds: 7.5 strides per 9 m

Copy and complete the following table and determine which rates are the most useful representations of fitness. Give the answers rounded to one decimal place. Justify your conclusions.

Running rates	Mrs M	Mr P	Mr A	World record 10 km runners	Your family member or friend
Seconds per 100 m					
Seconds per km					
Metres per minute					
Km per hour					
Strides per 100 m					
Strides per minute					
Strides per hour					

Fitness investigation

1 Using a stopwatch, measure your resting heart rate in beats per minute.

2 Run on the spot for one minute and then measure your working heart rate in beats per minute.

3 Using a stopwatch, time yourself for a 100 m run and also count your strides. At the end, measure your heart rate in beats per minute. Also calculate the following rates.
- Your running rate in m/s, m/min and km/h
- Your running rate in time per 100 m and time per km
- Your rate of strides per minute, strides per km and seconds per stride

4 Run 100 m four times without stopping and, using a stopwatch, record your cumulative time after each 100 m.
- Organise these results into a table.
- Draw a graph of the distance ran (vertical) vs time taken (horizontal).
- Calculate your running rate in m/min for each 100 m section.
- Calculate your overall running rate in m/min for the 400 m.
- Explain how and why your running rates changed over the 400 m.

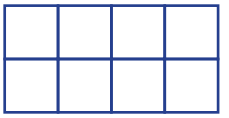
5 Try sprinting fast over a measured distance and record the time. Calculate your sprinting rate in each of the following units:
- minutes per 100 m
- metres per minute
- time per km
- km per hour

6 Research the running rate of the fastest schoolboy and schoolgirl in Australia. How do their sprinting rates compare to the running rates of Australian Olympian athletes?

Problems and challenges

Up for a challenge? If you get stuck on a question, check out the 'Working with Unfamiliar Questions' poster at the end of the book to help you.

1 This diagram is made up of 8 equal-sized squares.



How many squares need to be shaded if the ratio of shaded squares to unshaded squares is:

a 1 : 3? **b** 2 : 3? **c** 1 : 2?

2 Bottle A has 1 L of cordial drink with a cordial to water ratio of 3 : 7. Bottle B has 1 L of cordial drink with a cordial to water ratio of 1 : 4. The drink from both bottles is combined to form a 2 L drink. What is the new cordial to water ratio?

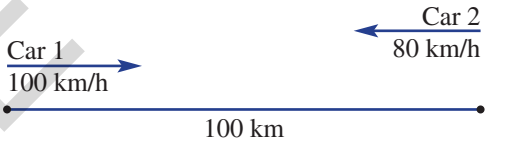
3 Brothers Marco and Matthew start riding from home into town, which is 30 km away. Marco rode at 10 km/h and Matthew took 20 minutes longer to complete the trip. Assuming that they both rode at a constant speed, how fast was Matthew riding?

4 **a** If 1 person takes 1 hour to dig 1 post hole, how long will it take 2 people to dig 2 post holes?

b If 3 people take 3 hours to make 3 wooden train sets, how many train sets can 6 people make in 6 hours?

5 At a market you can trade 2 cows for 3 goats or 3 goats for 8 sheep. How many sheep are 3 cows worth?

6 Two cars travel toward each other on a 100 km straight stretch of road. They leave at opposite ends of the road at the same time. The cars' speeds are 100 km/h and 80 km/h. How long does it take for the cars to pass each other?



7 A river is flowing downstream at a rate of 1 km/h. In still water Michael can swim at a rate of 2 km/h. Michael dives into the river and swims downstream then turns around and swims back to the starting point, taking 0.5 hours in total. How far did he swim?

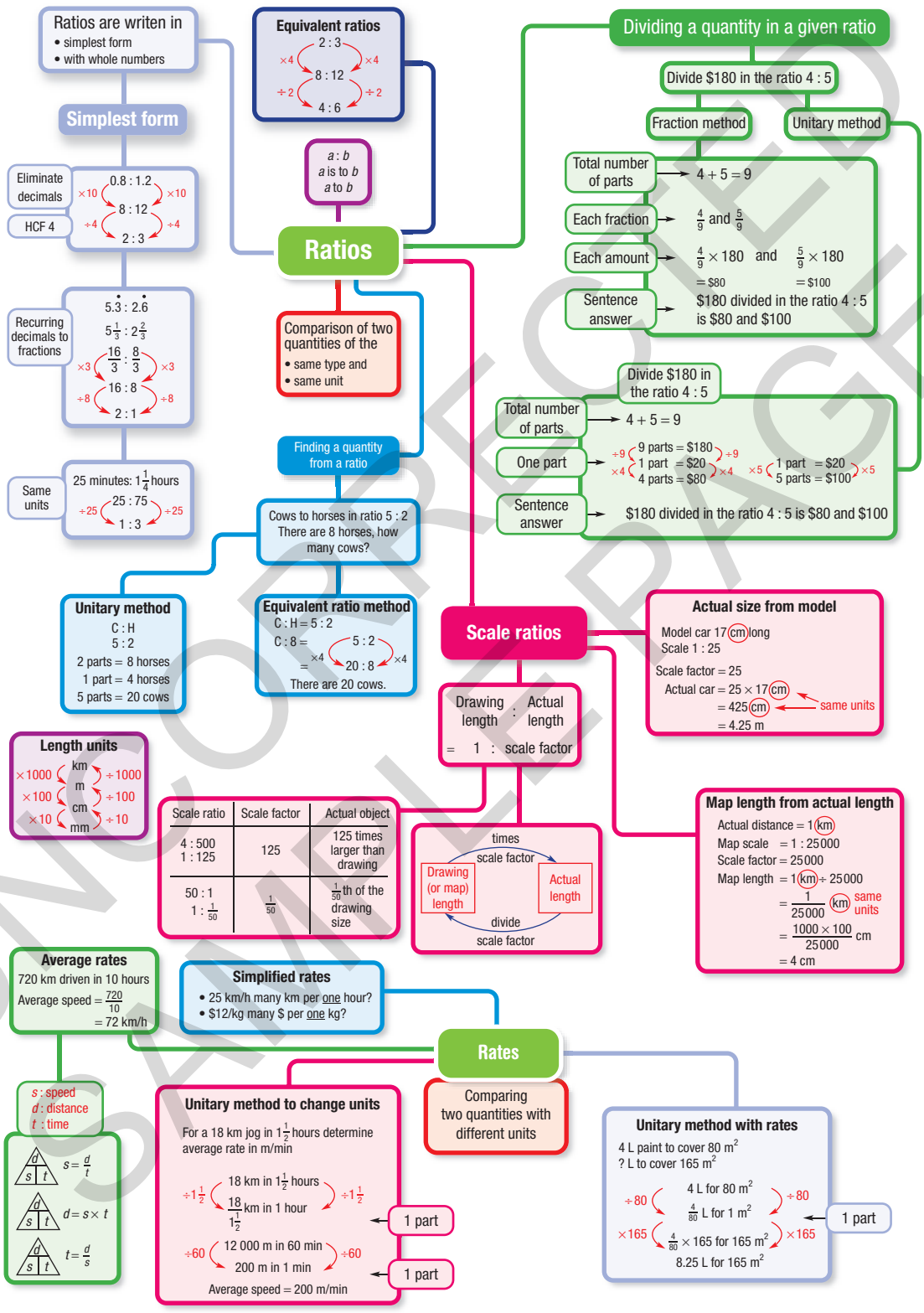
8 A fitness fanatic walks at 4 km/h for time t_1, and then runs at 7 km/h for time t_2. He travels a total of 26 km. If he had run for the same time that he had walked (t_1) and walked for the same time that he had run (t_2), then he would have travelled 29 km. What was the total time spent walking and running?

9 A top secret goo has been developed so that it doubles in volume every 60 seconds. At midnight a small amount is placed in a beaker and its growth observed. At 1 a.m. the beaker is full. At exactly what time did the goo fill only $\frac{1}{8}$ of the beaker?

10 A car averages 60 km/h for 20 km and then increases its speed averaging 80 km/h for the next 20 km. Calculate the car's average speed (to two decimal places) for the full 40 km trip.

11 Max travels at 50 km/h for 10 km. For the next 10 km Max wants to drive fast enough to make his average speed 100 km/h over the 20 km trip. Advise Max about reaching this average speed.

Chapter summary



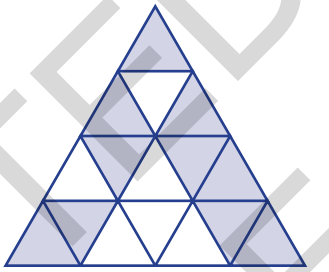
Chapter review

Multiple-choice questions

6A **1** A school has 315 boys, 378 girls and 63 teachers. The ratio of students to teachers is:

A 11 : 1 **B** 1 : 11 **C** 5 : 6 **D** 6 : 5

6A **2** Find the ratio of the shaded area to the unshaded area in this triangle.



A 3 : 5 **B** 8 : 5 **C** 5 : 3 **D** 5 : 8

6B **3** The ratio 500 mm to $\frac{1}{5}$ m is the same as:

A 50 : 2 **B** 2500 : 1 **C** 2 : 5 **D** 5 : 2

6B **4** The ratio $1\frac{1}{2} : \frac{3}{4}$ simplifies to:

A 2 : 1 **B** 1 : 2 **C** 3 : 4 **D** 4 : 3

6C Ext **5** \$750 is divided in the ratio 1 : 3 : 2. The smallest share is:

A \$250 **B** \$125 **C** \$375 **D** \$750

6C Ext **6** The ratio of the areas of two triangles is 5 : 2. The area of the larger triangle is 60 cm^2. What is the area of the smaller triangle?

A 12 cm^2 **B** 24 cm^2 **C** 30 cm^2 **D** 17 cm^2

6E **7** Callum fills his car with 28 litres of petrol at 142.7 cents per litre. His change from \$50 cash is:

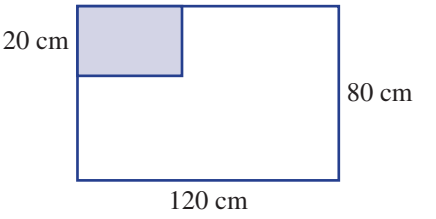
A \$10 **B** \$39.95 **C** \$10.05 **D** \$40

6F **8** 45 km/h is the same as:

A 0.25 m/s **B** 25 m/s **C** 12.5 m/s **D** 75 m/s

6G Ext **9** A flag is created by enlarging the shaded rectangle as shown. What is the length of the original rectangle?

A 20 cm **B** 30 cm
C 40 cm **D** 13 cm



6D **10** On a map, Sydney and Melbourne are 143.2 mm apart. If the cities are 716 km apart, what scale has been used?

A 1 : 5 **B** 1 : 5000 **C** 1 : 50 000 **D** 1 : 5 000 000

Short-answer questions

6A **1** In Lao's pencil case there are 6 coloured pencils, 2 black pens, 1 red pen, 5 textas, 3 lead pencils and a ruler. Find the ratio of:

- **a** lead pencils to coloured pencils
- **b** black pens to red pens
- **c** textas to all pencils

6B **2** True or false?

- **a** 1 : 4 = 3 : 6
- **b** The ratio 2 : 3 is the same as 3 : 2.
- **c** The ratio 3 : 5 is written in simplest form.
- **d** 40 cm : 1 m is written as 40 : 1 in simplest form.
- **e** $1\frac{1}{4} : 2 = 5 : 8$

6B **3** Copy and complete.

- **a** 4 : 50 = 2 : ☐
- **b** 1.2 : 2 = ☐ : 20
- **c** $\frac{2}{3} : 4 = 1 : ☐$
- **d** 1 : ☐ : 5 = 5 : 15 : 25

6B **4** Simplify the following ratios.

- **a** 10 : 40
- **b** 36 : 24
- **c** 75 : 100
- **d** 8 : 64
- **e** 27 : 9
- **f** 5 : 25
- **g** 6 : 4
- **h** 52 : 26
- **i** $6b : 9b$
- **j** $8a : 4$
- **k** $\frac{2}{7} : \frac{5}{7}$
- **l** $1\frac{1}{10} : \frac{2}{10}$
- **m** $2\frac{1}{2} : \frac{3}{4}$
- **n** 12 : 36 : 72

6B **5** Simplify the following ratios by first changing to the same units.

- **a** 2 cm : 8 mm
- **b** 5 mm : 1.5 cm
- **c** 3 L : 7500 mL
- **d** 30 min : 1 h
- **e** 400 kg : 2 t
- **f** 6 h : 1 day
- **g** 120 m : 1 km
- **h** 45 min : $2\frac{1}{2}$ h

6C Ext **6** Divide:

- **a** $80 in the ratio 7 : 9
- **b** 200 kg in the ratio 4 : 1
- **c** 40 m in the ratio 6 : 2
- **d** $1445 in the ratio 4 : 7 : 6
- **e** $1 in the ratio 3 : 1 : 1

6C Ext **7**

- **a** The ratio of the cost price of a TV to its retail price is 5 : 12. If its cost price is $480, calculate its retail price.
- **b** The ratio of Sally's height to Ben's height is 12 : 17. If the difference in their heights is 60 cm, how tall is Sally?
- **c** Orange juice, pineapple juice and guava juice are mixed in the ratio 4 : 3 : 2. If 250 mL of guava juice is used, how many litres of drink does this make?

6E **8** Express each rate in simplest form.

- **a** 10 km in 2 hours
- **b** $650 for 13 hours
- **c** 2800 km in 20 days

Chapter review

6E **9** Copy and complete.

a 400 km on 32 litres = ____ km/L
= ____ L/100 km

b 5 grams in 2 min = ____ g/min
= ____ g/h

c \$1200 in $\frac{1}{2}$ day = ____ \$/day
= ____ \$/h

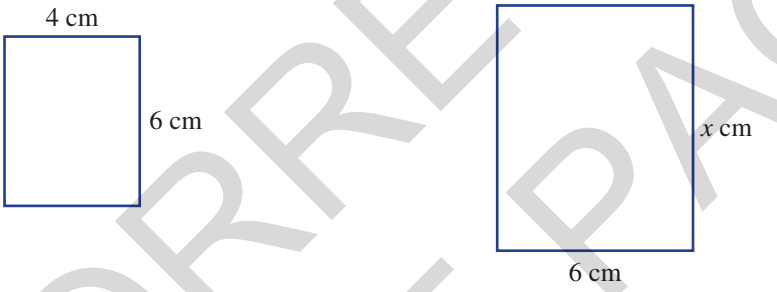
6D **10** For a scale of 1 : 1000, find the real length (in metres) if the scale length is given as:

a 0.0002 m **b** 2.7 cm **c** 140 mm

6D **11** Two cities are 50 km apart. How many millimetres apart are they on a map that has a scale of 1 : 100 000?

6D **12** Find the scale factor for the following pairs of similar figures, and find the value of x.

a



b

6F **13** Copy and complete.

a \$120/h = ____ c/min **b** 6 m/s = ____ km/h **c** 720 km/h = ____ m/s

6G Ext **14** **a** A truck uses 12 litres of petrol to travel 86 km. How far will it travel on 42 litres of petrol?

b Samira earns \$67.20 for a 12-hour shift. How much will she earn for a 7-hour shift?

c Tap 1 fills the pool in 12 hours, while tap 2 fills the same pool in 15 hours. How long does it take to fill this pool if both taps are used?

6H Ext **15** **a** Sandra drives to her mother's house. It takes 45 minutes. Calculate Sandra's average speed if her mother lives 48 km away.

b How long does it take Ari to drive 180 km along the freeway to work if he manages to average 100 km/h for the trip?

c How far does Siri ride his bike if he rides at 4.5 km/h for 90 minutes?

Extended-response questions

1 The Harrison family and the Nguyen family leave Wollongong at 8 a.m. on Saturday morning for a holiday in Melbourne. The Harrisons' 17-year-old son drives for the first 2 hours at 80 km/h. They then stop for a rest of $1\frac{1}{2}$ hours. Mr Harrison drives the rest of the way.

The Nguyen family drives straight to Melbourne with no stops. It takes them 6 hours and 15 minutes to drive the 627 km to Melbourne.

- **a** At what time did the Nguyen family arrive in Melbourne?
- **b** Calculate the average speed of the Nguyen's car. Round your answer to the nearest whole number.
- **c** At what time did the Harrisons resume their journey after their rest stop?
- **d** How many kilometres did the Harrisons still have to cover after their rest break before arriving in Melbourne?
- **e** If the Harrisons arrive in Melbourne at 4:30 p.m., for how long did Mr Harrison drive and at what speed?
- **f** Calculate the cost of the petrol for each family's trip if petrol cost 125c/L and the Harrison car's consumption is 36 L/100 km, while the Nguyen's car uses 40 L/100 km.