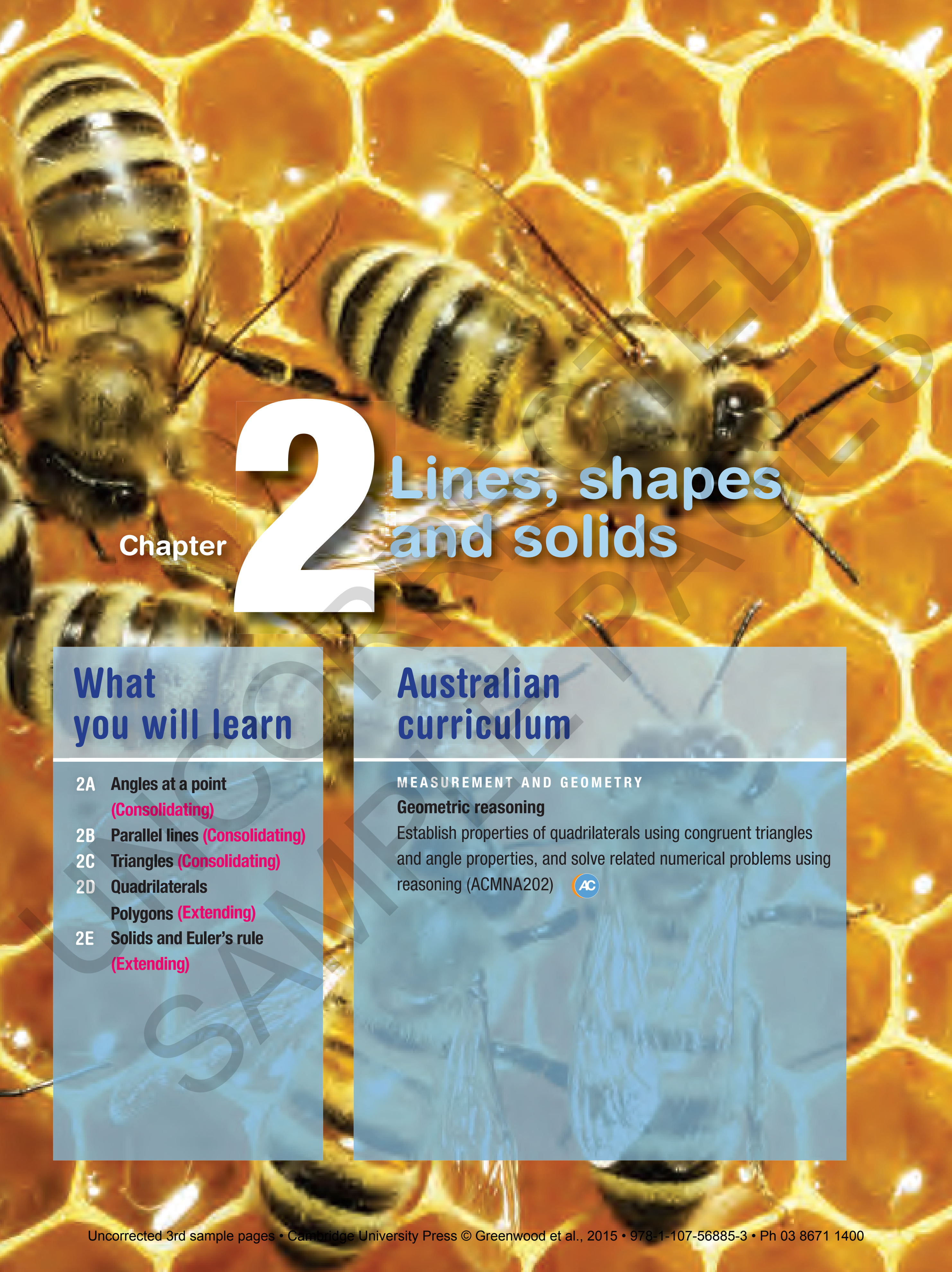


Chapter

2 Lines, shapes and solids

What you will learn

Australian curriculum

MEASUREMENT AND GEOMETRY

Geometric reasoning

Establish properties of quadrilaterals using congruent triangles and angle properties, and solve related numerical problems using reasoning (ACMNA202) AC

Online resources

- Chapter pre-test
- Videos of all worked examples
- Interactive widgets
- Interactive walkthroughs
- Downloadable HOTsheets
- Access to HOTmaths Australian Curriculum courses

The geometry of honey

The cells in honeycomb made by bees are hexagonal in shape, but each cell is not exactly a hexagonal prism. Each cell is actually a dodecahedron (12-faced polyhedron) with 6 rectangular sides (giving the hexagonal appearance) and 3 faces at each end. Each of the end faces is a rhombus and forms angles of 120° with the other sides of the honeycomb cell. The shape of each cell is not just a random arrangement of quadrilateral faces.

Instead, the geometry of the cells allows the cells to fit neatly together to form a very efficient geometrical construction. The structure allows for a minimum surface area for a given volume, maximising the use of space in which to store honey.

2A Angles at a point CONSOLIDATING

Interactive
Widgets
HOTsheets
Walkthroughs

For more than 2000 years, geometry has been based on the work of Euclid, the Greek mathematician who lived in Egypt in about 300 BCE. Before this time, the ancient civilisations had demonstrated and documented an understanding of many aspects of geometry, but Euclid was able to produce a series of 13 books called *Elements*, which contained a staggering 465 propositions. This great work is written in a well-organised and structured form, carefully building on solid mathematical foundations. The most basic of these foundations, called axioms, are fundamental geometric principles from which all other geometry can be built. There are five axioms described by Euclid:

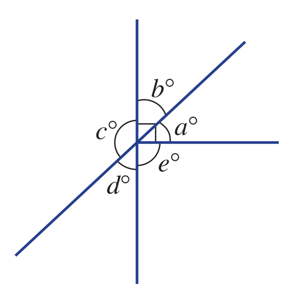
- Any two points can be joined by a straight line.
- Any finite straight line (segment) can be extended in a straight line.
- A circle can be drawn with any centre and any radius.
- All right angles are equal to each other.
- Given a line and a point not on the line, there is only one line through the given point and in the same plane that does not intersect the given line.

Euclid produced a series of 13 books called *Elements*.

These basic axioms are considered to be true without question and do not need to be proven. All other geometrical results can be derived from these axioms.

Let's start: Create a sentence or definition

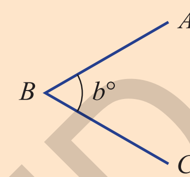
The five pronumerals a, b, c, d and e represent the size of five angles in this diagram. Can you form a sentence using two or more of these pronumerals and one of the following words? Using simple language, what is the meaning of each of your sentences?



- Supplementary
- Adjacent
- Vertically opposite
- Revolution
- Complementary
- Right

Key ideas

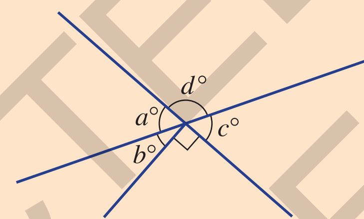
- The angle shown (to the right) could be named $\angle ABC$, $\angle CBA$, $\angle B$ or $A\hat{B}C$.
  - The size of the angle is $b°$.



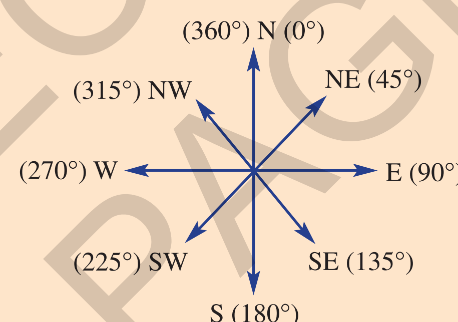
- Types of angles

Acute (0–90°) **Right** (90°) **Obtuse** (90–180°)
Straight (180°) **Reflex** (180–360°) **Revolution** (360°)

- Special pairs of angles at a point include:
  - **Complementary** angles (sum to 90°) $a + b = 90$
  - **Supplementary** angles (sum to 180°) $a + d = 180$
  - **Vertically opposite** angles (are equal) $a = c$



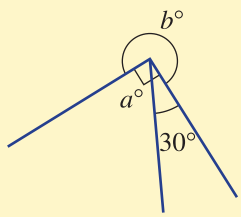
- Angles in a **revolution** sum to 360°.
- Two lines are **perpendicular** if they are at right angles (90°).
- Eight point compass bearing
  - Bearings are usually measured clockwise from north.



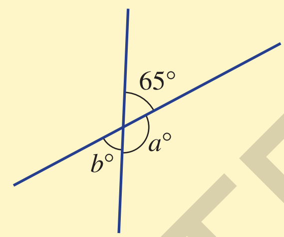
Example 1 Finding angles at a point

Determine the value of the pronumerals in these diagrams.

a



b



SOLUTION

a $a + 30 = 90$

$a = 60$

$b + 90 = 360$

$b = 270$

b $a + 65 = 180$

$a = 115$

$b = 65$

EXPLANATION

$a°$ and 30° are the sizes of two angles which make a complementary pair, adding to 90°.

Angles in a revolution add to 360°.

$a°$ and 65° are the sizes of two angles which make a supplementary pair, adding to 180°.

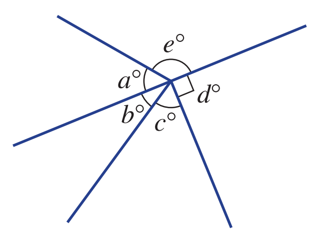
The $b°$ and 65° angles are vertically opposite.

Exercise 2A

1–3	3	—

UNDERSTANDING

1 Complete these sentences for this diagram.



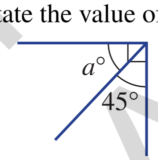
a $b°$ and $c°$ are__________ angles.

b $a°$ and $e°$ are__________ angles.

c $a°$, $b°$, $c°$, $d°$ and $e°$ form a __________.

2 State the value of the pronumeral (letter) in these diagrams.

a



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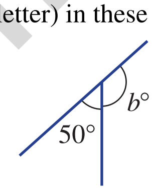
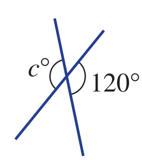
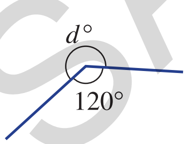


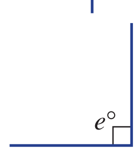
c



d



e



2A

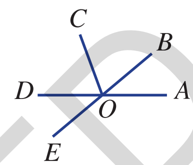
UNDERSTANDING

3 Estimate the size of these angles.

a $\angle AOB$

b $\angle AOC$

c Reflex $\angle AOE$



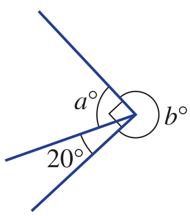
FLUENCY

4–6(½) | 4(½), 5, 6–7(½) | 4–7(½)

Example 1

4 Determine the value of the pronumerals in these diagrams.

a



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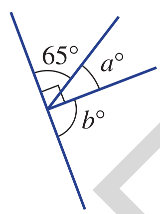

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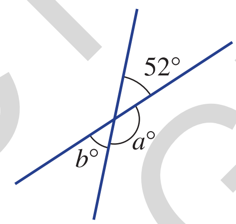

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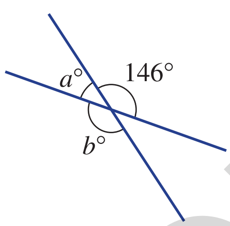

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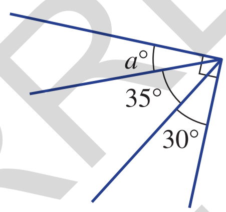

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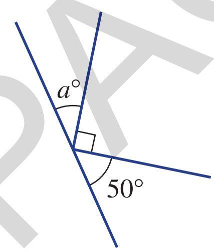

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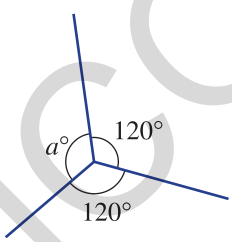

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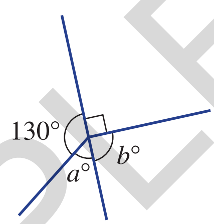
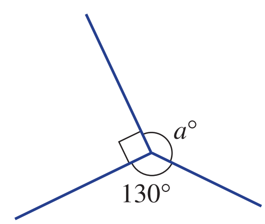


i



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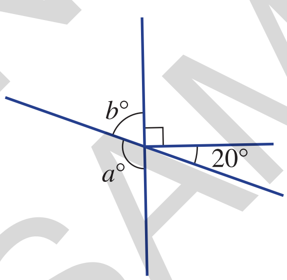

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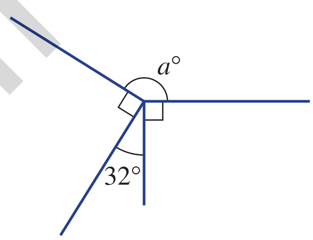

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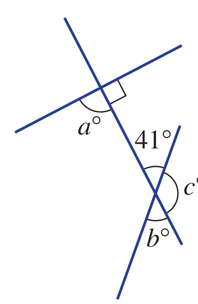


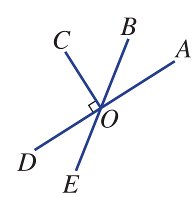
5 For the angles in this diagram, name an angle that is:

a vertically opposite to $\angle AOB$

b complementary to $\angle BOC$

c supplementary to $\angle AOE$

d supplementary to $\angle AOC$



2A

FLUENCY

6 Give the compass bearing, in degrees, for these directions.

a West (W)	b East (E)	c North (N)	d South (S)
e NW	f SE	g SW	h NE

7 In which direction (e.g. north-east or NE) would you be walking if you were headed on these compass bearings?

a 180°	b 360°	c 270°	d 90°
e 45°	f 315°	g 225°	h 135°

PROBLEM-SOLVING

8, 9(½) | 8, 9–10(½) | 9–10(½)

8 A round birthday cake is cut into sectors for nine friends (including Jack) at Jack's birthday party. After the cake is cut there is no cake remaining. What will be the angle at the centre of the cake for Jack's piece if:

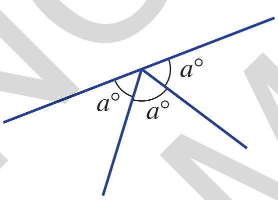
a everyone receives an equal share?

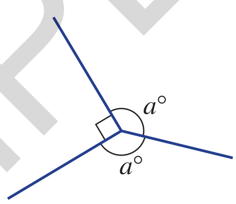
b Jack receives twice as much as everyone else? (In parts **b**, **c** and **d** assume his friends have equal shares of the rest.)

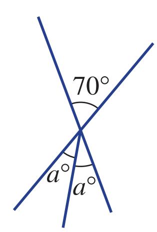
c Jack receives four times as much as everyone else?

d Jack receives ten times as much as everyone else?

9 Find the value of a in these diagrams.

a


b


c


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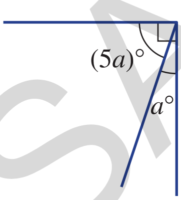

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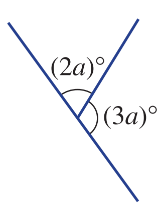

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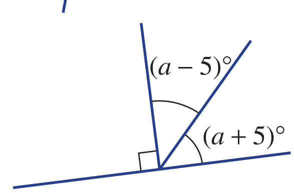


10 What is the angle between the hour hand and minute hand on a clock at these times?

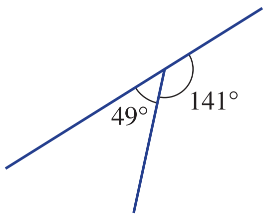
a 2 : 30 p.m.	b 5 : 45 a.m.	c 1 : 40 a.m.	d 10 : 20 p.m.
e 2 : 35 a.m.	f 12 : 05 p.m.	g 4 : 48 p.m.	h 10 : 27 a.m.

11	11, 12	12, 13

REASONING

11 Explain, with reasons, what is wrong with these diagrams.

a



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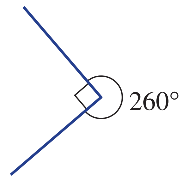

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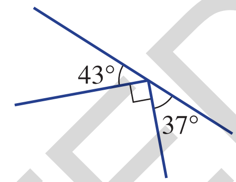
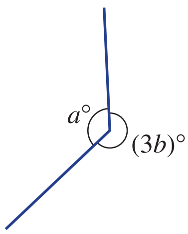


12 Write down an equation (e.g. $2a + b = 90$) for these diagrams.

a



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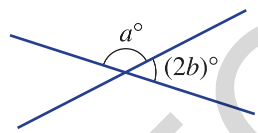

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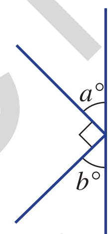
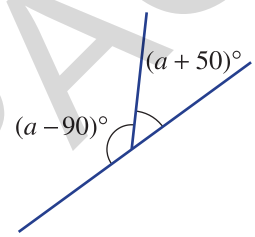


13 Consider this diagram (not drawn to scale).

a Calculate the value of a.

b Explain what is wrong with the way the diagram is drawn.



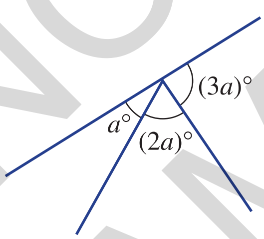
Geometry with equations

—	—	14

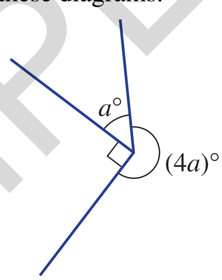
ENRICHMENT

14 Equations can be helpful in solving geometric problems in which more complex expressions are involved. Find the value of a in these diagrams.

a



b



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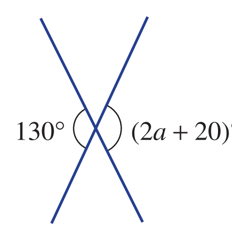

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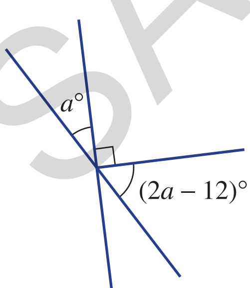

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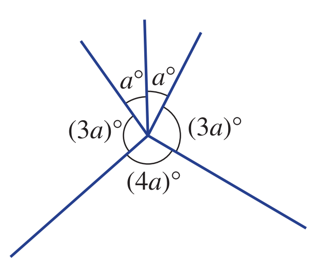

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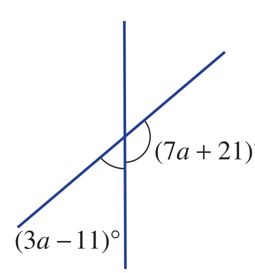
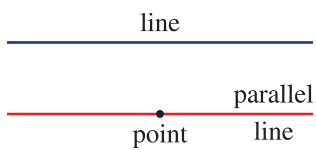


2B Parallel lines CONSOLIDATING

Interactive
Widgets
HOTsheets
Walkthroughs

Euclid's 5th axiom is: Given a line (shown in blue) and a point not on the line, there is only one line (shown in red) through the given point and in the same plane that does not intersect the given line.



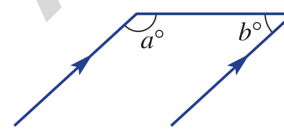
In simple language, Euclid's 5th axiom says that parallel lines do not intersect.


Parallel lines never intersect.

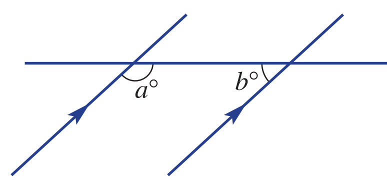
All sorts of shapes and solids both in the theoretical and practical worlds can be constructed using parallel lines. If two lines are parallel and are cut by a third line called a transversal, special pairs of angles are created.

Let's start: Hidden transversals

This diagram can often be found as part of a shape such as a parallelogram or another more complex diagram. To see the relationship between a and b more easily, you can extend the lines to form this second diagram. In this new diagram you can now see the pair of parallel lines and the relationships between all the angles.

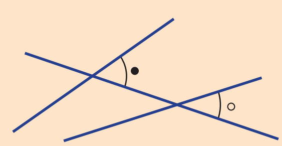


- Copy the new diagram.
- Label each of the eight angles formed with the pronumeral a or b, whichever is appropriate.
- What is the relationship between a and b? Can you explain why?

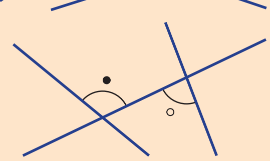


Key ideas

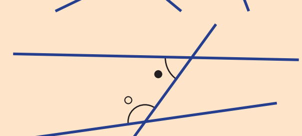
- A **transversal** is a line cutting at least two other lines.
- Pairs of angles formed by transversals can be:
  - **corresponding** (in corresponding positions)



  - **alternate** (on opposite sides of the transversal and inside the other two lines)



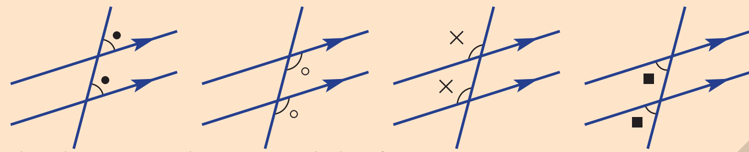
  - **co-interior** (on the same side of the transversal and inside the other two lines).



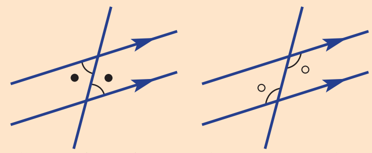
- Lines are parallel if they do not intersect.
  - Parallel lines are marked with the same number of arrows.

Key ideas

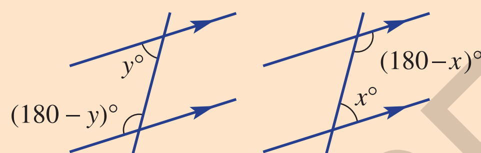
- If two parallel lines are cut by a transversal:
  - the corresponding angles are equal (4 pairs)



  - the alternate angles are equal (2 pairs)



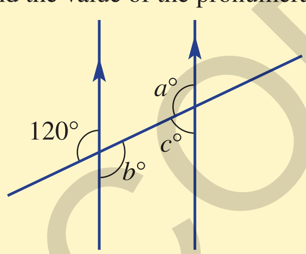
  - the co-interior angles are supplementary (sum to 180°) (2 pairs).



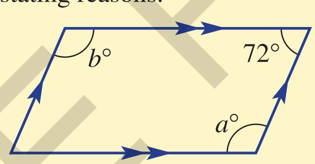
Example 2 Finding angles involving parallel lines

Find the value of the pronumerals in these diagrams, stating reasons.

a



b



SOLUTION	EXPLANATION
a $a = 120$ The angles of size $a°$ and 120° are corresponding and lines are parallel.	Corresponding angles on parallel lines are equal.
$b = 120$ The angles of size $a°$ and $b°$ are alternate and lines are parallel.	Alternatively, the angle of size $b°$ is vertically opposite to the angle marked 120°.
$c + 120 = 180$ $c = 60$	The angles of size $b°$ and $c°$ are co-interior and sum to 180°. Alternatively, look at the angles of size $a°$ and $c°$, which are supplementary.
b $a + 72 = 180$ $a = 108$ $b + 72 = 180$ $b = 108$ Co-interior angles in parallel lines are supplementary.	The pairs of angles are co-interior, which are supplementary if the lines are parallel.

Exercise 2B

1, 2(½)	2(½)	—

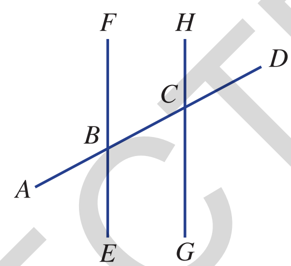
UNDERSTANDING

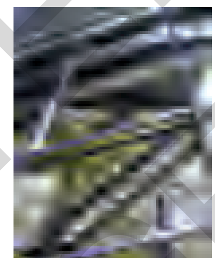
1 Two parallel lines are cut by a transversal. Write the missing word (*equal* or *supplementary*).

- **a** Corresponding angles are_____.
- **b** Co-interior angles are_____.
- **c** Alternate angles are_____.

2 Name the angle that is:

- **a** corresponding to $\angle ABF$
- **b** corresponding to $\angle BCG$
- **c** alternate to $\angle FBC$
- **d** alternate to $\angle CBE$
- **e** co-interior to $\angle HCB$
- **f** co-interior to $\angle EBC$
- **g** vertically opposite to $\angle ABE$
- **h** vertically opposite to $\angle HCB$





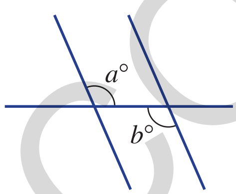
All sorts of shapes and solids can be constructed using parallel lines and transversals.

3–5(½)	3–6(½)	3–6(½)

FLUENCY

3 State whether the following marked angles are corresponding, alternate or co-interior.

a



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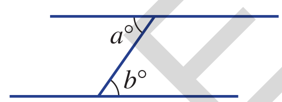
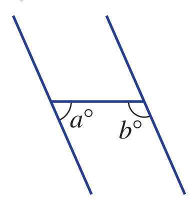
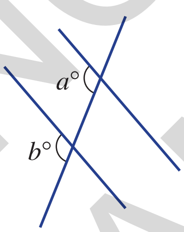


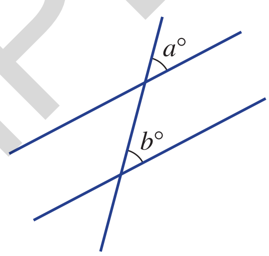
c



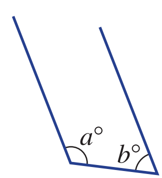
d



e



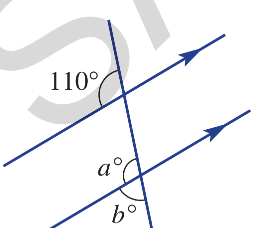
f



Example 2a

4 Find the value of the pronumerals in these diagrams, stating reasons.

a



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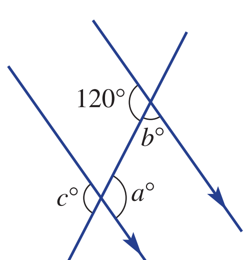
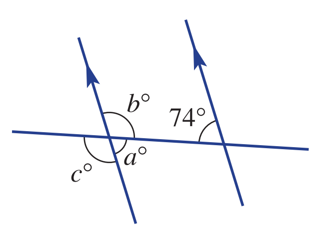
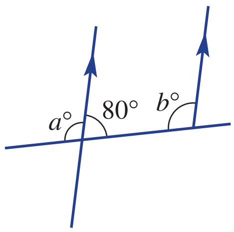


c



d


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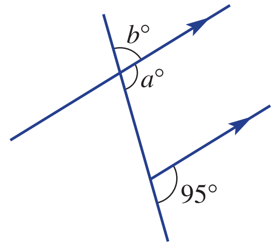

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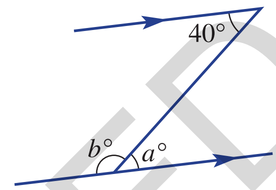
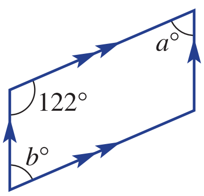
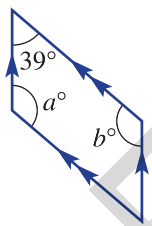
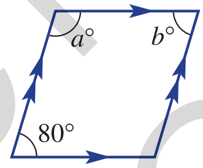


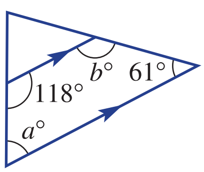
Example 2b

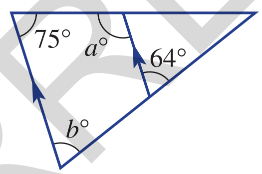
5 Find the value of the pronumerals in these diagrams, stating reasons.

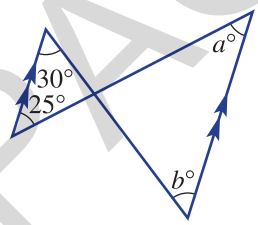
a


b


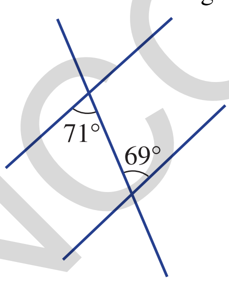
c


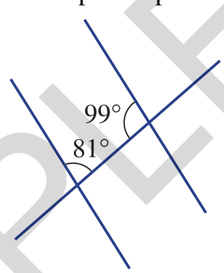
d


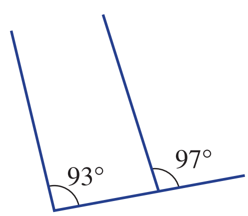
e


f


6 Decide if the following diagrams include a pair of parallel lines. Give a reason for each answer.

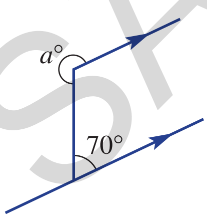
a


b


c


7(½)	7(½)	7

7 Find the value of a in these diagrams.

a


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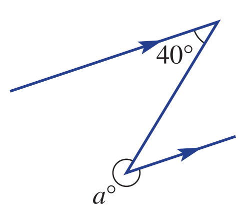

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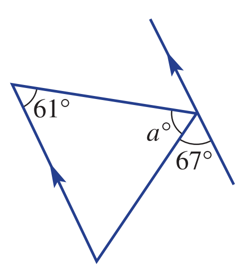
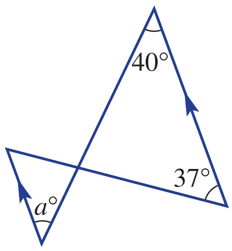
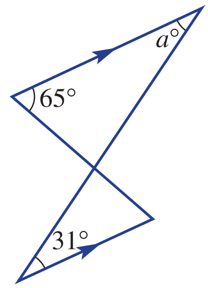


2B

PROBLEM-SOLVING

d


e


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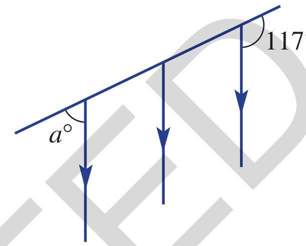
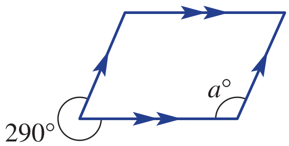
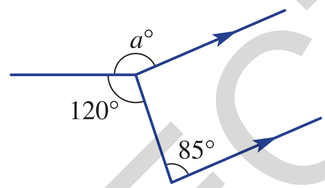
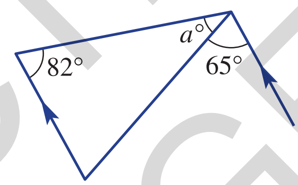


g


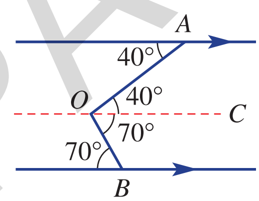
h


i


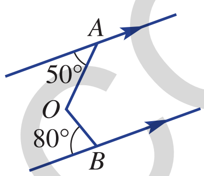
8(½)	8(½), 9	8(½), 9

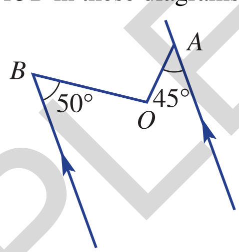
REASONING

8 Sometimes parallel lines can be added to a diagram to help find an unknown angle. For example, $\angle AOB$ can be found in this diagram by first drawing the dashed line and finding $\angle AOC$ (40°) and $\angle COB$ (70°). So $\angle AOB = 40° + 70° = 110°$.



Apply a similar technique to find $\angle AOB$ in these diagrams.

a


b


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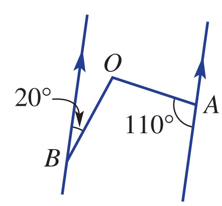
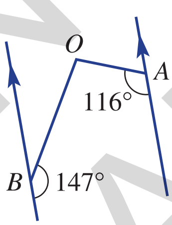
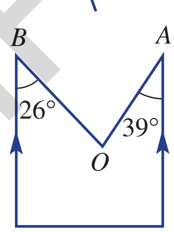
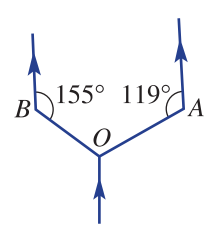
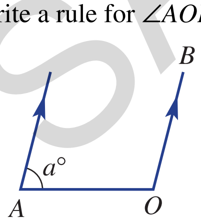


d


e


f


9 Write a rule for $\angle AOB$ using the given pronumerals, e.g. $\angle AOB = (90 - a°)$.

a


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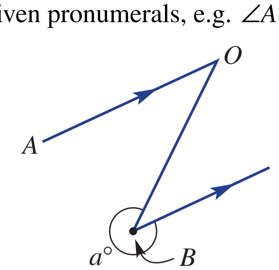

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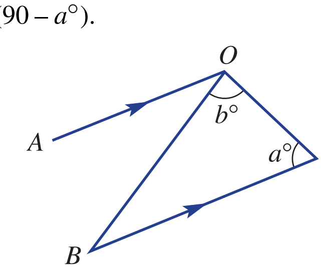
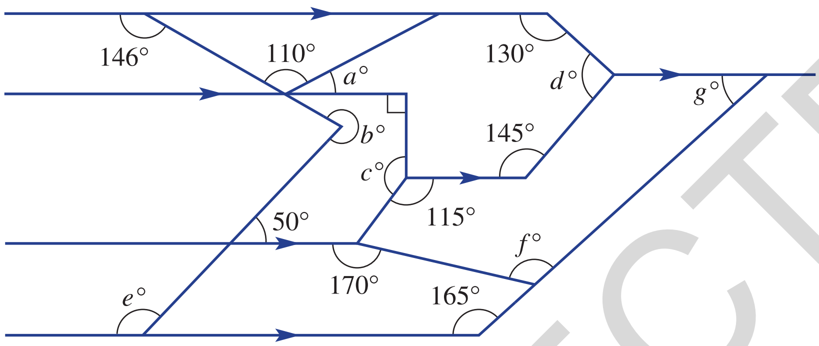


Pipe networks

—	—	10

ENRICHMENT

10 A plan for a natural gas plant includes many intersecting pipe lines some of which are parallel. Help the designers finish the plans by calculating the values of *a*–*g*.



2C Triangles CONSOLIDATING

Interactive
Widgets
HOTsheets
Walkthroughs

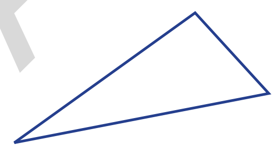
A triangle is a shape with three straight sides. As a real life object, the triangle is a very rigid shape and this leads to its use in the construction of houses and bridges. It is one of the most commonly used shapes in design and construction.

Knowing the properties of triangles can help to solve many geometrical problems and this knowledge can also be extended to explore other more complex shapes.

Triangular shapes are often used to striking effect in architecture, as shown by part of the National Gallery of Canada.

Let's start: Illustrating the angle sum

You can complete this task using a pencil and ruler or using computer dynamic geometry.

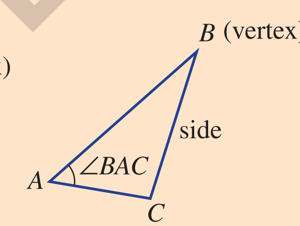


- Draw any triangle and measure each interior angle.
- Add all three angles to find the angle sum of your triangle.
- Compare your angle sum with the results of others. What do you notice?

If dynamic geometry is used, drag one of the vertices to alter the interior angles. Now check to see if your conclusions remain the same.

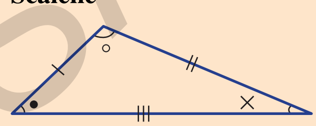
Key ideas

- A **triangle** has:
  - 3 sides
  - 3 vertices (vertices is the plural of vertex)
  - 3 interior angles.

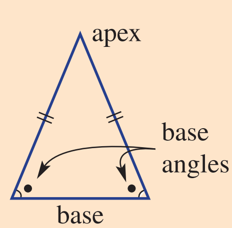


- Triangles classified by side lengths
  - Sides with the same number of dashes are of equal length.

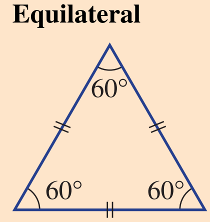
Scalene



Isosceles



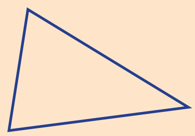
Equilateral



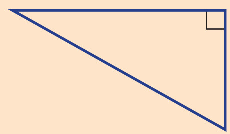
Key ideas

- Triangles classified by interior angles

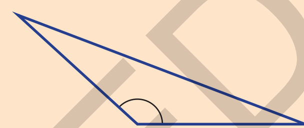
Acute (All angles acute)



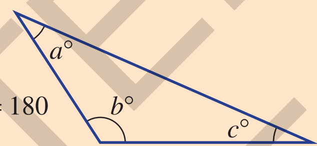
Right (1 right angle)



Obtuse (1 obtuse angle)

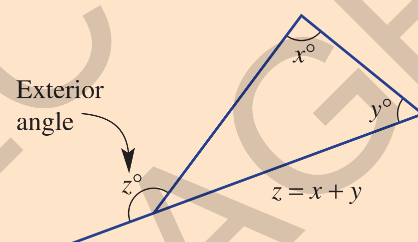


- The angle sum of a triangle is 180°.



$a + b + c = 180$

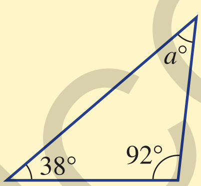
- The **exterior angle theorem:**
The exterior angle of a triangle is equal to the sum of the two opposite interior angles.



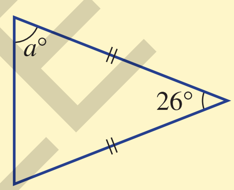
Example 3 Using the angle sum of a triangle

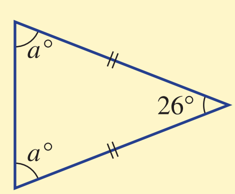
Find the value of a in these triangles.

a



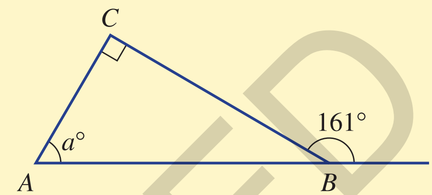
b



SOLUTION	EXPLANATION
a $a + 38 + 92 = 180$ $a = 180 - 130$ $= 50$	The angle sum of the three interior angles of a triangle is 180°. Also $38 + 92 = 130$ and $180 - 130 = 50$.
b $2a = 180 - 26$ $= 154$ $\therefore a = 77$	The two base angles in an isosceles triangle are equal. 

Example 4 Using the exterior angle theorem

Find the value of a in this diagram.



SOLUTION	EXPLANATION
$a + 90 = 161$ $\therefore a = 71$	Use the exterior angle theorem for a triangle. The exterior angle (161°) is equal to the sum of the two opposite interior angles.
or $\angle ABC = 180° - 161° = 19°$ so $a = 180 - (19 + 90)$ $= 71$	Alternatively find $\angle ABC$ (19°), then use the triangle angle sum to find the value of a.

Exercise 2C

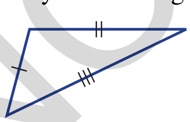
1–3	2, 3	—

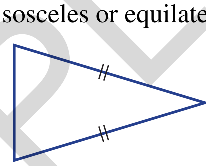
UNDERSTANDING

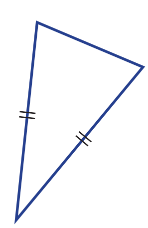
1 Give the common name of a triangle with these properties.

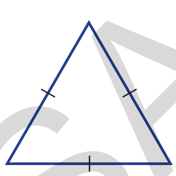
- **a** One right angle
- **b** 2 equal side lengths
- **c** All angles acute
- **d** All angles 60°
- **e** One obtuse angle
- **f** 3 equal side lengths
- **g** 2 equal angles
- **h** 3 different side lengths

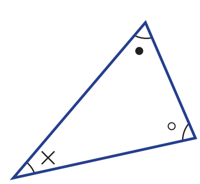
2 Classify these triangles as scalene, isosceles or equilateral.

a


b


c


d


e


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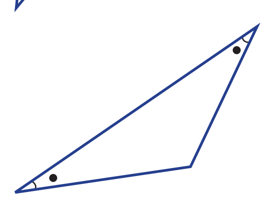
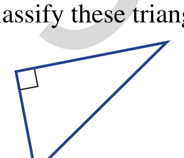


3 Classify these triangles as acute, right or obtuse.

a


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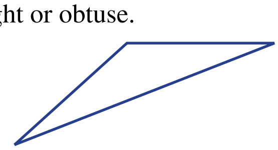
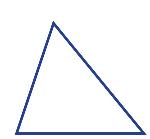


c


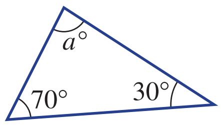
4–6, 7(½) | 4(½), 5–6, 7(½) | 4(½), 5–6, 7(½)

FLUENCY

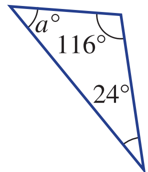
Example 3a

4 Use the angle sum of a triangle to help find the value of *a* in these triangles.

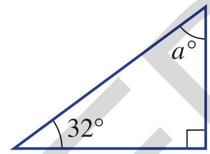
a



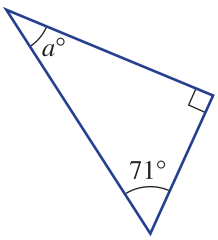
b



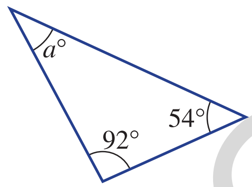
c



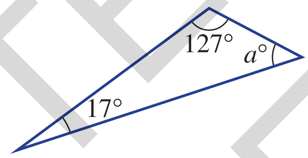
d



e

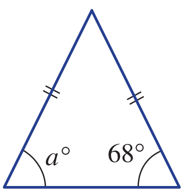


f

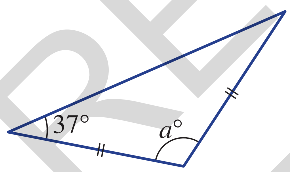


5 These triangles are isosceles. Find the value of *a*.

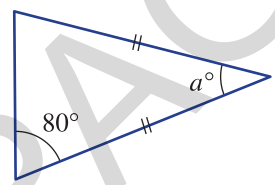
a



b



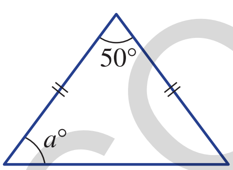
c



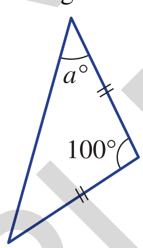
Example 3b

6 Find the value of *a* in these isosceles triangles.

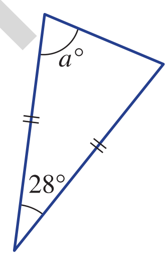
a



b



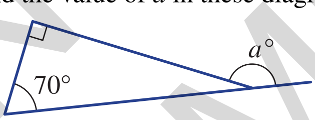
c



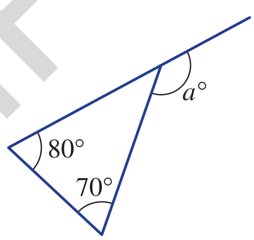
Example 4

7 Find the value of *a* in these diagrams.

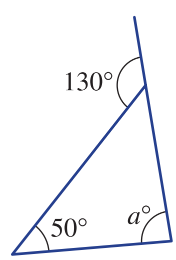
a



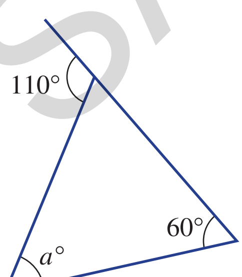
b



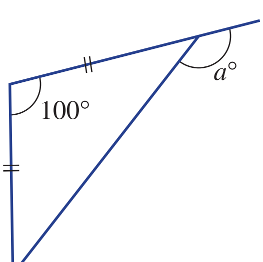
c



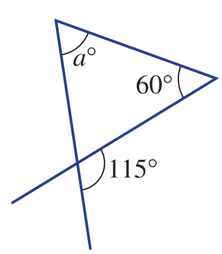
d



e



f



2C

8, 9 | 8, 9 | 9, 10

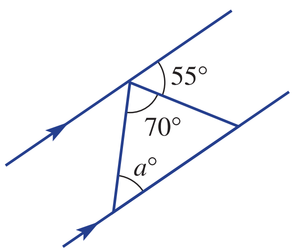
PROBLEM-SOLVING

8 Decide if it is possible to draw a triangle with the given description. Draw a diagram to support your answer.

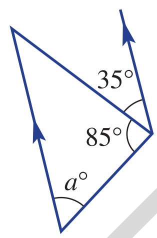
- **a** Right and scalene
- **b** Obtuse and equilateral
- **c** Right and isosceles
- **d** Acute and isosceles
- **e** Acute and equilateral
- **f** Obtuse and isosceles

9 Use your knowledge of parallel lines and triangles to find the unknown angle a.

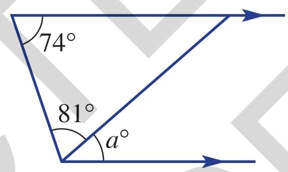
a



b

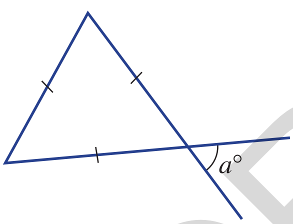


c

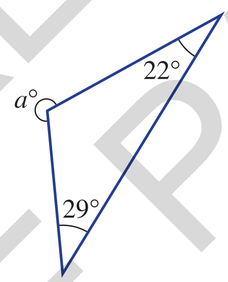


10 Find the value of a in these diagrams.

a



b



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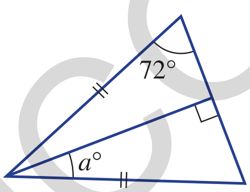
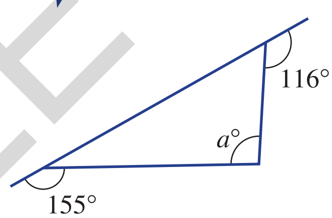
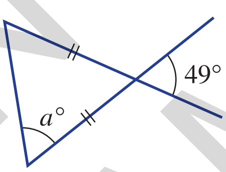


d



e



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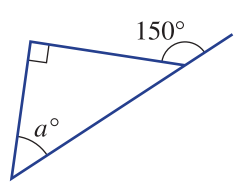
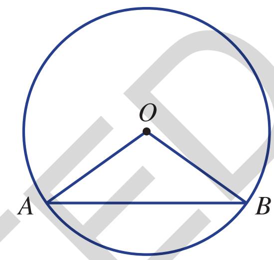


11	11, 12	12–15

REASONING

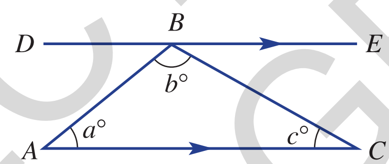
11 A triangle is constructed using a circle and two radius lengths.



a What type of triangle is ΔAOB and why?
b Name two angles that are equal.
c Find $\angle ABO$ if $\angle BAO$ is 30°.
d Find $\angle AOB$ if $\angle OAB$ is 36°.
e Find $\angle ABO$ if $\angle AOB$ is 100°.

12 To prove that the angle sum of a triangle is 180°, work through these steps with the given diagram.

a Using the pronumerals a, b or c, give the value of these angles and state a reason.
i $\angle ABD$ ii $\angle CBE$

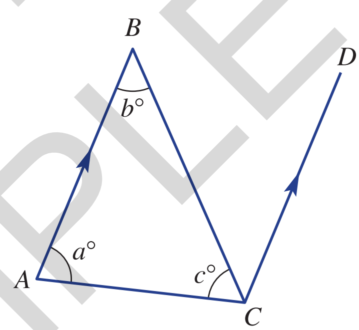


b What is true about the three angles $\angle ABD$, $\angle ABC$ and $\angle CBE$ and why?
c What do parts **a** and **b** above say about the pronumerals a, b and c, and what does this say about the angle sum of the triangle ABC?

13 Prove that a triangle cannot have two right angles.

14 Prove that an equilateral triangle must have 60° angles.

15 A different way of proving the angle sum of a triangle is to use this diagram.

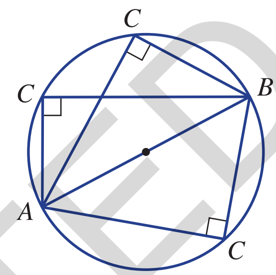


a Give a reason why $\angle BCD = b^\circ$.
b What do you know about the two angles $\angle BAC$ and $\angle ACD$ and why?
c What do parts **a** and **b** above say about the pronumerals a, b and c, and what does this say about the triangle ABC?

2C **Angle in a semicircle** — — 16

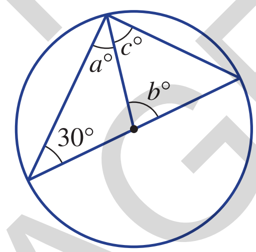
ENRICHMENT

16 The angle sum of a triangle can be used to prove other theorems, one of which relates to the angle in a semicircle. This theorem says that $\angle ACB$ in a semicircle is always 90° where AB is a diameter.

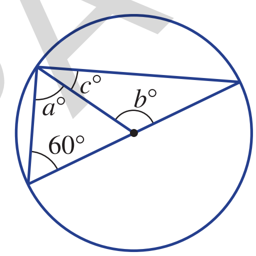


a Use your knowledge of isosceles triangles to find the value of a, b and c in this circle.

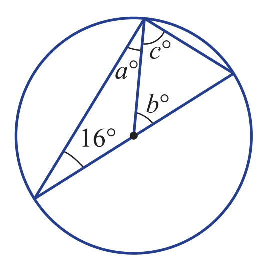
b What do you notice about the sum of the values of a and c?



c Repeat parts **a** and **b** above for this circle.



d Repeat parts **a** and **b** above for this circle.

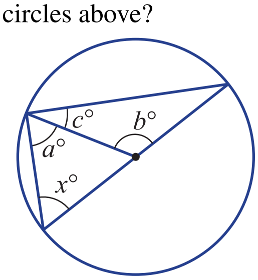


e What do you notice about the sum of the values of a and c for all the circles above?

f Prove this result generally by finding:

i a, b and c in terms of x

ii the value of $a + c$.



2D Quadrilaterals

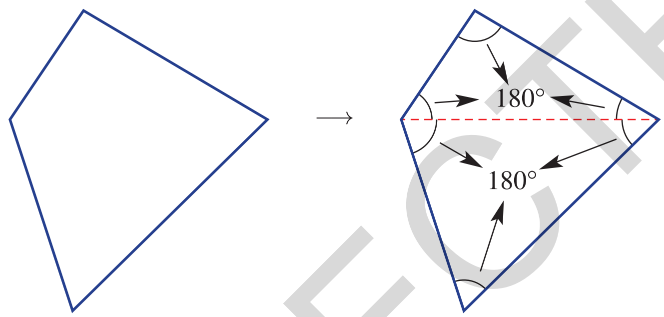
Interactive

Widgets

HOTsheets

Walkthroughs

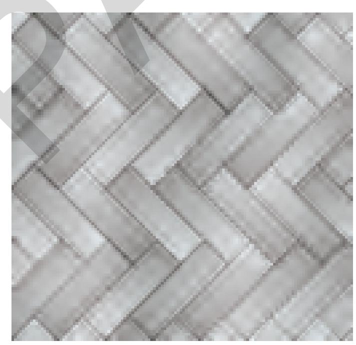
Quadrilaterals are four-sided shapes with four interior angles. All quadrilaterals have the same angle sum, but other properties depend on such things as pairs of sides of equal length, parallel sides and lengths of diagonals. All quadrilaterals can be drawn as two triangles and, since the six angles inside the two triangles make up the four angles of the quadrilateral, the angle sum is $2 \times 180^\circ = 360^\circ$.



Let's start: Which quadrilateral?

Name all the different quadrilaterals you can think of that have the properties listed below. There may be more than one quadrilateral for each property listed. Draw each quadrilateral to illustrate the shape and its features.

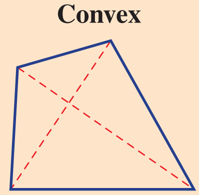
- 4 equal length sides
- 2 pairs of parallel sides
- Equal length diagonals
- 1 pair of parallel sides
- 2 pairs of equal length sides
- 2 pairs of equal opposite angles



Key ideas

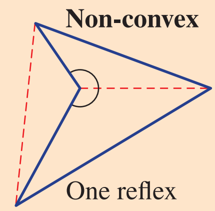
- **Quadrilaterals** can be convex or non-convex.
  - Convex quadrilaterals have all vertices pointing outwards.
    - The diagonals of a convex quadrilateral lie inside the figure.
  - Non-convex (or concave) quadrilaterals have one vertex pointing inwards.

Convex



All interior angles less than 180°

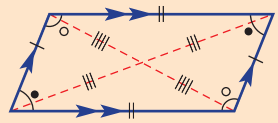
Non-convex



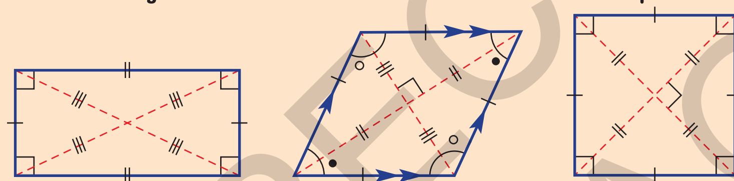
One reflex interior angle

Key ideas

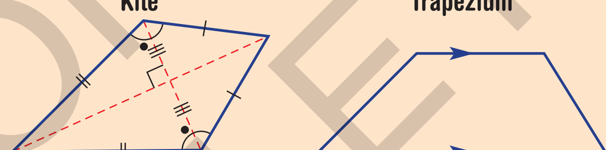
- Parallelograms are quadrilaterals with two pairs of parallel sides.
  - Other properties are illustrated in this diagram.



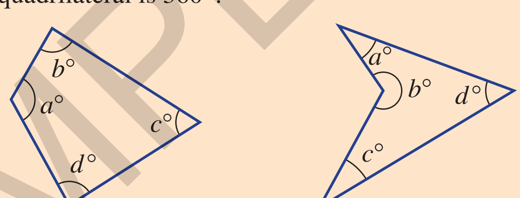
- Special parallelograms include:
  - Rectangle: Parallelogram with all angles 90.
  - Rhombus: Parallelogram with all sides equal.
  - Square: Rhombus with all angles 90°.



- Other special quadrilaterals include:
  - Kite: Quadrilateral with two adjacent pairs of equal sides.
  - Trapezium: Quadrilateral with at least one pair of parallel sides.

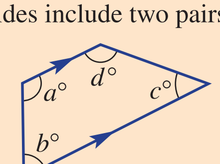


- The angle sum of any quadrilateral is 360°.



$a + b + c + d = 360$

- Quadrilaterals with parallel sides include two pairs of co-interior angles.



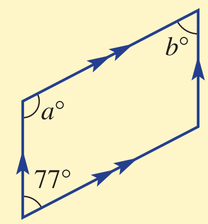
$c + d = 180$

$a + b = 180$

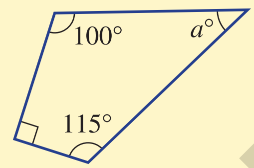
Example 5 Using the angle sum of a quadrilateral

Find the value of the pronumerals in these quadrilaterals.

a



b



SOLUTION	EXPLANATION
a $a + 77^\circ = 180$	Two angles inside parallel lines are co-interior and therefore sum to 180°.
$a = 103$	
$b = 77$	Opposite angles in a parallelogram are equal.
b $a + 100 + 90 + 115 = 360$	The sum of angles in a quadrilateral is 360°.
$a = 360 - 305$	
$= 55$	

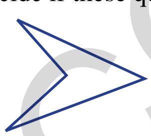
Exercise 2D

1, 2	2	—

UNDERSTANDING

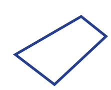
1 Decide if these quadrilaterals are convex or non-convex.

a



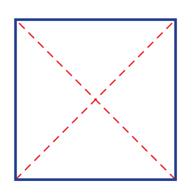
b

c



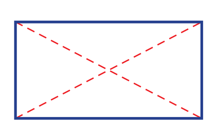
2 Refer to the diagrams in the **Key ideas** or accurately draw your own shapes, including the diagonals, to answer true or false to these statements.

a Square



- **i** All sides are of equal length.
- **ii** Diagonals are not equal in length.
- **iii** All sides are parallel to each other.
- **iv** Diagonals intersect at right angles.

b Rectangle

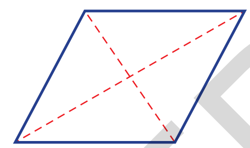


- **i** Diagonals always intersect at right angles.
- **ii** All interior angles are 90°.
- **iii** All sides are always of equal length.
- **iv** There are two pairs of parallel sides.

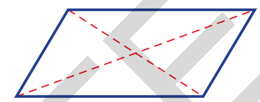
2D

UNDERSTANDING

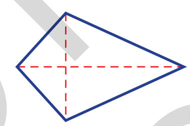
c Rhombus
- **i** All interior angles are always equal.
- **ii** All sides are of equal length.
- **iii** Diagonals intersect at right angles.



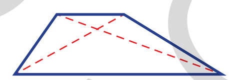
d Parallelogram
- **i** There are two pairs of equal length and parallel sides.
- **ii** Diagonals are always equal in length.
- **iii** Diagonals always intersect at right angles.



e Kite
- **i** There are two pairs of sides of equal length.
- **ii** There are always two pairs of parallel sides.
- **iii** Diagonals intersect at right angles.



f Trapezium
- **i** Diagonals are always equal in length.
- **ii** There are always two pairs of parallel sides.



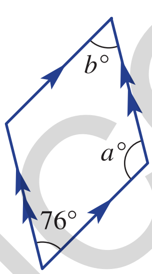
3, 4	3, 4(½), 5	3, 4(½), 5

FLUENCY

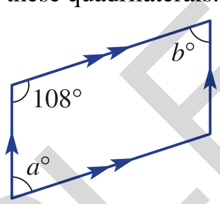
Example 5a

3 Find the value of the pronumerals in these quadrilaterals.

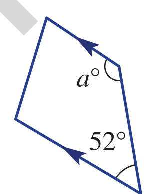
a



b



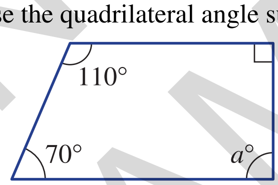
c



Example 5b

4 Use the quadrilateral angle sum to find the value of a in these quadrilaterals.

a



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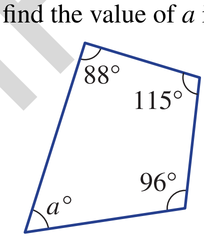

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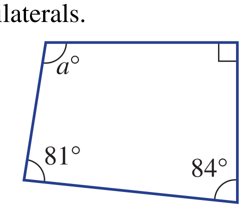

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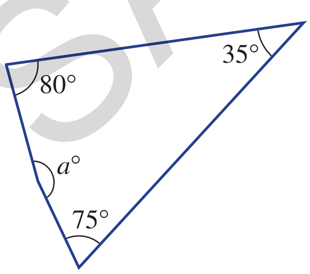

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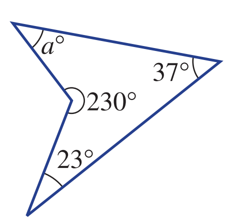
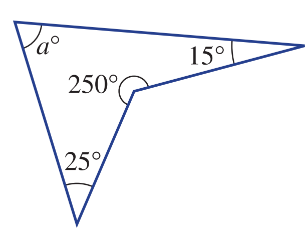


f

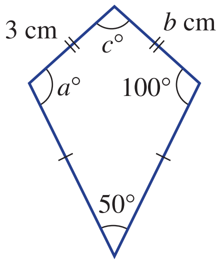


2D

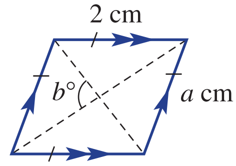
FLUENCY

5 By considering the properties of the given quadrilaterals, give the values of the pronumerals.

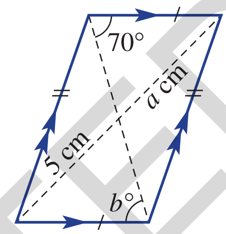
a



b



c

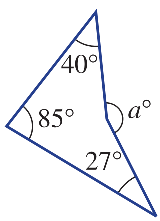


6	6	6(½), 7

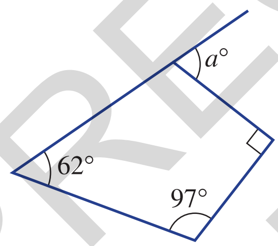
PROBLEM-SOLVING

6 Use your knowledge of geometry from the previous sections to find the values of a.

a



b



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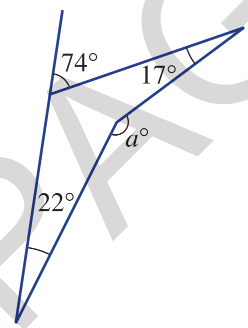
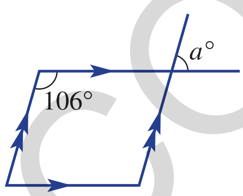
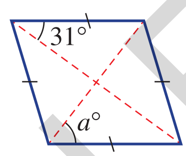


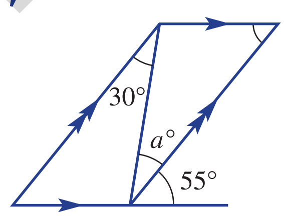
d



e

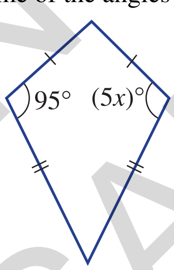


f

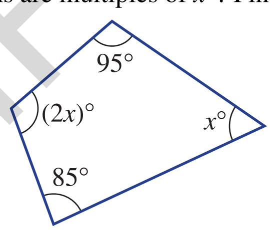


7 Some of the angles in these diagrams are multiples of $x°$. Find the value of x in each case.

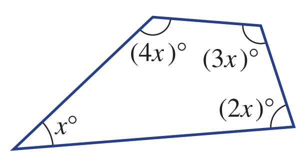
a



b



c



2D

8	8, 9	9, 10

REASONING

8 The word ‘bisect’ means to cut in half.

a Which quadrilaterals have diagonals that bisect each other?

b Which quadrilaterals have diagonals that bisect all their interior angles?

9 By considering the properties of special quadrilaterals, decide if the following are always true.

a A square is a type of rectangle.

b A rectangle is a type of square.

c A square is a type of rhombus.

d A rectangle is a type of parallelogram.

e A parallelogram is a type of square.

f A rhombus is a type of parallelogram.

10 Is it possible to draw a non-convex quadrilateral with two or more interior reflex angles? Explain and illustrate.

Quadrilateral Proofs

—	—	11

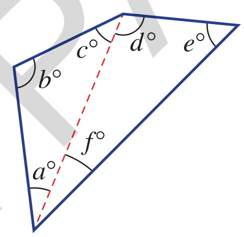
ENRICHMENT

11 Complete these proofs of two different angle properties of quadrilaterals.

a Angle sum = $a + b + c +$ _______ + _______ + _______

= _______ + _______(angle sum of a triangle)

= _______

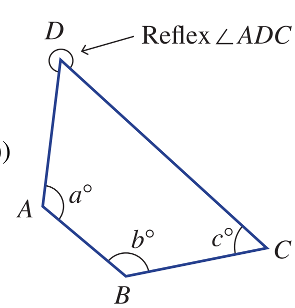


b $\angle ADC = 360° -$ (_______ + _______ + _______) (angle sum of a quadrilateral)

Reflex $\angle ADC = 360° - \angle ADC$

$= 360° - (360° -$ (_______ + _______ + _______))

= _______ + _______ + _______



2E Polygons EXTENDING

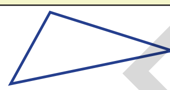
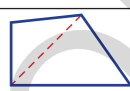
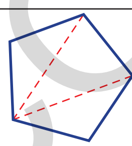
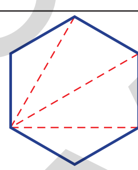
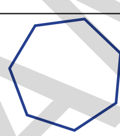
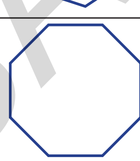
Interactive
Widgets
HOTsheets
Walkthroughs

The word 'polygon' comes from the Greek words *poly*, meaning 'many', and *gonia*, meaning 'angles'. The number of interior angles equals the number of sides and the angle sum of each type of polygon depends on this number. Also, there exists a general rule for the angle sum of a polygon with n sides, which we will explore in this section.

The Pentagon is a famous government office building in Washington, USA.

Let's start: Developing the rule

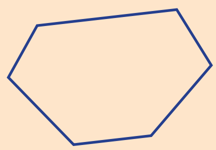
The following procedure uses the fact that the angle sum of a triangle is 180°, which was proved in an earlier section. Complete the table and try to write in the final row the general rule for the angle sum of a polygon.

Shape		Number of sides	Number of triangles	Angle sum
Triangle		3	1	1 × 180° = 180°
Quadrilateral		4	2	____ × 180° = ____
Pentagon		5		
Hexagon		6		
Heptagon		7		
Octagon		8		
n-sided polygon		n		(_____) × 180°

Key ideas

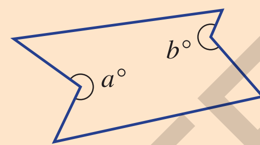
- **Polygons** are shapes with straight sides and can be convex or non-convex.
  - Convex polygons have all vertices pointing outwards.
  - Non-convex (or concave) polygons have at least one vertex pointing inwards.

Convex



All interior angles
less than 180°

Non-convex

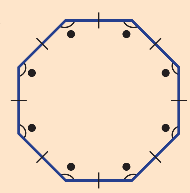


At least one reflex
interior angle

- Polygons are named according to their number of sides.

Number of sides	Name
3	Triangle
4	Quadrilateral
5	Pentagon
6	Hexagon
7	Heptagon
8	Octagon
9	Nonagon
10	Decagon
11	Undecagon
12	Dodecagon

- The angle sum S of a polygon with n sides is given by the rule:
  $S = (n - 2) \times 180°$.
- A **regular polygon** has sides of equal length and equal interior angles.



Regular
octagon

Example 6 Finding the angle sum

Find the angle sum of a heptagon.

SOLUTION

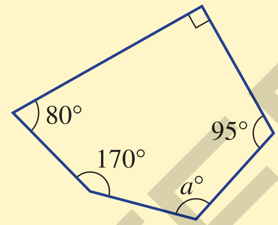
$S = (n - 2) \times 180°$
$= (7 - 2) \times 180°$
$= 5 \times 180°$
$= 900°$

EXPLANATION

A heptagon has 7 sides so $n = 7$.
Simplify $(7 - 2)$ before multiplying by 180°.

Example 7 Finding angles in polygons

Find the value of a in this pentagon.



SOLUTION

$S = (n - 2) \times 180^\circ$
$= (5 - 2) \times 180^\circ$
$= 540^\circ$

$a + 170 + 80 + 90 + 95 = 540$
$a + 435 = 540$
$a = 105$

EXPLANATION

First calculate the angle sum of a pentagon using $n = 5$.

Sum all the angles and set this equal to the angle sum of 540°. The difference between 540 and 435 is 105.

Example 8 Finding interior angles of regular polygons

Find the size of an interior angle in a regular octagon.

SOLUTION

$S = (n - 2) \times 180^\circ$
$= (8 - 2) \times 180^\circ$
$= 1080^\circ$

Angle size $= 1080^\circ \div 8$
$= 135^\circ$

EXPLANATION

First calculate the angle sum of a octagon using $n = 8$.

All 8 angles are equal in size so divide the angle sum by 8.

Exercise 2E

1–4	4	—

UNDERSTANDING

1 State the number of sides on these polygons.
a Hexagon
b Quadrilateral
c Decagon
d Heptagon
e Pentagon
f Dodecagon

2 Evaluate $(n - 2) \times 180^\circ$ if:
a $n = 6$
b $n = 10$
c $n = 22$

3 What is the common name given to these polygons?
a Regular quadrilateral
b Regular triangle

2E

UNDERSTANDING

4 Regular polygons have equal interior angles. Find the size of an interior angle for these regular polygons with the given angle sum.

a Pentagon (540°) **b** Decagon (1440°) **c** Octagon (1080°)

FLUENCY

5–7(½)	5–6(½), 7	5–7(½)

Example 6

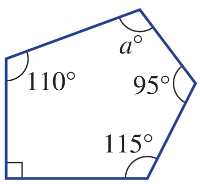
5 Find the angle sum of these polygons.

a Hexagon **b** Nonagon **c** Heptagon
d 15-sided polygon **e** 45-sided polygon **f** 102-sided polygon

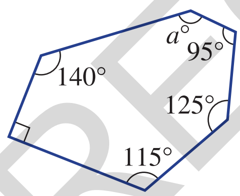
Example 7

6 Find the value of a in these polygons.

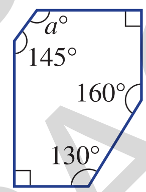
a



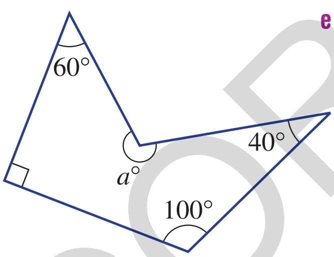
b



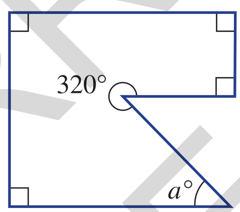
c



d



e



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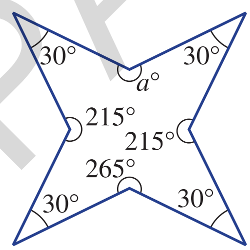


Example 8

7 Find the size of an interior angle of these regular polygons. Round the answer to one decimal place where necessary.

a Regular pentagon **b** Regular heptagon
c Regular undecagon **d** Regular 32-sided polygon

PROBLEM-SOLVING

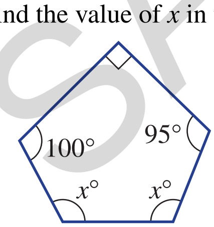
8	8, 9(½)	8, 9(½)

8 Find the number of sides of a polygon with the given angle sums.

a 1260° **b** 2340° **c** 3420° **d** 29 700°

9 Find the value of x in these diagrams.

a



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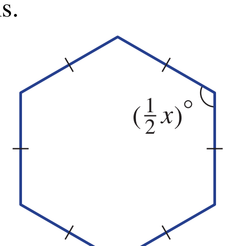
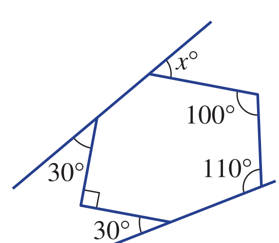
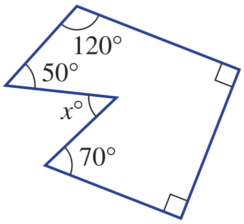


c



2E

PROBLEM-SOLVING

d


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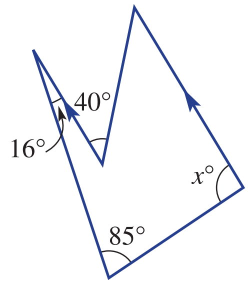

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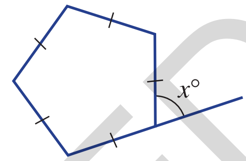


REASONING

10	10	10, 11

10 Consider a regular polygon with a very large number of sides (n).

- **a** What shape does this polygon look like?
- **b** Is there a limit to the size of a polygon angle sum or does it increase to infinity as n increases?
- **c** What size does each interior angle approach as n increases?

11 Let S be the angle sum of a regular polygon with n sides.

- **a** Write a rule for the size of an interior angle in terms of S and n.
- **b** Write a rule for the size of an interior angle in terms of n only.
- **c** Use your rule to find the size of an interior angle of these polygons. Round to two decimal places where appropriate.
  - **i** Regular dodecagon
  - **ii** 82-sided regular polygon

Unknown challenges

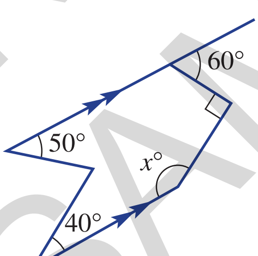
ENRICHMENT

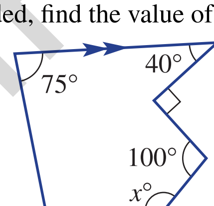
—	—	12

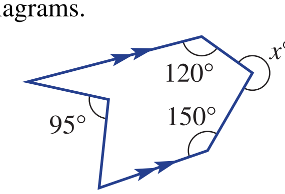
12 Find the number of sides of a regular polygon if each interior angle is:

- **a** 120°
- **b** 162°
- **c** 147.272727…°

13 With the limited information provided, find the value of x in these diagrams.

a


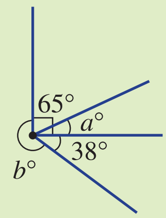
b


c


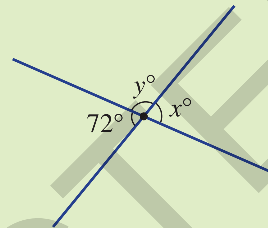
Progress quiz

2A/B **1** Determine the value of the pronumerals in these diagrams.

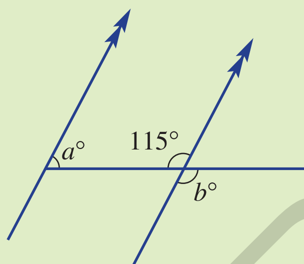
a



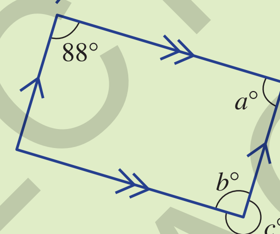
b



c



d

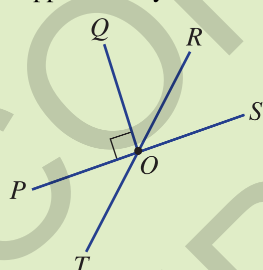


2A **2** For the angles in this diagram name the angle that is:

a vertically opposite to $\angle ROS$

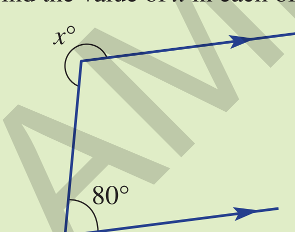
b complementary to $\angle QOR$

c supplementary to $\angle POT$

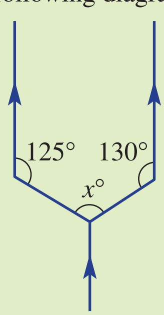


2B **3** Find the value of x in each of the following diagrams.

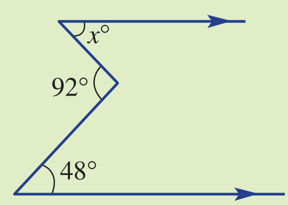
a



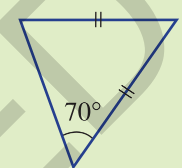
b



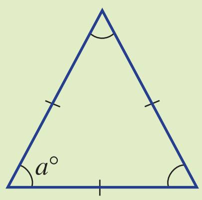
c

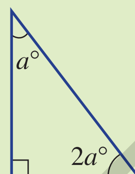


4 Choose two words from: Isosceles, scalene, equilateral, obtuse, acute and right, to describe this triangle.



2C **5** Find the value of a in each of the following diagrams.

a


b


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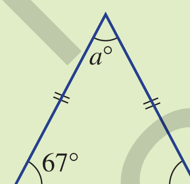
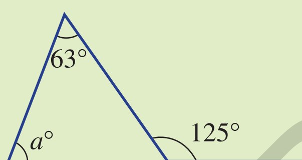
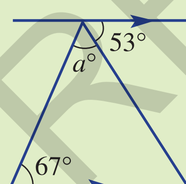
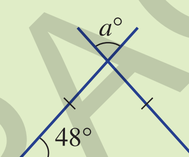
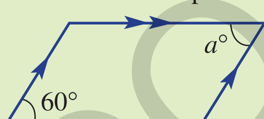


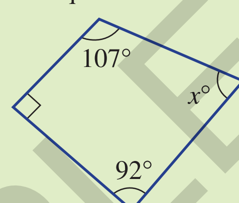
d


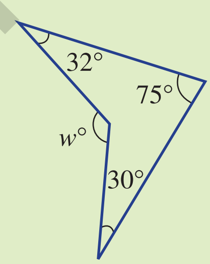
e


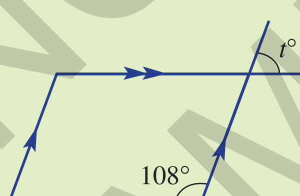
f


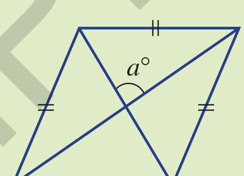
2D **6** Find the value of the pronumerals in these quadrilaterals.

a


b


c


d


e


2F Solids and Euler's rule EXTENDING

Interactive Widgets HOTsheets Walkthroughs

A solid occupies three-dimensional space and can take on all sorts of shapes. The outside surfaces could be flat or curved and the number of surfaces will vary depending on the properties of the solid. A solid with all flat surfaces is called a polyhedron, plural *polyhedra* or *polyhedrons*. The word 'polyhedron' comes from the Greek words *poly*, meaning 'many', and *hedron*, meaning 'faces'.

The top of this Canary Wharf building in London (left) is a large, complex polyhedron. Polyhedra also occur in nature, particularly in rock or mineral crystals such as quartz (right).

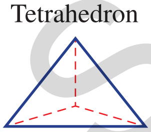
Let's start: Developing Euler's rule

Create a table with each polyhedron listed below in its own row in column 1 (see below). Include the name and a drawing of each polyhedron. Add columns to the table for faces (F), vertices (V), edges (E) and faces plus vertices added together ($F + V$).

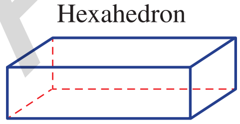
Polyhedron	Drawing	Faces (F)	Vertices (V)	Edges (E)	$F + V$

Count the faces, vertices and edges for each polyhedron and record the numbers in the table.

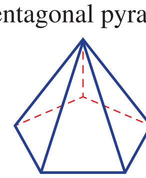
Tetrahedron



Hexahedron



Pentagonal pyramid

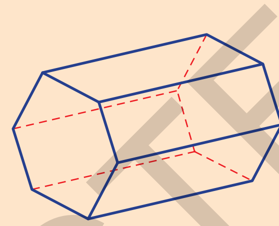


- What do you notice about the numbers in the columns for E and $F + V$?
- What does this suggest about the variables F, V and E? Can you write a rule?
- Add rows to the table, draw your own polyhedra and test your rule by finding the values for F, V and E.

Key ideas

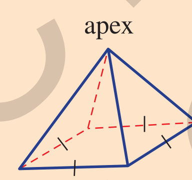
- A **polyhedron** (plural: polyhedra) is a closed solid with flat surfaces (faces), vertices and edges.
  - Polyhedra can be named by their number of faces.
    For example, tetrahedron (4 faces), pentahedron (5 faces) and hexahedron (6 faces).
- **Euler's rule** for polyhedra with F faces, V vertices and E edges is given by:
  $E = F + V - 2$
- **Prisms** are polyhedra with two identical (congruent) ends. The congruent ends define the **cross-section** of the prism and also its name. The other faces are parallelograms. If these faces are rectangles, as shown, then the solid is called a **right prism**.

Hexagonal prism



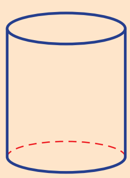
- **Pyramids** are polyhedra with a base face and all other triangular faces meeting at the same vertex point called the apex. They are named by the shape of the base.

Square pyramid

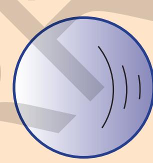


- Some solids have **curved** surfaces. Common examples include:

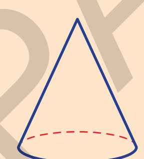
Cylinder



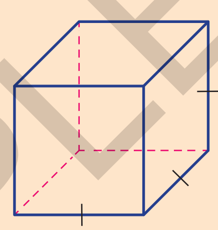
Sphere



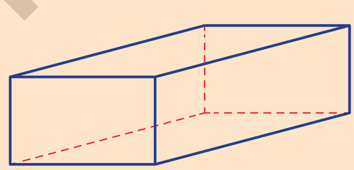
Cone



- A cube is a hexahedron with six square faces.



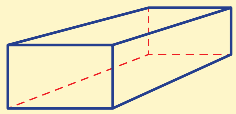
- A cuboid is a common name used for a rectangular prism.



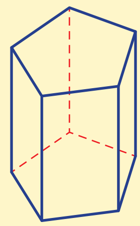
Example 9 Classifying solids

a Classify these solids by considering the number of faces, e.g. octahedron.

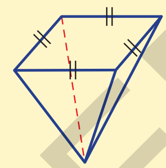
i



ii



iii



b Name these solids as a type of prism or pyramid, e.g. hexagonal prism or hexagonal pyramid.

SOLUTION	EXPLANATION
a **i** Hexahedron	The solid has 6 faces.
ii Heptahedron	The solid has 7 faces.
iii Pentahedron	The solid has 5 faces.
b **i** Rectangular prism	It has two rectangular ends with rectangular sides.
ii Pentagonal prism	It has two pentagonal ends with rectangular sides.
iii Square pyramid	It has a square base and four triangular faces meeting at an apex.

Example 10 Using Euler's rule

Use Euler's rule to find the number of faces on a polyhedron that has 10 edges and 6 vertices.

SOLUTION

$$E = F + V - 2$$
$$10 = F + 6 - 2$$
$$10 = F + 4$$
$$F = 6$$

EXPLANATION

Write down Euler's rule and make the appropriate substitutions. Solve for F, which represents the number of faces.

Exercise 2F

1–4	4	—

UNDERSTANDING

1 Write the missing word in these sentences.

- **a** A polyhedron has faces, _____ and edges.
- **b** A heptahedron has _____ faces.
- **c** A prism has two _____ ends.
- **d** A pentagonal prism has _____ faces.
- **e** The base of a pyramid has 8 sides. The pyramid is called a _____ pyramid.

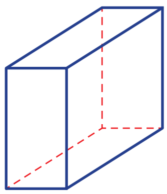
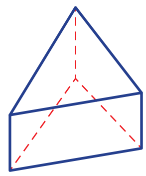
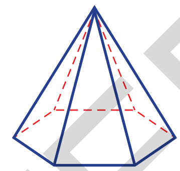
2 Find the value of the pronumeral in these equations.

- **a** $E = 10 + 16 - 2$
- **b** $12 = F + 7 - 2$
- **c** $12 = 6 + V - 2$

2F

UNDERSTANDING

3 Count the number of faces, vertices and edges (in that order) on these polyhedra.

a

b

c


4 Which of these solids are polyhedra (i.e. have only flat surfaces)?

A Cube B Pyramid C Cone D Sphere
E Cylinder F Rectangular prism G Tetrahedron H Hexahedron

FLUENCY

5–6(½), 7–10 | 5–6(½), 7–11 | 5–6(½), 7–11

Example 9a

5 Name the polyhedron that has the given number of faces.

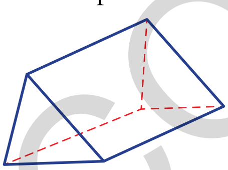
a 6 b 4 c 5 d 7
e 9 f 10 g 11 h 12

6 How many faces do you think these polyhedra have?

a Octahedron b Hexahedron c Tetrahedron d Pentahedron
e Heptahedron f Nonahedron g Decahedron h Undecahedron

Example 9b

7 Name these prisms.

a

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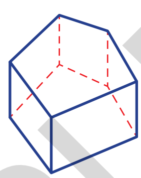
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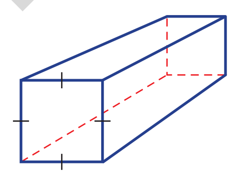
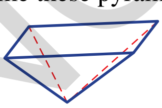


Example 9b

8 Name these pyramids.

a

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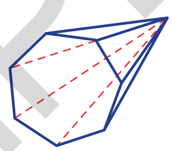
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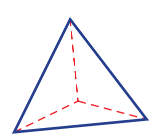


9 a Copy and complete this table.

Solid	Number of faces (F)	Number of vertices (V)	Number of edges (E)	$F + V$
Cube				
Square pyramid				
Tetrahedron				
Octahedron				

b Compare the number of edges (E) with the value $F + V$ for each polyhedron. What do you notice?

2F

FLUENCY

Example 10

10 Use Euler's rule to calculate the missing numbers in this table.

Faces (*F*)	Vertices (*V*)	Edges (*E*)
6	8	__
__	5	8
5	__	9
7	__	12
__	4	6
11	11	__

11 **a** A polyhedron has 16 faces and 12 vertices. How many edges does it have?
b A polyhedron has 18 edges and 9 vertices. How many faces does it have?
c A polyhedron has 34 faces and 60 edges. How many vertices does it have?

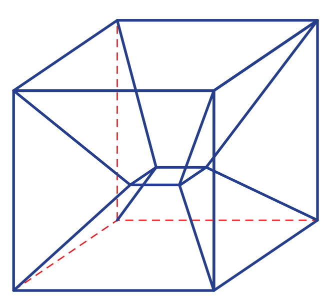
PROBLEM-SOLVING

12	12, 13	13, 14

12 Decide if the following statements are true or false. Make drawings to help.
a A tetrahedron is a pyramid.
b All solids with curved surfaces are cylinders.
c A cube and a rectangular prism are both hexahedrons.
d A hexahedron can be a pyramid.
e There are no solids with 0 vertices.
f There are no polyhedra with 3 surfaces.
g All pyramids will have an odd number of faces.

13 Decide if it is possible to cut the solid using a single straight cut, to form the new solid given in the brackets.
a Cube (rectangular prism)
b Square based pyramid (tetrahedron)
c Cylinder (cone)
d Octahedron (pentahedron)
e Cube (heptahedron)

14 This solid is like a cube but is open at the top and bottom and there is a square hole in the middle forming a tunnel. Count the number of faces (*F*), vertices (*V*) and edges (*E*) then decide if Euler's rule is true for such solids.



REASONING	15	15, 16	16–18

15 **a** A cuboid is a common name for a solid with six rectangular faces. Name the solid in two other different ways.
b A pyramid has base with 10 sides. Name the solid in two ways.

16 Rearrange Euler's rule.
a Write V in terms of F and E.
b Write F in terms of V and E.

17 Show that Euler's rule applies for these solids.
a Heptagonal pyramid
b Octagonal prism
c Octahedron

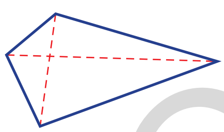
18 Decide if the following statements are true or false.
a For all pyramids, the number of faces is equal to the number of vertices.
b For all convex polyhedra, the sum $E + V + F$ is even.

Convex solids

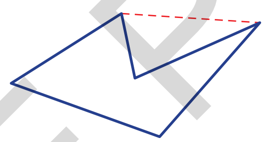
ENRICHMENT	—	—	19

19 Earlier you learned that a convex polygon will have all interior angles less than 180°. Notice also that all diagonals in a convex polygon are drawn *inside* the shape.

Convex polygon

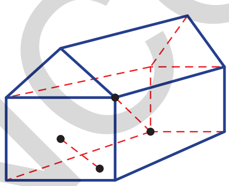


Non-convex polygon

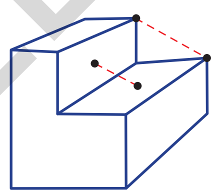


Solids can also be classified as convex or non-convex.

Convex solid



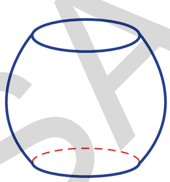
Non-convex solid



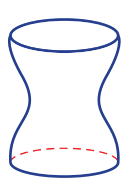
To test for a non-convex solid, join two vertices or two faces with a line segment that passes outside the solid.

a Decide if these solids are convex or non-convex.

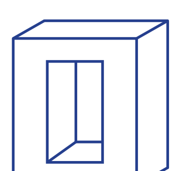
i



ii



iii



b Draw your own non-convex solids and check by connecting any two vertices or faces with a line segment outside the solid.

Investigation

Constructions

Geometric construction involves a precise set of mathematical and geometric operations that do not involve any approximate measurements or other guess work. The basic tools for geometric construction are a pair of compasses, a straight edge and a pencil or drawing pen. Computer geometry or drawing packages can also be used, and include digital equivalents of these tools.

For the following constructions use only a pair of compasses, a straight edge and a pencil. Alternatively, use computer geometry software and adapt the activities where required.

Perpendicular line

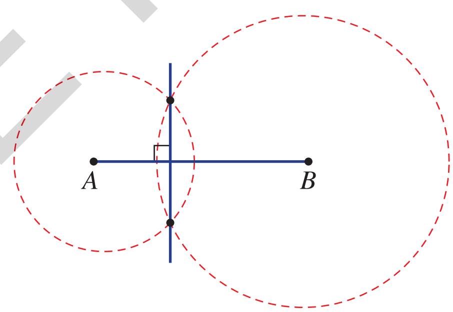
1 Construct:

a a segment AB

b a circle with centre A

c a circle with centre B

d a line joining the intersection points of the two circles



Perpendicular bisector

2 Repeat the construction for a perpendicular line, but ensure that the two circles have the same radius. If computer geometry is used, use the length of the segment AB for the radius of both circles.

A 60° angle

3 Construct:

a a ray AB

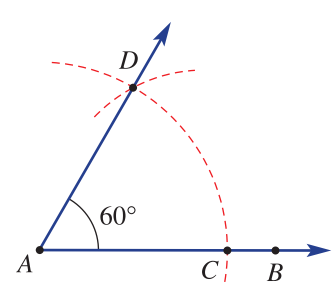
b an arc with centre A

c the intersection point C

d an arc with centre C and radius AC

e a point D at the intersection of the two arcs

f a ray AD

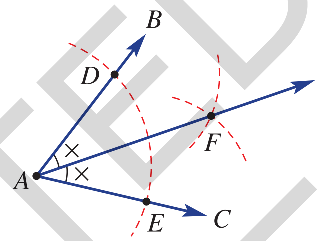


Equilateral triangle

4 Repeat the construction for a 60° angle then construct the segment CD.

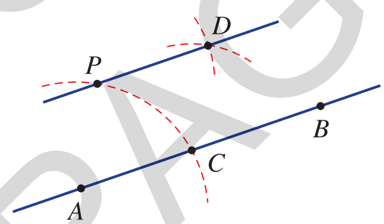
Angle bisector

5 Construct:
  a any angle $\angle BAC$
  b an arc with centre A
  c the two intersection points D and E
  d two arcs of equal radius with centres at D and E
  e the intersection point F
  f the ray AF



Parallel line through a point

6 Construct:
  a a line AB and point P
  b an arc with centre A and radius AP
  c the intersection point C
  d an arc with centre C and radius AP
  e an arc with centre P and radius AP
  f the intersection point D
  g the line PD



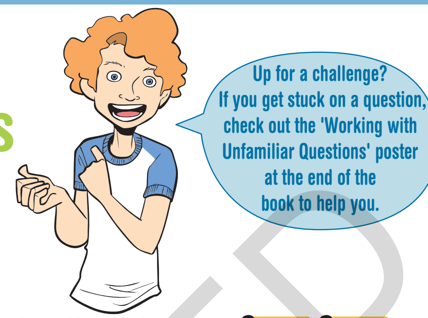
Rhombus

7 Repeat the construction for a parallel line through a point and construct the segments AP and CD.

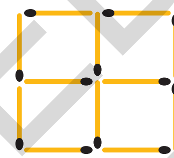
Construction challenges

8 For a further challenge try to construct these objects. No measurement is allowed.
  a 45° angle
  b Square
  c Perpendicular line at the end of a segment
  d Parallelogram
  e Regular hexagon

Problems and challenges



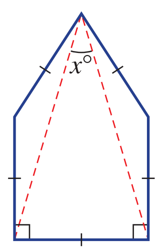
1 This shape includes 12 matchsticks. (To solve these puzzles all matches remaining must connect to other matches at both ends.)
  a Remove 2 matchsticks to form 2 squares.
  b Move 3 matchsticks to form 3 squares.



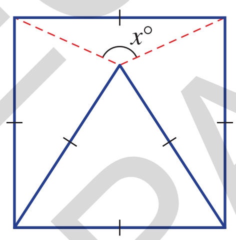
2 a Use 9 matchsticks to form 5 equilateral triangles.
  b Use 6 matchsticks to form 4 equilateral triangles.

3 Find the value of x in these diagrams.

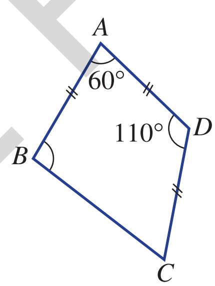
a



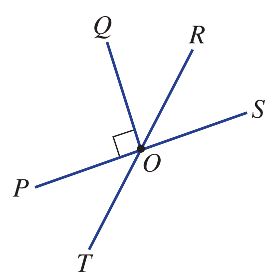
b



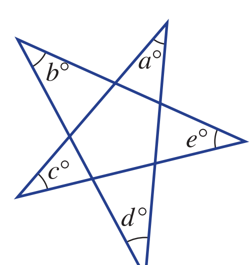
4 Find the size of $\angle ABC$ in this quadrilateral.



5 If $\angle ROS = 75°$ find the size of all other angles.



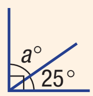
6 Find the value of $a + b + c + d + e$ in this star. Give reasons for your answer.



Chapter summary

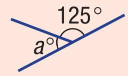
Angles at a point

Complementary angles



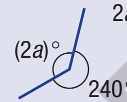
$a + 25 = 90$
$a = 65$

Supplementary angles



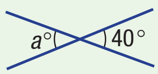
$a + 125 = 180$
$a = 55$

Revolution



$2a + 240 = 360$
$2a = 120$
$a = 60$

Vertically opposite angles

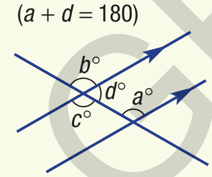


$a = 40$

Lines, shapes and solids

Parallel lines

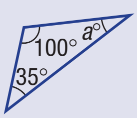
- Corresponding angles are equal ($a = b$)
- Alternate angles are equal ($a = c$)
- Co-interior angles are supplementary ($a + d = 180$)



Triangles

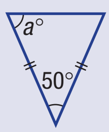
Angle sum = 180°

Scalene



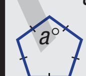
$a + 135 = 180$
$a = 45$

Isosceles



$2a + 50 = 180$
$2a = 130$
$a = 65$

Polygons (Ext)

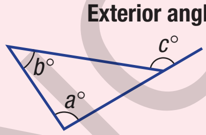


$S = (n - 2) \times 180°$
$= (5 - 2) \times 180°$
$= 540°$

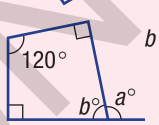
Regular pentagon

$a = \frac{540}{5} = 108$

Exterior angles



$c = a + b$



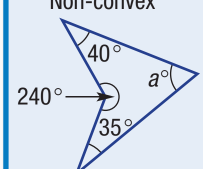
$b + 300 = 360$
$b = 60$
$a = 120$

Quadrilaterals

(Square, Rectangle, Rhombus, Parallelogram, Kite, Trapezium)

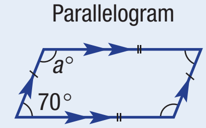
Angle sum = 360°

Non-convex



$a + 315 = 360$
$a = 45$

Parallelogram



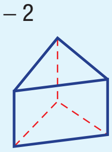
$a + 70 = 180$
$a = 110$

Polyhedra (Ext)

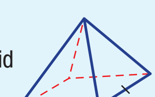
$E = F + V - 2$

Hexahedron

Triangular prism



Square pyramid



Multiple-choice questions

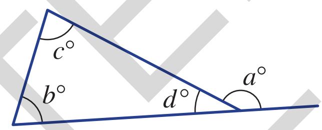
2A

1 What is the name given to two angles that sum to $90°$?

A Right **B** Supplementary
C Revolutionary **D** Complementary
E Vertically opposite

2C

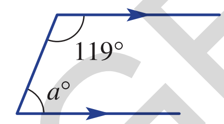
2 The value of a in this diagram is equal to:



A $b + c$ **B** $c + d$ **C** $b + d$ **D** $180 - a$
E $d + 180$

2B

3 The value of a in this diagram is equal to:



A 45 **B** 122 **C** 241 **D** 119 **E** 61

2D

4 The most general quadrilateral whose diagonals intersect at right angles and has 2 pairs of equal
length sides is a:

A square **B** rhombus
C kite **D** parallelogram
E rectangle

2E Ext

5 The rule for the angle sum S of a polygon with n sides is:

A $S = n \times 180°$ **B** $S \times n = 180°$
C $S = (n - 1) \times 180°$ **D** $S = (n - 2) \times 180°$
E $S = (n + 2) \times 180°$

2C

6 The size of an exterior angle on an equilateral triangle is:

A $60°$ **B** $120°$ **C** $180°$ **D** $100°$ **E** $45°$

2E Ext

7 The name given to an eleven-sided polygon is:

A heptagon **B** elevenagon **C** decagon **D** dodecagon **E** undecagon

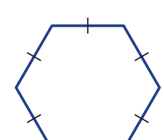
2F Ext

8 A polyhedron has 8 faces and 8 vertices. Its number of edges is:

A 14 **B** 16 **C** 18 **D** 15 **E** 10

2E Ext

9 The size of one interior angle of a regular hexagon is:



A $135°$ **B** $180°$ **C** $120°$ **D** $720°$ **E** $108°$

2F Ext

10 How many edges does a rectangular prism have?

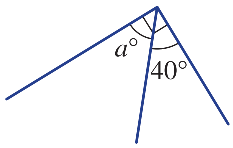
A 10 **B** 4 **C** 6 **D** 12 **E** 8

Short-answer questions

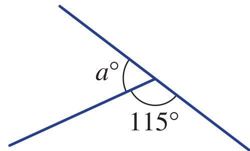
2A

1 Find the value of a in these diagrams.

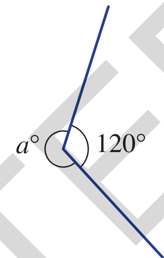
a



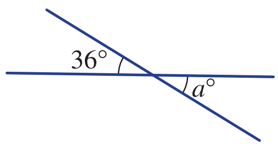
b



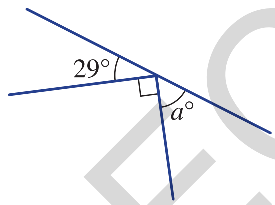
c



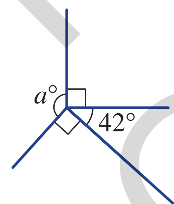
d



e



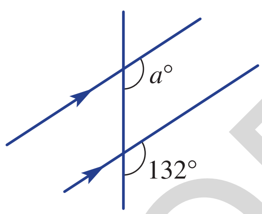
f



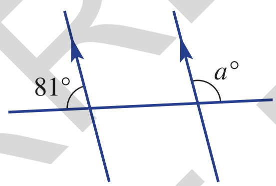
2B

2 These diagrams include parallel lines. Find the value of a.

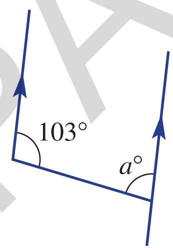
a



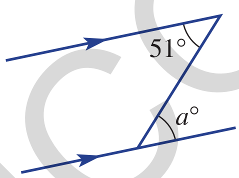
b



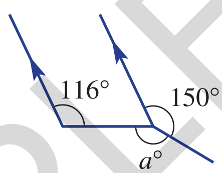
c



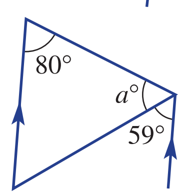
d



e

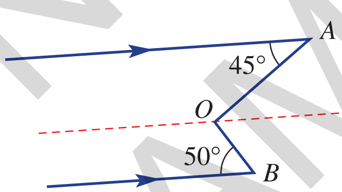


f



2B

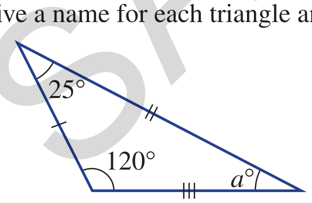
3 Use the dashed construction line to help find the size of $\angle AOB$ in this diagram.



2C

4 Give a name for each triangle and find the value of a.

a



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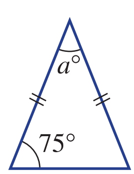

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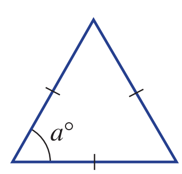
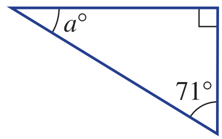
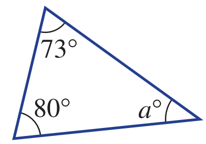
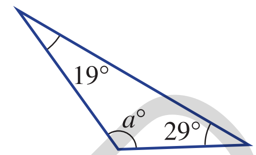


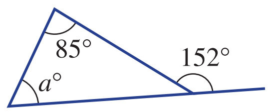
d 

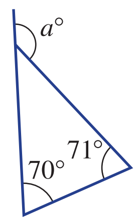
e 

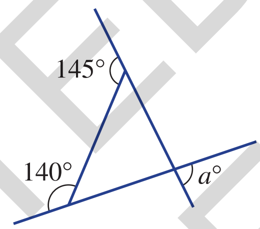
f 

2C

5 These triangles include exterior angles. Find the value of a.

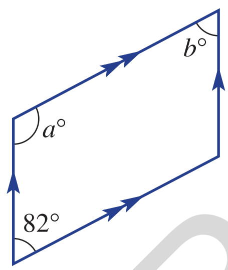
a 

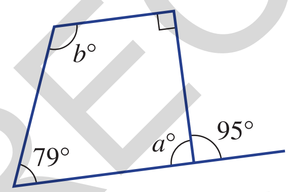
b 

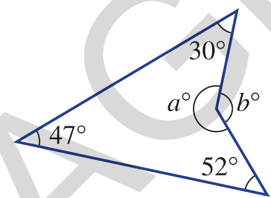
c 

2D

6 Find the value of a and b in these quadrilaterals.

a 

b 

c 

2E Ext

7 Find the angle sum of these polygons.

a Heptagon **b** Nonagon **c** 62-sided polygon

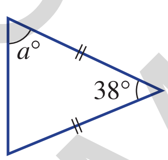
2E Ext

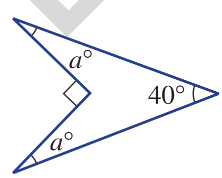
8 Find the size of an interior angle of these regular polygons.

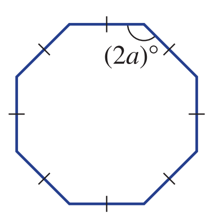
a Regular pentagon **b** Regular dodecagon

2E Ext

9 Find the value of a.

a 

b 

c 

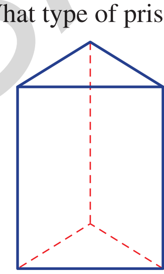
2F Ext

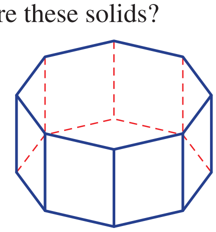
10 Name the polyhedron that has:

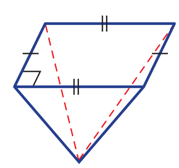
a 6 faces **b** 10 faces **c** 11 faces

2F Ext

11 What type of prism or pyramid are these solids?

a 

b 

c 

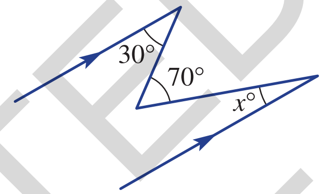
2F Ext

12 Complete this table for polyhedra with number of faces F, vertices V and edges E.

F	V	E
5	5	
9		21
	10	15

2B

13 Find the value of x in this diagram.



Extended-response questions

1 A regular polygon has 26 sides.

Ext

- **a** Find the angle sum.
- **b** Find the size of its interior angles correct to the nearest degree.
- **c** Find the size of its exterior angles correct to the nearest degree.
- **d** If there is one exterior angle showing for each vertex, find the sum of all the exterior angles.
- **e** The polygon is used to form the ends of a prism. For this prism find the number of:
  - **i** faces
  - **ii** vertices
  - **iii** edges

2 A modern house plan is shown here.

- **a** List the names of at least three different polygons that you see.
- **b** Find the values of the pronumerals a–f.