# Semester review 1

Linear relations

Multiple-choice questions

1 The simplified form of $2x(4 - 3y) - 3(3x - 4xy)$ is:

   A $6xy - x$ **B** $2xy$ **C** $x - 18xy$ **D** $12xy - 7x$ **E** $2x - 3y - 4xy$

2 The point that is not on the line $y = 3x - 2$ is:

   A $(-1, -5)$ **B** $(1, 1)$ **C** $(-2, -4)$ **D** $(4, 10)$ **E** $(0, -2)$

3 The length, d, and midpoint, M, of the line segment joining the points $(-2, 4)$
and $(3, -2)$ are:

   A $d = \sqrt{5}, M = (0.5, 1)$ **B** $d = \sqrt{61}, M = (2.5, 3)$ **C** $d = \sqrt{29}, M = (1, 1)$

   D $d = \sqrt{61}, M = (0.5, 1)$ **E** $d = \sqrt{11}, M = (1, 2)$

4 The equation of the line that is perpendicular to the line with equation $y = -2x - 1$ and
passes through the point $(1, -2)$ is:

   A $y = -\dfrac{1}{2}x + \dfrac{3}{2}$ **B** $y = 2x - 2$ **C** $y = -2x - 4$

   D $y = x - 2$ **E** $y = \dfrac{1}{2}x - \dfrac{5}{2}$

Ext

5 The graph of $3x + 2y < 6$ is:
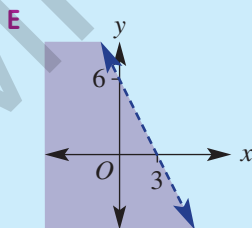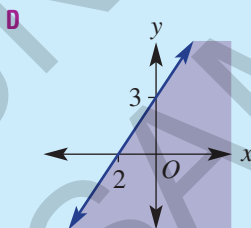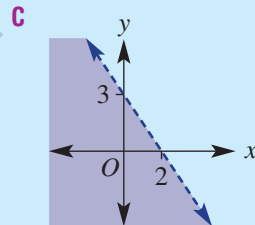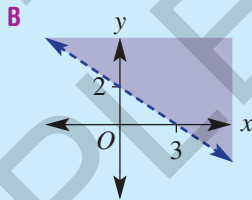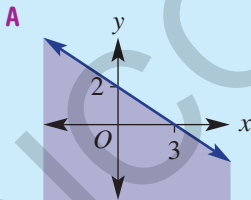
Short-answer questions

1 Simplify:

   a $\dfrac{12 - 8x}{4}$ **b** $\dfrac{5x - 10}{3} \times \dfrac{12}{x - 2}$

   c $\dfrac{3}{4} - \dfrac{2}{a}$ **d** $\dfrac{4}{x + 2} + \dfrac{5}{x - 3}$

2 **a** Solve these equations for x.

   i $2 - 3x = 14$ **ii** $2(2x + 3) = 7x$

   iii $\dfrac{x - 3}{2} = 5$ **iv** $\dfrac{3x - 2}{4} = \dfrac{2x + 1}{5}$

   b Solve these inequalities for x and graph their solutions on a number line.

   i $3x + 2 \leq 20$ **ii** $2 - \dfrac{x}{3} > 1$

3 **a** Find the gradient and y-intercept for these linear relations and sketch each graph.

   i $y = 3x - 2$ **ii** $4x + 3y = 6$

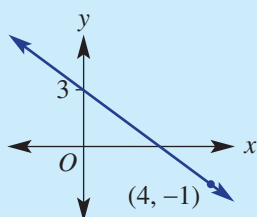   **b** Sketch by finding the x- and y-intercepts where applicable.

   i $y = 2x - 6$ **ii** $3x + 5y = 15$ **iii** $x = 3$ **iv** $y = -2x$
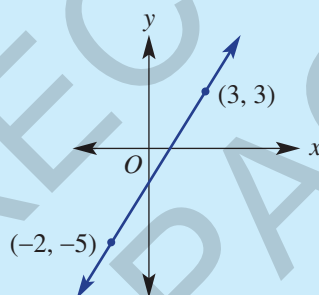
4 Find the equation of the straight lines shown.

   a

   b

5 Find the value(s) of a in each of the following when:

   a The lines $y = ax - 3$ and $y = -3x + 2$ are parallel.

   b The gradient of the line joining the points $(3, 2)$ and $(5, a)$ is -3.

   c The distance between $(3, a)$ and $(5, 4)$ is $\sqrt{13}$.

   d The lines $y = ax + 4$ and $y = \dfrac{1}{4}x - 3$ are perpendicular.

6 Solve these pairs of simultaneous equations.

   a $y = 2x - 1$ **b** $2x - 3y = 8$ **c** $2x + y = 2$ **d** $3x - 2y = 19$

      $y = 5x + 8$ $y = x - 2$ $5x + 3y = 7$ $4x + 3y = -3$

7 At a fundraising event, two hot dogs and three cans of soft drink cost \$13, and four hot dogs and two cans of soft drink cost \$18. What are the individual costs of a hot dog and a can of soft drink?

Ext

8 Sketch the half planes for these linear inequalities.

   a $y \geq 3 - 2x$ **b** $3x - 2y < 9$ **c** $y > -3$

Extended-response question

A block of land is marked on a map with coordinate axes and with boundaries given by the equations $y = 4x - 8$ and $3x + 2y = 17$.

a Solve the two equations simultaneously to find their point of intersection.

b Sketch each equation on the same set of axes, labelling axis intercepts and the point of intersection.

The block of land is determined by the intersecting region $x \geq 0$, $y \geq 0$, $y \geq 4x - 8$ and $3x + 2y \leq 17$.
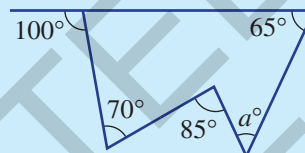
c Shade the area of the block of land (i.e. the intersecting region on the graph in part **b**).

d Find the area of the block of land if 1 unit represents 100 metres.
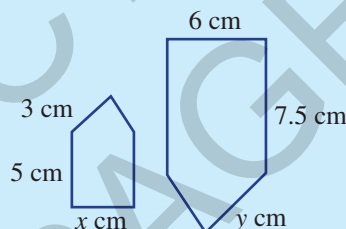
Geometry

Multiple-choice questions

1 The value of a in the diagram shown is:
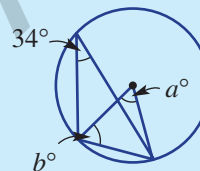 **A** 40 **B** 25 **C** 30
 D 50 **E** 45

2 The values of x and y in these similar figures are:
 A $x = 2.6, y = 5$
 B $x = 4, y = 4.5$
 C $x = 4, y = 7.5$
 D $x = 3, y = 6$
 E $x = 3.5, y = 4.5$

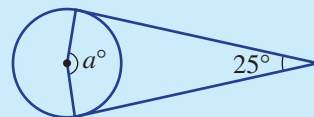**3** The values of the pronumerals in this diagram are:
 A $a = 17, b = 56$ **B** $a = 34, b = 73$
 C $a = 68, b = 56$ **D** $a = 34, b = 34$
 E $a = 68, b = 34$
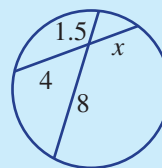
4 The value of angle a in this diagram is:
 A 115 **B** 165 **C** 140
 D 130 **E** 155

5 The value of x in this diagram is:
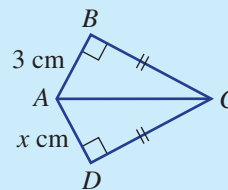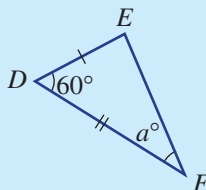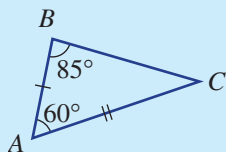 **A** 5.5 **B** 0.75 **C** 3 **D** 4 **E** 8

Short-answer questions

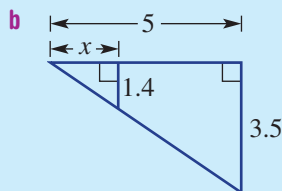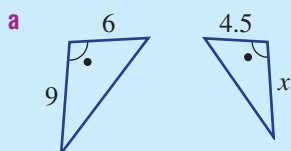**1** Prove the following congruence statements, giving reasons, and use this to find the value of the pronumerals.
 a $\triangle ABC \equiv \triangle DEF$ **b** $\triangle ABC \equiv \triangle ADC$

2 Use congruence to prove that a parallelogram (with opposite parallel sides) has equal opposite sides.

3 Find the value of the pronumeral, given these pairs of triangles are similar.

a
6   4.5
9   x

b
|← 5 →|
|← x →|
1.4
3.5

10A 4 Use the chord and circle theorems to find the value of each pronumeral.

a x cm, 8 cm

b x, 10

c $a°$, 83°, $b°$, 65°

d 50°, $x°$

e $a°$, 110°

f $b°$ $a°$, $c°$, 30°, 20°
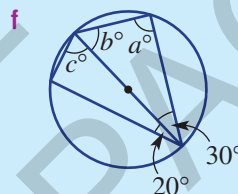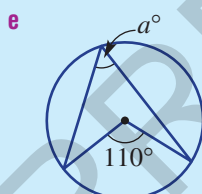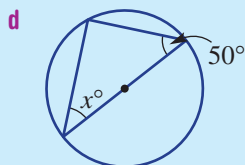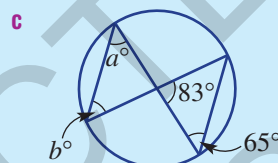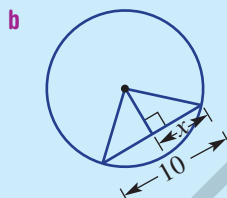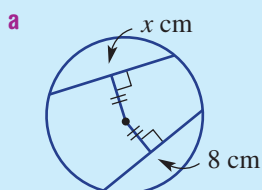
Ext 5 Use tangent properties to find the value of the pronumerals.

a 70°, $x°$

b 8 m, 2 m, x m

c 55°, 63°, $a°$, $b°$
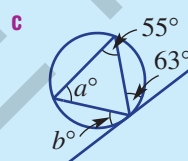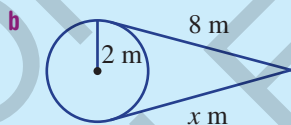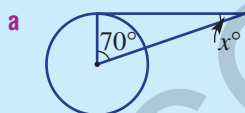
Ext 6 Find the value of x in each figure.

a 6, 12, 5, x

b 3, 5, 4, x

c x, 5, 8
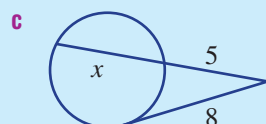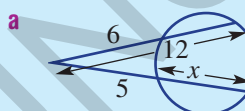
10A **Extended-response question**

A logo for a car manufacturing company is silver and blue and shaped as shown, with O indicating the centre of the circle.

C A O D B
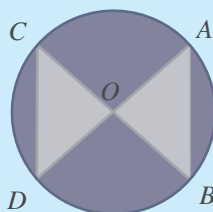
The radius of the logo is 5 cm and chord *AB* is 6 cm. Given the two chords are equidistant from the centre of the circle, complete the following.

a What is the length of *CD*? Give a reason.

b Hence, prove that $\triangle OAB \equiv \triangle OCD$.

c By first finding the length of *OM*, where *M* is the point such that $OM \perp AB$, find the area of $\triangle OAB$.

d Hence, determine what percentage of the logo is occupied by the silver portion, given the area of a circle is πr^2. Answer correct to one decimal place.

e Given that $\angle OCD = 53.1°$, what is the angle between the two triangles (i.e. $\angle BOD$)?

Indices and surds

Multiple-choice questions

1 The simplified form of $2\sqrt{45}$ is:

A $\sqrt{90}$ **B** $6\sqrt{5}$ **C** $10\sqrt{3}$ **D** $18\sqrt{5}$ **E** $6\sqrt{15}$

2 $7\sqrt{3} - 4\sqrt{2} + \sqrt{12} + \sqrt{2}$ simplifies to:

A $7\sqrt{3} - 3\sqrt{2}$ **B** $5\sqrt{3} + 3\sqrt{2}$ **C** $\sqrt{3} - 3\sqrt{2}$
D $9\sqrt{3} - 3\sqrt{2}$ **E** $3\sqrt{3} + 5\sqrt{2}$

3 The expanded form of $(2\sqrt{6} + 1)(3 - 4\sqrt{6})$ is:

A 3 **B** $\sqrt{6} - 33$ **C** $6\sqrt{6} + 3$ **D** $2\sqrt{6}$ **E** $2\sqrt{6} - 45$

4 The simplified form of $\dfrac{12(a^3)^{-2}}{(2ab)^2 \times a^2 b^{-1}}$, when written using positive indices, is:

A $\dfrac{6}{a^2 b}$ **B** $3a^2$ **C** $\dfrac{6a}{b}$ **D** $\dfrac{3}{a^2 b^3}$ **E** $\dfrac{3}{a^{10} b}$

5 0.00032379 in scientific notation, using three significant figures, is:

A 3.23×10^{-4} **B** 3.24×10^4 **C** 3.24×10^{-4}
D 32.4×10^3 **E** 0.324×10^{-5}

Short-answer questions

1 Simplify:

a $\sqrt{54}$ **b** $4\sqrt{75}$ **c** $\dfrac{3\sqrt{24}}{2}$

d $\sqrt{5} \times \sqrt{2}$ **e** $3\sqrt{7} \times \sqrt{7}$ **f** $3\sqrt{6} \times 4\sqrt{8}$

g $\sqrt{15} \div \sqrt{5}$ **h** $\dfrac{3\sqrt{30}}{9\sqrt{6}}$ **i** $\sqrt{\dfrac{200}{49}}$

2 Simplify fully.

a $2\sqrt{5} + 3\sqrt{7} + 5\sqrt{5} - 4\sqrt{7}$ **b** $\sqrt{20} - 2\sqrt{5}$ **c** $\sqrt{18} - 4 + 6\sqrt{2} - 2\sqrt{50}$

3 Expand and simplify these expressions.

a $2\sqrt{3}(\sqrt{5} - 2)$ **b** $(3\sqrt{5} - 2)(1 - 4\sqrt{5})$
c $(\sqrt{7} - \sqrt{3})(\sqrt{7} + \sqrt{3})$ **d** $(3\sqrt{3} + 4\sqrt{2})^2$

10A **4** Rationalise the denominator.

    a $\dfrac{3}{\sqrt{2}}$ **b** $\dfrac{2\sqrt{3}}{5\sqrt{6}}$ **c** $\dfrac{2-\sqrt{5}}{\sqrt{5}}$

5 Use the index laws to simplify the following. Express all answers with positive indices.

    a $(2x^2)^3 \times 3x^4y^2$ **b** $\left(\dfrac{3a}{b^4}\right)^2 \times \dfrac{2b^{10}}{6(2a^5)^0}$

    c $3a^{-5}b^2$ **d** $\dfrac{4x^{-2}y^3}{10x^{-4}y^6}$

6 Convert:

    a to a basic numeral

      i 3.72×10^4 **ii** 4.9×10^{-6}

    b to scientific notation, using three significant figures

      i 0.000072973 **ii** $4\,725\,400\,000$

10A **7** **a** Express in index form.

      i $\sqrt{10}$ **ii** $\sqrt{7x^6}$ **iii** $4\sqrt[5]{x^3}$ **iv** $15\sqrt{15}$

    b Express in surd form.

      i $6^{\frac{1}{2}}$ **ii** $20^{\frac{1}{5}}$ **iii** $7^{\frac{3}{4}}$

10A **8** Evaluate without using a calculator.

    a 5^{-1} **b** 2^{-4} **c** $81^{\frac{1}{4}}$ **d** $8^{-\frac{1}{3}}$

10A **9** Solve these exponential equations for x.

    a $4^x = 64$ **b** $7^{-x} = \dfrac{1}{49}$

    c $9^x = 27$ **d** $5^{5x+1} = 125^x$

10 Determine the final amount after 3 years if:

    a $2000 is compounded annually at 6%

    b $7000 is compounded annually at 3%.

Extended-response question

Lachlan's share portfolio is rising at 8% per year and is currently valued at $80\,000.

a Determine a rule for the value of Lachlan's share portfolio (V dollars) in n years' time.

b What will be the value of the portfolio, to the nearest dollar:

    i next year?   **ii** in 4 years' time?

c Use trial and error to find when, to two decimal places, the share portfolio will be worth $200\,000.

d After 4 years, however, the market takes a downwards turn and the share portfolio begins losing value. Two years after the downturn, Lachlan sells his shares for $96\,170. If the market was declining in value at a constant percentage per year, what was this rate of decline, to the nearest percentage?

Trigonometry

Multiple-choice questions

1 The value of x in the triangle shown is approximately:

   **A** 7.6   **B** 12.0   **C** 10.4

    D 6.5   **E** 8.2
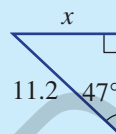
2 A bird 18 m up in a tree spots a worm on the ground 12 m from the base of the tree. The angle of depression from the bird to the worm is closest to:

    A 41.8°   **B** 56.3°   **C** 61.4°   **D** 33.7°   **E** 48.2°

3 A walker travels due south for 10 km and then on a true bearing of 110° for 3 km. The total distance south from the starting point to the nearest kilometre is:

    A 11 km   **B** 1 km   **C** 9 km   **D** 13 km   **E** 15 km
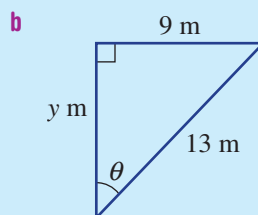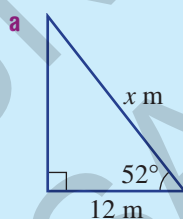
4 The area of the triangle shown is closest to:

    8 cm

    68°

    7 cm
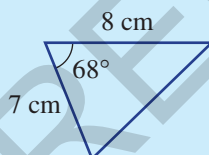
    A 69 cm²   **B** 52 cm²   **C** 28 cm²   **D** 26 cm²   **E** 10 cm²

5 Choose the *incorrect* statement.

    A $\theta = 290°$ is in quadrant 4   **B** $\sin 120° = \dfrac{\sqrt{3}}{2}$

    C $\cos 110° = -\cos 20°$ **D** $\tan \theta$ is positive for $200° < \theta < 250°$.

    E $\sin 230° = -\sin 50°$

Short-answer questions

1 Find the value of the pronumeral in each right-angled triangle, correct to one decimal place.

    a

    x m

    52°

    12 m

    b 9 m

    y m

    θ

    13 m

2 For the following bearings, give the true bearing of:

    i A from O **ii** O from A

    a

    N

    W O 30°   E

    S A
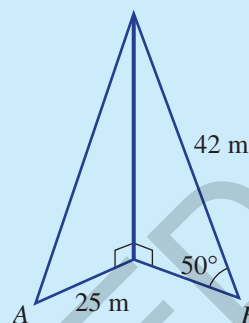
    b

    N

    A

    W   40° O E

    S
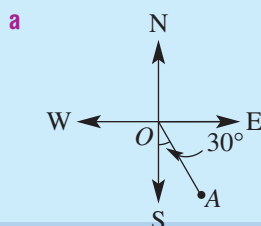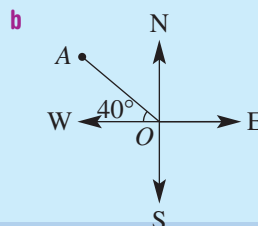
10A

3 Two wires reach from the top of an antenna to points *A* and *B* on the ground, as shown. Point *A* is 25 m from the base of the antenna, and the wire from point *B* is 42 m long and makes an angle of 50° with the ground.
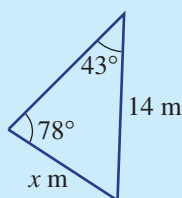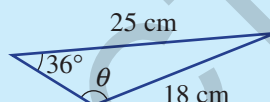 **a** Find the height of the antenna, to three decimal places.
  b Find the angle the wire at point *A* makes with the ground, to one decimal place.

10A

4 Find the value of the pronumeral, correct to one decimal place.
  a
  43°
  14 m
  78°
  x m

  b θ is obtuse
  25 cm
  36°
  θ
  18 cm

10A

5 Find the largest angle, correct to one decimal place, in a triangle with side lengths 8 m, 12 m and 15 m.

10A

6 a If $\theta = 223°$, state which of $\sin\theta$, $\cos\theta$ and $\tan\theta$ are positive?
  b Choose the angle θ to complete each statement.
    i $\sin 25° = \sin\theta$, where θ is obtuse.
    ii $\tan 145° = -\tan\theta$, where θ is acute.
    iii $\cos 318° = \cos\theta$, where θ is the reference angle.
  c State the exact value of:
    i $\cos 60°$ **ii** $\sin 135°$ **iii** $\tan 330°$

10A

7 Use this graph of $\sin\theta$ to answer the following.
  a Estimate the value of $\sin\theta$ for $\theta = 160°$.
  b Estimate the two values of θ for which $\sin\theta = -0.8$.
  c Is $\sin 40° < \sin 120°$?
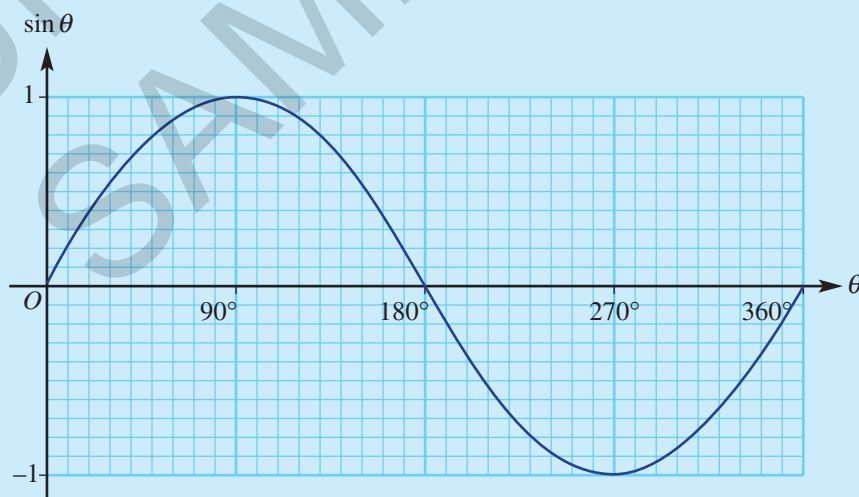
Extended-response question

A group of walkers set out on a trek to get to the base of a mountain range. The mountains have two peaks, which are 112 m and 86 m above ground level from the base. The angle of elevation from the peak of the smaller mountain to the peak of the taller mountain is 14°.

a   Find the horizontal distance between the two mountain peaks, correct to one decimal place.

To get to the base of the mountain range, the walkers set out from the national park entrance on a bearing of 52°T for a distance of 13 km and then turn on a bearing of 340°T for the last 8 km of the trek.

b   Draw a diagram representing the trek. Label all known measurements.

c   If the walkers are able to trek directly from their start location to their endpoint, what distance would they cover? Round your answer to three decimal places.

d   After they have explored the mountains, the group will be taken by bus back along the direct path from their end location to the park entrance. Determine the true bearing on which they will travel. Round your answer to the nearest degree.

Quadratic equations

Multiple-choice questions

1   The expanded form of $2(2x - 3)(3x + 2)$ is:
   A $12x^2 - 5x - 6$ **B** $12x^2 - 12$ **C** $12x^2 - 10x - 12$
   D $24x^2 - 20x - 24$ **E** $12x^2 - x - 6$

2   The factorised form of $25y^2 - 9$ is:
   A $(5y - 3)^2$ **B** $(5y - 3)(5y + 3)$ **C** $(25y - 3)(y + 3)$
   D $(5y - 9)(5y + 1)$ **E** $5(y + 1)(y - 9)$

3 $\dfrac{x^2 - 4}{x^2 - x - 6} \times \dfrac{x^2 - 4x + 3}{4x - 8}$ simplifies to:
   A $x - 2$ **B** $\dfrac{x + 3}{12}$ **C** $\dfrac{x^2 + 1}{2x}$ **D** $\dfrac{x - 1}{4}$ **E** $\dfrac{x^2}{x - 2}$

4   The solution(s) to the quadratic equation $x^2 - 4x + 4 = 0$ is/are:
   A $x = 0, 4$ **B** $x = 2$ **C** $x = 1, 4$ **D** $x = 2, -2$ **E** $x = -1, 4$

5   A quadratic equation $ax^2 + bx + c = 0$ has a discriminant equal to 17. This tells us that:
   A   The equation has a solution $x = 17$.
   B   The equation has no solutions.
   C $a + b + c = 17$
   D   The equation has two solutions.
   E   The equation has one solution.

Short-answer questions

1 Expand and simplify.

   a $(3x+1)(3x-1)$ b $(2x-5)^2$ c $(2x+3)(x+5)-(3x-5)(x-4)$

2 Factorise fully these quadratics. Remember to take out any common factors first.

   a $4x^2-y^2$ b $(x+2)^2-7$ c $3x^2-48$

   d $x^2+5x-14$ e $x^2-10x+25$ f $2x^2-16x+24$

10A 3 Factorise these non-monic quadratics.

   a $3x^2-2x-8$ b $6x^2+7x-3$ c $10x^2-23x+12$

4 Solve these quadratic equations using the null factor law.

   a $2x(x-3)=0$ b $(x+4)(2x-1)=0$

   c $x^2+5x=0$ d $x^2-16=0$

   e $x^2-7=0$ f $x^2-4x+4=0$

   g $x^2-5x-24=0$ **10A** h $3x^2+5x-2=0$

5 Solve these quadratic equations by first writing them in standard form.

   a $x^2=40-3x$ b $x(x-6)=4x-21$ c $\dfrac{x+20}{x}=x$

6 a Factorise by completing the square.

     i x^2-6x+4 ii x^2+4x+7 iii x^2+3x+1

   b Use your answers to part a to solve these, if possible.

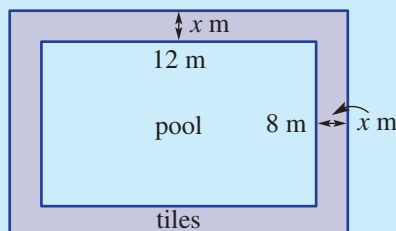   i $x^2-6x+4=0$ ii $x^2+4x+7=0$ iii $x^2+3x+1=0$

10A 7 Solve these quadratic equations using the quadratic formula. Leave your answers in exact surd form.

   a $2x^2+3x-6=0$ b $x^2-4x-6=0$

Extended-response question

A rectangular backyard swimming pool, measuring 12 metres by 8 metres, is surrounded by a tiled path of width x metres, as shown.

a Find a simplified expression for the area of the tiled path.

b If $x=1$, what is the tiled area?

c Solve an appropriate equation to determine the width, x metres, if the tiled area is 156 m^2.

10A d Find the width, x metres, if the tiled area is 107.36 m^2. Use the quadratic formula.