Chapter 1 Integers

What you will learn

- 1A Whole number addition and subtraction (Consolidating)
- 1B Whole number multiplication and division (Consolidating)
- 1C Number properties (Consolidating)
- 1D Divisibility and prime factorisation (Consolidating)
- 1E Negative numbers (Consolidating)
- 1F Addition and subtraction of negative integers (Consolidating)
- 1G Multiplication and division of integers
- 1H Order of operations and substitution

Australian curriculum

NUMBER AND ALGEBRA

Number and place value

Carry out the four operations with integers, using efficient mental and written strategies and appropriate digital technologies (ACMNA183)

Online resources

- Chapter pre-test
- Videos of all worked examples
- Interactive widgets
- Interactive walkthroughs
- Downloadable HOTsheets
- Access to HOTmaths Australian Curriculum courses

Public key encryption

Today, most of the world's electronic commercial transactions are encrypted so that important information does not get into the wrong hands. Public key encryption uses the RSA algorithm named after its inventors Rivest, Shamir and Ademan, who invented the mathematical procedure in 1977. The algorithm creates public and private number keys that are used to encrypt and decrypt information. These keys are generated using products of prime numbers. Because prime factors of large numbers are very difficult to find (even for today's powerful computers) it is virtually impossible to decrypt the code without a private key. The algorithm uses prime numbers, division and remainders, equations and the 2300-year-old Euclidean division algorithm to complete the task. If it wasn't for Euclid (about 300 BCE) and the prime numbers, today's electronic transactions would not be secure.

1A Whole number addition and subtraction CONSOLIDATING

Interactive
Widgets
HOTsheets
Walkthroughs


A child or an adult ticket can be purchased for fairground rides.

The number system that we use today is called the Hindu–Arabic or decimal system and uses the digits 0, 1, 2, 3, 4, 5, 6, 7, 8 and 9. The value of each digit depends on its place in the number, so, for example, the 4 in 3407 has a place value of 400. Whole numbers include 0 (zero) and the counting (natural) numbers 1, 2, 3, 4, . . . Two numbers can be added to find a sum or subtracted to find a difference. If 22 child tickets and 13 adult tickets were purchased for fairground rides, the sum of the number of tickets (35) is found using addition and the difference between the number of child and adult tickets (9) is found using subtraction.

Let's start: Sum and difference

Use a guess-and-check method to try to find a pair of numbers described by these sentences.

- The sum of two numbers is 88 and their difference is 14.
- The sum of two numbers is 317 and their difference is 3.

Describe the meaning of the words 'sum' and 'difference' and discuss how you found the pair of numbers in each case.

Key ideas

- Two numbers can be added in any order. This is called the **commutative law** for addition. The commutative law does not hold for subtraction.
  $a + b = b + a$ For example: $7 + 11 = 11 + 7$
  $a - b \neq b - a$ For example: $5 - 2 \neq 2 - 5$
- Three or more numbers can be added in any order. This uses the **associative law** for addition. The associative law does not hold for subtraction.
  $(a + b) + c = a + (b + c)$ For example: $(2 + 5) + 4 = 2 + (5 + 4)$
  $(a - b) - c \neq a - (b - c)$ For example: $(9 - 5) - 2 \neq 9 - (5 - 2)$
- Addition and subtraction **algorithms** can be used for larger numbers.
  For example:

$$\begin{array}{r} {}^{1}4\,{}^{1}3\,9 \\ +\,1\,8\,2 \\ \hline 6\,2\,1 \end{array} \qquad \begin{array}{r} {}^{2}\cancel{3}\,{}^{1}1\,4 \\ -\,1\,4\,2 \\ \hline 1\,7\,2 \end{array}$$

Key ideas

- Strategies for mental arithmetic include:
  - **Partitioning** For example: 247 + 121 = (200 + 100) + (40 + 20) + (7 + 1) = 368
    85 – 22 = (80 – 20) + (5 – 2) = 63
  - **Compensating** For example: 134 + 29 = 134 + 30 – 1 = 163
    322 – 40 = 320 – 40 + 2 = 282
  - **Doubling or halving** For example: 35 + 37 = 2 × 35 + 2 = 72
    240 – 123 = 240 – 120 – 3 = 117

Example 1 Using mental arithmetic

Evaluate this difference and sum mentally.

a 347 – 39 **b** 125 + 127

SOLUTION	EXPLANATION
a 347 – 39 = 308	347 – 39 = 347 – 40 + 1 = 307 + 1 = 308
b 125 + 127 = 252	125 + 127 = 2 × 125 + 2 = 250 + 2 = 252

Example 2 Using an algorithm

Use an algorithm to find this sum and difference.

a $\begin{array}{r} 938 \\ +\ 217 \end{array}$ **b** $\begin{array}{r} 141 \\ -\ \ 86 \end{array}$

SOLUTION	EXPLANATION
a $\begin{array}{r} 9\,^{1}3\,8 \\ +\ 2\,1\,7 \\ \hline 1\,1\,5\,5 \end{array}$	8 + 7 = 15 (carry the 1 to the tens column) 1 + 3 + 1 = 5 9 + 2 = 11
b $\begin{array}{r} 1\,^{3}\cancel{4}\,^{1}1 \\ -\ \ 8\,6 \\ \hline 5\,5 \end{array}$	Borrow from the tens column then subtract 6 from 11. Now borrow from the hundreds column and then subtract 8 from 13.

Exercise 1A

UNDERSTANDING	1–2(½)	2(½)	—

1 Write the number that is:

a 26 plus 17 **b** 43 take away 9
c 134 minus 23 **d** 451 add 50
e the sum of 19 and 29 **f** the sum of 111 and 236
g the difference between 59 and 43 **h** the difference between 339 and 298
i 36 more than 8 **j** 142 more than 421
k 32 less than 49 **l** 120 less than 251

2 Write the digit missing from these sums and differences.

a $$\begin{array}{r} 237 \\ +\ 4\square \\ \hline 279 \end{array}$$
b $$\begin{array}{r} 49 \\ +38 \\ \hline 8\square \end{array}$$
c $$\begin{array}{r} 493 \\ +214 \\ \hline 7\square 7 \end{array}$$
d $$\begin{array}{r} 1\square 4 \\ +392 \\ \hline 556 \end{array}$$
e $$\begin{array}{r} 38 \\ -19 \\ \hline 1\square \end{array}$$
f $$\begin{array}{r} 128 \\ -\ 8\square \\ \hline 39 \end{array}$$
g $$\begin{array}{r} 3\square 4 \\ -162 \\ \hline 142 \end{array}$$
h $$\begin{array}{r} 251 \\ -1\square 4 \\ \hline 87 \end{array}$$

FLUENCY	3–4(½)	3–4(½), 5	3–4(½), 5

Example 1

3 Evaluate these sums and differences mentally.

a 94 – 62 **b** 146 + 241 **c** 1494 – 351
d 36 + 19 **e** 138 + 25 **f** 251 – 35
g 99 – 20 **h** 441 – 50 **i** 350 + 351
j 115 + 114 **k** 80 – 41 **l** 320 – 159

Example 2

4 Use an algorithm to find these sums and differences.

a $$\begin{array}{r} 128 \\ +\ 46 \\ \hline \end{array}$$
b $$\begin{array}{r} 94 \\ +337 \\ \hline \end{array}$$
c $$\begin{array}{r} 9014 \\ +\ 927 \\ +\ 421 \\ \hline \end{array}$$
d $$\begin{array}{r} 814 \\ +1439 \\ +\ 326 \\ \hline \end{array}$$
e $$\begin{array}{r} 94 \\ -36 \\ \hline \end{array}$$
f $$\begin{array}{r} 421 \\ -204 \\ \hline \end{array}$$
g $$\begin{array}{r} 1726 \\ -1699 \\ \hline \end{array}$$
h $$\begin{array}{r} 14072 \\ -\ 328 \\ \hline \end{array}$$
i $$\begin{array}{r} 428 \\ +324 \\ +107 \\ +\ 29 \\ \hline \end{array}$$
j $$\begin{array}{r} 1004 \\ +\ 2407 \\ +\ 9116 \\ +10494 \\ \hline \end{array}$$
k $$\begin{array}{r} 3017 \\ -2942 \\ \hline \end{array}$$
l $$\begin{array}{r} 10024 \\ -\ 936 \\ \hline \end{array}$$

FLUENCY

5 A racing bike's odometer shows 21 432 km at the start of a race and 22 110 km at the end of the race. What was the total distance of the race?

Casey Stoner racing at the Malaysian Grand Prix.

6, 7	7, 8	7–9

PROBLEM-SOLVING

6 The sum of two numbers is 39 and their difference is 5. What is the larger number?

7 Find the missing digits in these sums and differences.

a
$$\begin{array}{cccc} & 2 & 3 & \square \\ + & \square & 9 & 4 \\ \hline & 6 & \square & 1 \end{array}$$

b
$$\begin{array}{cccc} & \square & 3 & \square \\ + & & \square & 2 \\ \hline & 2 & 1 & 9 \end{array}$$

c
$$\begin{array}{cccc} & \square & 3 & 7 \\ + & 4 & 9 & \square \\ \hline & 7 & \square & 2 \end{array}$$

d
$$\begin{array}{ccc} & \square & 3 \\ - & 2 & 9 \\ \hline & 6 & \square \end{array}$$

e
$$\begin{array}{cccc} & 3 & \square & 2 \\ - & \square & 3 & \square \\ \hline & 1 & 0 & 4 \end{array}$$

f
$$\begin{array}{ccccc} & 2 & \square & \square & 5 \\ - & & 6 & 8 & \square \\ \hline & \square & 3 & 1 & 8 \end{array}$$

8 Wally has two more marbles than Ashan and together they have 88 marbles. How many marbles does Ashan have?

9 Evaluate the following without the use of a calculator.

a $1 + 2 + 3 + 4 + \ldots + 99 + 100 - 99 - 98 - \ldots - 2 - 1$

b $1 - 2 + 3 - 4 + 5 - 6 + \ldots - 98 + 99$

1A

10	10, 11	10–12	REASONING

10 Explain why these number puzzles cannot be solved.

a
$$\begin{array}{cccc} & \square & 2 & \square \\ + & 3 & \square & 6 \\ \hline 2 & 3 & 4 & 9 \end{array}$$

b
$$\begin{array}{cccc} & 3 & \square & 6 \\ - & 3 & 2 & \square \\ \hline & & 8 & 2 \end{array}$$

11 x, y and z represent any three numbers. Complete these statements.

a $x + y + z = \_\_ + x + y$

b $x - y + z = z - \_\_ + \_\_ = x + \_\_ - \_\_$

12 How many different combinations of numbers make the following true? List the combinations and explain your reasoning.

a
$$\begin{array}{cccc} & 1 & \square & 4 \\ + & 2 & \square & \square \\ \hline & 4 & 2 & 7 \end{array}$$

b
$$\begin{array}{cccc} & 3 & \square & \square \\ - & 1 & \square & 4 \\ \hline & 1 & 4 & 3 \end{array}$$

Magic triangles

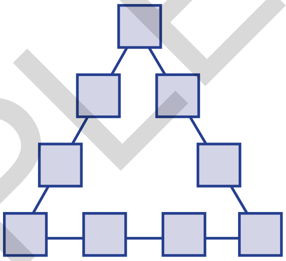
—	—	13	ENRICHMENT

13 The sides of a magic triangle all sum to the same total.

a Show how it is possible to arrange all the digits from 1 to 9 so that each side adds to 17.

b Show how it is possible to arrange the same digits to a different total. How many different totals can you find?

c In how many different ways can you obtain each total? Switching the two middle numbers on each side does not count as a new combination.



1B Whole number multiplication and division CONSOLIDATING

Interactive

Widgets

HOTsheets

Walkthroughs

It is useful to know how to multiply and divide numbers without the use of technology. Mental strategies can be used in some problems, and algorithms can be used for more difficult cases. Calculating the cost of 9 tickets at $51 each, for example, can be done mentally, but the short division algorithm might be useful when calculating the number of trips a dump truck with a capacity of 140 tonnes will need to shift 1000 tonnes of coal.


A typical large mining truck has a capacity of 140 tonnes.

Let's start: Multiplication or division?

In solving many problems it is important to know whether multiplication or division should be used. Decide if the following situations require the use of multiplication or division. Discuss them in a group or with a partner.

- The number of cookies 4 people get if a packet of 32 cookies is shared equally between them
- The cost of paving 30 square metres of courtyard at a cost of $41 per square metre
- The number of sheets of paper in a shipment of 4000 boxes of 5 reams each (1 ream is 500 sheets)
- The number of hours I can afford a plumber at $75 per hour if I have a fixed budget of $1650.

Make up your own situation that requires the use of multiplication and another for division.

Key ideas

- Finding a **product** is the result of using multiplication. We say the product of 11 and 9 is 99.
- The **multiplication algorithm** can be used when products cannot be found mentally.
- Using division results in finding a **quotient** and a **remainder**.

$$\begin{array}{rl} 217 & \\ \times\ \ 26 & \\ \hline 1302 & \leftarrow\ 217\times 6 \\ 4340 & \leftarrow\ 217\times 20 \\ \hline 5642 & \leftarrow\ 1302+4340 \end{array}$$

For example: 38 ÷ 11 = 3 and 5 remainder

dividend divisor quotient

$$\begin{array}{r} 7\ 3\ \ 2 \\ 7\,\overline{)\,51^{2}2^{1}4} \end{array}$$

- The short division algorithm can be used when quotients cannot be found mentally.
- The **commutative law** holds for multiplication but not division.
  $7 \times 5 = 5 \times 7$ but $21 \div 3 \neq 3 \div 21$
- The **associative law** holds for multiplication but not division.
  $(5 \times 4) \times 2 = 5 \times (4 \times 2)$ but $(5 \div 4) \div 2 \neq 5 \div (4 \div 2)$
- Mental strategies for multiplication
  - Using the commutative and associative laws
    For example: $5 \times 17 \times 4 = 5 \times 4 \times 17 = 20 \times 17 = 340$
  - Using the **distributive law**
    For example: $4 \times 87 = (4 \times 80) + (4 \times 7) = 320 + 28 = 348$
    or $4 \times 87 = (4 \times 90) - (4 \times 3) = 360 - 12 = 348$
  - Doubling and halving
    For example: $4 \times 74 = 2 \times 148 = 296$
- Mental strategies for division
  - Halving both numbers
    For example: $132 \div 4 = 66 \div 2 = 33$
  - Using the distributive law
    For example: $96 \div 3 = (90 \div 3) + (6 \div 3) = 30 + 2 = 32$
    or $147 \div 3 = (150 \div 3) - (3 \div 3) = 50 - 1 = 49$

Example 3 Using mental strategies

Use a mental strategy to evaluate the following.

a 5×160 **b** 7×89 **c** $464 \div 8$

SOLUTION	EXPLANATION
a $5 \times 160 = 800$	Double one and halve the other so 5×160 becomes 10×80
b $7 \times 89 = 623$	Use the distributive law so 7×89 becomes $(7 \times 90) - (7 \times 1) = 630 - 7$
c $464 \div 8 = 58$	Halve both numbers repeatedly so $464 \div 8$ becomes $232 \div 4 = 116 \div 2$

Example 4 Using multiplication and division algorithms

Use an algorithm to evaluate the following.

a
$$\begin{array}{r} 412 \\ \times\ 25 \end{array}$$

b $938 \div 13$

SOLUTION	EXPLANATION
a $\begin{array}{r} 412 \\ \times\ \ 25 \\ \hline 2060 \\ 8240 \\ \hline 10300 \end{array}$	$412 \times 5 = 2060$ and $412 \times 20 = 8240$ Add these two products to get the final answer.
b $\begin{array}{r} 7\ \ 2\ \text{ Rem } 2 \\ 13\,\overline{)\,93^{2}8} \end{array}$ So $938 \div 13 = 72$ and 2 remainder.	$93 \div 13 = 7$ and 2 remainder $28 \div 13 = 2$ and 2 remainder

Exercise 1B

1–3(½) 3(½) —

UNDERSTANDING

1 Find the results for the following.

- **a** The product of 7 and 8
- **b** The product of 13 and 100
- **c** The remainder when 2 is divided into 19
- **d** The remainder when 9 is divided into 104
- **e** The quotient of 13 divided by 4
- **f** The quotient of 120 divided by 59

2 Use your knowledge of the multiplication table to write down the answers to the following.

a 5×8	**b** 11×9	**c** 6×7	**d** 9×8
e 11×6	**f** 12×11	**g** 8×4	**h** 7×9
i $100 \div 10$	**j** $88 \div 8$	**k** $121 \div 11$	**l** $144 \div 12$
m $56 \div 7$	**n** $33 \div 3$	**o** $65 \div 5$	**p** $78 \div 6$

3 Decide if these simple equations are true or false.

a $4 \times 13 = 13 \times 4$	**b** $2 \times 7 \times 9 = 7 \times 9 \times 2$
c $6 \div 3 = 3 \div 6$	**d** $60 \div 20 = 30 \div 10$
e $14 \div 2 \div 7 = 7 \div 2 \div 14$	**f** $51 \times 7 = (50 \times 7) + (1 \times 7)$
g $79 \times 13 = (80 \times 13) - (1 \times 13)$	**h** $93 \div 3 = (90 \div 3) + (3 \div 3)$
i $133 \div 7 = (140 \div 7) - (7 \div 7)$	**j** $33 \times 4 = 66 \times 8$

1B

4–6(½), 7	4–6(½), 7–8	4–6(½), 8

FLUENCY

Example 3

4 Use a mental strategy to evaluate the following.

a $5 \times 13 \times 2$ **b** $2 \times 26 \times 5$ **c** 4×35 **d** 17×4
e 17×1000 **f** 136×100 **g** 59×7 **h** 119×6
i 9×51 **j** 6×61 **k** 4×252 **l** 998×6
m $128 \div 8$ **n** $252 \div 4$ **o** $123 \div 3$ **p** $508 \div 4$
q $96 \div 6$ **r** $1016 \div 8$ **s** $5 \times 12 \times 7$ **t** $570 \div 5 \div 3$

Example 4a

5 Use a multiplication algorithm to evaluate the following.

a $\begin{array}{r} 67 \\ \times\ \ 9 \\ \hline \end{array}$ **b** $\begin{array}{r} 129 \\ \times\ \ \ 4 \\ \hline \end{array}$ **c** $\begin{array}{r} 294 \\ \times\ \ 13 \\ \hline \end{array}$ **d** $\begin{array}{r} 1004 \\ \times\ \ \ 90 \\ \hline \end{array}$

e $\begin{array}{r} 690 \\ \times\ \ 14 \\ \hline \end{array}$ **f** $\begin{array}{r} 4090 \\ \times\ \ 101 \\ \hline \end{array}$ **g** $\begin{array}{r} 246 \\ \times 139 \\ \hline \end{array}$ **h** $\begin{array}{r} 1647 \\ \times\ \ 209 \\ \hline \end{array}$

Example 4b

6 Use the short division algorithm to evaluate the following.

a $3\overline{)85}$ **b** $7\overline{)214}$ **c** $10\overline{)4167}$ **d** $11\overline{)143}$
e $15\overline{)207}$ **f** $19\overline{)3162}$ **g** $28\overline{)196}$ **h** $31\overline{)32690}$

7 A university student earns \$550 for 22 hours work. What is the student's pay rate per hour?

8 Packets of biscuits are purchased by a supermarket in boxes of 18. The supermarket orders 220 boxes and sells 89 boxes in one day. How many packets of biscuits remain in the supermarket?

9, 10	9, 10	10, 11

PROBLEM-SOLVING

9 Riley buys a fridge, which he can pay for by the following options.

A 9 payments of \$183 **B** \$1559 up front

Which option is cheaper and by how much?

10 The shovel of a giant mechanical excavator can move 13 tonnes of rock in each load. How many loads are needed to shift 750 tonnes of rock?

PROBLEM-SOLVING

11 Find the missing digits in these problems.

a
$$\begin{array}{rrr} & 2 & \square \\ \times & 2 & 1 \\ \hline & 2 & 3 \\ 4 & \square & 0 \\ \hline 4 & 8 & \square \end{array}$$

b
$$\begin{array}{rrrr} & 1 & \square & 7 \\ \times & & 2 & \square \\ \hline & \square & 4 & 8 \\ 2 & \square & 4 & 0 \\ \hline \square & \square & 8 & 8 \end{array}$$

c
$$\begin{array}{r} 9\;4 \text{ with 6 remainder} \\ 7\overline{)6\,\square\,4} \end{array}$$

d
$$\begin{array}{r} 1\;9 \text{ with 3 remainder} \\ 17\overline{)\square\,\square\,6} \end{array}$$

12	12, 13	13, 14

REASONING

12 If a represents any number except 0, simplify the following.

a $a \div a$ **b** $a \div 1$ **c** $0 \div a$ **d** $25 \times a \div a$

13 A mental strategy for division involves separately dividing a pair of factors of the divisor.
For example:

$$\begin{aligned} 114 \div 6 &= 144 \div 2 \div 3 \qquad \text{(Note: 2 and 3 are factors of 6)} \\ &= 57 \div 3 \\ &= 19 \end{aligned}$$

Use this technique to evaluate the following.

a $204 \div 6$ **b** $144 \div 8$ **c** $261 \div 9$ **d** $306 \div 18$

14 Evaluate the following without using an algorithm.

a $(99 \times 17) + (1 \times 17)$ **b** $(82 \times 7) - (2 \times 7)$

c $(493 \times 12) + (507 \times 12)$ **d** $(326 \times 15) - (306 \times 15)$

Maximum ticket numbers

—	—	15

ENRICHMENT

15 **a** Gen spends exactly $80 to buy child tickets at $7 each and adult tickets at $12 each. Find the maximum number of tickets that could be purchased.

b Alfred spends exactly $141 to buy child tickets at $9 each and adult tickets at $15 each. Find the maximum number of tickets that could be purchased.

c Explain your method for solving the above two questions. Make up your own similar question and test it on a friend.

1C Number properties CONSOLIDATING

Interactive

Widgets

HOTsheets

Walkthroughs

The properties of numbers are at the foundation of mathematical problem-solving. A class of 63 students, for example, can be divided into 7 equal groups of 9, but a class of 61 cannot be divided into smaller equal groups greater than 1. This is because 61 is a prime number with no other factors apart from 1 and itself; 63 is a multiple of 9 and the numbers 9 and 7 are factors of 63.

Into how many equal groups could these people be divided?

Let's start: How many in 60 seconds?

In 60 seconds, write down as many numbers as you can that fit each description.

- Multiples of 7
- Factors of 144
- Prime numbers

Compare your lists with the results of the class. For each part decide if there are any numbers less than 100 that you missed.

Key ideas

- A **multiple** of a number is obtained by multiplying the number by the **counting numbers** 1, 2, 3, . . .
  For example: Multiples of 9 include 9, 18, 27, 36, 45, . . .
- The **lowest common multiple** (LCM) is the smallest multiple of two or more numbers that is common.
  For example: Multiples 3 are 3, 6, 9, 12, 15, 18, . . .
  For example: Multiples of 5 are 5, 10, 15, 20, 25, . . .
  The LCM of 3 and 5 is therefore 15.
- A **factor** of a number has a remainder of zero when divided into the given number.
  For example, 11 is a factor of 77 since $77 \div 11 = 7$ with 0 remainder.
- The **highest common factor** (HCF) is the largest factor of two or more numbers that is common.
  For example: Factors of 24 are 1, 2, 3, 4, 6, 8, 12, 24.
  For example: Factors of 36 are 1, 2, 3, 4, 6, 9, 12, 18, 36.
  The HCF of 24 and 36 is therefore 12.
- **Prime numbers** have only two factors, the number itself and 1.
  - 2, 13 and 61 are examples of prime numbers.
  - 1 is not considered to be a prime number.
- **Composite numbers** have more than two factors.
  - 6, 20 and 57 are examples of composite numbers.
- The **square** of a number x is $x^2 = x \times x$.
  - We say x^2 as 'x squared' or 'the square of x' or 'x to the power 2'.
  - $3^2 = 9$ and $11^2 = 121$ ($3^2 = 3 \times 3$ and $11^2 = 11 \times 11$)
  - If x is a whole number then x^2 is called a perfect square. 4 and 121 are examples of perfect squares.
- The **square root** of a number is written with the symbol $\sqrt{\ }$.
  - $\sqrt{b} = a$ if $a^2 = b$ (and a is positive), for example, $\sqrt{9} = 3$ since $3^2 = 9$.
    Note: $\sqrt{16}$ is a positive number only and is equal to 4 not ± 4.
- The **cube** of a number x is $x^3 = x \times x \times x$.
  - We say x^3 as 'x cubed or 'the cube of x' or 'x to the power 3'.
  - $2^3 = 2 \times 2 \times 2 = 8$ and $5^3 = 5 \times 5 \times 5 = 125$
- The **cube root** of a number is written with the symbol $\sqrt[3]{\ }$.
  - $\sqrt[3]{b} = a$ if $a^3 = b$, for example, $\sqrt[3]{8} = 2$ since $2^3 = 8$.

Example 5 Finding the LCM and HCF

a Find the LCM of 6 and 8.

b Find the HCF of 36 and 48.

SOLUTION	EXPLANATION
a Multiples of 6 are: 6, 12, 18, 24, 30, . . . Multiples of 8 are: 8, 16, 24, 32, 40, . . .	First, list some multiples of 6 and 8. Continue the lists until there is at least one in common.
The LCM is 24.	Choose the smallest number that is common to both lists.
b Factors of 36 are: 1, 2, 3, 4, 6, 9, 12, 18, 36 Factors of 48 are: 1, 2, 3, 4, 6, 8, 12, 16, 24, 48	First, list factors of 36 and 48.
The HCF is 12.	Choose the largest number that is common to both lists.

Example 6 Finding squares, cubes, square roots and cube roots

Evaluate the following.

a 6^2 **b** $\sqrt{81}$ **c** 2^3 **d** $\sqrt[3]{64}$

SOLUTION	EXPLANATION
a $6^2 = 6 \times 6$ $= 36$	Find the product of 6 with itself.
b $\sqrt{81} = 9$	$9^2 = 9 \times 9 = 81$ so $\sqrt{81} = 9$ Note: $\sqrt{x}$ $(x \geq 0)$ is a positive number so $\sqrt{81} \neq -9$.
c $2^3 = 2 \times 2 \times 2$ $= 8$	In general $x^3 = x \times x \times x$.
d $\sqrt[3]{64} = 4$	$4^3 = 4 \times 4 \times 4 = 64$ so $\sqrt[3]{64} = 4$

Exercise 1C

1–3, 4(½)	2, 4(½)	—

UNDERSTANDING

1 Circle or write down the number in each list which is not a multiple of the first number listed.

a 3, 6, 9, 12, 14, 18, 21
b 11, 22, 33, 45, 55, 66
c 21, 43, 63, 84, 105
d 13, 26, 40, 52, 65.

2 Write down the missing factor from each list.

a Factors of 18: 1, 2, 3, 9, 18
b Factors of 24: 1, 2, 4, 6, 8, 12, 24

3 Classify these numbers as prime or composite.

a 7
b 12
c 29
d 69
e 105
f 117
g 221
h 1 046 734

4 Evaluate the following.

a $2 \times 2 \times 2$
b $3 \times 3 \times 3$
c $4 \times 4 \times 4$
d $5 \times 5 \times 5$
e $6 \times 6 \times 6$
f $7 \times 7 \times 7$
g $8 \times 8 \times 8$
h $9 \times 9 \times 9$

5–8(½)	5–8(½), 9	5–9(½)

FLUENCY

Example 5a

5 Find the LCM of these pairs of numbers.

a 2, 3
b 5, 9
c 8, 12
d 4, 8
e 25, 50
f 4, 18
g 8, 60
h 12, 20

Example 5b

6 Find the HCF of these pairs of numbers.

a 6, 8
b 18, 9
c 16, 24
d 24, 30
e 7, 13
f 19, 31
g 72, 36
h 108, 64

Example 6a, 6b

7 Evaluate these squares and square roots.

a 4^2
b 10^2
c 13^2
d 15^2
e 100^2
f 20^2
g $\sqrt{25}$
h $\sqrt{49}$
i $\sqrt{121}$
j $\sqrt{900}$
k $\sqrt{1600}$
l $\sqrt{256}$

Example 6c, 6d

8 Evaluate these cubes and cube roots.

a 2^3
b 4^3
c 7^3
d 5^3
e 6^3
f 10^3
g $\sqrt[3]{27}$
h $\sqrt[3]{8}$
i $\sqrt[3]{125}$
j $\sqrt[3]{512}$
k $\sqrt[3]{729}$
l $\sqrt[3]{1\,000\,000}$

9 Find:

a the LCM of 8, 12 and 6
b the LCM of 7, 3 and 5
c the HCF of 20, 15 and 10
d the HCF of 32, 60 and 48

PROBLEM-SOLVING

10, 11	11–13	12–14

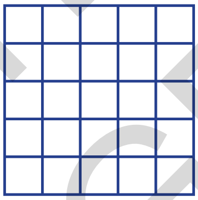
10 A teacher has 64 students to divide into small equal groups of greater than 2 with no remainder. In how many ways can this be done?

11 Three sets of traffic lights (A, B and C) all turn red at 9 a.m. exactly. Light set A turns red every 2 minutes, light set B turns red every 3 minutes and light set C turns red every 5 minutes. How long does it take for all three lights to turn red again at the same time?

12 How many prime numbers less than 100 are there?

13 **a** How many squares of any size are there on this grid?

b What do you notice about the number of squares of each size? Do you notice a pattern?



14 Cyclist A rides a lap of a circular course every 3 minutes. Cyclist B rides a lap of the same course every 5 minutes. If both cyclists start at the same place at the same time, how long will it take before they are both back together at the starting position?

REASONING

15	15, 16	16–18

15 Using the definitions (descriptions) in the **Key ideas**, decide if the number one (1) is a prime, a composite or neither.

16 Explain why all prime numbers except the number 2 are odd.

17 Explain why all square numbers (1, 4, 9, 16, . . .) have an odd number of factors.

18 Decide if the following statements are always true. If they are not, give an example that shows that the statement is not always true. Assume that a and b are different numbers.

a The LCM of two numbers a and b is $a \times b$.

b The LCM of two prime numbers a and b is $a \times b$.

c The HCF of two prime numbers a and b is 1.

Goldbach's conjecture and twin primes

— — 19, 20

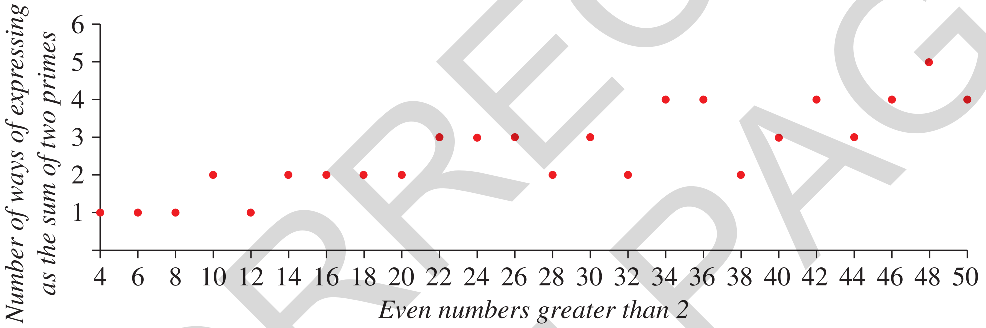
ENRICHMENT

19 In 1742, Goldbach wrote a letter to Euler suggesting that every even number greater than 5 is the sum of three primes. Euler replied saying that this was equivalent to saying that every even number greater than 2 is the sum of two primes. If the number 1 is not considered to be prime (the modern convention), the idea becomes *Every even number greater than 2 is the sum of two primes*. This is known today as Goldbach's conjecture.

a Show ways in which the following numbers can be written as a sum of two primes.

i 28 **ii** 62 **iii** 116

b Goldbach's conjecture does not discuss the odd numbers. Are there any odd numbers greater than 4 and less than 20 which cannot be written as a sum of two primes? If there are any, list them.



A graph illustrating Goldbach's conjecture up to and including 50, is obtained by plotting the number of ways of expressing even numbers greater than 2 as the sum of two primes.

20 Twin primes are pairs of prime numbers that differ by 2. It has been conjectured that there are infinitely many twin primes. List the pairs of twin primes less than 100.

1D Divisibility and prime factorisation

CONSOLIDATING

Interactive

Widgets

HOTsheets

Walkthroughs

The fundamental theorem of arithmetic says that every whole number greater than 1 can be written as a product of prime numbers, for example, $6 = 3 \times 2$ and $20 = 2 \times 2 \times 5$. For this reason it is often said that prime numbers are the building blocks of all other whole numbers. Writing numbers as a product of prime numbers can help to simplify expressions and determine other properties of numbers or pairs of numbers.

Prime numbers can be thought of as the building blocks or foundations of all other whole numbers.

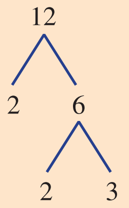
Let's start: Remembering divisibility tests

To test if a number is divisible by 2, we simply need to see if the number is even or odd. All even numbers are divisible by 2. Try to remember the divisibility tests for each of the following. As a class, can you describe all the tests for the following?

- Divisible by 3
- Divisible by 6
- Divisible by 10
- Divisible by 4
- Divisible by 8
- Divisible by 5
- Divisible by 9

Key ideas

- **Prime factorisation** involves writing a number as a product of prime numbers.
  For example: $12 = 2 \times 2 \times 3$
  - $2^2 \times 3$ is the **prime factor** form of 12.
  - The prime numbers are usually written in ascending order.
  - A prime factor tree can help to determine the prime factor form.



$\therefore 12 = 2^2 \times 3$

- The **lowest common multiple** (LCM) of two numbers in their prime factor form is the product of all the different primes raised to their highest power.
  For example: $12 = 2^2 \times 3$ and $30 = 2 \times 3 \times 5$
  So the LCM of 12 and 30 is $2^2 \times 3 \times 5 = 60$.
- The **highest common factor** (HCF) of two numbers in their prime factor form is the product of all the common primes raised to their smallest power.
  For example: $12 = 2^2 \times 3$ and $30 = 2 \times 3 \times 5$
  So the HCF of 12 and 30 is $2 \times 3 = 6$.

Key ideas

- **Divisibility tests**

  A number is divisible by:

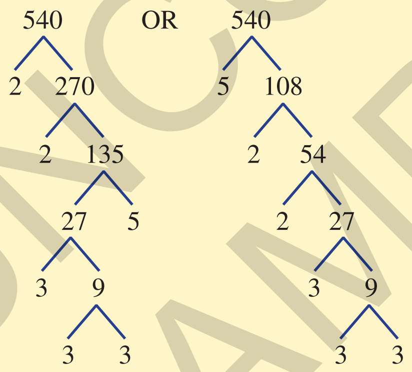
  - 2 if it ends with the digit 0, 2, 4, 6 or 8, for example, 384 is even
  - 3 if the sum of all the digits is divisible by 3
    For example, 162 where 1 + 6 + 2 = 9, which is divisible by 3
  - 4 if the number formed by the last two digits is divisible by 4
    For example, 148 where 48 is divisible by 4
  - 5 if the last digit is a 0 or 5
    For example, 145 or 2090
  - 6 if it is divisible by both 2 and 3
    For example, 456 where 6 is even and 4 + 5 + 6 = 15, which is divisible by 3
  - 8 if the number formed from the last 3 digits is divisibly by 8
    For example, 2112 where 112 is divisible by 8.
  - 9 if the sum of all the digits is divisible by 9
    For example, 3843 where 3 + 8 + 4 + 3 = 18, which is divisible by 9
  - 10 if the last digit is a 0
    For example, 4230

  There is no simple test for 7.

Example 7 Finding prime factor form

Use a factor tree to write 540 as a product of prime factors.

SOLUTION



So $540 = 2^2 \times 3^3 \times 5$

EXPLANATION

First, divide 540 into the product of any two factors.

Since 540 is even it is easy to choose 2 as one of the factors but 3 or 5 could also be chosen.

Continue dividing numbers into prime factors until all the factors are prime numbers.

Write the factors in ascending order.

Example 8 Testing for divisibility

Use divisibility tests to decide if the number 627 is divisible by 2, 3, 4, 5, 6, 8 or 9.

SOLUTION	EXPLANATION
Not divisible by 2 since 7 is odd.	The last digit needs to be even.
Divisible by 3 since $6 + 2 + 7 = 15$ and this is divisible by 3.	The sum of all the digits needs to be divisible by 3.
Not divisible by 4 as 27 is not divisible by 4.	The number formed from the last two digits needs to be divisible by 4.
Not divisible by 5 as the last digit is not a 0 or 5.	The last digit needs to be a 0 or 5.
Not divisible by 6 as it is not divisible by 2.	The number needs to be divisible by both 2 and 3.
Not divisible by 8 as the last 3 digits together are not divisible by 8.	The number formed from the last three digits needs to be divisible by 8.
Not divisible by 9 as $6 + 2 + 7 = 15$ is not divisible by 9.	The sum of all the digits needs to be divisible by 9.

Example 9 Finding the LCM and HCF

Find the LCM and HCF of 105 and 90, using prime factorisation.

SOLUTION	EXPLANATION
$105 = 3 \times 5 \times 7$ $90 = 2 \times 3^2 \times 5$	First, express each number in prime factor form. Note that 3 and 5 are common primes.
$\text{LCM} = 2 \times 3^2 \times 5 \times 7$ $= 630$	For the LCM include all the different primes, raising the common primes to their highest power.
$\text{HCF} = 3 \times 5$ $= 15$	For the HCF include only the common primes raised to the smallest power.

Exercise 1D

	1–4	4	—

UNDERSTANDING

1 Write down all the factors of these numbers.

a 15 **b** 24 **c** 40 **d** 84

2 Write down the first 10 prime numbers. Note that 1 is not a prime number.

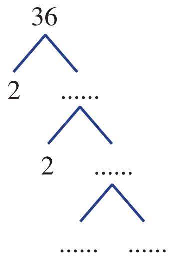
3 Evaluate the following.

a $2^2 \times 3$ **b** $2 \times 3^2 \times 5$ **c** $2^3 \times 5 \times 7$ **d** $3^3 \times 11$

Example 7

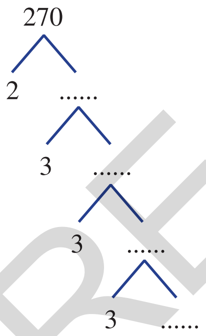
4 Copy and complete these factor trees to help write the prime factor form of the given numbers.

a



$\therefore$ $36 = 2^2 \times$ ......

b



$\therefore$ $270 = 2 \times$ ...... $\times$ ......

	5–7(½)	5–7(½)	5–7(½)

FLUENCY

5 Use a factor tree to find the prime factor form of these numbers.

a 20 **b** 28 **c** 40 **d** 90
e 280 **f** 196 **g** 360 **h** 660

6 How many different primes make up the prime factor form of these numbers?

a 30 **b** 63 **c** 180 **d** 2695

Example 8

7 Use divisibility tests to decide if these numbers are divisible by 2, 3, 4, 5, 6, 8 or 9.

a 51 **b** 126 **c** 248 **d** 387
e 315 **f** 517 **g** 894 **h** 3107

	8, 9(½)	9(½), 10, 11	9(½), 10, 11

PROBLEM-SOLVING

8 Find the highest common prime factors of these pairs of numbers.

a 10, 45 **b** 42, 72 **c** 24, 80 **d** 539, 525

Example 9

9 Find the LCM and the HCF of these pairs of numbers, using prime factorisation.

a 10, 12 **b** 14, 28 **c** 15, 24 **d** 12,15
e 20, 28 **f** 13, 30 **g** 42, 9 **h** 126, 105

10 Aunt Elly's favourite nephew visits her every 30 days. The other nephew visits her every 42 days. If both nephews visit Aunt Elly on one particular day, how long will it be before they both visit her again on the same day?

11 Two armies face each other for battle. One army has 1220 soldiers and the other has 549 soldiers. Both armies are divided into smaller groups of equal size called platoons. Find the largest possible number of soldiers in a platoon if the platoon size is equal for the two armies.

12	12, 13	13, 14

12 Decide if the following statements are true or false. If a statement is false, give an example to show this.

a All numbers divisible by 9 are divisible by 3.

b All numbers divisible by 3 are divisible by 9.

c All numbers divisible by 8 are divisible by 4.

d All numbers divisible by 4 are divisible by 8.

13 If a number is divisible by 2 and 3, then it must be divisible by 6. Use this idea to complete these sentences.

a A number is divisible by 14 if it is divisible by ____ and ____.

b A number is divisible by 22 if it is divisible by ____ and ____.

c A number is divisible by 15 if it is divisible by ____ and ____.

d A number is divisible by 77 if it is divisible by ____ and ____.

14 Powers higher than 3 can be used in prime factorisation.

e.g. $48 = 2 \times 2 \times 2 \times 2 \times 3 = 2^4 \times 3$

Write the prime factor form of each number.

a 162 **b** 96 **c** 5625 **d** 1792

Divisibility by 11

—	—	15

15 There is a more complex test to see if a number is divisible by 11.

a Divide each of these numbers by 11.

i 22	**ii** 88	**iii** 121	**iv** 165
v 308	**vi** 429	**vii** 1034	**viii** 9020

b For the number 2035 (which is divisible by 11):

i Find the sum of the first and third digits.

ii Find the sum of the second and fourth digits.

iii Subtract your answer to part **ii** from your answer to part **i**. What do you notice?

c Repeat all the tasks in part **b** for the number 8173 (which is divisible by 11).

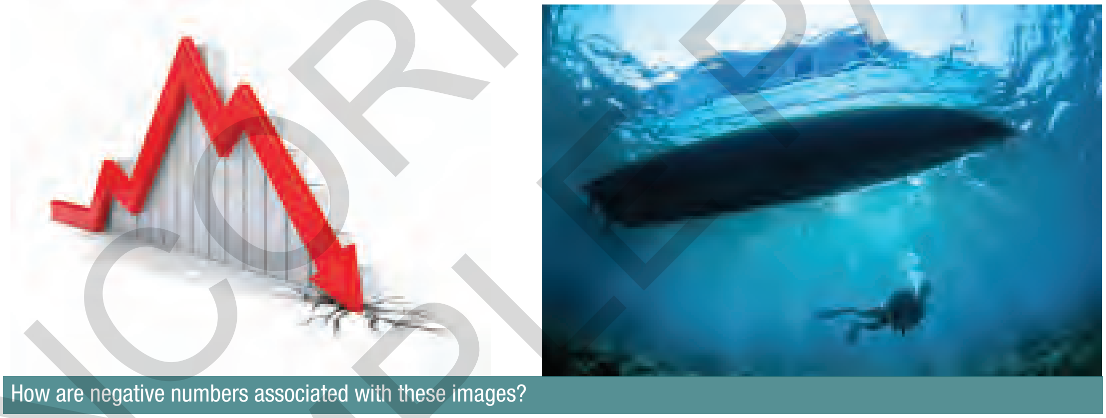
d Now find the sum of all the alternate digits for these numbers which are divisible by 11. Subtract the second sum from the first. What do you notice?

i 4092	**ii** 913
iii 2475	**iv** 77

e Can you now write down a divisibility test for dividing by 11? Test it on some numbers.

1E Negative numbers CONSOLIDATING

Although the Indian mathematician Brahmagupta set out the rules for the use of negative numbers in the 7th century, a British mathematician Maseres claimed in 1758 that negative numbers ‘darken the very whole doctrines of the equations and make dark of the things which are in their nature excessively obvious and simple’. Despite this view that negative numbers were unnatural and had little meaning, they have found their way into the practical world of science, engineering and commerce. We can use negative numbers to distinguish between left and right, up and down, financial profits and losses, warm and cold temperatures, and the clockwise and anticlockwise rotation of a wheel.



How are negative numbers associated with these images?

Let’s start: A negative world

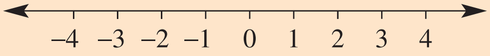
Describe how to use negative numbers to describe these situations.

- 6°C below zero
- A loss of $4200
- 150 m below sea level
- A turn of 90° anticlockwise
- The solution to the equation $x + 5 = 3$

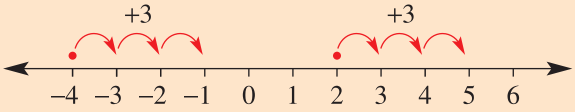
Can you describe another situation in which you might make use of negative numbers?

Key ideas

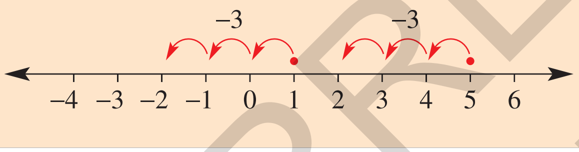
- Negative numbers are numbers less than zero.
- The integers are . . . , –4, –3, –2, –1, 0, 1, 2, 3, 4, . . .
  These include positive integers (natural numbers), zero and negative integers. These are illustrated clearly on a number line.



- Adding or subtracting a positive integer can result in a positive or negative number.



  - Adding a positive integer
    For example: $2 + 3 = 5$
    $-4 + 3 = -1$
  - Subtracting a positive integer
    For example: $1 - 3 = -2$
    $5 - 3 = 2$



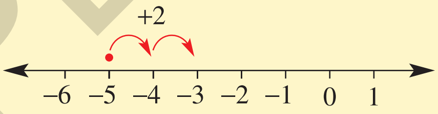
Example 10 Adding and subtracting a positive integer

Evaluate the following.

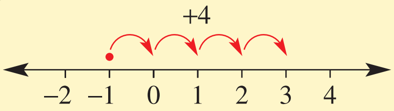
a $-5 + 2$ **b** $-1 + 4$ **c** $3 - 7$ **d** $-2 - 3$

SOLUTION **EXPLANATION**

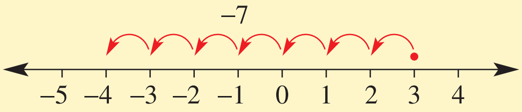
a $-5 + 2 = -3$



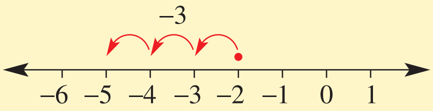
b $-1 + 4 = 3$



c $3 - 7 = -4$



d $-2 - 3 = -5$



Exercise 1E

1–3 3 —

UNDERSTANDING

1 Write the symbol < (less than) or > (greater than) to make these statements true.

a 5 ____ –1 **b** –3 ____ 4 **c** –10 ____ 3 **d** –1 ____ –2

e –20 ____ –24 **f** –62 ____ –51 **g** 2 ____ –99 **h** –61 ____ 62

2 Write the missing numbers in these patterns.

a –3, –2, ____, 0, 1, ____, 3 **b** 1, 0, ____, –2, –3, ____, –5

c –10, –8, –6, ____, ____, 0, 2 **d** 20, 10, ____, ____, –20, –40

3 What is the final temperature?

a 10°C is reduced by 12°C **b** 32°C is reduced by 33°C

c –11°C is increased by 2°C **d** –4°C is increased by 7°C

4–7(½) 4–8(½) 4–8(½)

FLUENCY

Example 10a, 10b

4 Evaluate the following.

a –1 + 2 **b** –3 + 7 **c** –10 + 11 **d** –4 + 12

e –20 + 35 **f** –100 + 202 **g** –7 + 2 **h** –15 + 8

i –26 + 19 **j** –38 + 24 **k** –173 + 79 **l** –308 + 296

Example 10c, 10d

5 Evaluate the following.

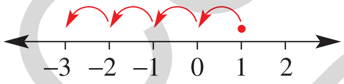
a 4 – 5 **b** 10 – 15 **c** 0 – 26 **d** 14 – 31

e 103 – 194 **f** 316 – 390 **g** –4 – 7 **h** –11 – 20

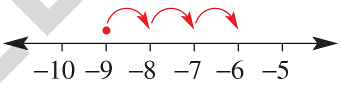
i –14 – 15 **j** –10 – 100 **k** –400 – 37 **l** –348 – 216

6 Write the sum (e.g. –3 + 4 = 1) or difference (e.g. 1 – 5 = –4) to match these number lines.

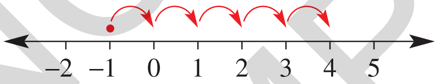
a



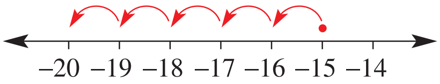
b



c



d



7 Write the missing number.

a –1 + ____ = 5 **b** ____ + 30 = 26

c ____ + 11 = –3 **d** –32 + ____ = –21

e 5 – ____ = –10 **f** ____ – 17 = –12

g ____ – 4 = –7 **h** –26 – ____ = –38

8 Work from left to right to evaluate the following.

a –3 + 4 – 8 + 6 **b** 0 – 10 + 19 – 1

c 26 – 38 + 14 – 9 **d** 9 – 18 + 61 – 53

9, 10	10–12	10–12

PROBLEM-SOLVING

9 In a high-rise building there are 25 floors above ground floor (floor 1, floor 2, . . .) and 6 floors below ground floor. A lift starts at floor 3 and moves 5 floors down then 18 floors up, 4 more floors up, 26 floors down and finally 6 floors up. At which floor does the lift finish?

10 Insert a + and/or a – sign into these statements to make them true.

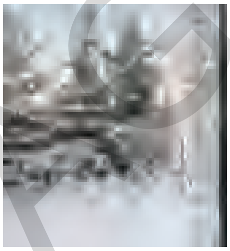
a 5____7 = –2

b 4____6____3 = 1

c –2____5____4 = –11

11 On Monday Milly borrows \$35 from a friend. On Tuesday she pays her friend \$40. On Friday she borrows \$42 and pays back \$30 that night. How much does Milly owe her friend in the end?

12 The temperature in Greenland on a sunny day rises 19°C from its minimum temperature to a maximum of – 4°C. What was the minimum temperature on the day?



13	13, 14	14–16

REASONING

13 If a and b are positive integers, decide if the following are *always* true.

a $a + b > 0$

b $a - b < 0$

c $b - a < 0$

d $-a - b < 0$

e $-a + b > 0$

f $-b + a < 0$

14 If a and b are positive integers and $a > b$, decide if the following are true or false.

a $b < a$

b $a - b < 0$

c $b - a < 0$

15 For what value of a is $a = -a$?

16 Find a method to evaluate the following without using a calculator or algorithm. Explain your method.

$$-1 + 2 - 3 + 4 - 5 + 6 - 7 + 8 - \ldots - 997 + 998 - 999 + 1000$$

Simultaneous integers

—	—	17

ENRICHMENT

17 Find the pairs of integers (a, b) that satisfy both equations in each part.

a $a + b = 5$ and $a - b = -3$

b $a + b = -4$ and $a - b = -10$

c $a + 2b = -1$ and $a - 2b = -9$

d $a + b = -8$ and $a - 2b = -14$

1F Addition and subtraction of negative integers

CONSOLIDATING

Interactive

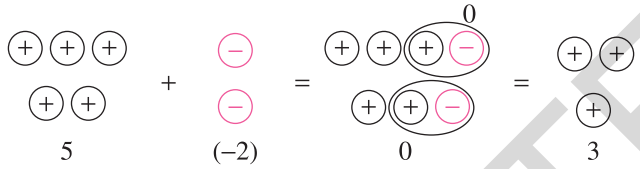
Widgets

HOTsheets

Walkthroughs

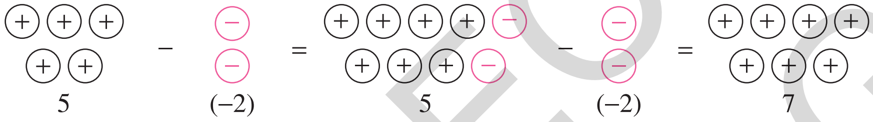
If ⊕ represents +1 and ⊖ represents –1 then ⊕⊖ added together has a value of zero.

Using these symbols 5 + (–2) = 3 could be illustrated as the addition of 2⊖, leaving a balance of 3.



So 5 + (–2) is the same a 5 – 2.

Also 5 – (–2) = 7 could be illustrated first as 7⊕ and 2⊖ together then subtracting the 2⊖.



So 5 – (–2) is the same a 5 + 2.

When adding or subtracting negative integers, we follow the rules set out by the above two illustrations.

Let's start: Circle arithmetic

Use ⊕ and ⊖ as shown in the introduction to illustrate and calculate the answers to these additions and subtractions.

- 3 + (–2)
- –2 + (–4)
- –5 + (–2)
- 3 – (–2)
- –3 – (–2)
- –1 – (–4)

Negative numbers can be used to describe the depth of divers below sea level. The deeper the dive, the larger the negative number.

Key ideas

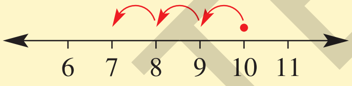
- Adding a negative number is the same as subtracting its opposite.
  For example:
  2 + (–3) = 2 – 3 = –1
  –4 + (–7) = –4 – 7 = –11
- Subtracting a negative number is the same as adding its opposite.
  For example:
  2 – (–5) = 2 + 5 = 7
  –6 – (–4) = –6 + 4 = –2

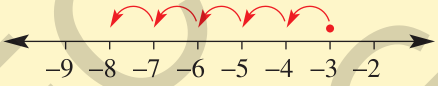
Example 11 Adding and subtracting negative integers

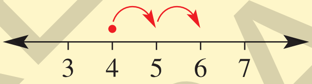
Evaluate the following.

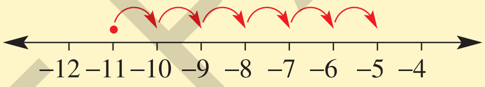
a 10 + (–3) **b** –3 + (–5) **c** 4 – (–2) **d** –11 – (–6)

SOLUTION	EXPLANATION
a 10 + (–3) = 10 – 3 = 7	Adding –3 is the same as subtracting 3.
b –3 + (–5) = –3 – 5 = –8	Adding –5 is the same as subtracting 5.
c 4 – (–2) = 4 + 2 = 6	Subtracting –2 is the same as adding 2.
d –11 – (–6) = –11 + 6 = –5	Subtracting –6 is the same as adding 6.









Exercise 1F

1(½), 2, 3(½) 3(½) —

UNDERSTANDING

1 –3 and 3 are opposites. Write down the opposites of these numbers.

a –6 **b** 10 **c** 38 **d** – 46

e –32 **f** 88 **g** 673 **h** –349

2 Write the words ‘add’ or ‘subtract’ to suit each sentence.

a To add a negative number _________ its opposite.

b To subtract a negative number _________ its opposite.

3 Decide if the following statements are true or false.

a 5 + (–2) = 5 + 2 **b** 3 + (– 4) = 3 – 4 **c** –6 + (– 4) = –6 – 4

d –1 + (–3) = 1 – 3 **e** 8 – (–3) = 8 + 3 **f** 2 – (–3) = 2 – 3

g –3 – (–1) = 3 + 1 **h** –7 – (–5) = –7 + 5 **i** –6 – (–3) = 6 + 3

1F

4–5(½) 4–6(½) 4–6(½)

FLUENCY

Example 11a, 11b

4 Evaluate the following.

a $6 + (-2)$ **b** $4 + (-1)$ **c** $7 + (-12)$ **d** $20 + (-5)$
e $2 + (-4)$ **f** $26 + (-40)$ **g** $-3 + (-6)$ **h** $-16 + (-5)$
i $-18 + (-20)$ **j** $-36 + (-50)$ **k** $-83 + (-22)$ **l** $-120 + (-139)$

Example 11c, 11d

5 Evaluate the following.

a $2 - (-3)$ **b** $4 - (-4)$ **c** $15 - (-6)$ **d** $24 - (-14)$
e $59 - (-13)$ **f** $147 - (-320)$ **g** $-5 - (-3)$ **h** $-8 - (-10)$
i $-13 - (-16)$ **j** $-10 - (-42)$ **k** $-88 - (-31)$ **l** $-125 - (-201)$

6 Write down the missing number.

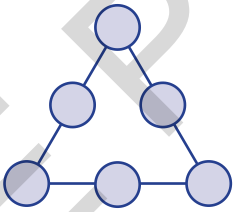
a $4 + \_\_\_\_ = 1$ **b** $6 + \_\_\_\_ = 0$ **c** $-2 + \_\_\_\_ = -1$
d $\_\_\_\_ + (-8) = 2$ **e** $\_\_\_\_ + (-5) = -3$ **f** $\_\_\_\_ + (-3) = -17$
g $12 - \_\_\_\_ = 14$ **h** $8 - \_\_\_\_ = 12$ **i** $-1 - \_\_\_\_ = 29$
j $\_\_\_\_ - (-7) = 2$ **k** $\_\_\_\_ - (-2) = -4$ **l** $\_\_\_\_ - (-436) = 501$

7, 8 8–10 10–12

PROBLEM-SOLVING

7 Place the integers from –3 to 2 in this magic triangle so that each side adds to the given number.

a –3 **b** 0



8 A magic square has each row, column and main diagonal adding to the same magic sum. Complete these magic squares.

a

		1
0	–2	–4

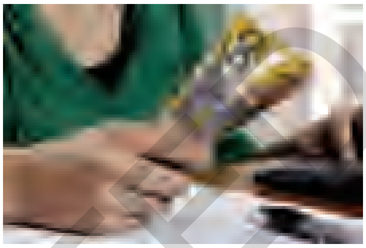
b

–12		
	–15	
	–11	–18

9 Find a pair of negative integers a and b that satisfy both equations in each part.

a $a + b = -8$ and $a - b = 2$
b $a + b = -24$ and $a - b = -6$

PROBLEM-SOLVING



10 A bank account has an initial balance of \$150. Over a one-week period the following occurred.

- \$180 was spent on shoes.
- \$300 of debt was added to the account as a cash advance.
- \$250 of debt was repaid.
- \$110 of debt was added because of a bank fee.
- \$150 of debt was removed with a cash deposit.

What was the balance of the account at the end of the week?

11 The sum of two integers is –5 and their difference is 11. What are the two numbers?

12 The sum of two integers is 11 and their difference is 19. What are the two numbers?

13	13, 14	14, 15

REASONING

13 Describe the error made in the working shown.

a $5 - (-2) = 5 - 2$
$= 3$

b $-2 + (-3) = 2 - 3$
$= -1$

14 If a and b are both negative integers and $a > b$, decide if the following are *always* less than zero.

a $a + b$ **b** $b + a$ **c** $a - b$ **d** $b - a$

15 If a is a negative number, decide if the following are equal to zero.

a $a + a$ **b** $a - a$ **c** $a + (-a)$ **d** $a - (-a)$

Applying rules

—	—	16, 17

ENRICHMENT

16 A rule linking two integers x and y is given by $y = 5 - x$.

a Complete this table.

x	–2	–1	0	1	2	3
y						

b Find a value for y if $x = -13$. **c** Find a value for x if $y = 50$.

17 A rule linking two integers x and y is given by $x - y = -3$.

a Complete this table.

x	–3	–2	–1	0	1	2	3
y							

b Find a value for y if $x = 12$. **c** Find a value for x if $y = -6$.

Progress quiz

1A **1** Evaluate these sums and differences mentally.

a 86 – 53 **b** 28 + 14 **c** 213 + 145 **d** 462 – 70

1A **2** Use an algorithm to find these sums and differences.

a $\begin{array}{r} 58 \\ +265 \\ \hline \end{array}$ **b** $\begin{array}{r} 82 \\ -45 \\ \hline \end{array}$ **c** $\begin{array}{r} 378 \\ +26 \\ +139 \\ \hline \end{array}$ **d** $\begin{array}{r} 5024 \\ -2957 \\ \hline \end{array}$

1B **3** Use a mental strategy to evaluate the following.

a 5×140 **b** 6×49 **c** $128 \div 8$ **d** $1692 \div 4$

1B **4** Use an algorithm to evaluate the following.

a $\begin{array}{r} 37 \\ \times 6 \\ \hline \end{array}$ **b** $\begin{array}{r} 307 \\ \times 219 \\ \hline \end{array}$ **c** $7\overline{)427}$ **d** $15\overline{)347}$

1C **5** **a** Find the LCM of 8 and 12. **b** Find the HCF of 24 and 30.

1C **6** Evaluate these squares and square roots.

a 6^2 **b** 30^2 **c** $\sqrt{64}$ **d** $\sqrt{2500}$

1C **7** Evaluate these cubes and cube roots.

a 2^3 **b** 100^3 **c** $\sqrt[3]{27}$ **d** $\sqrt[3]{125}$

1D **8** Use a factor tree to write 360 as a product of prime factors.

1D **9** Use divisibility tests to decide if the number 126 is divisible by 2, 3, 4, 5, 6, 8 or 9. State a reason for each answer.

1D **10** Find the HCF and LCM of these pairs of numbers, using prime factorisation.

a 42 and 18 **b** 105 and 90

1E **11** Evaluate the following.

a –6 + 20 **b** –5 – 12 **c** –206 + 132

d –218 – 234 **e** –5 + 7 – 9 – 6 **f** 12 – 46 + 27 – 63

1F **12** Evaluate the following.

a 8 + (–14) **b** –220 + (–146) **c** 17 – (–8) **d** –12 – (–61)

1D **13** Three schools are competing at a sports carnival. Each school has a different coloured sports uniform. The numbers of Year 8 students competing are: 162 with a green uniform, 108 with a red uniform and 144 with a blue uniform. All the Year 8 students are to be split up into equal sized teams.

a What is the largest possible team size so every Year 8 student is in a team of students all from their own school?

b How many of these Year 8 teams will be formed from each school?

1G Multiplication and division of integers

Interactive
Widgets
HOTsheets
Walkthroughs

As a repeated addition, $3 \times (-2)$ can be written as $(-2) + (-2) + (-2) = -6$. So $3 \times (-2) = -6$ and, since $a \times b = b \times a$ for all numbers a and b, then -2×3 is also equal to -6.

For division we can write the product $3 \times 2 = 6$ as a quotient $6 \div 2 = 3$. Similarly, if $3 \times (-2) = -6$ then $-6 \div (-2) = 3$. Also if $-2 \times 3 = -6$ then $-6 \div 3 = -2$.

These observations suggest that the quotient of two negative numbers results in a positive number and the product or quotient of two numbers of opposite sign is a negative number.

$6 \div (-2) = -3$ can also be rearranged to $-3 \times (-2) = 6$, which also suggests that the product of two negative numbers is a positive number.

Let's start: Logical patterns

Complete the patterns in these tables to discover the rules for the product of integers.

□	△	□ × △
3	2	6
2	2	
1	2	
0	2	
–1	2	
–2	2	
–3	2	

□	△	□ × △
3	–2	–6
2	–2	–4
1	–2	
0	–2	
–1	–2	
–2	–2	
–3	–2	

Use the table results to complete these sentences.

- $3 \times 2 = 6$ so $6 \div$ ____ $= 3$
- $-3 \times 2 =$ ____ so $-6 \div 2 =$ ____
- $3 \times (-2) =$ ____ so ____ $\div (-2) = 3$
- $-3 \times (-2) =$ ____so $6 \div (-2) =$ ____

What do these observations tell us about multiplying and dividing positive and negative numbers?

Key ideas

- The product or quotient of two integers of the same sign is a positive integer.
  - Positive × Positive = Positive
  - Positive ÷ Positive = Positive
  - Negative × Negative = Positive
  - Negative ÷ Negative = Positive
- The product or quotient of two integers of opposite signs is a negative integer.
  - Positive × Negative = Negative
  - Positive ÷ Negative = Negative
  - Negative × Positive = Negative
  - Negative ÷ Positive = Negative

Example 12 Finding products and quotients of integers

Evaluate the following.

a $3 \times (-7)$ **b** $-4 \times (-12)$ **c** $-63 \div 7$ **d** $-121 \div (-11)$

SOLUTION	EXPLANATION
a $3 \times (-7) = -21$	The product of two numbers of opposite sign is negative.
b $-4 \times (-12) = 48$	-4 and -12 are both negative and so the product will be positive.
c $-63 \div 7 = -9$	The two numbers are of opposite sign so the answer will be negative.
d $-121 \div (-11) = 11$	-121 and -11 are both negative so the quotient will be positive.

Example 13 Combining multiplication and division

Work from left to right to evaluate $-2 \times 9 \div (-3) \times (-5)$

SOLUTION	EXPLANATION
$-2 \times 9 \div (-3) \times (-5) = -18 \div (-3) \times (-5)$	First, evaluate $-2 \times 9 = -18$
$= 6 \times (-5)$	$-18 \div (-3) = 6$
$= -30$	$6 \times (-5) = -30$

Exercise 1G

1–3 3 —

UNDERSTANDING

1 Write the missing numbers in these tables. You should create a pattern in the third column.

a

□	△	□ × △
3	5	15
2	5	
1	5	
0	5	
–1	5	
–2	5	
–3	5	

b

□	△	□ × △
3	–5	–15
2	–5	–10
1	–5	
0	–5	
–1	–5	
–2	–5	
–3	–5	

2 Write the missing numbers in these sentences. Use the tables in Question **1** to help.

a $3 \times 5 =$ ____ so $15 \div 5 =$ ____

b $-3 \times 5 =$ ____ so $-15 \div 5 =$ ____

c $3 \times (-5) =$ ____ so $15 \div (-5) =$ ____

d $-3 \times (-5) =$ ____ so $15 \div (-5) =$ ____

UNDERSTANDING

3 Decide if these statements are true or false.

a Any integer multiplied by zero is equal to zero.
b The product of two positive integers is negative.
c The product of two positive integers is positive.
d The quotient of two integers of opposite sign is negative.
e The quotient of two integers of the same sign is negative.

4–6(½)	4–6(½), 7	4–7(½)

FLUENCY

Example 12a, 12b

4 Evaluate the following.

a $4 \times (-5)$ **b** $6 \times (-9)$ **c** -4×10 **d** -11×9
e $-2 \times (-3)$ **f** $-5 \times (-21)$ **g** $-20 \times (-20)$ **h** $-100 \times (-3)$
i -4×38 **j** $41 \times (-3)$ **k** $-18 \times (-3)$ **l** $-51 \times (-15)$

Example 12c, 12d

5 Evaluate the following.

a $-10 \div 2$ **b** $-38 \div 19$ **c** $-60 \div 15$ **d** $-120 \div 4$
e $32 \div (-16)$ **f** $52 \div (-2)$ **g** $180 \div (-4)$ **h** $900 \div (-25)$
i $-6 \div (-2)$ **j** $-30 \div (-10)$ **k** $-45 \div (-5)$ **l** $-300 \div (-50)$

6 Write the missing number.

a $\_\_\_\_ \times 3 = -9$ **b** $\_\_\_\_ \times (-7) = 35$ **c** $\_\_\_\_ \times (-4) = -28$
d $-3 \times \_\_\_\_ = -18$ **e** $-19 \times \_\_\_\_ = 57$ **f** $\_\_\_\_ \div (-9) = 8$
g $\_\_\_\_ \div 6 = -42$ **h** $85 \div \_\_\_\_ = -17$ **i** $-150 \div \_\_\_\_ = 5$

Example 13

7 Evaluate the following by working from left to right.

a $-4 \times 2 \div (-8)$ **b** $30 \div (-15) \times (-7)$ **c** $48 \div (-3) \times (-10)$
d $-1 \times 58 \times (-2) \div (-4)$ **e** $-110 \div (-11) \times 12 \div (-1)$ **f** $-15 \times (-2) \div (-3) \times (-2)$

8, 9	9–11	10–12

PROBLEM-SOLVING

8 Insert × signs and/or ÷ signs to make these equations true.

a $-2 \,\_\_\_\_\, 3 \,\_\_\_\_\, (-6) = 1$ **b** $10 \,\_\_\_\_\, (-5) \,\_\_\_\_\, (-2) = 25$
c $6 \,\_\_\_\_\, (-6) \,\_\_\_\_\, 20 = -20$ **d** $-14 \,\_\_\_\_\, (-7) \,\_\_\_\_\, (-2) = -1$
e $-32 \,\_\_\_\_\, (-3) \,\_\_\_\_\, (-2) = -48$ **f** $130 \,\_\_\_\_\, (-4) \,\_\_\_\_\, (-8) = 65$

9 The average of three numbers is -4. A new number is added to the list making the average -3. What is the new number?

10 The average of 10 numbers is -5. A new number is added to the list making the average -6. What is the new number?

11 The product of two numbers is -24 and their sum is -5. What are the two numbers?

12 The quotient of two numbers is -4 and their difference is 10. What are the two numbers?

13 | 13, 14 | 13–15

REASONING

13 Remember that $a^2 = a \times a$ and $a^3 = a \times a \times a$.

a Evaluate these expressions.

i $(-2)^2$ **ii** $(-3)^3$ **iii** $(-4)^3$ **iv** $(-5)^2$

b Will the square of a negative number always be positive? Explain why.

c Will the cube of a negative number always be negative? Explain why.

14 a and b are both positive integers with $a > b$. Decide if the following are true or false.

a $-a < b$ **b** $-a \times b > 0$ **c** $-a \div b < 0$

15 Consider the rule $y = -2x - 4$.

a Find the value of y if $x = -3$.

b Find the value of x if $y = -2$.

c Find the value of x that makes $y = 0$.

d Find the value of x that makes $y = -100$.

16 We know that $3^2 = 9$ and $(-3)^2 = 9$. Explain why $\sqrt{-9}$ is not a real number.

17 Is it possible to find the cube root of a negative number? Explain why and give some examples. $\sqrt[3]{-1} = ?$

What's my integer rule?

— | — | 18

ENRICHMENT

18 Find a rule linking x and y for these tables. Start your rules by making y the subject, e.g. $y = -2x + 1$.

a

x	y
–3	8
–2	5
–1	2
0	–1
1	–4
2	–7

b

x	y
–3	18
–2	11
–1	4
0	–3
1	–10
2	–17

c

x	y
–4	17
–2	5
0	1
2	5
4	17
6	37

1H Order of operations and substitution

Interactive

Widgets

HOTsheets

Walkthroughs

An expression such as $a + 2b$ can be evaluated if we know the values of a and b. The expression includes the operations addition (listed first) and multiplication ($2b$ is $2 \times b$); however, by convention, we know that multiplication is done before the addition. In this section we will deal with order of operations, using both positive and negative integers.

Let's start: Equal to 1

Maria makes up a difficult problem for which the answer is equal to 1 but forgets to bring the piece of paper she wrote it on to class. Maria remembers the numbers but not the extra sets of brackets that were involved.

$$-5 \times (-4) + 2 + (-40) \div 5 + 3 - 4 = 1$$

She remembers that there were 2 extra sets of brackets that should be inserted. Can you decide where they should go?

Key ideas

- The rules for order of operations are:
  - Deal with operations inside brackets first.
  - Do multiplication and division next, working from left to right.
  - Do addition and subtraction last, working from left to right.

For example:

$$\begin{aligned}(-2 + 3) \times 4 + 6 - 5 \div (-5) &= 1 \times 4 + 6 - 5 \div (-5)\\ &= 4 + 6 - (-1)\\ &= 10 + 1 = 11\end{aligned}$$

- Expressions can be evaluated by substituting numbers for the given pronumerals.

For example: If $a = -2$ and $b = -3$ then
$$\begin{aligned}a + 5b &= -2 + 5 \times (-3)\\ &= -2 + (-15)\\ &= -17\end{aligned}$$

  - Remember, for example, that $5b$ means $5 \times b$ and $\frac{a}{3}$ means $a \div 3$.

Example 14 Using order of operations

Evaluate the following.

a $5 - 6 \times (-2)$ **b** $-21 \div (5 - (-2))$

SOLUTION	EXPLANATION
a $5 - 6 \times (-2) = 5 - (-12)$ $= 17$	Do the multiplication before the addition and remember that $5 - (-12) = 5 + 12$.
b $-21 \div (5 - (-2)) = -21 \div 7$ $= -3$	Deal with brackets first and remember that $5 - (-2) = 5 + 2$.

Example 15 Substituting integers

Substitute the given integers to evaluate the expressions.

a $a - 3b$ with $a = -2$ and $b = -4$

b $(a + b) \div (-5)$ with $a = -7$ and $b = 2$

c $a^2 - b^3$ with $a = -2$ and $b = -3$

SOLUTION	EXPLANATION
a $\begin{aligned} a - 3b &= -2 - 3 \times (-4) \\ &= -2 - (-12) \\ &= 10 \end{aligned}$	Substitute $a = -2$ and $b = -4$ and then evaluate, noting that $-2 - (-12) = -2 + 12$.
b $\begin{aligned} (a + b) \div (-5) &= (-7 + 2) \div (-5) \\ &= -5 \div (-5) \\ &= 1 \end{aligned}$	Substitute $a = -7$ and $b = 2$ and then deal with the brackets before the division.
c $\begin{aligned} a^2 - b^3 &= (-2)^2 - (-3)^3 \\ &= 4 - (-27) \\ &= 4 + 27 \\ &= 31 \end{aligned}$	Use brackets when substituting into expressions with powers. $(-2)^2 = -2 \times (-2) = 4$ $(-3)^3 = -3 \times (-3) = -27$

Exercise 1H

1–3(½)	2–3(½)	—

UNDERSTANDING

1 Decide if both sides of these simple statements are equal.

a $(2 + 3) - 1 = 2 + 3 - 1$

b $(3 + (-2)) - (-1) = 3 + (-2) - (-1)$

c $5 \times (2 + (-3)) = 5 \times 2 + (-3)$

d $-8 \times 2 - (-1) = -8 \times (2 - (-1))$

e $-10 \div 2 - 4 = -10 \div (2 - 4)$

f $-2 \times 3 + 8 \div (-2) = (-2 \times 3) + (8 \div (-2))$

2 Copy and complete the working for each problem.

a $\begin{aligned} -12 \div (6 + (-2)) &= -12 \div \underline{\qquad} \\ &= \underline{\qquad} \end{aligned}$

b $\begin{aligned} (-8 + 2) \times (-3) &= \underline{\quad} \times (-3) \\ &= \underline{\qquad} \end{aligned}$

c $\begin{aligned} &(-2 + (-1)) \div (15 \div (-5)) \\ &= \underline{\qquad} \div (15 \div (-5)) \\ &= \underline{\qquad} \div (-3) \\ &= \underline{\qquad} \end{aligned}$

d $\begin{aligned} 6 \times (-1 - 5) \div 9 &= 6 \times \underline{\qquad} \div 9 \\ &= \underline{\qquad} \div 9 \\ &= \underline{\qquad} \end{aligned}$

3 Complete the working for these substitutions.

a $a + 2b \ (a = -3, b = 4)$

$\begin{aligned} a + 2b &= -3 + 2 \times 4 \\ &= \underline{\qquad} + \underline{\qquad} \\ &= \underline{\qquad} \end{aligned}$

b $6a - 2b \ (a = 3, b = -4)$

$\begin{aligned} 6a - 2b &= 6 \times 3 - 2 \times (-4) \\ &= 18 - \underline{\qquad} \\ &= \underline{\qquad} \end{aligned}$

UNDERSTANDING

c $3 \times (a - b)\ (a = 5, b = -1)$

$3 \times (a - b) = 3 \times (5 - (-1))$

$= 3 \times \_\_\_\_$

$= \_\_\_\_$

d $-2 \times (a - 1) \div b\ (a = 11, b = -5)$

$-2 \times (a - 1) \div b = -2 \times (\_\_\_\_ - 1) \div \_\_\_\_$

$= -2 \times \_\_\_\_ \div \_\_\_\_$

$= \_\_\_\_ \div \_\_\_\_$

$= \_\_\_\_$

4–6(½) 4–9(½) 5–10(½)

FLUENCY

Example 14a

4 Evaluate the following. Remember to use the normal order of operations.

a $-2 \times 3 \times 5$ **b** $-6 - 2 \times 3$ **c** $4 - 8 \times (-1)$

d $-3 \div (-1) + 7 \times (-2)$ **e** $6 \times (-2) - 10 \div (-5)$ **f** $4 + 8 \times (-2) \div (-16)$

g $20 - 10 \div (-5) \times 2$ **h** $0 \times (-3) + 2 \times (-30)$ **i** $35 - 10 \div (-2) + 0$

Example 14b

5 Use order of operations to evaluate the following.

a $3 \times (2 - 4)$ **b** $(7 - (-1)) \times 3$ **c** $(-8 + (-2)) \div (-5)$

d $40 \div (8 - (-2)) + 3$ **e** $0 \times (38 - (-4)) \times (-6)$ **f** $-6 \times (-1 + 3) \div (-4)$

g $((-2) + 1) \times (8 - (-3))$ **h** $(-6 - 4) \div (50 \div (-10))$ **i** $-2 \times (8 - 7 \times (-2))$

j $(3 - (-2) \times 2) \div 7$ **k** $-4 \times (2 - (-6) \div 6)$ **l** $-5 \div (1 - 3 \times 2)$

Example 15a, 15b

6 Evaluate these expressions using $a = -2$ and $b = 1$.

a $a + b$ **b** $a - b$ **c** $2a - b$

d $b - a$ **e** $a - 4b$ **f** $3b - 2a$

g $b \times (2 + a)$ **h** $(2b + a) - (b - 2a)$

7 Evaluate these expressions using $a = -3$ and $b = 5$.

a ab **b** ba **c** $a + b$ **d** $a - b$

e $b - a$ **f** $3a + 2b$ **g** $(a + b) \times (-2)$ **h** $(a + b) - (a - b)$

Example 15c

8 Evaluate these expressions using $a = -3$ and $b = 5$.

a $a + b^2$ **b** $a^2 - b$ **c** $b^2 - a$ **d** $b^3 + a$

e $a^3 - b$ **f** $a^2 - b^2$ **g** $b^3 - a^3$ **h** $(b - a^2)^2$

9 Evaluate these expressions using $a = -4$ and $b = -3$.

a $3a + b$ **b** $b - 2a$ **c** $4b - 7a$ **d** $-2a - 2b$

e $4 + a - 3b$ **f** $ab - 4a$ **g** $-2 \times (a - 2b) + 3$ **h** $ab - ba$

i $3a + 4b + ab$ **j** $a^2 - b$ **k** $a^2 - b^2$ **l** $b^3 - a^3$

10 Evaluate the following.

a $3 \times (-2)^2$ **b** $-2 \times (-2)^3$ **c** $-16 \div (-2)^3$

d $-4 + \sqrt{25}$ **e** $7 - \sqrt{16}$ **f** $-26 + \sqrt[3]{27}$

g $-4 + 2 \times \sqrt[3]{8}$ **h** $-8 \div \sqrt[3]{-64} + 1$ **i** $-3 \times (-2)^3 + 4$

j $(3 - (-4)^2) \times (-2)$ **k** $(\sqrt[3]{-27} + 3) \div (-1)$ **l** $\sqrt[3]{-8} \times (\sqrt[3]{1000} + 1)$

PROBLEM-SOLVING	11	11, 12	11, 12

11 The temperature in a mountain hut is 15°C at 9 p.m. on Monday night. It drops by 2°C per hour for 11 hours and then the next morning rises by 1°C per hour for the next 4 hours. What is the temperature at midday on Tuesday?

A mountain hut in Tasmania

12 Insert brackets in these statements to make them true.

a $-2 + 1 \times 3 = -3$
b $-10 \div 3 - (-2) = -2$
c $-8 \div (-1) + 5 = -2$
d $-1 - 4 \times 2 + (-3) = 5$
e $-4 + (-2) \div 10 + (-7) = -2$
f $20 + 2 - 8 \times (-3) = 38$
g $1 - (-7) \times 3 \times 2 = 44$
h $4 + (-5) \div 5 \times (-2) = -6$

REASONING	13	13, 14	13–15

13 If a, b and c are integers, decide whether or not the following equations are always true.

a $(a + b) + c = a + (b + c)$
b $(a - b) - c = a - (b - c)$
c $(a \times b) \times c = a \times (b \times c)$
d $(a \div b) \div c = a \div (b \div c)$
e $a - b = b - a$
f $-(a - b) = b - a$

14 We can write $(a + b) \div c$ without brackets in the form $\frac{a+b}{c}$. Evaluate these expressions if $a = -5$, $b = -3$ and $c = -2$.

a $\frac{a+b}{c}$
b $\frac{a-b}{c}$
c $\frac{2c-5a}{b}$
d $\frac{-c-2a}{b}$

15 We can use brackets within brackets for more complex expressions. The inside brackets are dealt with first. Evaluate these.

a $(-6 \times (-2 + 1) + 3) \times (-2)$
b $(2 - (3 - (-1))) \times (-2)$
c $-10 \div (2 \times (3 - (-2)))$

ENRICHMENT	—	—	16–18

Tricky brackets

16 Insert one or more sets of brackets to make these statements true.

a $1 - 3 \times (-4) \div (-13) = -1$
b $4 \div 3 + (-7) \times (-5) = 5$
c $6 - 7 \div (-7) + 6 = 1$
d $-1 - 5 + (-2) \times 1 - 4 = 8$

17 By inserting one extra set of brackets, how many different answers could be obtained from $-4 \times 3 - (-2) + 8$?

18 Make up your own statement like that in Questions **16** and then remove any brackets. Ask a friend to see if they can find where the brackets should go.

Investigation

Euclidean division algorithm

This is a fragment of an Egyptian papyrus from a nearly 2000-year-old copy of Euclid's *Elements*, written in ancient Greek. The diagram shows that the text concerns the relationship of squares and rectangles derived from a straight line divided into unequal parts.

Source: Bill Casselman

The Euclidean division algorithm is a method for finding the highest common factor (also called the greatest common divisor) of two numbers. It is a method that can be performed by hand or programmed into a computer to quickly find the result. Euclid, the famous Greek mathematician, first published the algorithm in his well-known books titled *Elements* in about 300 BCE. The algorithm is used today in many mathematical situations. It is also an important part of today's public key encryption method that is used to code and decipher electronic information in the world of commerce.

In simple terms this is how the algorithm works.

- Let the two numbers be a and b where $a > b$.
- Let $c = a - b$.
- Let the new a and b be the smallest pair from the previous a, b and c. Make $a > b$.
- Repeat the above two steps until $a = b$. The HCF is the value of a (or b) at this point.
- If $a - b = 1$ then the HCF = 1.

The algorithm uses the fact that if two numbers a and b have a common divisor then $a - b$ will also have the same common divisor.

Examples

1 Find the HCF of 12 and 30.

Step	a	b	$a - b = c$
1	30	12	$30 - 12 = 18$
2	18	12	$18 - 12 = 6$
3	12	6	$12 - 6 = 6$
4	6	6	0

The HCF of 12 and 30 is therefore 6.

2 Find the HCF of 7 and 15.

Step	a	b	$a - b = c$
1	15	7	15 − 7 = 8
2	8	7	8 − 7 = 1

The HCF of 17 and 15 is therefore 1.

Using the algorithm

3 Use the Euclidean division algorithm to find the HCF of these pairs of numbers. Set your steps out in a table similar to the example above.

a 12 and 8
b 13 and 29
c 42 and 24
d 184 and 136
e 522 and 666
f 91 and 137

Using a spreadsheet

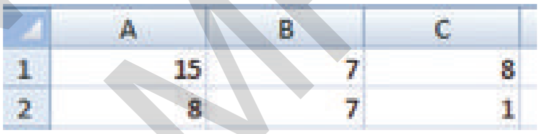
4 **a** Set up a spreadsheet using the formulas shown. Leave the cells A1 and B1 empty, as this is where you will enter your two starting numbers.

	A	B	C	D
1			=A1-B1	
2	=MEDIAN(A1,B1,C1)	=MIN(A1,B1,C1)		

b Enter the two numbers 30 and 12 into cells A1 and B1. Put the larger number into A1. Now fill down each formula until the value in column C is 0. The HCF is therefore 6.

	A	B	C
1	30	12	18
2	18	12	6
3	12	6	6
4	6	6	0

c Enter the two numbers 15 and 7 into cells A1 and B1. Put the larger number into A1. Now fill down each formula until the value in column C is 1. The HCF is therefore 1.



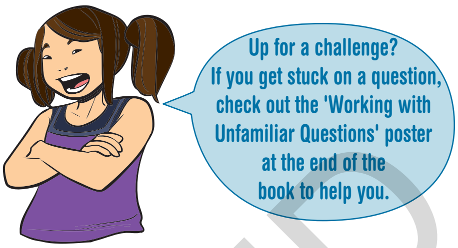
	A	B	C
1	15	7	8
2	8	7	1

d Test your spreadsheet on the pairs of numbers that you worked out by hand in Question **3** above. Here they are again.

i 12 and 8
ii 13 and 29
iii 42 and 24
iv 184 and 136
v 522 and 666
vi 91 and 137

e Now choose a pair of large numbers and use your spreadsheet to find the HCF.

Problems and challenges



1 List the numbers less than 50 that are the product of two prime numbers.

2 **a** Two squares have side lengths 5 cm and 12 cm. Determine the side length of a single square with an area equal to the combined area of these two squares.

b Three cubes have side lengths 1 cm, 6 cm and 8 cm. Determine the side length of a single cube equal in volume to the combined volume of these three cubes.

3 What is the smallest number divisible by all the digits 2, 3, 4, 5, 6, 7, 8 and 9?

4 Evaluate the following expressions given $x = -2$ and $y = -5$.

a $y + y^2 + y^3$ **b** $10 - 2(y - x)$ **c** $60 + 3(x^3 - y^2)$

5 The brackets are missing from these statements. Insert brackets to make them true.

a $-5 \times 3 \div (-3) + 2 - 4 + (-3) = -6$ **b** $-100 \div 4 \times (-2) - 2 \times 3 - (-2) = 32$

6 $n!(n \text{ factorial}) = n \times (n-1) \times (n-2) \times \ldots \times 3 \times 2 \times 1$, so $5! = 5 \times 4 \times 3 \times 2 \times 1$. Evaluate these without the use of a calculator.

a $6! \div 5!$ **b** $1000! \div 999!$ **c** $15! \div 13!$ **d** $5! - 4!$

7 Find a rule linking y and x in each table. Make y the subject of each, e.g. $y = -2x + 3$.

a

x	y
−7	10
−6	9
−5	8
−4	7

b

x	y
−4	13
−3	6
−2	1
−1	−2

c

x	y
−5	−121
−3	−23
−1	3
1	5

d

x	y
−27	−7
−8	−5
−1	−3
1	1

8 Determine the remainder when each of the following numbers is divided by 5.

a $4^{567} + 1$ **b** $4^{678} + 1$

9 Two different prime numbers a and b, are both less than 8. Determine which values of a and b give the largest HCF of $3a^2b$ and $2ab^2$ and state the value of the HCF.

Chapter summary

Integers {..., –3, –2, –1, 0, 1, 2, 3, ...}

Whole numbers 0, 1, 2, 3, ...

Addition and subtraction

$$\begin{array}{r} 2^{1}47 \\ +\,108 \\ \hline 355 \end{array} \qquad \begin{array}{r} {}^{8}\cancel{9}\,\overset{13}{\cancel{4}}\,2^{1} \\ -\,368 \\ \hline 574 \end{array}$$

Multiplication and division

$$\begin{array}{rl} 167 & \\ \times\,15 & \\ \hline 835 & 5 \times 167 \\ 1670 & 10 \times 167 \\ \hline 2505 & 835 + 1670 \end{array} \qquad \begin{array}{r} 5\,9 \\ 7\overline{)4\,1\,{}^{6}6} \end{array}$$

$\therefore 416 \div 7 = 59$ and 3 remainder

Mental strategies

- $156 + 79 = 156 + 80 - 1 = 235$
- $45 + 47 = 45 + 45 + 2 = 92$
- $3 \times 22 = 3 \times 20 + 3 \times 2 = 66$
- $4 \times 88 = 2 \times 176 = 352$
- $164 \div 4 = 82 \div 2 = 41$
- $297 \div 3 = (300 \div 3) - (3 \div 3) = 99$

Negative number operations

- $-2 + 4 = 2$
- $-3 - 7 = -10$
- $4 + (-7) = 4 - 7 = -3$
- $3 - (-2) = 3 + 2 = 5$
- $-2 \times 3 = -6$
- $-5 \times (-7) = 35$
- $10 \div (-2) = -5$
- $-28 \div (-4) = 7$

Substitution

$a = -2,\ b = 5,\ c = -4$

$$\begin{aligned} 2c - ab &= 2 \times (-4) - (-2) \times 5 \\ &= -8 - (-10) \\ &= -8 + 10 \\ &= 2 \end{aligned}$$

Order of operations

Brackets, × and ÷ then + and –

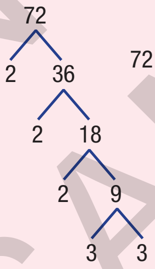
$$\begin{aligned} &-5 \times (-3 - 3) \div 6 + (-1) \\ &= -5 \times (-6) \div 6 + (-1) \\ &= 30 \div 6 + (-1) \\ &= 5 + (-1) \\ &= 4 \end{aligned}$$

Properties

Primes (2 factors)

2, 3, 5, 7, 11, 13, ...

Prime factorisation



$72 = 2^3 \times 3^2$

Squares and cubes

$4^2 = 16,\ \sqrt{16} = 4$

$3^3 = 27,\ \sqrt[3]{27} = 3$

LCM

Multiples of 3: 3, 6, 9, 12, ...
Multiples of 4: 4, 8, 12, ...
∴ LCM = 12

HCF

Factors of 8: 1, 2, 4, 8
Factors of 28: 1, 2, 4, 7, 14, 28
∴ HCF = 4

Divisibility

- 2 Even number.
- 3 Sum of digits divisible by 3.
- 4 Number from last 2 digits divisible by 4.
- 5 Last digit 0 or 5.
- 6 Divisible by 2 and 3.
- 8 Number from last 3 digits divisible by 8.
- 9 Sum of digits divisible by 9.
- 10 Last digit 0.

Multiple-choice questions

1A **1** 127 – 79 is the same as:

A 127 – 80 – 1 **B** 127 – 80 + 1 **C** 127 – 100 + 19
D 127 – 70 + 9 **E** 130 – 80 + 1

1A **2** The sum and difference of 291 and 147 are:

A 448 and 154 **B** 428 and 156 **C** 438 and 144
D 338 and 144 **E** 438 and 154

1A/B **3** Which of these four statements is/are true?

i $3 - 1 = 1 - 3$ **ii** $15 \div 5 = 5 \div 15$
iii $89 \times 3 = 90 \times 3 - 1 \times 3$ **iv** $171 + 50 = 170 + 50 - 1$

A i and iii **B** ii and iv **C** i, ii and iii
D iv only **E** iii only

1B **4** This division problem gives no remainder.

$$\begin{array}{r} 4\ \ 6 \\ 6\overline{)2\,\boxed{}\,6} \end{array}$$

The missing number is:

A 2 **B** 3 **C** 4 **D** 6 **E** 7

1C **5** The HCF and LCM (in that order) of 21 and 14 are:

A 7 and 14 **B** 14 and 21 **C** 42 and 7 **D** 7 and 28 **E** 7 and 42

1E **6** The temperatures of two countries on a particular day are –13°C and 37°C. The difference between the two temperatures is:

A 40°C **B** 36°C **C** 50°C **D** 46°C **E** 24°C

1F **7** The missing number in the statement $-4 - \_\_\_\_ = -1$ is:

A –3 **B** 3 **C** 5 **D** –6 **E** –5

1G **8** The missing number in the statement $\_\_\_\_ \div (-7) = 8$ is:

A 42 **B** –42 **C** –6 **D** 56 **E** –56

1H **9** $-9 \times (-6 + (-2)) \div -12$ is equal to:

A 6 **B** –6 **C** –3 **D** –3 **E** –4

1G **10** Two negative numbers add to –5 and their product is 6. The two numbers are:

A –3, 2 **B** –4, –1 **C** –5, –1 **D** –3, –2 **E** –7, –2

Short-answer questions

1A **1** Use a mental strategy to evaluate the following.

a 324 + 173 **b** 592 – 180 **c** 89 + 40 **d** 135 – 68

e 55 + 57 **f** 280 – 141 **g** 1001 + 998 **h** 10 000 – 4325

1A **2** Use a mental strategy to find these sums and differences.

a $\begin{array}{r} 392 \\ +\ 147 \\ \hline \end{array}$ **b** $\begin{array}{r} 1031 \\ +999 \\ \hline \end{array}$ **c** $\begin{array}{r} 147 \\ -86 \\ \hline \end{array}$ **d** $\begin{array}{r} 3970 \\ -896 \\ \hline \end{array}$

1B **3** Use a mental strategy for these products and quotients.

a $2 \times 17 \times 5$ **b** 3×99 **c** 8×42 **d** 141×3

e $164 \div 4$ **f** $357 \div 3$ **g** $618 \div 6$ **h** $1005 \div 5$

1B **4** Find these products and quotients using an algorithm.

a $\begin{array}{r} 139 \\ \times\ 12 \\ \hline \end{array}$ **b** $\begin{array}{r} 507 \\ \times\ 42 \\ \hline \end{array}$ **c** $3\overline{)843}$ **d** $7\overline{)854}$

1B **5** Find the remainder when 673 is divided by these numbers.

a 5 **b** 3 **c** 7 **d** 9

1C **6** Evaluate:

a $\sqrt{81}$ **b** $\sqrt{121}$ **c** 7^2 **d** 20^2

e $\sqrt[3]{27}$ **f** $\sqrt[3]{64}$ **g** 5^3 **h** 10^3

1C **7** **a** Find all the factors of 60.

b Find all the multiples of 7 between 110 and 150.

c Find all the prime numbers between 30 and 60.

d Find the LCM of 8 and 6.

e Find the HCF of 24 and 30.

1D **8** Write these numbers in prime factor form. You may wish to use a factor tree.

a 36 **b** 84 **c** 198

1D **9** Use divisibility tests to decide if these numbers are divisible by 2, 3, 4, 5, 6, 8 or 9.

a 84 **b** 155 **c** 124 **d** 621

1D **10** Write the numbers 20 and 38 in prime factor form and then use this to help find the following.

a LCM of 20 and 38 **b** HCF of 20 and 38

1E **11** Evaluate:

a –6 + 9 **b** –24 + 19 **c** 5 – 13 **d** –7 – 24

e –62 – 14 **f** –194 – 136 **g** –111 + 110 **h** –328 + 426

1F **12** Evaluate:

a 5 + (–3) **b** –2 + (–6) **c** –29 + (–35) **d** 162 + (–201)

e 10 – (–6) **f** –20 – (–32) **g** –39 – (–19) **h** 37 – (–55)

1F/G **13** a and b are both negative integers with $a > b$. Classify these as true or false.

a $b < a$ **b** $a + b > 0$ **c** $a \times b < 0$ **d** $a \div b > 0$

1G **14** Evaluate:

a -5×2 **b** $-11 \times (-8)$ **c** $9 \times (-7)$ **d** $-100 \times (-2)$

e $-10 \div (-5)$ **f** $48 \div (-16)$ **g** $-32 \div 8$ **h** $-81 \div (-27)$

1H **15** Evaluate using the order of operations.

a $2 + 3 \times (-2)$ **b** $-3 \div (11 + (-8))$

c $-2 \times 3 + 10 \div (-5)$ **d** $-20 \div 10 - 4 \times (-7)$

e $5 \times (-2 - (-3)) \times (-2)$ **f** $0 \times (-2 + 11 \times (-3)) + (-1)$

g $-19 \div (-18 - 1) \div (-1)$ **h** $15 \div (-2 + (-3)) + (-17)$

1H **16** Let $a = -2,\ b = 3$ and $c = -5$ and evaluate these expressions.

a $ab + c$ **b** $a^2 - b$ **c** $ac - b$ **d** abc

e $a^3 - bc$ **f** $c^3 - b$ **g** $bc \div b$ **h** $5b^3 - 2c$

Extended-response questions

1 A monthly bank account show deposits as positive numbers and purchases and withdrawals (P + W) as negative numbers.

Details	P + W	Deposits	Balance
Opening balance	–	–	\$250
Water bill	–\$138	–	a
Cash withdrawal	–\$320	–	b
Deposit	–	c	\$115
Supermarket	d	–	–\$160
Deposit	–	\$400	e

a Find the values of a, b, c, d and e.

b If the water bill amount was \$150, what would be the new value for letter e?

c What would the final deposit need to be if the value for e was \$0? Assume the original water bill amount is \$138 as in the table above.

2 Two teams compete at a club games night. Team A has 30 players while team B has 42 players.

a How many players are there in total?

b Write both 30 and 42 in prime factor form.

c Find the LCM and HCF of the number of players representing the two teams.

d Teams are asked to divide into groups with equal numbers of players. What is the largest group size possible if team A and team B must have groups of the same size?

e In a game of 'scissors, paper, rock', each team forms a line in single file. Player 1 from team A plays against player 1 from team B, then the second pair play against each other, and so on. Once each game is complete, the players go to the back of their line. How many games are played before the first pair plays each other again?