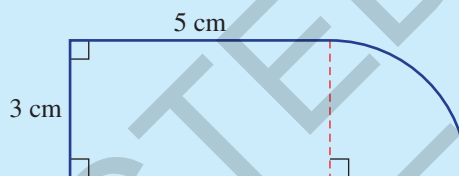# Semester review 2

Measurement

Multiple-choice questions

1 Correct to two decimal places, the perimeter and area, respectively, for this shape are:

  A   12.71 cm, 16.77 cm^2

  B   20.71 cm, 22.07 cm^2

  C   25.42 cm, 29.14 cm^2

  D   18.36 cm, 43.27 cm^2

  E   17.71 cm, 17.25 cm^2

5 cm

3 cm

2   0.04 m^2 is equivalent to:

  A   4 cm^2 **B**   40 mm^2 **C**   0.0004 cm^2 **D**   400 cm^2 **E**   0.00004 km^2

3 A square-based pyramid has base area 30 m^2 and height 7 m. Its volume is:

  A   105 m^3 **B**   70 m^3 **C**   210π m^3 **D**   210 m^3 **E**   140 m^3

4 The curved surface area of this half cylinder, in exact form, is:

  A   80π cm^2 **B**   105π cm^2 **C**   92.5π cm^2
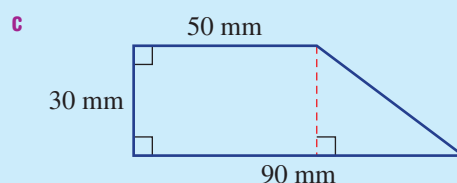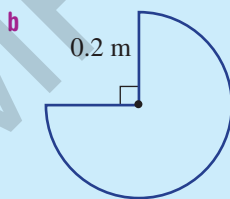
  D   120π cm^2 **E**   160π cm^2

10 cm   16 cm
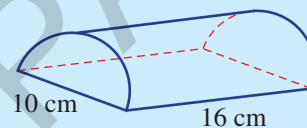
5   The volume of a sphere of diameter 30 cm is closest to:

  A   113 097 cm^3 **B**   2827 cm^3 **C**   11 310 cm^3 **D**   14 137 cm^3 **E**   7069 cm^3

Short-answer questions

1 Find the perimeter and area of these shapes. Give answers correct to one decimal place where necessary. You will need to use Pythagoras' theorem for part **c**.

  a   3 cm

     4 cm

     10 cm

  b   0.2 m

  c   50 mm

     30 mm

     90 mm
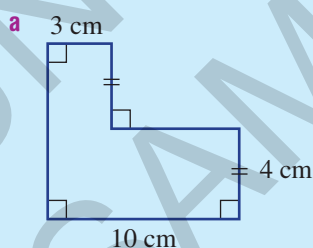
2 Find the surface area and volume for these solids. Give your answers to one decimal place.

  a   7 m   6 m

  b   2 cm   6 cm

  c   1 m
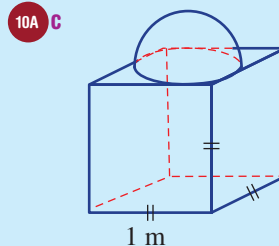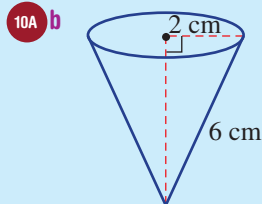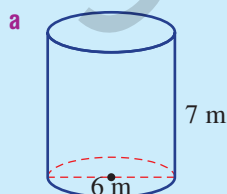
3 A rectangular prism has length 5 cm, width 3 cm and volume 27 cm^3.
   a Find the height of the prism.
   b Find the total surface area of the prism.

10A **4** A cone has volume 90 cm^3 and height 10 cm. Find the exact radius of the cone.

Extended-response question

A cylindrical glass vase is packaged inside a box that is a rectangular prism, so that the vase touches the box on all four sides and is the same height as the box. The vase has a diameter of 8 cm and height 15 cm. Round your answers to two decimal places where necessary.
   **a** Find the volume of the vase.
   b Find the volume of space inside the box but outside the vase.

10A **c** A glass stirring rod is included in the vase. Find the length of the longest rod that can be packaged inside the vase.
   d Find the difference in the length of rod in part **c** and the longest rod that can fit inside the empty box. Round your answer to two decimal places.

Parabolas and other graphs

Multiple-choice questions

1 The graph of $y = (x - 2)^2 + 3$ could be:

A
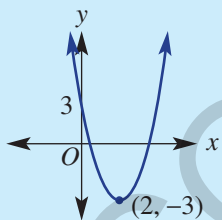
B
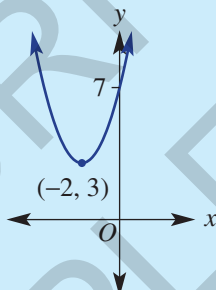
C
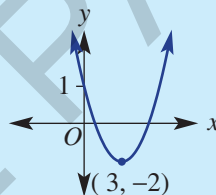
D
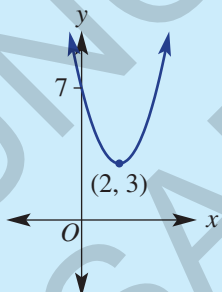
E
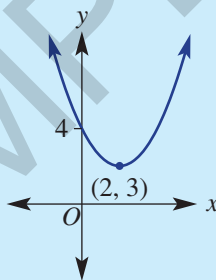
2 The graph of $y = x^2 - 4x$ has a turning point with coordinates:
   A $(2, -4)$ **B** $(0, -4)$ **C** $(4, 0)$ **D** $(-2, 12)$ **E** $(1, -3)$

10A **3** For the quadratic $y = ax^2 + bx + c$, $b^2 - 4ac < 0$, we know that the graph has:
   A a maximum turning point   **B** two x-intercepts
   C no y-intercept   **D** no x-intercepts
   E a minimum turning point

4 The graph with equation $x^2 + y^2 = 9$ is:
  A a circle with radius 9
  B a parabola with turning point $(0, 9)$
  C a circle with radius 3
  D a hyperbola with asymptote at $x = 3$
  E an exponential curve with y-intercept $(0, 3)$

10A

5 The equation of the asymptote of $y = 2^x + 3$ is:
  A $x = 3$ **B** $y = 3$ **C** $x = 2$ **D** $y = 2$ **E** $y = 2x$

Short-answer questions

1 Sketch the following parabolas and state the transformations from $y = x^2$.
  a $y = 3x^2$ **b** $y = -(x + 2)^2$ **c** $y = x^2 + 5$

2 Consider the quadratic $y = x^2 + 4x - 5$.
  a Find the y-intercept.
  b Find the x-intercepts by factorising.
  c Use symmetry to find the turning point.
  d Sketch the graph.

3 Consider the quadratic $y = -2(x - 3)^2 + 8$.
  a State the coordinates of the turning point and whether it is a maximum or minimum.
  b Find the y-intercept.
  c Find the x-intercepts by factorising.
  d Sketch the graph.

4 Sketch the following quadratics by first completing the square.
  a $y = x^2 + 6x + 2$ **b** $y = x^2 - 5x + 8$

10A

5 Consider the quadratic $y = 2x^2 - 4x - 7$.
  a Use the discriminant to determine the number of x-intercepts of the graph.
  b Sketch its graph using the quadratic formula. Round x-intercepts to one decimal place.
  c Find the points of intersection of the quadratic and the line $y = -6x - 3$ by solving simultaneously.

6 Sketch the following graphs, labelling key features.
  a $x^2 + y^2 = 4$ **b** $x^2 + y^2 = 10$ **c** $y = 3^x$

  d $y = 5^{-x}$ **e** $y = \dfrac{2}{x}$ **f** $y = -\dfrac{6}{x}$

7 Find the coordinates of the points of intersection of the graphs of the following.
  a $x^2 + y^2 = 15$ and $y = 2x$
  b $y = 2^x$ and $y = 16$
  c $y = \dfrac{2}{x}$ and $y = 8x$

10A

8 Sketch the graphs of the following relations. Label important features.
  a $(x - 2)^2 + (y + 1)^2 = 16$ **b** $y = 2^{x+3} + 1$ **c** $y = \dfrac{1}{x + 2} - 3$

Extended-response question

A rollercoaster has a section modelled by the equation $h = \dfrac{1}{40}(x^2 - 120x + 1100)$, where h is

the height above the ground and x is the horizontal distance from the start of the section.

All distances are measured in metres and x can take all values between 0 and 200 metres.

a   Sketch the graph of h vs x for $0 \le x \le 200$, labelling the endpoints.

b   What is the height above ground at the start of the section?

c   The rollercoaster travels through an underground tunnel. At what positions from
the start will it enter and leave the tunnel?

d   What is the maximum height the rollercoaster reaches?

e   What is the maximum depth the rollercoaster reaches?

Probability

Multiple-choice questions

1   The number of tails obtained from 100 tosses of two fair coins is shown in the table.

Number of tails	0	1	2
Frequency	23	57	20

From this table, the experimental probability of obtaining two tails is:

A   0.23   B   0.25   C   0.2   D   0.5   E   0.77

2   From the given two-way table $\Pr(A \cap B')$ is:

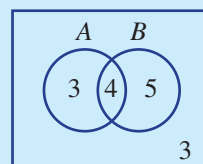|	A	A'	
B	2		
B'			5
	6		12

A $\dfrac{1}{2}$ B $\dfrac{2}{3}$ C $\dfrac{1}{4}$ D $\dfrac{4}{5}$ E $\dfrac{1}{3}$

3   Two events, A and B, are such that $\Pr(A) = 0.7$, $\Pr(B) = 0.4$ and $\Pr(A \cap B) = 0.3$.
$\Pr(A \cup B)$ is equal to:

A   1.4   B   0.8   C   0.6   D   0   E   0.58

4   From the information in the Venn diagram, $\Pr(A \mid B)$ is:

A $\dfrac{5}{12}$ B $\dfrac{4}{5}$ C $\dfrac{4}{7}$

D $\dfrac{4}{9}$ E $\dfrac{1}{3}$

5   A bag of 5 marbles contains 2 green ones. Two marbles are randomly selected without
replacement. The probability of selecting the 2 green marbles is:

A $\dfrac{9}{20}$ B $\dfrac{2}{25}$ C $\dfrac{1}{10}$ D $\dfrac{2}{5}$ E $\dfrac{4}{25}$

Short-answer questions

1 Consider events A and B. Event A is the set of letters in the word 'grape' and event B is the set of letters in the word 'apricot':

$A = \{g, r, a, p, e\}$ $B = \{a, p, r, i, c, o, t\}$

a Represent the two events A and B in a Venn diagram.

b If a letter is randomly selected from the alphabet, find:

   i $\Pr(A)$ **ii** $\Pr(A \cap B)$ **iii** $\Pr(A \cup B)$ **iv** $\Pr(B')$

c Are the events A and B mutually exclusive? Why or why not?

2 The Venn diagram shows the distribution of elements in two sets, A and B.

a Transfer the information in the Venn diagram to a two-way table.

b Find:

   i $n(A \cap B)$ **ii** $n(A' \cap B)$

   iii $n(B')$ **iv** $n(A \cup B)$

c Find:

   i $\Pr(A \cap B)$ **ii** $\Pr(A \cap B')$

   iii $\Pr(B)$ **iv** $\Pr(B|A)$

3 Two events, A and B, are such that $\Pr(A) = 0.24$, $\Pr(B) = 0.57$ and $\Pr(A \cup B) = 0.63$. Find:

a $\Pr(A \cap B)$ **b** $\Pr(A' \cap B')$

4 Two fair 4-sided dice numbered 1 to 4 are rolled and the total is noted.

a List the sample space as a table.

b State the total number of outcomes.

c Find the probability of obtaining:

   i a sum of 4

   ii a sum of at least 5

   iii a sum of 7, given the sum is at least 5.

5 In a group of 12 friends, 8 study German, 4 study German only and 2 study neither German nor Mandarin.

Let A be the event 'studies German'.

Let B be the event 'studies Mandarin'.

a Summarise the information in a Venn diagram.

b Find:

   i $\Pr(A)$ **ii** $\Pr(A|B)$

c State whether or not the events A and B are independent.
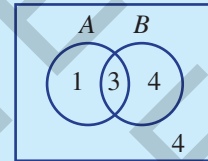
Extended-response question

Lindiana Jones selects two weights from her pocket to sit on a weight-sensitive trigger device after removing the goblet of fire. Her pocket contains three weights, each weighing 200 g, and five weights, each weighing 250 g. The two weights are selected randomly without replacement. Use a tree diagram to help answer the following.

a Find the probability that Lindiana selects two weights totalling:
  i 400 g   **ii** 450 g   **iii** 500 g

b If the total weight selected is less than 480 g, a poison dart will shoot from the wall. Find the probability that Lindiana is at risk from the poison dart.

c By feeling the weight of her selection, Lindiana knows that the total weight is more than 420 g. Given this information, what is the probability that the poison dart will be fired from the wall?

Statistics

Multiple-choice questions

1 For the given stem-and-leaf plot, the range and median, respectively, of the data are:

Stem	Leaf					
0	2	2	6	7		
1	0	1	2	3	5	8
2	3	3	5	7	9	

  $1 \mid 5$ means 15

  A 20, 12.5   **B** 7, 12   **C** 27, 12.5   **D** 29, 3   **E** 27, 13

2 The interquartile range (IQR) for the data set 2, 3, 3, 7, 8, 8, 10, 13, 15 is:
  A 5   **B** 8.5   **C** 7   **D** 13   **E** 8

3 The best description of the correlation between the variables for the scatter plot shown is:
  A weak, negative
  B strong, positive
  C strong, negative
  D weak, positive
  E no correlation

4 A line of best fit passes through the points (10, 8) and (15, 18). The equation of the line is:

  A $y = \frac{2}{3}x + 8$ **B** $y = 2x - 12$ **C** $y = -\frac{1}{2}x + 13$

  D $y = 2x + 6$ **E** $y = \frac{1}{2}x + 3$
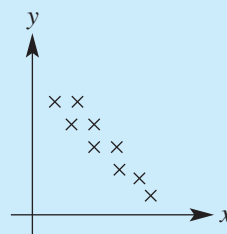
10A

5 The mean and sample standard deviation of the small data set 2, 6, 7, 10 and 12, correct to one decimal place, are:
  A $\bar{x} = 7.4$ and $\sigma = 3.8$ **B** $\bar{x} = 7$ and $\sigma = 3.7$ **C** $\bar{x} = 7.4$ and $\sigma = 3.4$
  D $\bar{x} = 7$ and $\sigma = 7.7$ **E** $\bar{x} = 27.1$ and $\sigma = 9.9$

Short-answer questions

1 Twenty people are surveyed to find out how many days in the past completed month they used public transport. The results are as follows.

 7, 16, 22, 23, 28, 12, 18, 4, 0, 5

 8, 19, 20, 22, 14, 9, 21, 24, 11, 10

 a Organise the data into a frequency table with class intervals of 5 and include a percentage frequency column.

 b Construct a histogram for the data, showing both the frequency and the percentage frequency on the one graph.

 c i State the frequency of people who used public transport on 10 or more days.

   ii State the percentage of people who used public transport on fewer than 15 days.

   iii State the most common interval of days for which public transport was used. Can you think of a reason for this?

2 By first finding quartiles and checking for outliers, draw box plots for the following data sets.

 a 8, 10, 2, 17, 6, 25, 12, 7, 12, 15, 4

 b 5.7, 4.8, 5.3, 5.6, 6.2, 5.7, 5.8, 5.1, 2.6, 4.8, 5.7, 8.3, 7.1, 6.8

3 Farsan's bank balance over 12 months is recorded below.

Month	J	F	M	A	M	J	J	A	S	O	N	D
Balance ($)	1500	2100	2300	2500	2200	1500	1200	1600	2000	2200	1700	2000

 a Plot the time-series for the 12 months.

 b Describe the way in which the bank balance has changed over the 12 months.

 c Between which consecutive months did the biggest change in the bank balance occur?

 d What is the overall change in the bank balance over the year?

4 Consider the variables x and y and the corresponding bivariate data below.

x	4	5	6	7	8	9	10
y	2.1	2.5	3.1	2.8	4	3.6	4.9

 a Draw a scatter plot for the data.

 b Describe the correlation between x and y as either positive, negative or none.

 c Fit a line of good fit by eye to the data on the scatter plot.

 d Use your line of good fit to estimate:

   i y when $x = 7.5$ ii x when $y = 5.5$

5 The back-to-back stem-and-leaf plot below shows the number of DVDs owned by people in two different age groups.

Over 40 Leaf	Stem	Under 40 Leaf
9 7 6 6 4 3 2	0	7 8
6 4 3 2 2 0	1	2 5 5 7 8
8 3	2	4 4 6
	3	2 6 9
	4	1 8

2 | 4 means 24

a By considering the centre and spread of the data, state with reasons:
  i Which data set will have the higher mean?
  ii Which data set will have the smaller standard deviation?

b Calculate the mean and standard deviation for each data set. Round your answers to one decimal place where necessary.

Extended-response question

After a month of watering with a set number of millilitres of water per day, the height of a group of the same species of plant is recorded below.

Water (mL)	8	5	10	14	12	15	18
Height (cm)	25	27	34	40	35	38	45

a Using 'Water' for the x-axis, find the equation of:
  i the least squares regression line
  ii the median–median regression line.

b Use your least squares regression line to estimate the height (to the nearest cm) of a plant watered with 16 mL of water per day.

c Use your median–median regression line to estimate the daily amount of water (to the nearest mL) given to a plant of height 50 cm.

Logarithms and polynomials

10A

Multiple-choice questions

1 The value of $\log_2 16$ is:

A 8   B 256   C 2^{16} D $\dfrac{1}{4}$ E 4

2 An equivalent statement to $3^x = 20$ is:

A $x = \log_3 20$ B $20 = \log_3 x$ C $x = \log_{20} 3$ D $3 = \log_x 20$ E $x = \log_{10}\left(\dfrac{20}{3}\right)$

3 The expression that is *not* a polynomial is:

A $3x^2 + 1$ B $2 - 5x^5 + x$ C $7x - 5$ D $4x^3 + 2x - \dfrac{1}{x}$ E $5x^6 + 2x^4 - 3x^2 - x$

4 When $P(x) = x^4 - 3x^3 + 2x + 1$ is divided by $(x - 2)$, the remainder is:
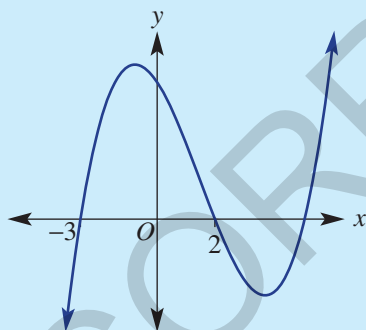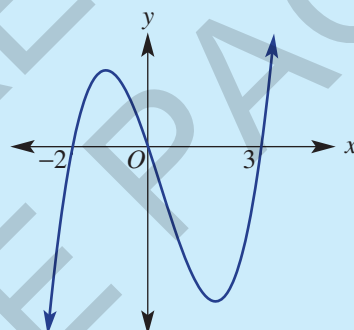
A 37   B 2   C -3 D 13   E 5
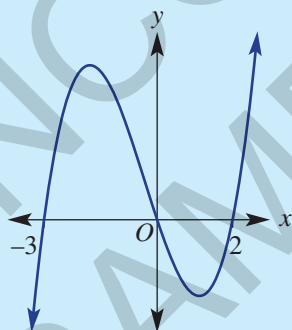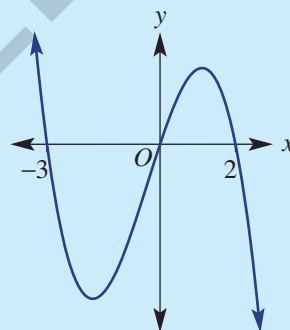
5 A possible graph of $y = -x(x + 3)(x - 2)$ is:
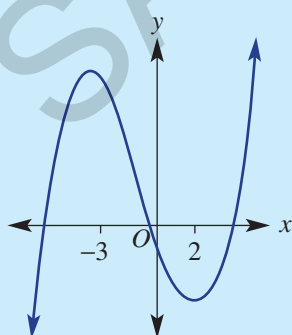
A

B

C

D

E

10A **Short-answer questions**

1   Simplify where necessary and evaluate.

   a $\log_4 64$ b $\log_5 \dfrac{1}{25}$ c $\log_{10} 1000$ d $\log_7 1$

   e $\log_4 2 + \log_4 8$ f $\log_3 54 - \log_3 6$ g $\log_8 8$ h $\log_a a^3$

2   Solve for x.

   a $\log_6 216 = x$ b $\log_x 27 = 3$ c $\log_3 x = 4$

3   a   Solve for x using the given base.

   i $3^x = 30$ ii $15 \times 2.4^x = 60$

   b   Solve for x using base 10 and evaluate, correct to three decimal places.

   i $7^x = 120$ ii $2000 \times 0.87^x = 500$

4   Consider the polynomials $P(x) = x^3 + 3x^2 - 4x - 6$ and $Q(x) = 2x^3 - 3x - 4$.

   a   Find:

   i $P(2)$ ii $P(-1)$ iii $Q(-3)$ iv $Q(1)$

   b   Expand and simplify.

   i $P(x) \times Q(x)$ ii $(Q(x))^2$

5   Divide $P(x) = x^3 - 4x^2 + 2x + 7$ by $(x - 3)$ and write in the form $P(x) = (x - 3)Q(x) + R$,
   where $Q(x)$ is the quotient and R is the remainder.

6   Find the remainder when $P(x) = x^3 - 2x^2 - 13x - 10$ is divided by each of the
   following and, hence, state if it is a factor.

   a $x - 1$ b $x + 2$ c $x - 3$

7   Solve for x.

   a $(x + 1)(x - 3)(x + 6) = 0$ b $-x(2x - 5)(3x + 2) = 0$
   c $x^3 + 5x^2 + 2x - 8 = 0$ d $2x^3 - 3x^2 - 3x + 2 = 0$

10A **Extended-response question**

A section of a train track that heads through a valley and then over a mountain is modelled
by the equation $P(x) = -2x^3 + 3x^2 + 23x - 12$ for $-5 \le x \le 6$.

a   Show that $(x + 3)$ is a factor of $P(x)$.

b   Hence, factorise $P(x)$ using division.

c   Sketch a graph of this section of the track, labelling axes, intercepts and endpoints.