Chapter **4**

Trigonometry

What you will learn

4A Reviewing algebra
4B Trigonometric ratios
4C Finding angles
4D Applications in two dimensions
4E Bearings
4F Applications in three dimensions (10A)
4G Obtuse angles and exact values (10A)
4H The sine rule (10A)
4I The cosine rule (10A)
4J Area of a triangle (10A)
4K The four quadrants (10A)
4L Graphs of trigonometric functions (10A)

Australian curriculum

MEASUREMENT AND GEOMETRY

Pythagoras and trigonometry

Solve right-angled triangle problems, including those involving direction and angles of elevation and depression.

(10A) Establish the sine, cosine and area rules for any triangle and solve related problems.

(10A) Use the unit circle to define trigonometric functions, and graph them with and without the use of digital technologies.

(10A) Solve simple trigonometric equations.

(10A) Apply trigonometry to solving three-dimensional problems in right-angled triangles. AC

Online resources

- Chapter pre-test
- Videos of all worked examples
- Interactive widgets
- Interactive walkthroughs
- Downloadable HOTsheets
- Access to HOTmaths Australian Curriculum courses

Global positioning systems

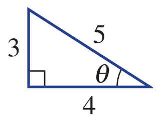
Global positioning systems, once a secret military technology, are now a commonplace navigational tool. The global positioning system (GPS) network relies on 24 satellites that orbit the Earth at about 3000 km/h, taking about 12 hours to complete one orbit. If a handheld or car GPS device receives radio signals from at least three satellites, then a process of triangulation can be used to pinpoint the position of the receiver. Triangulation involves knowing the position of the satellites and the distance between the satellites and the receiver. Trigonometry is used in these calculations.

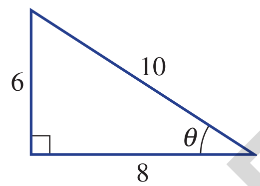
4A Trigonometric ratios

Interactive Widgets HOTsheets Walkthroughs

The study of trigonometry explores the relationship between the angles and side lengths of triangles. Trigonometry can be applied to simple problems, such as finding the angle of elevation of a kite, to solving complex problems in surveying and design.

Trigonometry is built upon the three ratios sine, cosine and tangent. These ratios do not change for triangles that are similar in shape.





$\sin\theta = \frac{3}{5}$ $\qquad$ $\sin\theta = \frac{6}{10} = \frac{3}{5}$

$\cos\theta = \frac{4}{5}$ $\qquad$ $\cos\theta = \frac{8}{10} = \frac{4}{5}$

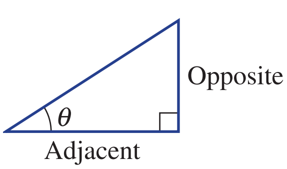
$\tan\theta = \frac{3}{4}$ $\qquad$ $\tan\theta = \frac{6}{8} = \frac{3}{4}$

Trigonometry is the basis for surveying.

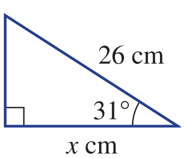
Let's start: Which ratio?

In a group or with a partner, see if you can recall some facts from Year 9 Trigonometry to answer the following questions.

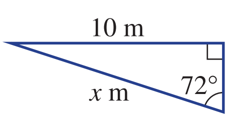
- What is the name given to the longest side of a right-angled triangle?
- $\tan\theta = \frac{\text{Opposite}}{\text{Adjacent}}$ is one trigonometric ratio. What are the other two?



- Which ratio would be used to find the value of x in this triangle? Can you also find the answer?

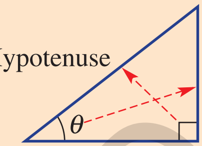


- Which ratio would be used to find the value of x in this triangle? Can you also find the answer?



Key ideas

- The **hypotenuse** is the longest side of a right-angled triangle. It is opposite the right angle.
- Given a right-angled triangle and another angle θ, the three trigonometric ratios are:
  - sine of angle θ ($\mathbf{sin}\theta$) $= \dfrac{\text{length of the opposite side}}{\text{length of the hypotenuse}}$
  - cosine of angle θ ($\mathbf{cos}\theta$) $= \dfrac{\text{length of the adjacent side}}{\text{length of the hypotenuse}}$
  - tangent of angle θ ($\mathbf{tan}\theta$) $= \dfrac{\text{length of the opposite side}}{\text{length of the adjacent side}}$

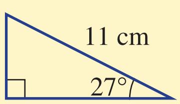


- Many people like to use SOHCAHTOA to help remember the three ratios.
  - $\sin\theta = \dfrac{O}{A}$ $\cos\theta = \dfrac{A}{H}$ $\tan\theta = \dfrac{O}{A}$
- To find an unknown length on a right-angled triangle:
  - Choose a trigonometric ratio that links one known angle and a known side length with the unknown side length.
  - Solve for the unknown side length.

Example 1 Solving for an unknown in the numerator

Find the value of x in these right-angled triangles, correct to two decimal places.

a



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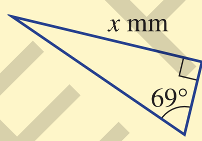
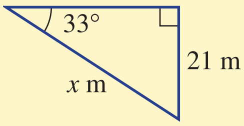


SOLUTION	EXPLANATION
a $\cos\theta = \dfrac{A}{H}$	Choose the ratio $\cos\theta = \dfrac{\text{Adjacent}}{\text{Hypotenuse}}$.
$\cos 27^\circ = \dfrac{x}{11}$	Multiply both sides by 11, then use a calculator.
$\therefore\ x = 11 \times \cos 27^\circ$	
$= 9.80$ (to 2 d.p.)	Round your answer as required.
b $\tan\theta = \dfrac{O}{A}$	The tangent ratio uses the opposite and the adjacent sides.
$\tan 69^\circ = \dfrac{x}{38}$	
$\therefore\ x = 38 \times \tan 69^\circ$	Multiply both sides by 38.
$= 98.99$ (to 2 d.p.)	

Example 2 Solving for an unknown in the denominator

Find the value of x in these right-angled triangles, rounding your answer to two decimal places.

a



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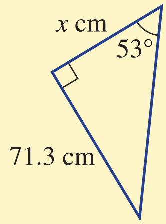


SOLUTION	EXPLANATION
a $\sin\theta = \frac{O}{H}$	Choose the sine ratio since the adjacent side is not marked.
$\sin 33^\circ = \frac{21}{x}$	
$x \times \sin 33^\circ = 21$	Multiply both sides by x to remove the fraction, then divide both sides by sin 33°.
$x = \frac{21}{\sin 33^\circ}$	
$= 38.56$ (to 2 d.p.)	Evaluate using a calculator and round your answer as required.
b $\tan\theta = \frac{O}{A}$	The hypotenuse is unmarked, so use the tangent ratio.
$\tan 53^\circ = \frac{71.3}{x}$	
$x \times \tan 53^\circ = 71.3$	Multiply both sides by x, then solve by dividing both sides by tan 53°.
$x = \frac{71.3}{\tan 53^\circ}$	
$= 53.73$ (to 2 d.p.)	

Exercise 4A

1(½), 2, 3(½)	3(½)	—

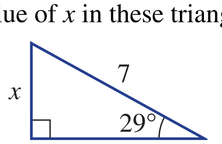
UNDERSTANDING

1 Use a calculator to evaluate the following, correct to three decimal places.

a cos 37° **b** sin 72° **c** tan 50° **d** cos 21.4°
e sin 15.9° **f** tan 85.1° **g** cos 78.43° **h** sin 88.01°

2 Decide which ratio $\left(\text{i.e. } \sin\theta = \frac{O}{H}, \cos\theta = \frac{A}{H} \text{ or } \tan\theta = \frac{O}{A}\right)$ would be best to help find the value of x in these triangles. Do not find the value of x.

a



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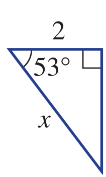

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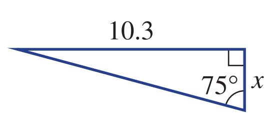

4A

UNDERSTANDING

3 Solve for x in these equations, correct to two decimal places.

a $\tan 31° = \frac{x}{3}$ **b** $\cos 54° = \frac{x}{5}$ **c** $\sin 15.6° = \frac{x}{12.7}$

d $\sin 57° = \frac{2}{x}$ **e** $\cos 63.4° = \frac{10}{x}$ **f** $\tan 71.6° = \frac{37.5}{x}$

4–5(½)	4–5(½), 6	4–5(½), 6

FLUENCY

Example 1

4 Use trigonometric ratios to find the values of the pronumerals, to two decimal places.

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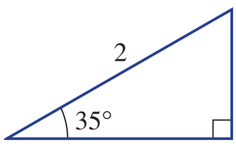

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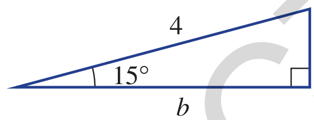

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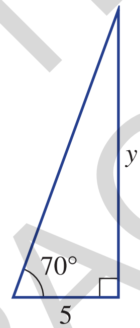

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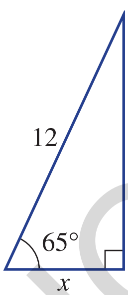

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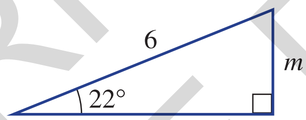

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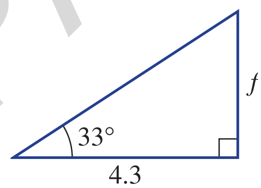

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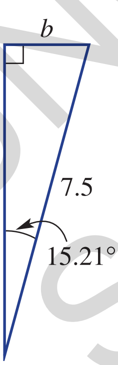

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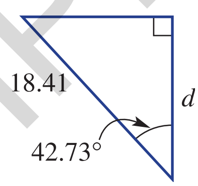

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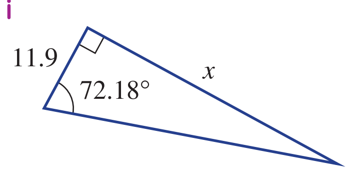

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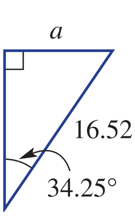

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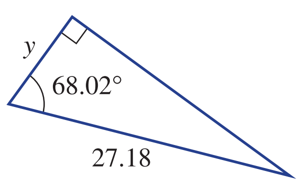

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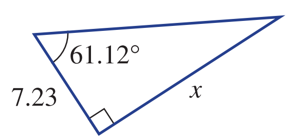

4A

FLUENCY

Example 2

5 Use trigonometric ratios to find the values of the pronumerals, to two decimal places, for these right-angled triangles.

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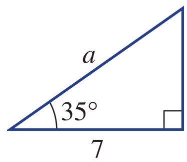

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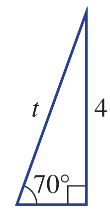

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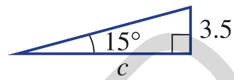

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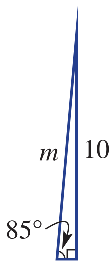

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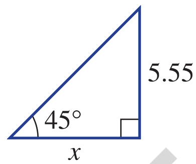

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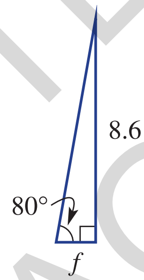

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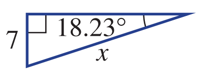

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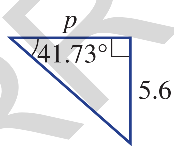

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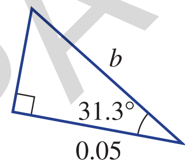

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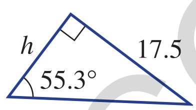

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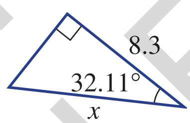

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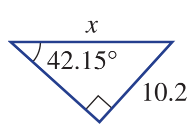

6 Find the unknown side lengths for these right-angled triangles, correct to two decimal places where necessary.

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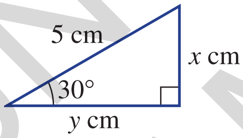

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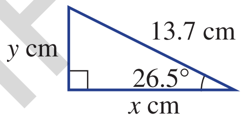

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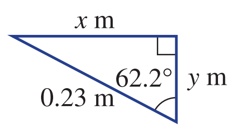
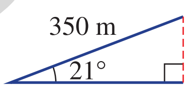


7, 8	9–11	10–13

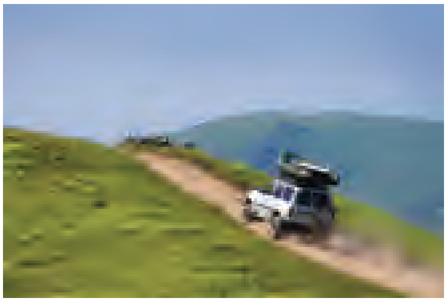
PROBLEM-SOLVING

7 A 4WD climbs a 350 m straight slope at an angle of 21° to the horizontal.



a Find the vertical distance travelled, correct to the nearest metre.

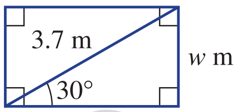
b Find the horizontal distance travelled, correct to the nearest metre.



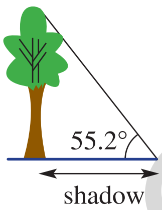
4A

PROBLEM-SOLVING

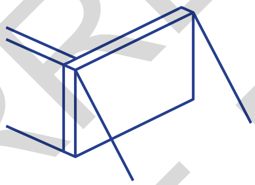
8 A diagonal wall brace of length 3.7 metres is at an angle of 30° to the horizontal. Find the width (w m) of the face of the wall, to the nearest centimetre.



9 The angle from the horizontal of the line of sight from the end of a tree's shadow to the top of the tree is 55.2°. The length of the shadow is 15.5 m. Find the height of the tree, correct to one decimal place.



10 On a construction site, large concrete slabs of height 5.6 metres are supported at the top by steel beams positioned at an angle of 42° from the vertical. Find the length of the steel beams, to two decimal places.

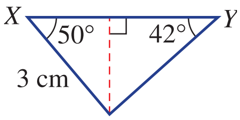


11 By measuring the diagonals, a surveyor checks the dimensions of a rectangular revegetation area of length 25 metres. If the angle of the diagonal to the side length is 28.6°, find the length of the diagonals, correct to one decimal place.

12 A right-angled triangular flag is made for the premiers of a school competition. The second-longest edge of the flag is 25 cm and the largest non-right angle on the flag is 71°. Find the length of the longest edge of the flag, to the nearest millimetre.

13 Find the length XY in these diagrams, correct to one decimal place.

a



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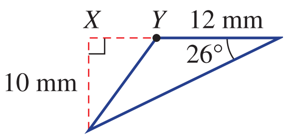
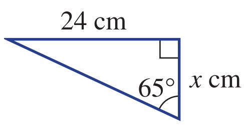


4A

	14	14	14, 15

REASONING

14 A student solves for x, to two decimal places, in the given triangle and gets 11.21, as shown. But the answer is 11.19. Explain the student's error.



$$\tan 65° = \frac{24}{x}$$

$$x \tan 65° = 24$$

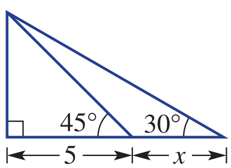
$$x = \frac{24}{\tan 65°}$$

$$= \frac{24}{2.14}$$

$$= 11.21$$

15 Find the value of x, correct to one decimal place, in these triangles.

a



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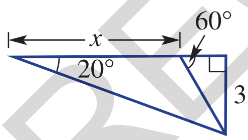

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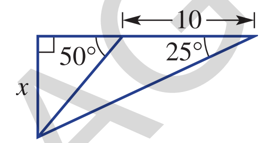

Exploring identities

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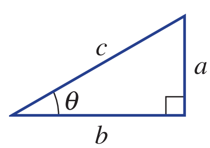
ENRICHMENT

16 For the following proofs, consider the right-angled triangle shown.



a Show that $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$ by completing these steps.

i Write a in terms of c and θ.

ii Write b in terms of c and θ.

iii Write $\tan\theta$ in terms of a and b.

iv Substitute your expressions from parts **i** and **ii** into your expression for $\tan\theta$ in part **iii**.

Simplify to prove $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$.

v Can you find a different way of proving the rule described above?

b Show that $(\sin\theta)^2 + (\cos\theta)^2 = 1$ by completing these steps.

i Write a in terms of c and θ.

ii Write b in terms of c and θ.

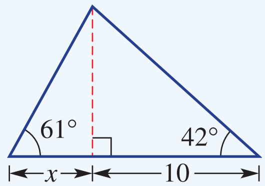
iii State Pythagoras' theorem using a, b and c.

iv Use your results from parts **i**, **ii** and **iii** to show that $(\sin\theta)^2 + (\cos\theta)^2 = 1$.

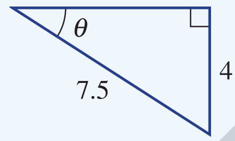
Using calculators in trigonometry

1 Find the value of the unknowns in these triangles, correct to two decimal places.

a

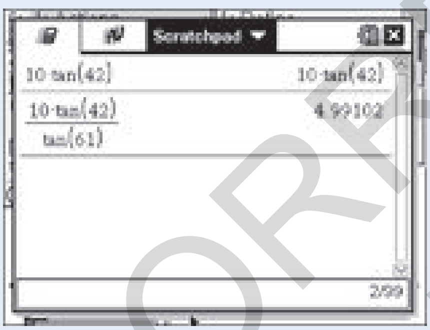


b



Using the TI-Nspire:

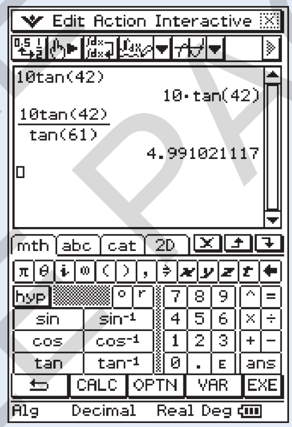
a First, find the height of the triangle using tan. Do not round this value before using it for the next step. Then use this result to find the value of x also using tan. Ensure your settings include degree mode.



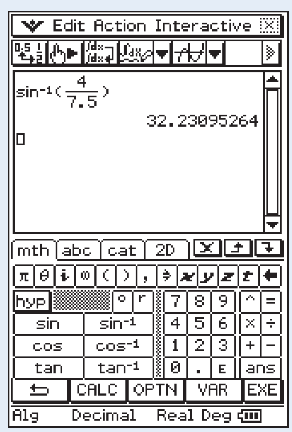
b Use the inverse sine function in degree mode. **Control, enter** gives the result in decimal form.

Using the ClassPad:

a In **Standard Degree** mode, first find the height of the triangle using tan. Use this result to find the value of x also using tan. Do this calculation in **Decimal Degree** mode.



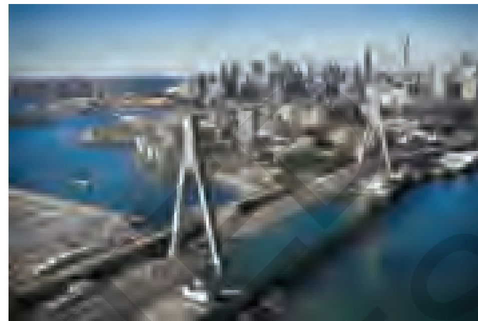
b Use the inverse sine function in **Decimal Degree** mode.



4B Finding angles

Interactive
Widgets
HOTsheets
Walkthroughs

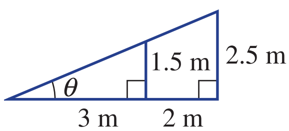
The three trigonometric ratios discussed earlier can also be used to find unknown angles in right-angled triangles if at least two side lengths are known. If, for example, $\cos\theta = \frac{1}{2}$ then we use the inverse trigonometric function for cosine, $\cos^{-1}\left(\frac{1}{2}\right)$, to find θ. Calculators are used to obtain these values.



ANZAC Bridge in Sydney is a cable-stayed bridge in which each cable forms a triangle with the pylons and the bridge deck.

Let's start: The ramp

A ski ramp is 2.5 m high and 5 m long (horizontally) with a vertical strut of 1.5 m placed as shown.



- Discuss which triangle could be used to find the angle of incline, θ. Does it matter which triangle is used?
- Which trigonometric ratio is to be used and why?
- How does $\tan^{-1}$ on a calculator help to calculate the value of θ?
- Discuss how you can check if your calculator is in degree mode.

Key ideas

- **Inverse trigonometric functions** are used to find angles in right-angled triangles.

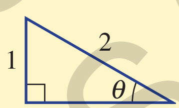
If $\sin\theta = k$	If $\cos\theta = k$	If $\tan\theta = k$
then $\theta = \sin^{-1}(k)$.	then $\theta = \cos^{-1}(k)$.	then $\theta = \tan^{-1}(k)$.

Where $-1 \le k \le 1$ for $\sin\theta$ and $\cos\theta$.

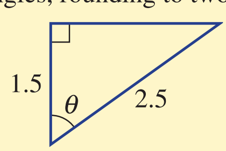
Example 3 Finding angles

Find the value of θ in the following right-angled triangles, rounding to two decimal places in part **b**.

a



b



SOLUTION	EXPLANATION
a $\sin\theta = \frac{1}{2}$	Use $\sin\theta$, as the opposite side and the hypotenuse are given.
$\therefore\ \theta = \sin^{-1}\left(\frac{1}{2}\right)$ $= 30°$	Use inverse sine on a calculator to find the angle.

b $\cos\theta = \dfrac{1.5}{2.5}$

$\therefore\ \theta = \cos^{-1}\left(\dfrac{1.5}{2.5}\right)$

$= 53.13°$ (to 2 d.p.)

The adjacent side and the hypotenuse are given, so use cos θ.

Use inverse cosine on a calculator to find the angle and round your answer to two decimal places.

Example 4 Working with simple applications

A long, straight mine tunnel is sunk into the ground. Its final depth is 120 m and the end of the tunnel is 100 m horizontally from the ground entrance. Find the angle the tunnel makes with the horizontal (θ), correct to one decimal place.

SOLUTION

$\tan\theta = \dfrac{120}{100}$

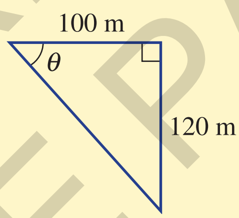
$\theta = \tan^{-1}\left(\dfrac{120}{100}\right)$

$= 50.2°$ (to 1 d.p.)

$\therefore$ 50.2° is the angle the tunnel makes with the thorizontal.

EXPLANATION

Start with a labelled diagram.



Exercise 4B

1–2(½), 3	3	—

UNDERSTANDING

1 Write the missing part in each sentence.

a If $\cos 60° = 0.5$, then $\cos^{-1}(0.5) =$ ____.

b If $\sin 30° = \frac{1}{2}$, then $\sin^{-1}($____$) = 30°$.

c If $\tan 37° \approx 0.75$, then $\tan^{-1}($____$) \approx 37°$.

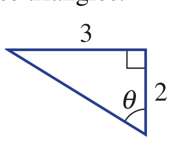
2 Find θ in the following, rounding your answer to two decimal places where necessary.

a $\sin\theta = 0.4$ **b** $\cos\theta = 0.5$ **c** $\tan\theta = 0.2$ **d** $\sin\theta = 0.1$

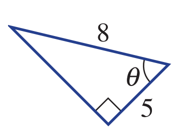
e $\cos\theta = 0.9$ **f** $\tan\theta = 1$ **g** $\sin\theta = 0.25$ **h** $\cos\theta = 0.85$

3 Decide which trigonometric ratio (i.e. sine, cosine or tangent) would be used to find θ in these triangles.

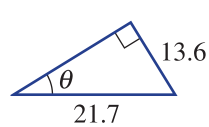
a



b



c



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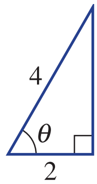
4B

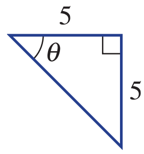
4–5(½) | 4–6(½) | 4–6(½)

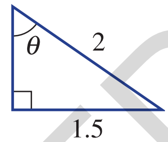
FLUENCY

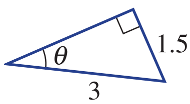
Example 3

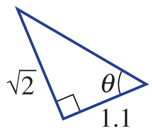
4 Find the value of θ in the following right-angled triangles, rounding your answer to two decimal places where necessary.

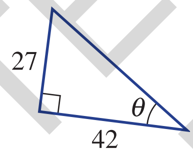
a


b


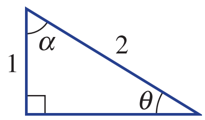
c


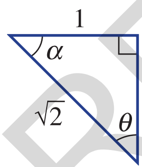
d


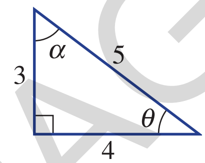
e


f


5 Find the value of α and θ, to one decimal place where necessary, for these special triangles.

a


b


c


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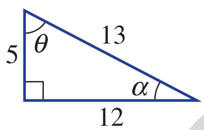
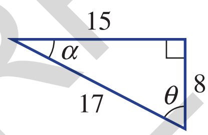
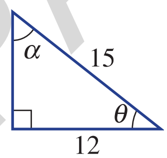


e


f


6 The lengths of two sides of a right-angled triangle are provided. Use this information to find the size of the two interior acute angles, and round each answer to one decimal place.

a hypotenuse 5 cm, opposite 3.5 cm
b hypotenuse 7.2 m, adjacent 1.9 m
c hypotenuse 0.4 mm, adjacent 0.21 mm
d opposite 2.3 km, adjacent 5.2 km
e opposite 0.32 cm, adjacent 0.04 cm
f opposite $\sqrt{5}$ cm, hypotenuse $\sqrt{11}$ cm

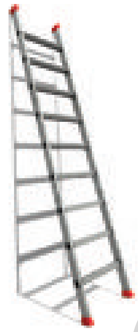
4B

7, 8	8, 9	9, 10

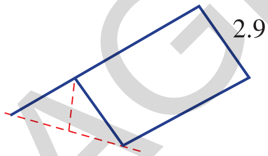
PROBLEM-SOLVING

Example 4

7 A ladder reaches 5.5 m up a wall and sits 2 m from the base of the wall. Find the angle the ladder makes with the horizontal, correct to two decimal places.

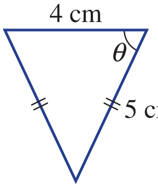


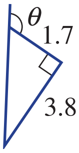
8 A tarpaulin with a simple A-frame design is set up as a shelter. The width of half of the tarpaulin is 2.9 metres, as shown. Find the angle to the ground that the sides of the tarpaulin make if the height at the middle of the shelter is 1.5 metres. Round your answer to the nearest 0.1 of a degree.

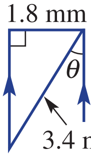


9 A diagonal cut of length 2.85 metres is to be made on a rectangular wooden slab from one corner to the other. The front of the slab measures 1.94 metres. Calculate the angle with the front edge at which the carpenter needs to begin the cut. Round your answer to one decimal place.

10 Find the value of θ in these diagrams, correct to one decimal place.

a


b


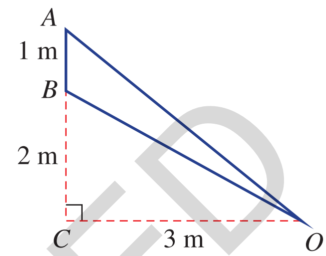
c


4B

11	11, 12	12, 13

REASONING

11 Consider $\triangle OAC$ and $\triangle OBC$.

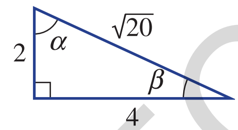


a Find, correct to one decimal place where necessary:

i $\angle AOC$ **ii** $\angle BOC$

b Hence, find the angle $\angle AOB$.

12 This triangle includes the unknown angles α and β.



a Explain why only one inverse trigonometric ratio needs to be used to find the values of both α and β.

b Find α and β, correct to one decimal place, using your method from part **a**.

13 **a** Draw a right-angled isosceles triangle and show all the internal angles.

b If one of the shorter sides is of length x, show that $\tan 45^\circ = 1$.

c Find the exact length of the hypotenuse in terms of x.

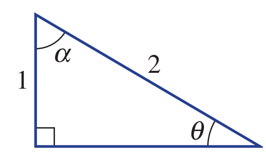
d Show that $\sin 45^\circ = \cos 45^\circ$.

A special triangle

—	—	14

ENRICHMENT

14 Consider this special triangle.



a Find the value of θ.

b Find the value of α.

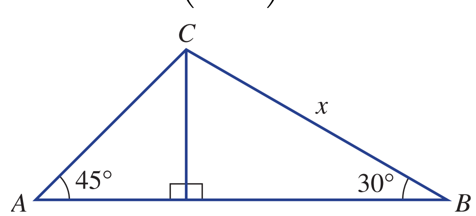
c Use Pythagoras' theorem to find the exact length of the unknown side, in surd form.

d Hence, write down the exact value for the following, in surd form.

i $\sin 30^\circ$ **ii** $\cos 60^\circ$ **iii** $\sin 60^\circ$

iv $\cos 30^\circ$ **v** $\tan 30^\circ$ **vi** $\tan 60^\circ$

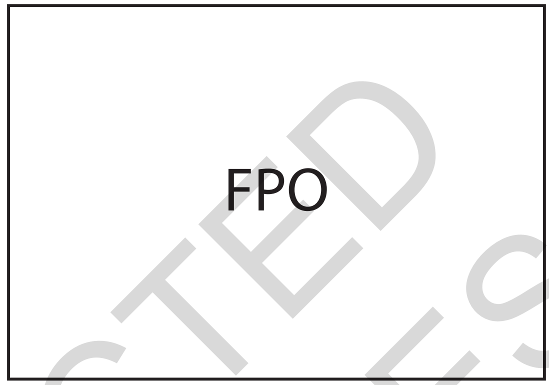
e For the diagram below, show that $AB = \left(\frac{\sqrt{3}+1}{2}\right)x$.



4C Applications in two dimensions

Interactive
Widgets
HOTsheets
Walkthroughs

A key problem-solving strategy for many types of problems in mathematics is to draw a diagram. This strategy is particularly important when using trigonometry to solve worded problems that include right-angled triangles.



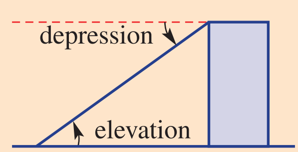
Let's start: Mountain peaks

Two mountain peaks in Victoria are Mt Stirling (1749 m) and Mt Buller (1805 m). A map shows a horizontal distance between them of 6.8 km.

- Discuss if you think there is enough information to find the angle of elevation of Mt Buller from Mt Stirling.
- What diagram can be used to summarise the information?
- Show how trigonometry can be used to find this angle of elevation.
- Discuss what is meant by the words *elevation* and *depression* in this context.

Key ideas

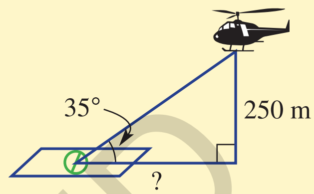
- The **angle of elevation** is measured *up* from the horizontal.



- The **angle of depression** is measured *down* from the horizontal.
  - On the same diagram, the angle of elevation and the angle of depression are equal. They are alternate angles in parallel lines.
- To solve more complex problems involving trigonometry:
  - Visualise and draw a right-angled triangle with the relevant information.
  - Use a trigonometric ratio to find the unknown.
  - Answer the question in words.

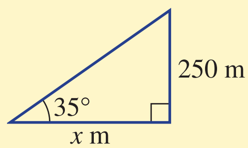
Example 5 Applying trigonometry in worded problems

A helicopter is hovering at an altitude of 250 metres. The angle of elevation from the helipad to the helicopter is 35°. Find the horizontal distance of the helicopter from the helipad, to the nearest centimetre.



SOLUTION

Let x metres be the horizontal distance from the helicopter to the helipad.



$$\tan 35^\circ = \frac{250}{x}$$

$$\therefore\ x \times \tan 35^\circ = 250$$

$$x = \frac{250}{\tan 35^\circ}$$

$$= 357.04$$

The horizontal distance from the helicopter to the helipad is 357.04 m.

EXPLANATION

Use $\tan\theta = \frac{\text{O}}{\text{A}}$ since the opposite and adjacent sides are being used. Solve for x.

There are 100 cm in 1 m, so round to two decimal places for the nearest centimetre.

Answer the question in words.

Example 6 Combining trigonometry with problem solving

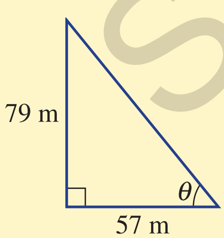
Two vertical buildings 57 metres apart are 158 metres and 237 metres high, respectively. Find the angle of elevation from the top of the shorter building to the top of the taller building, correct to two decimal places.

SOLUTION

Let θ be the angle of elevation from the top of the shorter building to the top of the taller building.

Height difference = 237 – 158

= 79 m



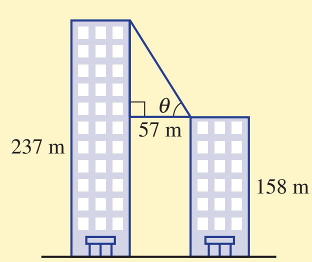
$$\tan\theta = \frac{79}{57}$$

$$\theta = \tan^{-1}\left(\frac{79}{57}\right)$$

$$= 54.19^\circ$$

The angle of elevation from the top of the shorter building to the top of the taller building is 54.19°.

EXPLANATION



Draw the relevant right-angled triangle separately. We are given the opposite (O) and the adjacent (A) sides; hence, use tan.

Use the inverse tan function to find θ, correct to two decimal places.

Answer the question in words.

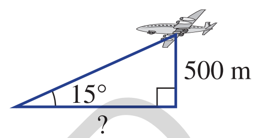
Exercise 4C

1–4	4	—

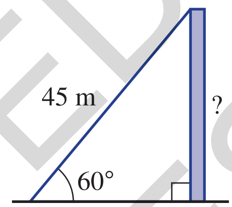
UNDERSTANDING

Example 5

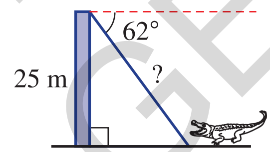
1 The altitude of an aeroplane is 500 metres, and the angle of elevation from the runway to the aeroplane is 15°. Find the horizontal distance from the aeroplane to the runway, to the nearest centimetre.



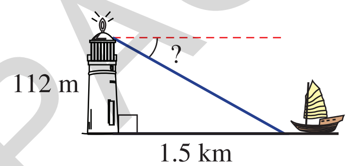
2 A cable of length 45 metres is anchored from the ground to the top of a communications mast. The angle of elevation of the cable to the top of the mast is 60°. Find the height of the communications mast, to the nearest metre.



3 The angle of depression from the top of a 25 metre tall viewing tower to a crocodile on the ground is 62°. Find the direct distance from the top of the tower to the crocodile, to the nearest centimetre.



4 Find the angle of depression from a lighthouse beacon that is 112 metres above sea level to a boat that is at a horizontal distance of 1.5 kilometres from the lighthouse. Round your answer to 0.1 of a degree.

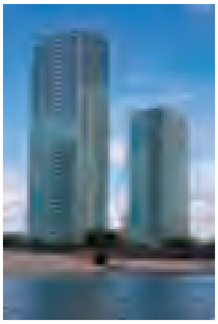


5–7	5–8	6–8

FLUENCY

5 The distance between two buildings is 24.5 metres. Find the height of the taller building, to the nearest metre, if the angle of elevation from the top of the shorter building to the top of the taller building is 85° and the height of the shorter building is 40 m.

6 The angle of depression from one mountain summit to another is 15.9°. If the two mountains differ in height by 430 metres, find the horizontal distance between the two summits, to the nearest centimetre.

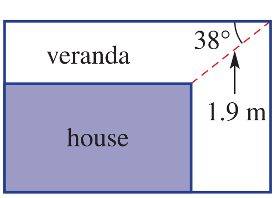


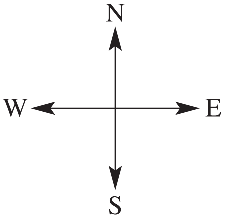
Example 6

7 Two vertical buildings positioned 91 metres apart are 136 metres and 192 metres tall, respectively. Find the angle of elevation from the top of the shorter building to the top of the taller building, to the nearest degree.

8 An L-shaped veranda has dimensions as shown. Find the width, to the nearest centimetre, of the veranda for the following sides of the house.

a north side **b** east side



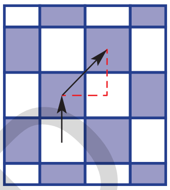


4C

	9, 10	9, 10	10, 11

PROBLEM-SOLVING

9 A knight on a chessboard is moved forward 3.6 cm from the centre of one square to another, then diagonally across at 45° to the centre of the destination square. How far did the knight move in total? Give your answer to two decimal places.



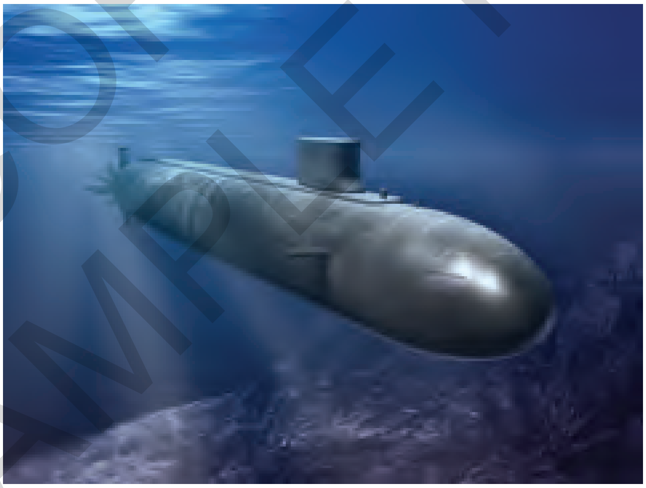
10 Two unidentified flying discs are detected by a receiver. The angle of elevation from the receiver to each disc is 39.48°. The discs are hovering at a direct distance of 826 m and 1.296 km from the receiver. Find the difference in height between the two unidentified flying discs, to the nearest metre.

11 Initially a ship and a submarine are stationary at sea level, positioned 1.78 kilometres apart. The submarine then manoeuvres to position A, 45 metres directly below its starting point. In a second manoeuvre, the submarine dives a further 62 metres to position B. Give all answers to two decimal places.

a Find the angle of elevation of the ship from the submarine when the submarine is at position A.

b Find the angle of elevation of the ship from the submarine when the submarine is at position B.

c Find the difference in the angles of elevation from the submarine to the ship when the submarine is at positions A and B.



	12	12, 13	14, 15

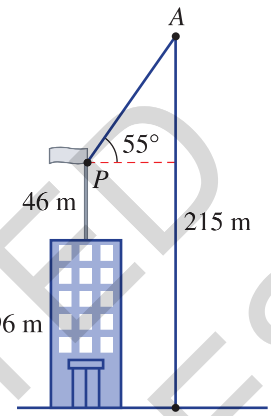
REASONING

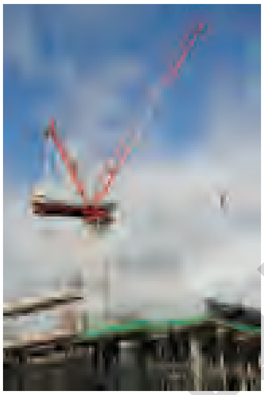
12 A communications technician claims that when the horizontal distance between two television antennas is less than 12 metres, then an interference problem will occur. The heights of two antennas above ground level are 7.5 metres and 13.9 metres, respectively, and the angle of elevation from the top of the shorter antenna to the top of the taller antenna is 29.5°. According to the technician's claim, will there be an interference problem for these two antennas?

4C

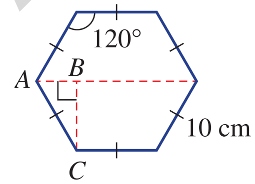
REASONING

13 The pivot point (P) of the main supporting arm (AP) of a construction crane is 46 metres above the top of a 96 metre tall office building. When the supporting arm is at an angle of 55° to the horizontal, the length of cable dropping from the point A to the ground is 215 metres. Find the length of the main supporting arm (AP), to the nearest centimetre.





14 Consider a regular hexagon with internal angles of 120° and side lengths of 10 cm.

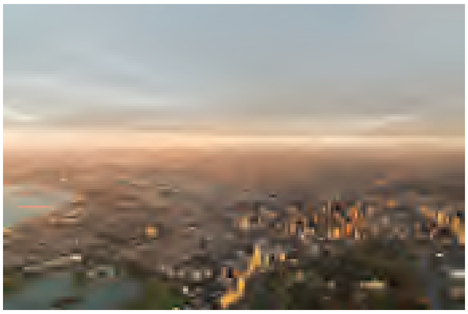


a For the given diagram find, to the nearest millimetre, the lengths:
   i BC ii AB

b Find the distance, to the nearest millimetre, between:
   i two parallel sides ii two opposite vertices.

c Explore and describe how changing the side lengths of the hexagon changes the answers to part **b**.

15 An aeroplane is flying horizontally, directly towards the city of Melbourne at an altitude of 400 metres. At a given time the pilot views the city lights of Melbourne at an angle of depression of 1.5°. Two minutes later the angle of depression of the city lights is 5°. Find the speed of the aeroplane in km/h, correct to one decimal place.



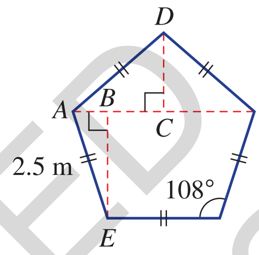
4C

Vegetable garden design

—	—	16

ENRICHMENT

16 A vegetable garden is to be built in the shape of a regular pentagon using redgum sleepers of length 2.5 metres, as shown. It is known that the internal angles of a regular pentagon are 108°.



a Find the values of the following angles.

i $\angle AEB$ **ii** $\angle EAB$ **iii** $\angle CAD$ **iv** $\angle ADC$

b Find these lengths, to two decimal places.

i AB **ii** BE **iii** AC **iv** CD

c Find the distance between a vertex on the border of the vegetable garden and the centre of its opposite side, to two decimal places.

d Find the distance between any two non-adjacent vertices on the border of the vegetable garden, to two decimal places.

e Show that when the length of the redgum sleepers is x metres, the distance between a vertex and the centre of its opposite side of the vegetable garden will be $1.54x$ metres, using two decimal places.

4D Bearings

Interactive
Widgets
HOTsheets
Walkthroughs

True bearings are used to communicate direction and therefore are important in navigation. Ship and aeroplane pilots, bushwalkers and military personnel all use bearings to navigate and communicate direction.

Accurate navigation is vital to the military.

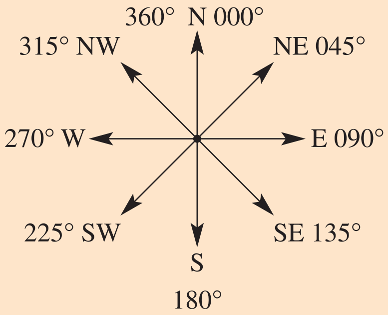
Let's start: Navigating a square

A mining surveyor starts walking from base camp to map out an area for soil testing. She starts by walking 2 km on a true bearing of 020° and wants to map out an area that is approximately square.

- Draw a diagram showing the first leg of the walk and the direction of north.
- If the surveyor turns right for the next leg, what will be the true bearing for this section?
- List the direction (as a true bearing) and the distance for all four legs of the walk. Remember that the mapped area must be a square.

Key ideas

- **True bearings** (°T) are measured clockwise from due north. Some angles and directions are shown in this diagram; for example, NE means north-east.
  - True bearings are usually written using three digits.
  - Opposite directions differ by 180°.



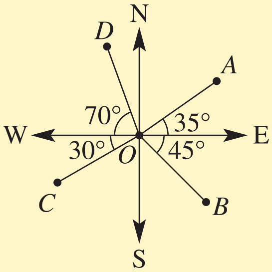
Example 7 Stating a direction

A, *B*, *C* and *D* are four points, as shown.

a Give the true bearing for each point from the origin, *O*, in this diagram.

b Give the true bearing of:

i *O* from *A* **ii** *O* from *D*



SOLUTION | **EXPLANATION**

a The bearing of A is $90° - 35° = 055°$T.
East is 090°, so subtract 35° from 90°.

The bearing of B is $90° + 45° = 135°$T.
B is 90° plus the additional 45° in a clockwise direction.

The bearing of C is $270° - 30° = 240°$T.
West is 270°, so subtract 30° from 270°.

The bearing of D is $270° + 70° = 340°$T.
Alternatively for D, subtract 20° from 360°.

b i The bearing of O from A is $180° + 55° = 235°$T
The bearing of A from O is 055°T and an opposite direction differs by 180°.

ii The bearing of O from D is $340° - 180° = 160°$T
Subtract 180° from the opposite direction (340°T).

Example 8 Using bearings with trigonometry

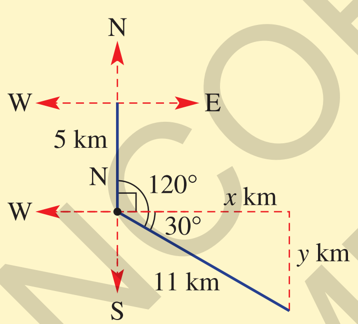
A ship travels due south for 5 km, then on a true bearing of 120° for 11 km.

a Find how far east the ship is from its starting point, correct to two decimal places.

b Find how far south the ship is from its starting point.

SOLUTION | **EXPLANATION**

a



Draw a clear diagram, labelling all relevant angles and lengths. Draw a compass at each change of direction. Clearly show a right-angled triangle, which will help to solve the problem.

$$\cos 30° = \frac{x}{11}$$

$$x = 11 \times \cos 30°$$
$$= 9.53 \text{ (to 2.d.p.)}$$

As x is adjacent to 30° and the hypotenuse has length 11 km, use cos.

The ship is 9.53 km east of its initial position.
Answer in words.

b $$\sin 30° = \frac{y}{11}$$

Use sine for opposite and hypotenuse. Use the value provided rather than your answer from part **a**.

$$y = 11 \times \sin 30°$$
$$= 5.5$$

Multiply both sides by 11.

Distance south = 5.5 + 5 = 10.5 km

The ship is 10.5 km south of its initial position.

Find total distance south by adding the initial 5 km. Answer in words.

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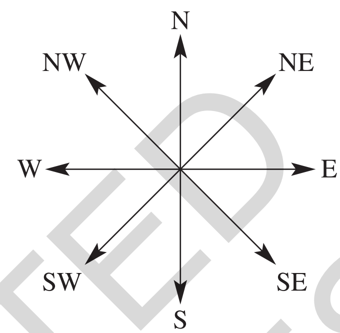
Exercise 4D

UNDERSTANDING	1–3(½)	2(½)	—

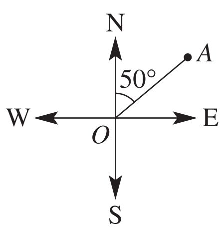
1 Give the true bearing for each of these directions.

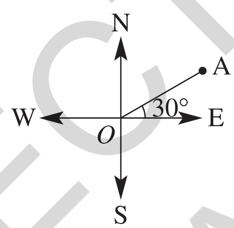
a N
b NE
c E
d SE
e S
f SW
g W
h NW

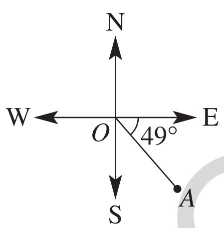


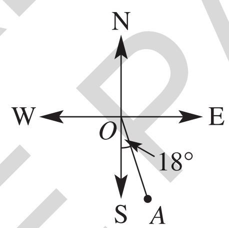
Example 7a

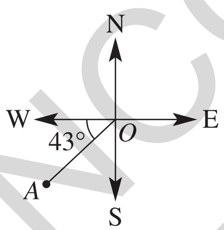
2 For each diagram, give the true bearing from O to A.

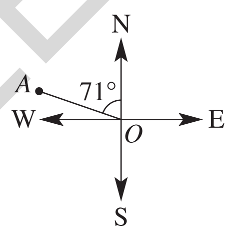
a


b


c


d


e


f


3 Write down the bearing that is the opposite direction to the following.

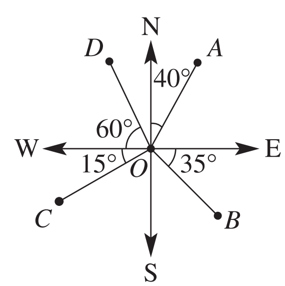
a 020°T
b 262°T
c 155°T
d 344°T

FLUENCY	4–7	4–8	4, 6, 8, 9

Example 7b

4 Find the bearing of O from each of the following points, shown in this simple map. *Hint:* First, find the bearing of each point from O.

a A
b B
c C
d D



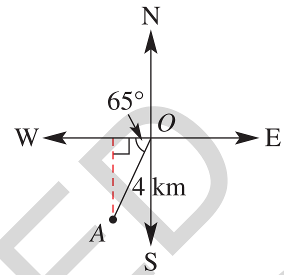
4D

FLUENCY

5 For this simple map, find the following, correct to one decimal place.

a How far west is point A from O?

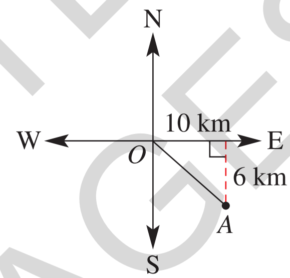
b How far south is point A from O?



6 Find the true bearing, correct to the nearest degree, of:

a point A from O

b point O from A.



Example 8

7 A ship travels due south for 3 km, then on a true bearing of 130° for 5 km.

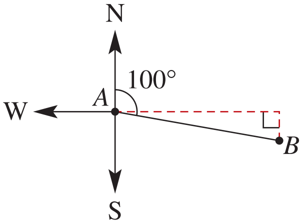
a Find how far east the ship is from its starting point, correct to two decimal places.

b Find how far south the ship is from its starting point, correct to two decimal places.

8 Two points, A and B, positioned 15 cm apart, are such that B is on a true bearing of 100° from A.

a Find how far east point B is from A, correct to two decimal places.

b Find how far south point B is from A, correct to the nearest millimetre.



FLUENCY

9 An aeroplane flies 138 km in a southerly direction from a military air base to a drop-off point. The drop-off point is 83 km west of the air base. Find the bearing, correct to the nearest degree, of:

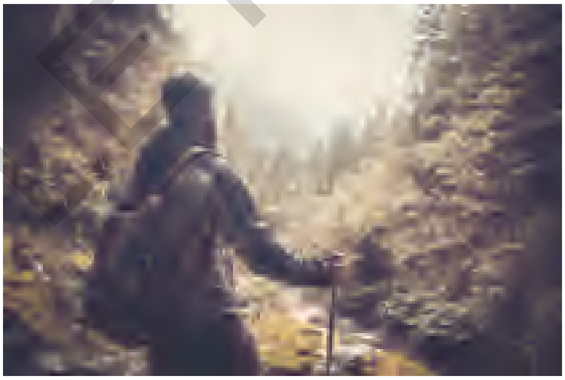
a the drop-off point from the air base
b the air base from the drop-off point.

10, 11	10–12	12, 13

PROBLEM-SOLVING

10 A bushwalker hikes due north from a resting place for 1.5 km to a waterhole and then on a true bearing of 315° for 2 km to base camp.

a Find how far west the base camp is from the waterhole, to the nearest metre.
b Find how far north the base camp is from the waterhole, to the nearest metre.
c Find how far north the base camp is from the initial resting place, to the nearest metre.



11 On a map, point C is 4.3 km due east of point B, whereas point B is 2.7 km on a true bearing of 143° from point A. Give your answer to two decimal places for the following.

a Find how far east point B is from A.
b Find how far east point C is from A.
c Find how far south point C is from A.

12 A military desert tank manoeuvres 13.5 km from point A on a true bearing of 042° to point B. From point B, how far due south must the tank travel to be at a point due east of point A? Give the answer correct to the nearest metre.

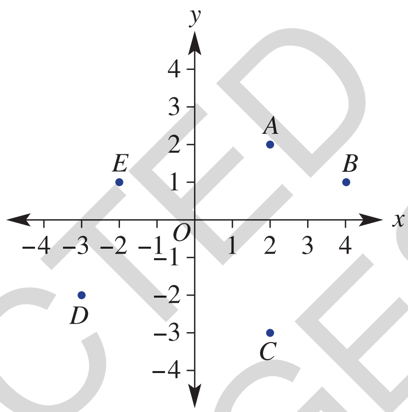
4D

PROBLEM-SOLVING

13 Consider the points O, A, B, C, D and E on this Cartesian plane. Round the answers to one decimal place.

a Find the true bearing of:
- **i** A from O
- **ii** D from O
- **iii** B from C
- **iv** E from C

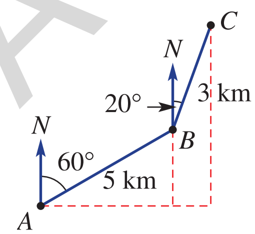
b Find the true bearing from:
- **i** O to E
- **ii** A to B
- **iii** D to C
- **iv** B to D



14	14, 15	14–16

REASONING

14 An overall direction and distance of a journey can be calculated by considering two (or more) smaller parts (or legs). Find the bearing of C from A and the length AC in this journey by answering these parts.



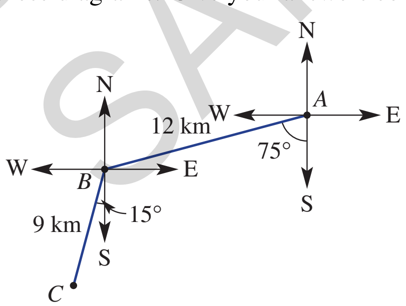
a Find, correct to two decimal places where necessary, how far north:
- **i** point B is from A
- **ii** point C is from B
- **iii** point C is from A.

b Find, correct to two decimal places, how far east:
- **i** point B is from A
- **ii** point C is from B
- **iii** point C is from A.

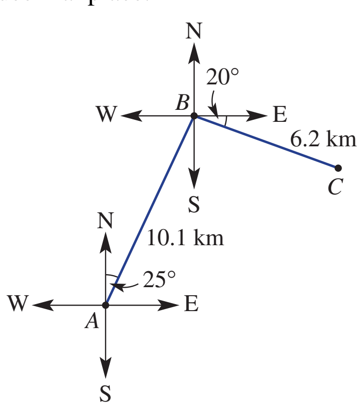
c Now use your answers above to find the following, correct to one decimal place.
- **i** the bearing of C from A
- **ii** the distance from A to C. (*Hint:* Use Pythagoras' theorem.)

15 Use the technique outlined in Question **14** to find the distance AC and the bearing of C from A in these diagrams. Give your answers correct to one decimal place.

a



b



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4D

REASONING

16 Tour groups A and B view a rock feature from different positions on a road heading east–west. Group A views the rock at a distance of 235 m on a bearing of 155° and group B views the rock feature on a bearing of 162° at a different point on the road. Round all answers to two decimal places in the following.

a Find how far south the rock feature is from the road.

b Find how far east the rock feature is from:

 i group A

 ii group B.

c Find the distance between group A and group B.

Navigation challenges	—	—	17, 18

ENRICHMENT

17 A light aeroplane is flown from a farm airstrip to a city runway that is 135 km away. The city runway is due north from the farm airstrip. To avoid a storm, the pilot flies the aeroplane on a bearing of 310° for 50 km, and then due north for 45 km. The pilot then heads directly to the city runway. Round your answers to two decimal places in the following.

a Find how far west the aeroplane diverged from the direct line between the farm airstrip and the city runway.

b Find how far south the aeroplane was from the city runway before heading directly to the city runway on the final leg of the flight.

c Find the bearing the aeroplane was flying on when it flew on the final leg of the flight.

18 A racing yacht sails from the start position to a floating marker on a bearing of 205.2° for 2.82 km, then to a finish line on a bearing of 205.9° for 1.99 km. Round each of the following to two decimal places.

a Find how far south the finish line is from the start position.

b Find how far west the finish line is from the start position.

c Use Pythagoras' theorem to find the distance between the finish line and the start position.

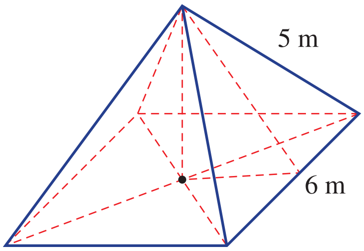
4E Applications in three dimensions 10A

Interactive
Widgets
HOTsheets
Walkthroughs

Although a right-angled triangle is a two-dimensional shape, it can also be used to solve problems in three dimensions. Being able to visualise right-angled triangles included in three-dimensional diagrams is an important part of the process of finding angles and lengths associated with three-dimensional objects.

Let's start: How many right-angled triangles?

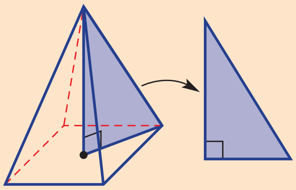
A right square-based pyramid has the apex above the centre of the base. In this example, the base length is 6 m and slant height is 5 m. Other important lines are dashed.



- Using the given dashed lines and the edges of the pyramid, how many different right-angled triangles can you draw?
- Is it possible to determine the exact side lengths of all your right-angled triangles?
- Is it possible to determine all the angles inside all your right-angled triangles?

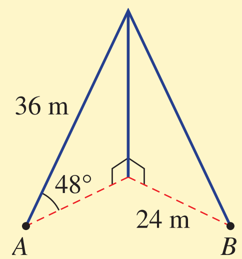
Key ideas

- Using trigonometry to solve problems in three dimensions involves:
  - visualising and drawing any relevant two-dimensional triangles
  - using trigonometric ratios to find unknowns
  - relating answers from two-dimensional diagrams to the original three-dimensional object.



Example 9 Applying trigonometry in 3D

A vertical mast is supported at the top by two cables reaching from two points, *A* and *B*. The cable reaching from point *A* is 36 metres long and is at an angle of 48° to the horizontal. Point *B* is 24 metres from the base of the mast.



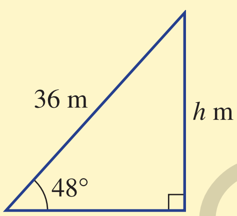
a Find the height of the mast, correct to three decimal places.

b Find the angle to the horizontal of the cable reaching from point *B*, to two decimal places.

SOLUTION / **EXPLANATION**

a Let h be the height of the mast, in metres.

First, draw the right-angled triangle, showing the information given.



$$\sin 48^\circ = \frac{h}{36}$$

The opposite (O) and hypotenuse (H) are given, so use sine.

$$h = 36 \times \sin 48^\circ$$
$$= 26.753 \text{ (to 3 d.p.)}$$

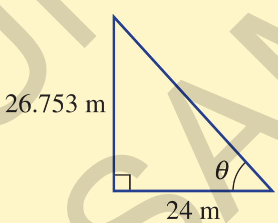
Multiply both sides by 36 and round to three decimal places.

The height of the mast is 26.753 m.

Answer the question in words.

b

Draw the second triangle, including the answer from part **a**.



$$\tan\theta = \frac{26.753\ldots}{24}$$

More precisely, use $\tan\theta = \dfrac{36 \times \sin 48^\circ}{24}$.

$$\theta = \tan^{-1}\left(\frac{26.753\ldots}{24}\right)$$

$$\theta = \tan^{-1}\left(\frac{36 \times \sin 48^\circ}{24}\right)$$

$$= 48.11^\circ \text{ (to 2 d.p.)}$$

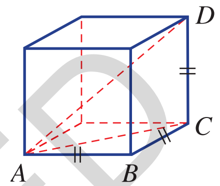
The cable reaching from point *B* is at an angle of 48.11° to the horizontal.

Answer the question in words, rounding your answer appropriately.

Exercise 4E

UNDERSTANDING — 1, 2 | 2 | —

1 The cube shown here has side length 2 m.



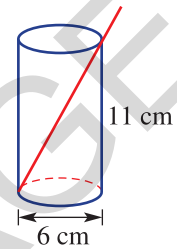
a Draw the right-angled triangle *ABC* and find and label all the side lengths. Pythagoras' theorem can be used. Answer using exact values (e.g. $\sqrt{5}$).

b Draw the right-angled triangle *ACD* and find and label all the side lengths. Pythagoras' theorem can be used. Answer using exact values.

c Use trigonometry to find $\angle DAC$, correct to one decimal place.

d Find the size of $\angle CAB$.

2 Find the angle of elevation this red drinking straw makes with the base of the can, which has diameter 6 cm and height 11 cm. Round your answer to one decimal place.



FLUENCY — 3–5 | 3–6 | 3, 5, 6

Example 9

3 A vertical mast is supported at the top by two cables reaching from two points, *A* and *B*. The cable reaching from point *A* is 43 metres long and is at an angle of 61° to the horizontal. Point *B* is 37 metres from the base of the mast.

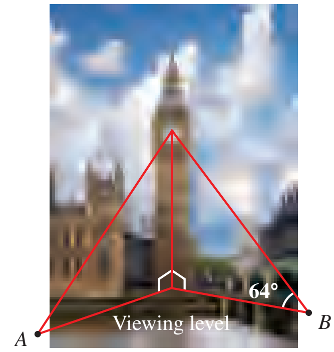
a Find the height of the mast, correct to three decimal places.

b Find the angle to the horizontal of the cable reaching from point *B*, to two decimal places.

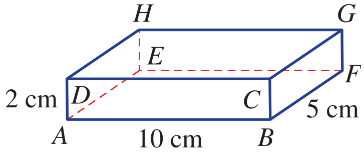
4 Viewing points *A* and *B* are at a horizontal distance from a clock tower of 36 metres and 28 metres, respectively. The viewing angle to the clockface at point *B* is 64°.

a Find the height of the clockface above the viewing level, to three decimal places.

b Find the viewing angle to the clockface at point *A*, to two decimal places.



5 A rectangular prism, *ABCDEFGH*, is 5 cm wide, 10 cm long and 2 cm high.



a By drawing the triangle *ABF* find, to two decimal places:

i $\angle BAF$ ii *AF*

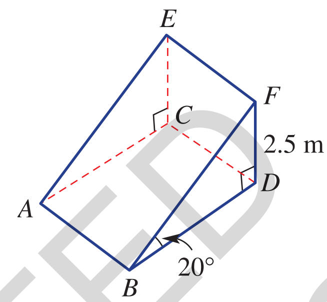
b By drawing the triangle *AGF*, find $\angle GAF$, to two decimal places.

FLUENCY

6 A ramp, *ABCDEF*, rests at an angle of 20° to the horizontal and the highest point on the ramp is 2.5 metres above the ground, as shown. Give your answers to two decimal places in the following questions.

a Find the length of the ramp *BF*.

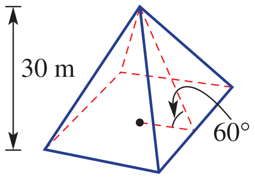
b Find the length of the horizontal *BD*.



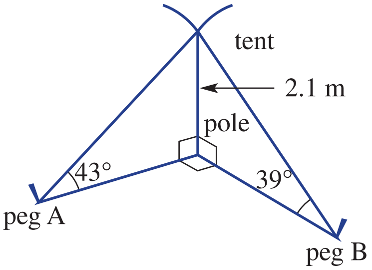
7, 8	8, 9	9, 10

PROBLEM-SOLVING

7 The triangular faces of a right square-based pyramid are at an angle of 60° to the base. The height of the pyramid is 30 m. Find the perimeter of the base of the pyramid, correct to one decimal place.



8 A tent pole 2.1 metres tall is secured by ropes in two directions. The ropes are held by pegs A and B at angles of 43° and 39°, respectively, from the horizontal. The line from the base of the pole to peg A is at right angles to the line from the base of the pole to peg B. Round your answers to two decimal places in these questions.



a Find the distance from the base of the tent pole to:

i peg A ii peg B

b Find the angle at peg B formed by peg A, peg B and the base of the pole.

c Find the distance between peg A and peg B.

4E

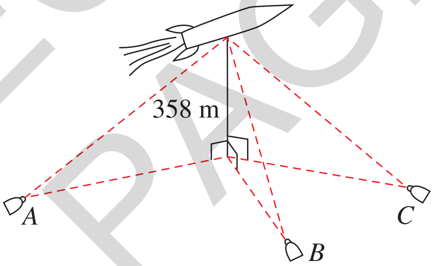
PROBLEM-SOLVING

9 The communities of Wood Town and Green Village live in a valley. Communication between the two communities is enhanced by a repeater station on the summit of a nearby mountain. It is known that the angles of depression from the repeater station to Wood Town and Green Village are 44.6° and 58.2°, respectively. Also, the horizontal, distances from the repeater to Wood Town and Green Village are 1.35 km and 1.04 km, respectively.

a Find the vertical height, to the nearest metre, between the repeater station and:
  i Wood Town **ii** Green Village

b Find the difference in height between the two communities, to the nearest metre.

10 Three cameras operated at ground level view a rocket being launched into space. At 5 seconds immediately after launch, the rocket is 358 m above ground level and the three cameras, *A*, *B* and *C*, are positioned at an angle of 28°, 32° and 36°, respectively, to the horizontal. At the 5 second mark, find:



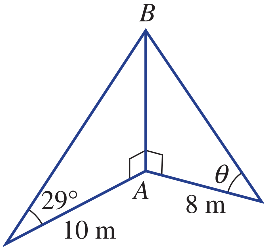
a which camera is closest to the rocket

b the distance between the rocket and the closest camera, to the nearest centimetre.

11	11	11, 12

REASONING

11 It is important to use a high degree of accuracy for calculations that involve multiple parts. For this 3D diagram complete these steps.



a Find *AB*, correct to one decimal place.

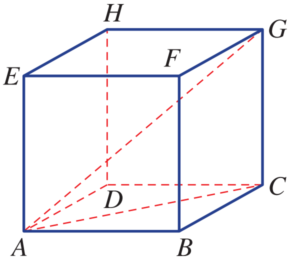
b Use your answer from part **a** to find θ, correct to one decimal place.

c Now recalculate θ using a more accurate value for *AB*. Round θ to one decimal place.

d What is the difference between the answers for parts **b** and **c**?

12 For a cube, *ABCDEFGH*, of side length 1 unit, as shown, use trigonometry to find the following, correct to two decimal places where necessary. Be careful that errors do not accumulate.

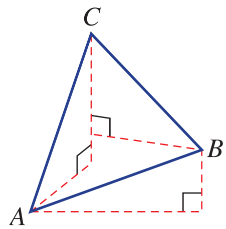
a $\angle BAC$ **b** AC **c** $\angle CAG$ **d** AG



Three points in 3D	—	—	13, 14

ENRICHMENT

13 Three points, A, B and C, in three-dimensional space are such that $AB = 6$, $BC = 3$ and $AC = 5$. The angles of elevation from A to B and from B to C are 15° and 25°, respectively. Round your answer to two decimal places in the following.



a Find the vertical difference in height between:
   i A and B
   ii B and C
   iii A and C

b Find the angle of elevation from A to C.

14 The points A, B and C in 3D space are such that:

- $AB = 10$ mm, $AC = 17$ mm and $BC = 28$ mm.
- The angle of elevation from A to B is 20°.
- The angle of elevation from A to C is 55°.

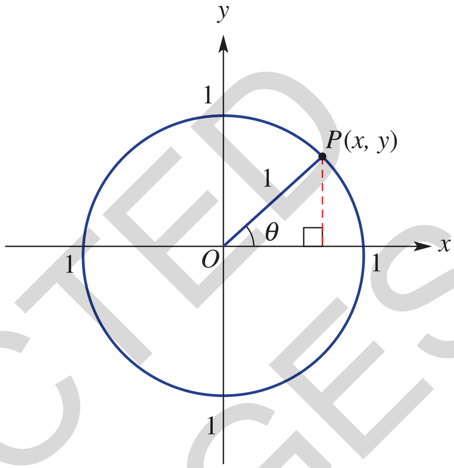
Find the angle of elevation from B to C, to the nearest degree.

Triangulation points or 'trig stations' such as this are used in geodetic surveying to mark points at which measurements are made to calculate local altitude. The calculations involved are similar to those in the enrichment questions above.

4F Obtuse angles and exact values 10A

Interactive
Widgets
HOTsheets
Walkthroughs

To explain how obtuse angles are used in trigonometry, we use a circle with radius 1 unit on a number plane. This is called the unit circle, in which angles are defined anticlockwise from the positive x-axis. The unit circle can be used to consider any angle, but for now we will consider angles between 0° and 180°.



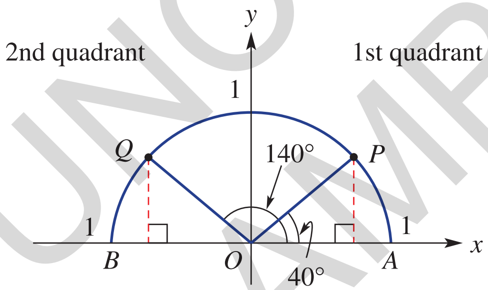
Using a point $P(x, y)$ on the unit circle, we define the three trigonometric ratios:

- $\sin\theta = \dfrac{\text{Opposite}}{\text{Hypotenuse}} = \dfrac{y}{1} = y$
- $\cos\theta = \dfrac{\text{Adjacent}}{\text{Hypotenuse}} = \dfrac{x}{1} = x$
- $\tan\theta = \dfrac{\text{Opposite}}{\text{Adjacent}} = \dfrac{y}{x} = \dfrac{\sin\theta}{\cos\theta}$

So $\sin\theta$ is the y-coordinate of point P and $\cos\theta$ is the x-coordinate of point P. The ratio $\tan\theta$ is the y-coordinate divided by the x-coordinate, which leads to the identity $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$. This is true for any point on the unit circle defined by any angle θ, which is measured anticlockwise from the positive x-axis.

Let's start: The first and second quadrants

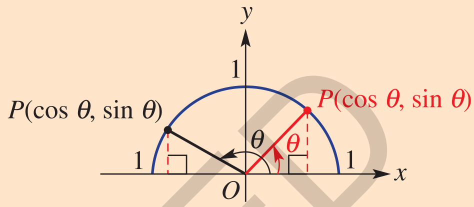
On the unit circle shown, the points P and Q are defined by the angles 40° and 140°.



- If $\cos\theta$ is the x-coordinate of a point on the unit circle, find, correct to two decimal places:
  - the x-coordinate of P
  - the x-coordinate of Q.
- What do you notice about the x-coordinates of P and Q? Discuss.
- If $\sin\theta$ is the y-coordinate of a point on the unit circle, find, correct to two decimal places:
  - the y-coordinate of P
  - the y-coordinate of Q.
- What do you notice about the y-coordinates of P and Q? Discuss.
- Use $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$ to compare the values of tan 40° and tan 140°. What do you notice?

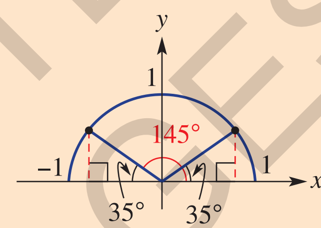
Key ideas

- The **unit circle** is a circle of radius 1 unit.
- The unit circle is used to define cos θ, sin θ and tan θ for all angles θ.
  - A point $P(x, y)$ is a point on the unit circle that is defined by an angle θ, which is measured anticlockwise from the positive x-axis.
  - **cos θ is the x-coordinate of P.**
  - **sin θ is the y-coordinate of P.**
  - **tan** $\theta = \frac{y}{x}$

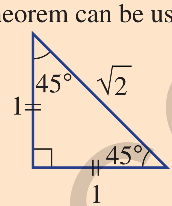


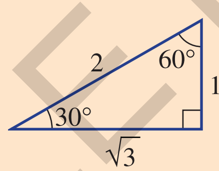
- For supplementary angles θ and $180° - \theta$:
  - $\cos(180° - \theta) = -\cos\theta$
  - $\sin(180° - \theta) = \sin\theta$
  - $\tan(180° - \theta) = -\tan\theta$

  For example:
  - $\cos 145° = -\cos 35°$
  - $\sin 145° = \sin 35°$
  - $\tan 145° = -\tan 35°$



- For acute angles ($0° < \theta < 90°$), all three trigonometric ratios are positive.
- For obtuse angles ($90° < \theta < 180°$), sin θ is positive, cos θ is negative and tan θ is negative.
- Exact values for sin θ, cos θ and tan θ can be obtained using two special triangles. Pythagoras' theorem can be used to confirm the length of each side.





$\sin 45° = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$ $\quad$ $\sin 30° = \frac{1}{2}$ $\quad$ $\sin 60° = \frac{\sqrt{3}}{2}$

$\cos 45° = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$ $\quad$ $\cos 30° = \frac{\sqrt{3}}{2}$ $\quad$ $\cos 60° = \frac{1}{2}$

$\tan 45° = \frac{1}{1} = 1$ $\quad$ $\tan 30° = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$ $\quad$ $\tan 60° = \frac{\sqrt{3}}{1} = \sqrt{3}$

- Exact values for sin θ, cos θ and tan θ for angles of 0°, 30°, 45°, 60° and 90° are given in this table.

θ	sin θ	cos θ	tan θ
0°	0	1	0
30°	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{3}$
45°	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1
60°	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$
90°	1	0	undefined

Example 10 Choosing supplementary angles

Choose an obtuse angle to complete each statement.

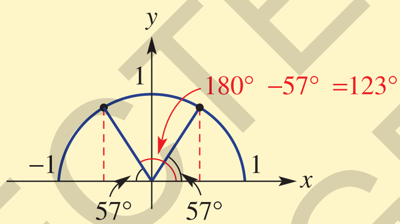
a $\sin 30° = \sin$ ____ **b** $\cos 57° = -\cos$ ____ **c** $\tan 81° = -\tan$ ____

SOLUTION / **EXPLANATION**

a $\sin 30° = \sin 150°$

Choose the supplement of 30°, which is $180° - 30° = 150°$.

b $\cos 57° = -\cos 123°$



c $\tan 81° = -\tan 99°$

The supplement of 81° is 99°.

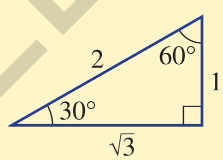
Example 11 Using exact values

Find the exact value of each of the following.

a $\cos 60°$ **b** $\sin 150°$ **c** $\tan 135°$

SOLUTION / **EXPLANATION**

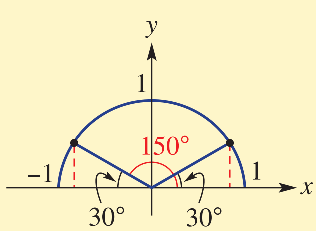
a $\cos 60° = \dfrac{1}{2}$



$$\cos 60° = \frac{\text{Adjacent}}{\text{Hypotenuse}} = \frac{1}{2}$$

b $\sin 150° = \sin 30°$

$= \dfrac{1}{2}$



The sine of supplementary angles are equal and the exact value of $\sin 30°$ is $\frac{1}{2}$.

c $\tan 135° = -\tan 45°$

$= -1$

45° and 135° are supplementary angles.

Also $\tan(180° - \theta) = -\tan\theta$ and $\tan 45° = 1$.

Exercise 4F

1, 2–4(½)	4(½)	—

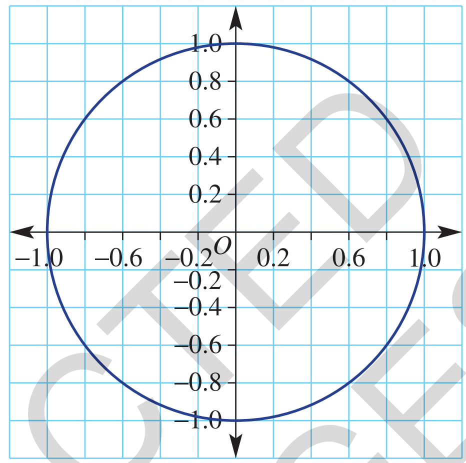
UNDERSTANDING

1 **a** Use a pair of compasses to draw a unit circle on graph paper and label the axes in increments of 0.2.

b Use a protractor to draw an angle of 135° and then use the axes scales to estimate the values of sin 135° and cos 135°.

c Use a protractor to draw an angle of 160° and then use the axes scales to estimate the values of sin 160° and cos 160°.

d Check your answers to parts **b** and **c** with a calculator.



2 What is the supplementary angle of each of the following angles?

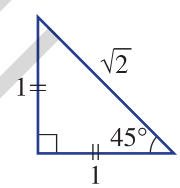
a 31° **b** 52° **c** 45° **d** 87°
e 139° **f** 124° **g** 151° **h** 111°

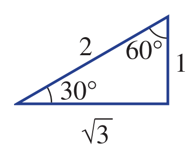
3 Use a calculator to evaluate each pair, correct to two decimal places.

a sin 20°, sin 160° **b** sin 80°, sin 100° **c** sin 39°, sin 141°
d cos 40°, cos 140° **e** cos 25°, cos 155° **f** cos 65°, cos 115°
g tan 50°, tan 130° **h** tan 70°, tan 110° **i** tan 12°, tan 168°

4 Use trigonometric ratios with these triangles to write down an exact value for:

a sin 45° **b** cos 45° **c** tan 45°
d cos 30° **e** sin 30° **f** tan 30°
g tan 60° **h** cos 60° **i** sin 60°





5–8(½)	5–8(½)	5–8(½)

FLUENCY

Example 10

5 Choose an obtuse angle to complete each statement.

a sin 40° = sin ____ **b** sin 20° = sin ____ **c** sin 65° = sin ____
d cos 25° = – cos ____ **e** cos 42° = – cos ____ **f** cos 81° = – cos ____
g tan37° = –tan ____ **h** tan 56° = – tan ____ **i** tan 8° = – tan ____

6 Choose an acute angle to complete each statement.

a sin 150° = sin ____ **b** sin 125° = sin ____ **c** sin 94° = sin ____
d – cos 110° = cos ____ **e** – cos 135° = cos ____ **f** – cos 171° = cos ____
g – tan 159° = tan ____ **h** – tan 102° = tan ____ **i** – tan 143° = tan ____

7 Decide whether the following will be positive or negative.

a sin 153° **b** tan 37° **c** cos 84° **d** cos 171°
e tan 136° **f** sin 18° **g** tan 91° **h** cos 124°

4F

FLUENCY

Example 11

8 Find an exact value for each of the following.

a $\cos 30^\circ$	**b** $\sin 45^\circ$	**c** $\tan 60^\circ$	**d** $\cos 45^\circ$
e $\cos 150^\circ$	**f** $\tan 120^\circ$	**g** $\sin 135^\circ$	**h** $\cos 135^\circ$
i $\sin 120^\circ$	**j** $\tan 150^\circ$	**k** $\cos 120^\circ$	**l** $\sin 150^\circ$
m $\tan 135^\circ$	**n** $\sin 90^\circ$	**o** $\cos 90^\circ$	**p** $\tan 90^\circ$

PROBLEM-SOLVING

9, 11	9–11	10–12

9 State the two values of θ if $0^\circ < \theta < 180^\circ$ and such that:

a $\sin\theta = \dfrac{1}{2}$ **b** $\sin\theta = \dfrac{\sqrt{2}}{2}$ **c** $\sin\theta = \dfrac{\sqrt{3}}{2}$

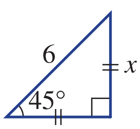
10 Find θ if $90^\circ < \theta < 180^\circ$.

a $\cos\theta = -\dfrac{1}{2}$ **b** $\cos\theta = -\dfrac{\sqrt{2}}{2}$ **c** $\cos\theta = -\dfrac{\sqrt{3}}{2}$

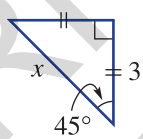
d $\tan\theta = -\sqrt{3}$ **e** $\tan\theta = -1$ **f** $\tan\theta = -\dfrac{\sqrt{3}}{3}$

11 Use trigonometric ratios to find the exact value of x. Calculators are not required.

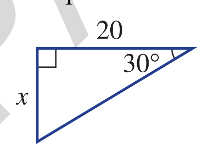
a



b



c



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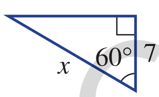
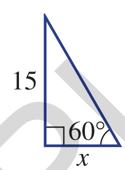
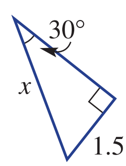


e

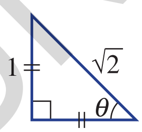


f



12 Find the exact value of θ without the use of a calculator.

a



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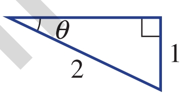

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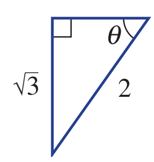
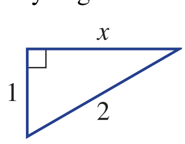


REASONING

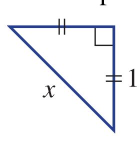
13	13, 14	15, 16

13 Use Pythagoras' theorem to find the exact value of x in these special triangles.

a



b



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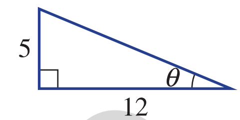
4F

REASONING

14 This right-angled triangle has its two shorter sides of length 5 and 12.



a Use Pythagoras' theorem to find the length of the hypotenuse.

b Find:

i $\sin\theta$ **ii** $\cos\theta$ **iii** $\tan\theta$

c Use $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$ to verify your result from part **b iii**.

15 If $0° < \theta < 90°$, find $\tan\theta$ when:

a $\sin\theta = \dfrac{1}{\sqrt{10}}$ and $\cos\theta = \dfrac{3}{\sqrt{10}}$

b $\sin\theta = \dfrac{2}{\sqrt{11}}$ and $\cos\theta = \dfrac{\sqrt{7}}{\sqrt{11}}$

c $\sin\theta = \dfrac{2}{5}$ and $\cos\theta = \dfrac{\sqrt{21}}{5}$

16 If $90° < \theta < 180°$, find $\tan\theta$ when:

a $\sin\theta = \dfrac{5}{\sqrt{34}}$ and $\cos\theta = -\dfrac{3}{\sqrt{34}}$

b $\sin\theta = \dfrac{\sqrt{20}}{6}$ and $\cos\theta = -\dfrac{2}{3}$

c $\sin\theta = \dfrac{1}{5\sqrt{2}}$ and $\cos\theta = -\dfrac{7}{5\sqrt{2}}$

Complementary ratios	—	—	17

ENRICHMENT

17 You will recall that complementary angles sum to 90°. Answer these questions to explore the relationship between sine and cosine ratios of complementary angles.

a Evaluate the following, correct to two decimal places.

i $\sin 10°$ **ii** $\cos 80°$ **iii** $\sin 36°$ **iv** $\cos 54°$

v $\cos 7°$ **vi** $\sin 83°$ **vii** $\cos 68°$ **viii** $\sin 22°$

b Describe what you notice from part **a**.

c Complete the following.

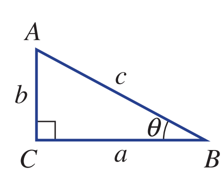
i $\cos\theta = \sin($ ____ $)$ **ii** $\sin\theta = \cos($ ____ $)$

d State the value of θ if θ is acute.

i $\sin 20° = \cos\theta$ **ii** $\sin 85° = \cos\theta$

iii $\cos 71° = \sin\theta$ **iv** $\cos 52° = \sin\theta$

e For this triangle $\angle B = \theta$.



i Write $\angle A$ in terms of θ.

ii Write a ratio for $\sin\theta$ in terms of b and c.

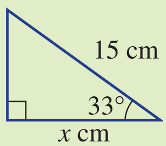
iii Write a ratio for $\cos(90° - \theta)$ in terms of b and c.

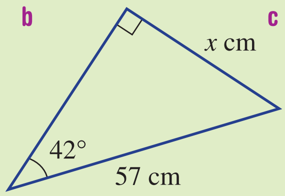
f If $\cos(90° - \theta) = \dfrac{2}{3}$, find $\tan\theta$.

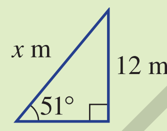
Progress quiz

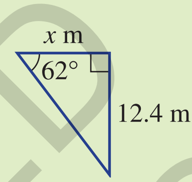
4A

1 Find the value of x in these right-angled triangles, rounding your answer to two decimal places.

a


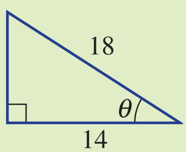
b


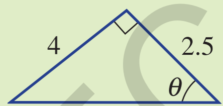
c


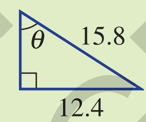
d


4B

2 Find the value of θ in the following right-angled triangles, correct to the nearest degree.

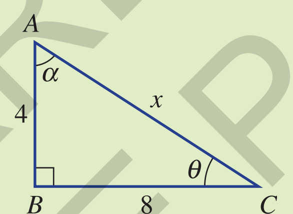
a


b


c


4A/B

3 For triangle ABC, find:



a the exact value of:

i x ii $\sin\alpha$ iii $\sin\theta$

b θ, correct to one decimal place.

4B

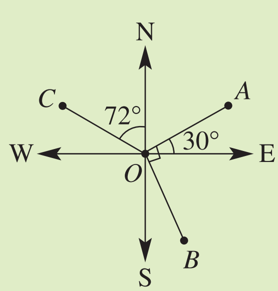
4 At what angle to the horizontal must a 4.5 metre ladder be placed against a wall if it must reach up to just below a window that is 4 metres above the level ground? Round your answer to the nearest degree.

4C

5 The angle of depression from the top of a 20 metre building to a worker standing on the ground below is 40°. Find the distance of the worker from the base of the building, correct to two decimal places.

4D

6 Give the true bearing of A, B and C from the origin, O, in the given diagram.



4D **7** A man leaves camp C at 11 a.m. and walks 12 km on a bearing of 200°. He then stops. A woman also leaves camp C at 11 a.m. However, she walks on a bearing of 110° for 6.5 km before stopping.

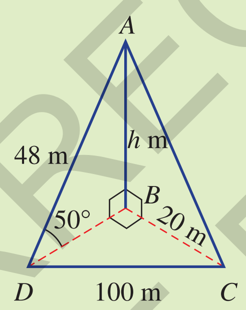
a How far apart are the man and the woman once they stop? Give your answer correct to two decimal places.

b If the man changes direction and walks to where the woman is waiting, on what bearing should he walk? Round your answer to one decimal place.

4E 10A **8** Consider the given 3D diagram.

a Find the value of h, correct to two decimal places.

b Find $\angle ACB$, to the nearest degree.

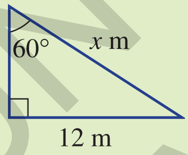


4F 10A **9** Write down the exact value of:

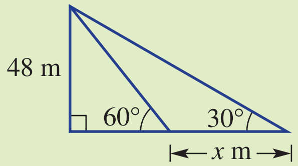
a sin 30° **b** sin 150° **c** cos 60°
d tan 45° **e** cos 135° **f** tan 120°

4F 10A **10** Find the exact value of x in each of these triangles.

a



b

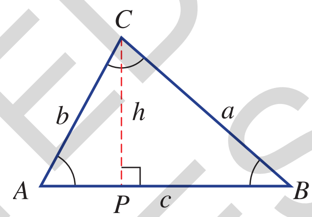


4G The sine rule 10A

Interactive Widgets HOTsheets Walkthroughs

The use of sine, cosine and tangent functions can be extended to non right-angled triangles.

First consider this triangle with sides a, b and c and with opposite angles $\angle A$, $\angle B$ and $\angle C$. Height h is also shown.



From $\triangle CPB$, $\quad \sin B = \dfrac{h}{a}$

so $\quad h = a\sin B$

From $\triangle CPA$, $\quad \sin A = \dfrac{h}{b}$

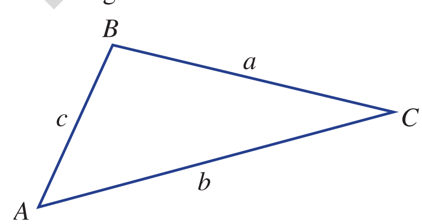
so $\quad h = b\sin A$

$\therefore a\sin B = b\sin A$ or $\dfrac{a}{\sin A} = \dfrac{b}{\sin B}$

Similarly, it can be shown that $\dfrac{a}{\sin A} = \dfrac{c}{\sin C}$ and $\dfrac{b}{\sin B} = \dfrac{c}{\sin C}$.

Let's start: Explore the sine rule

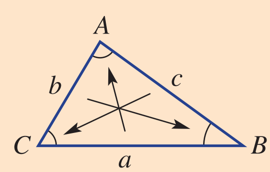
Use a ruler and a protractor to measure the side lengths (a, b and c) in centimetres, correct to one decimal place, and the angles (A, B and C), correct to the nearest degree, for this triangle.



- Calculate the following.

  a $\dfrac{a}{\sin A}$ **b** $\dfrac{b}{\sin B}$ **c** $\dfrac{c}{\sin C}$
- What do you notice about the three answers above?
- Draw your own triangle and check to see if your observations are consistent for any triangle.

Key ideas



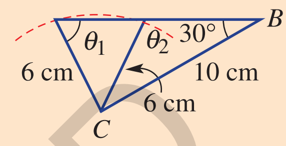
- When using the sine rule, label triangles with capital letters for vertices and the corresponding lower-case letter for the side opposite the angle.
- The **sine rule** states that the ratios of each side of a triangle to the sine of the opposite angle are equal.

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} \quad \text{or} \quad \frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

  - The sine rule holds true for both acute- and obtuse-angled triangles.
  - Use the sine rule when you know:
    - one side length and
    - one angle opposite that side length and
    - another side length or angle.

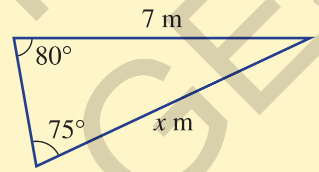
Key ideas

- The **ambiguous case** arises when we are given two sides and an angle that is not the included angle.
  - This example shows a diagram with two given side lengths and one angle. Two triangles are possible.
  - Using $\dfrac{6}{\sin 30^\circ} = \dfrac{10}{\sin \theta}$ could give two results for θ (i.e. θ_1 or θ_2). You will need to choose the correct angle (i.e. acute or obtuse) to suit your triangle (if known).



Example 12 Finding a side length using the sine rule

Find the value of x in this triangle, correct to one decimal place.



SOLUTION

$$\frac{x}{\sin 80^\circ} = \frac{7}{\sin 75^\circ}$$

$$x = \frac{7}{\sin 75^\circ} \times \sin 80^\circ$$

$$= 7.1 \text{ (to 1 d.p.)}$$

EXPLANATION

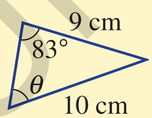
Use the sine rule $\dfrac{a}{\sin A} = \dfrac{b}{\sin B}$.

Multiply both sides by $\sin 80^\circ$.

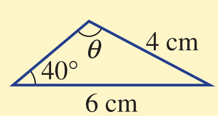
Example 13 Finding an angle using the sine rule

Find the value of θ in these triangles, correct to one decimal place.

a θ is acute



b θ is obtuse



SOLUTION

a

$$\frac{10}{\sin 83^\circ} = \frac{9}{\sin \theta}$$

$$10 \times \sin \theta = 9 \times \sin 83^\circ$$

$$\sin \theta = \frac{9 \times \sin 83^\circ}{10}$$

$$\theta = \sin^{-1}\left(\frac{9 \times \sin 83^\circ}{10}\right)$$

$$= 63.3^\circ \text{ (to 1 d.p.)}$$

EXPLANATION

Alternatively, use $\dfrac{\sin A}{a} = \dfrac{\sin B}{b}$.

So $\dfrac{\sin \theta}{9} = \dfrac{\sin 83^\circ}{10}$.

$$\sin \theta = \frac{9 \times \sin 83^\circ}{10}$$

Use $\sin^{-1}$ on your calculator to find the value of θ.

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b $\frac{4}{\sin 40^\circ} = \frac{6}{\sin \theta}$

$4 \times \sin \theta = 6 \times \sin 40^\circ$

$\sin \theta = \frac{6 \times \sin 40^\circ}{4}$

$\theta = 74.6^\circ$ or $180^\circ - 74.6^\circ = 105.4^\circ$

θ is obtuse, so $\theta = 105.4^\circ$ (to 1 d.p.).

Alternatively, use $\frac{\sin \theta}{6} = \frac{\sin 40^\circ}{4}$.

$\sin \theta = \frac{6 \times \sin 40^\circ}{4}$

This is an example of the ambiguous case of the sine rule but as θ is obtuse, you will need to choose the supplement of 74.6°.

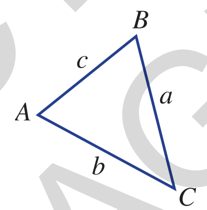
Exercise 4G

1–3	2, 3	—

UNDERSTANDING

1 Copy and complete the sine rule.

$\frac{a}{\quad} = \frac{\quad}{\sin B} = \frac{c}{\quad}$



2 Solve each equation for a, b or c, correct to one decimal place.

a $\frac{a}{\sin 47^\circ} = \frac{2}{\sin 51^\circ}$ **b** $\frac{b}{\sin 31^\circ} = \frac{7}{\sin 84^\circ}$ **c** $\frac{5}{\sin 63^\circ} = \frac{c}{\sin 27^\circ}$

3 Find θ, correct to one decimal place, if θ is acute.

a $\frac{4}{\sin 38^\circ} = \frac{5}{\sin \theta}$ **b** $\frac{11}{\sin 51^\circ} = \frac{9}{\sin \theta}$ **c** $\frac{1.2}{\sin \theta} = \frac{1.8}{\sin 47^\circ}$

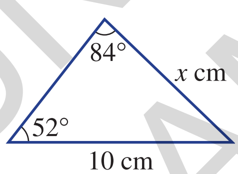
4–5(½)	4–5(½)	4–5(½)

FLUENCY

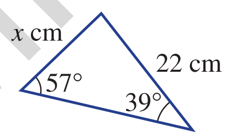
Example 12

4 Find the value of x in these triangles, correct to one decimal place.

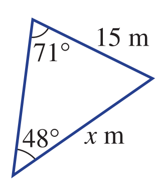
a



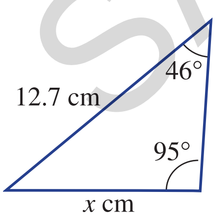
b



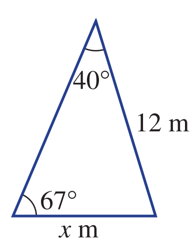
c



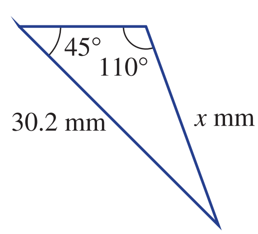
d



e



f

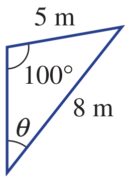


4G

FLUENCY

Example 13a

5 Find the value of θ, correct to one decimal place, if θ is acute.

a


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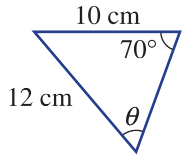

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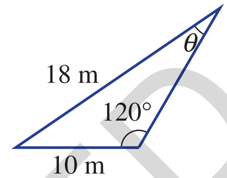

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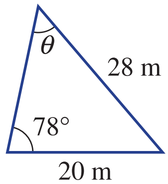

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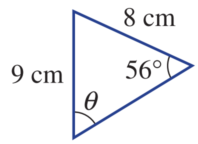

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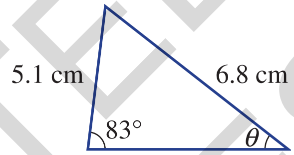
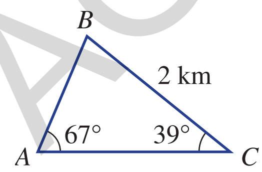


6, 7	7–9	8–10

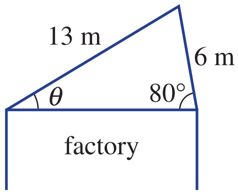
PROBLEM-SOLVING

6 Three markers, A, B and C, map out the course for a cross-country race. The angles at A and C are 67° and 39°, respectively, and BC is 2 km.



a Find the length AB, correct to three decimal places.
b Find the angle at B.
c Find the length AC, correct to three decimal places.

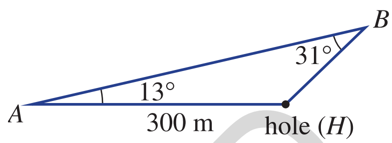
7 A factory roof has a steep 6 m section at 80° to the horizontal and another 13 m section. What is the angle of elevation of the 13 m section of roof? Give your answer to one decimal place.

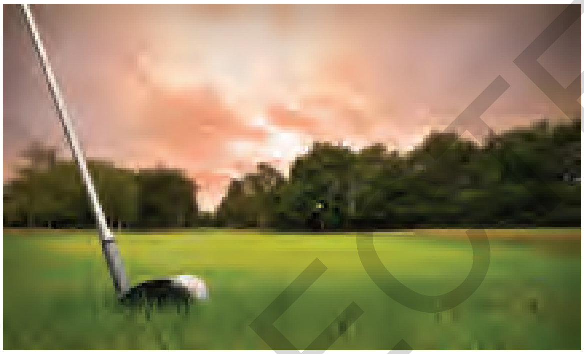


4G

PROBLEM-SOLVING

8 A golf ball is hit off-course by 13° to point B. The shortest distance to the hole is 300 m and the angle formed by the new ball position is 31°, as shown. Find the new distance to the hole (BH), correct to one decimal place.

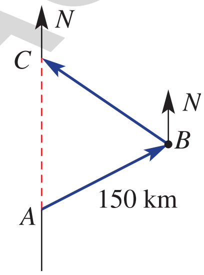




9 An aeroplane is flying due north but, to avoid a storm, it flies 150 km on a bearing of 060°T and then on a bearing of 320°T until it reaches its original course.

a Find the angles $\angle ABC$ and $\angle ACB$.

b As a result of the diversion, how much farther did the aeroplane have to fly? Round your answer to the nearest kilometre.

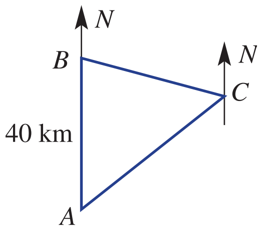


10 A ship heads due north from point A for 40 km to point B, and then heads on a true bearing of 100° to point C. The bearing from C to A is 240°.

a Find $\angle ABC$.

b Find the distance from A to C, correct to one decimal place.

c Find the distance from B to C, correct to one decimal place.



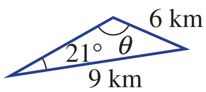
4G

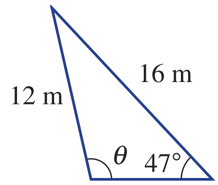
REASONING

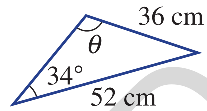
11	11, 12	12, 13

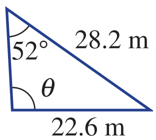
Example 13b

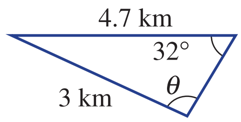
11 Find the value of θ, correct to 1 decimal place, if θ is obtuse.

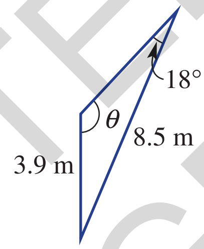
a


b


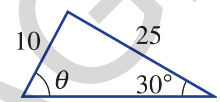
c


d


e


f


12 Try to find the angle θ in this triangle. What do you notice? Can you explain this result?



13 A triangle ABC has $\angle C = 25°$, $AC = 13$ cm and $AB = 9$ cm. Find all possible values of $\angle B$, correct to one decimal place.

More on the ambiguous case

ENRICHMENT

—	—	14

14 When finding a missing angle θ in a triangle, the number of possible solutions for θ can be one or two, depending on the given information.

Two solutions: A triangle ABC has $AB = 3$ cm, $AC = 2$ cm and $\angle B = 35°$.

a Find the possible values of $\angle C$, correct to one decimal place.
b Draw a triangle for each angle for $\angle C$ in part **a**.

One solution: A triangle ABC has $AB = 6$ m, $AC = 10$ m and $\angle B = 120°$.

c Find the possible values of $\angle C$, correct to one decimal place.
d Explain why there is only one solution for $\angle C$ and not the extra supplementary angle, as in parts **a** and **b** above.
e Draw a triangle for your solution to part **c**.

4H The cosine rule 10A

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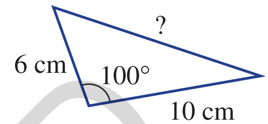
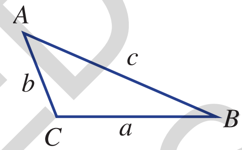
When a triangle is defined by two sides and the included angle, the sine rule is unhelpful in finding the length of the third side because at least one of the other two angles is needed.



In such situations a new rule called the cosine rule can be used. It relates all three side lengths and the cosine of one angle. This means that the cosine rule can also be used to find an angle inside a triangle when given all three sides.

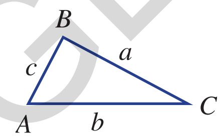


The proof of the cosine rule will be considered in the Enrichment question of this section.

Let's start: Cosine rule in three ways

One way to write the cosine rule is like this:

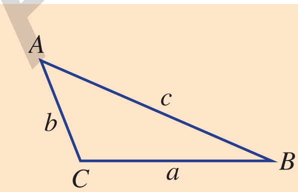
$c^2 = a^2 + b^2 - 2ab\cos C$, where c^2 is the subject of the formula.



- Rewrite the cosine rule by replacing c with a, a with c and C with A.
- Rewrite the cosine rule by replacing c with b, b with c and C with B.

Key ideas

- The **cosine rule** relates one angle and three sides of any triangle.
- The cosine rule is used to find:
  - the third side of a triangle when given two sides and the included angle
  - an angle when given three sides.

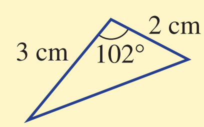


$$c^2 = a^2 + b^2 - 2ab\cos C \quad \text{or} \quad \cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

- If θ is obtuse, then note that $\cos\theta$ is negative.

Example 14 Finding a side length, using the cosine rule

Find the length of the third side in this triangle, correct to two decimal places.



SOLUTION

$c^2 = a^2 + b^2 - 2ab\cos C$

$= 3^2 + 2^2 - 2(3)(2)\cos 102°$

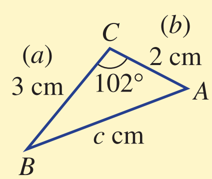
$= 13 - 12\cos 102°$

$= 15.49494\ldots$

$\therefore c = 3.94$ (to 2 d.p.)

The length of the third side is 3.94 cm.

EXPLANATION

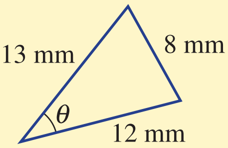


Let c be the length of the unknown side, so $a = 3$ and $b = 2$. Alternatively, let $b = 3$ and $a = 2$.

$c = \sqrt{15.49494\ldots}$

Example 15 Finding an angle, using the cosine rule

Find the angle θ in this triangle, correct to two decimal places.



SOLUTION

$$c^2 = a^2 + b^2 - 2ab\cos C$$
$$8^2 = 13^2 + 12^2 - 2(13)(12)\cos\theta$$
$$64 = 313 - 312\cos\theta$$

EXPLANATION

Choose θ to represent $\angle C$, so this makes $c = 8$.

Alternatively, use $\cos C = \dfrac{a^2 + b^2 - c^2}{2ab}$ to give the same result.

$313 - 64 = 249$

$$312\cos\theta = 249$$
$$\cos\theta = \frac{249}{312}$$
$$\theta = \cos^{-1}\left(\frac{249}{312}\right)$$
$$= 37.05^\circ \text{ (to 2 d.p.)}$$

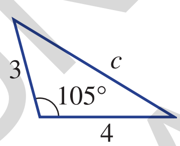
Exercise 4H

UNDERSTANDING	1, 2	1, 2(½)	—

1 Copy and complete the cosine rule for each triangle.

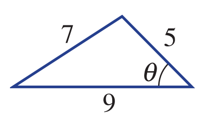
a $c^2 = a^2 + b^2 - 2ab\cos C$

$c^2 = 3^2 + \_\_\_\_ - 2 \times \_\_ \times \_\_ \times \cos \_\_$



b $c^2 = a^2 + b^2 - 2ab\cos C$

$\_\_\_\_ = 5^2 + 9^2 - 2 \times \_\_ \times \_\_ \times \cos\theta$



2 Simplify and solve for the unknown (i.e. c or θ) in these equations, correct to one decimal place.

a $c^2 = 4^2 + 7^2 - 2 \times 4 \times 7 \times \cos 120^\circ$

b $c^2 = 1.5^2 + 1.1^2 - 2 \times 1.5 \times 1.1 \times \cos 70^\circ$

c $10^2 = 7^2 + 6^2 - 2 \times 7 \times 6 \times \cos\theta$

d $18^2 = 21^2 + 30^2 - 2 \times 21 \times 30 \times \cos\theta$

4H

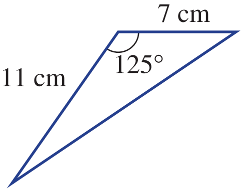
	Fluency		
	3–4(½)	3–4(½)	3–4(½)

FLUENCY

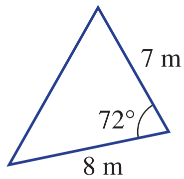
Example 14

3 Find the length of the third side, correct to two decimal places.

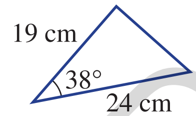
a



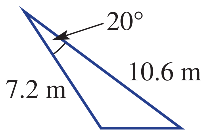
b



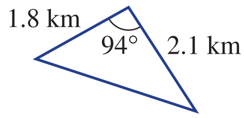
c



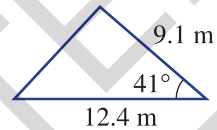
d



e



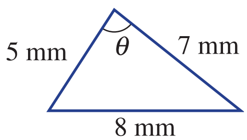
f



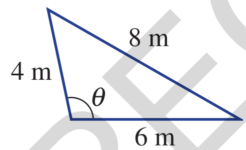
Example 15

4 Find the angle θ, correct to two decimal places.

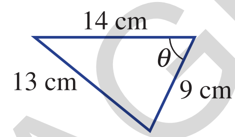
a



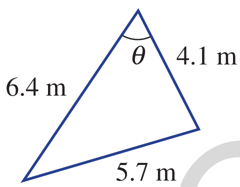
b



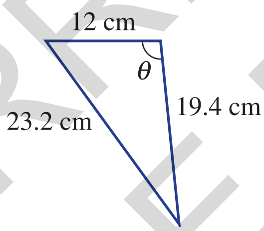
c



d



e



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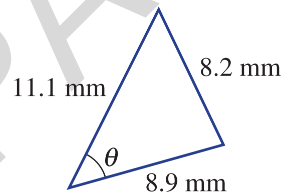


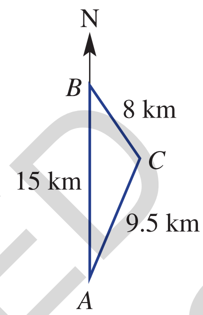
	Problem-solving		
	5–7	6–8	7–9

PROBLEM-SOLVING

5 A triangular goat paddock has two sides of lengths 320 m and 170 m, and a 71° angle between them. Find the length of the third side, correct to the nearest metre.

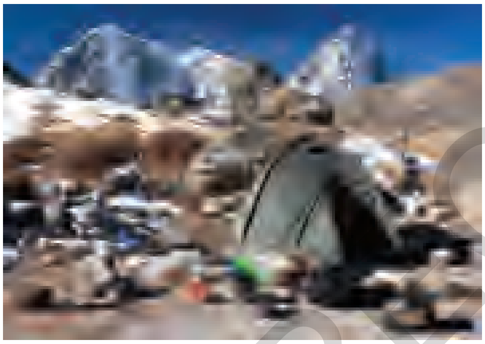
6 Find the size of all three angles in a triangle that has side lengths 10 m, 7 m and 13 m. Round each angle to one decimal place.

7 Three camp sites, A, B and C, are planned for a hike and the distances between the camp sites are 8 km, 15 km and 9.5 km, as shown. If camp site B is due north of camp site A, find the following, correct to one decimal place.



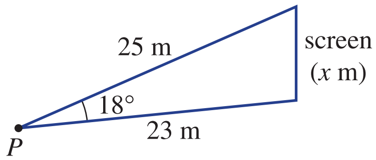
a the bearing from camp site B to camp site C

b the bearing from camp site C to camp site A



8 A helicopter on a joy flight over Kakadu National Park travels due east for 125 km, then on a bearing of 215°T for 137 km before returning to its starting point. Find the total length of the journey, correct to the nearest kilometre.

9 The viewing angle to a vertical screen is 18° and the distances between the viewing point, P, and the top and bottom of the screen are 25 m and 23 m, respectively. Find the height of the screen (x m), correct to the nearest centimetre.

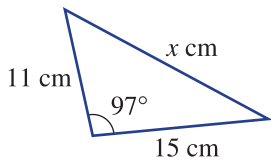


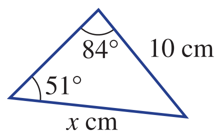
4H

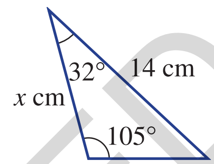
	10(½)	10(½), 11	10(½), 11, 12

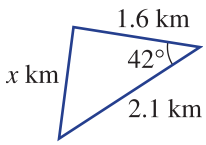
REASONING

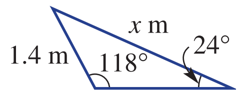
10 Decide whether the cosine rule or sine rule would be used to calculate the value of x in these triangles.

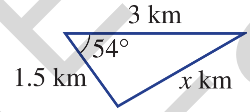
a


b


c


d


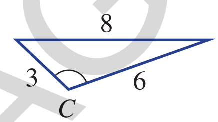
e


f


11 A student uses the cosine rule to find an angle in a triangle and simplifies the equation to $\cos\theta = -0.17$. Is the triangle acute or obtuse? Give a reason.

12 a Rearrange $c^2 = a^2 + b^2 - 2ab\cos C$ to make $\cos C$ the subject.

b Use your rule to find angle C in this triangle, correct to one decimal place.

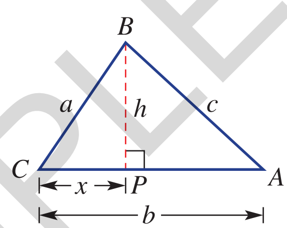


Proof of the cosine rule

	—	—	13

ENRICHMENT

13 Triangle ABC shown here includes point P such that $PB \perp CA$, $BP = h$ and $CP = x$.



a Write an expression for length AP.

b Use Pythagoras' theorem and $\triangle CBP$ to write an equation in a, x and h.

c Use Pythagoras' theorem and $\triangle APB$ to write an equation in b, c, x and h.

d Combine your equations from parts **b** and **c** to eliminate h. Simplify your result.

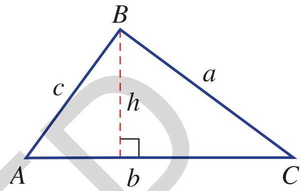
e Use $\cos\theta = \dfrac{\text{Adjacent}}{\text{Hypotenuse}}$ to write an expression for $\cos C$.

f Combine your equations from parts **d** and **e** to prove $c^2 = a^2 + b^2 - 2ab\cos C$.

4I Area of a triangle 10A

Interactive
Widgets
HOTsheets
Walkthroughs

We can use trigonometry to establish a rule for the area of a triangle using two sides and the included angle.

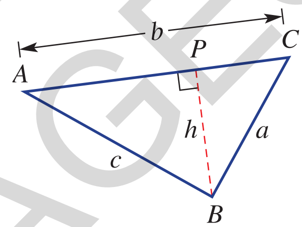


We can see in this triangle that $\sin C = \frac{h}{a}$, so $h = a\sin C$.

$\therefore A = \frac{1}{2}bh$ becomes $A = \frac{1}{2}ba\sin C$.

Let's start: Calculating area in two ways

Draw any triangle ABC and construct the height PB. Measure the following as accurately as possible.



i AC ii BC iii BP iv $\angle C$

Now calculate the area using:

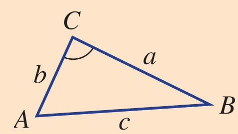
- Area $= \frac{1}{2}bh$
- Area $= \frac{1}{2}ab\sin C$

How close are your answers? They should be equal!

Key ideas

- The area of a triangle is equal to half the product of two sides and the sine of the included angle.

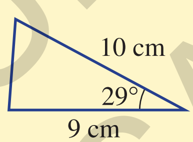
  Area $= \frac{1}{2}ab\sin C$



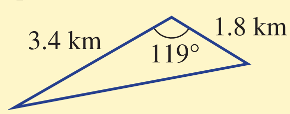
Example 16 Finding the area of a triangle

Find the area of these triangles, correct to one decimal place.

a



b



SOLUTION

a Area $= \frac{1}{2}ab\sin C$

$= \frac{1}{2} \times 9 \times 10 \times \sin 29°$

$= 21.8\text{ cm}^2$ (to 1 d.p.)

EXPLANATION

Substitute the two sides (a and b) and the included angle (C) into the rule.

b $\text{Area} = \frac{1}{2}ab\sin C$

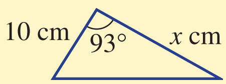
$= \frac{1}{2} \times 1.8 \times 3.4 \times \sin 119^\circ$

$= 2.7\ \text{km}^2$ (to 1 d.p.)

If C is obtuse, then $\sin C$ is positive and the rule can still be used for obtuse-angled triangles.

Example 17 Finding a side length given the area

Find the value of x, correct to two decimal places, given that the area of this triangle is $70\ \text{cm}^2$.



SOLUTION

$\text{Area} = \frac{1}{2}ab\sin C$

$70 = \frac{1}{2} \times 10 \times x \times \sin 93^\circ$

$14 = x\sin 93^\circ$

$x = \frac{14}{\sin 93^\circ}$

$= 14.02$ (to 2 d.p.)

EXPLANATION

Substitute all the given information into the rule, letting $a = 11$ and $b = x$. Use $\angle C = 93^\circ$ as the included angle.

$\frac{1}{2} \times 10 = 5$, so divide both sides by 5 and then solve for x.

Exercise 4I

1–3	3	—

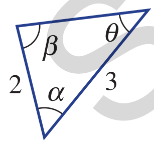
UNDERSTANDING

1 Evaluate $\frac{1}{2}ab\sin C$, correct to one decimal place, for the given values of a, b and C.

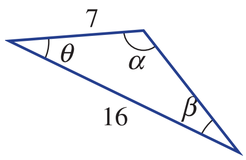
a $a = 3, b = 4, C = 38^\circ$ **b** $a = 6, b = 10, C = 74$ **c** $a = 15, b = 7, C = 114^\circ$

2 Which angle pronumeral (i.e. α, β or θ) represents the included angle between the two given sides in these triangles?

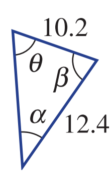
a



b



c



3 Solve these equations for C. Round your answer to two decimal places.

a $10 = \frac{1}{2} \times 4 \times 6 \times \sin C$ **b** $25 = \frac{1}{2} \times 7 \times 10 \times \sin C$ **c** $42 = \frac{1}{2} \times 11 \times 9 \times \sin C$

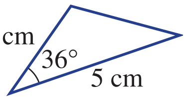
4(½), 5	4(½), 5, 6(½)	4(½), 5, 6(½)

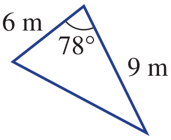
4I

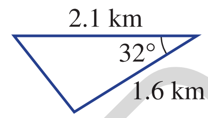
FLUENCY

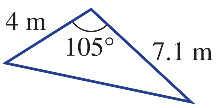
Example 16

4 Find the area of these triangles, correct to one decimal place.

a


b


c


d


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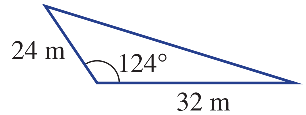

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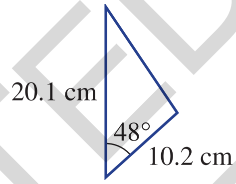


5 Find the area of these triangles, correct to one decimal place.

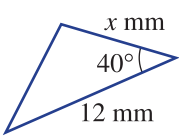
a $\triangle XYZ$ if $XY = 5$ cm, $XZ = 7$ cm and $\angle X = 43°$

b $\triangle STU$ if $ST = 12$ m, $SU = 18$ m and $\angle S = 78°$

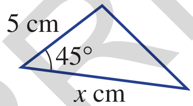
c $\triangle EFG$ if $EF = 1.6$ km, $FG = 2.1$ km and $\angle F = 112°$

Example 17

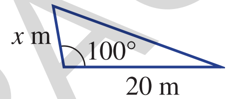
6 Find the value of x, correct to one decimal place, for these triangles with given areas.

a


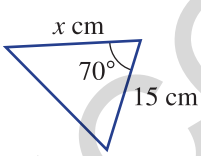
Area = 22 mm^2

b


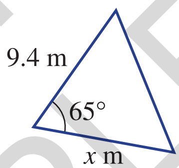
Area = 14 cm^2

c


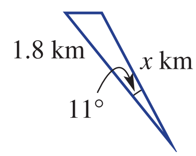
Area = 90 m^2

d


Area = 128 cm^2

e


Area = 45 m^2

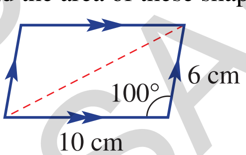
f


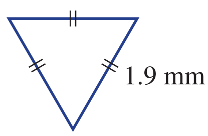
Area = 0.23 km^2

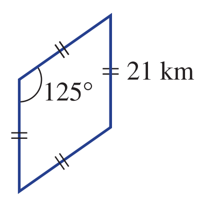
7, 8	8, 9	8–10

PROBLEM-SOLVING

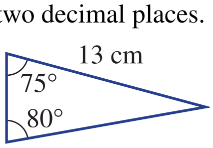
7 Find the area of these shapes, correct to two decimal places.

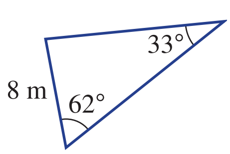
a


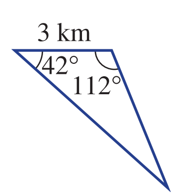
b


c


8 First use the sine rule to find another side length, and then find the area of these triangles, correct to two decimal places.

a


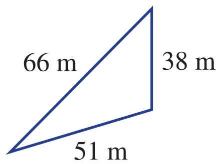
b


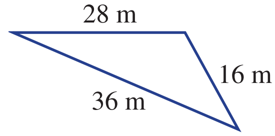
c


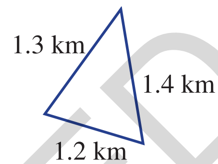
4I

PROBLEM-SOLVING

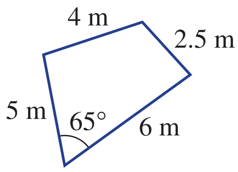
9 First use the cosine rule to find an angle, and then calculate the area of these triangles, correct to two decimal places.

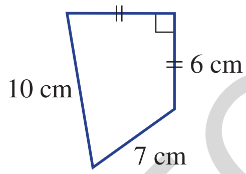
a


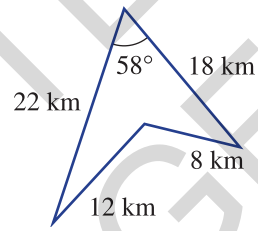
b


c


10 Find the area of these quadrilaterals, correct to one decimal place.

a


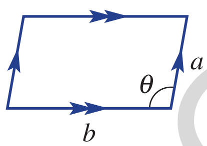
b


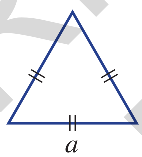
c


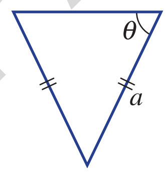
11	11, 12	11–13

REASONING

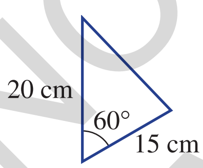
11 Write a rule for the area of these shapes, using the given pronumerals.

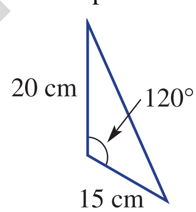
a


b


c


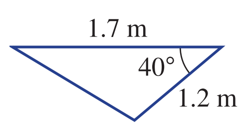
12 a Find the area of these two triangles, correct to one decimal place.

i


ii


b What do you notice about your answers in part **a**? How can you explain this?

c Draw another triangle that has the same two given lengths and area as the triangle on the right.



13 a Use the rule Area $= \frac{1}{2}ab\sin C$ to find the two possible values of θ in this triangle. Round your answer to one decimal place.

$$\triangle ABC \text{ with } AB = 11 \text{ m},\ AC = 8 \text{ m}, \text{ included angle } \theta \text{ and Area } = 40 \text{ m}^2.$$

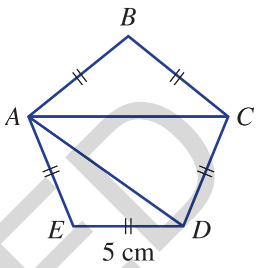
b Draw the two triangles for the two sets of results found in part **a**.

Polygon areas — — 14

4I

ENRICHMENT

14 You will recall that the sum (S) of the interior angles of a polygon with n sides is given by $S = 180(n - 2)$.



- **a** This regular pentagon has each side measuring 5 cm.
  - **i** Calculate the angle sum of a pentagon.
  - **ii** Calculate the size of one interior angle of a regular pentagon.
  - **iii** Find the area of $\triangle AED$, correct to two decimal places.
  - **iv** Find the length AD, correct to two decimal places.
  - **v** Find $\angle ADC$ and $\angle DAC$.
  - **vi** Find the area of $\triangle ADC$, correct to two decimal places.
  - **vii** Find the total area of the pentagon, correct to one decimal place.
- **b** Use a similar approach to find the area of a regular hexagon of side length 5 cm, correct to one decimal place.
- **c** Can this method be used for other regular polygons? Explore and give examples.

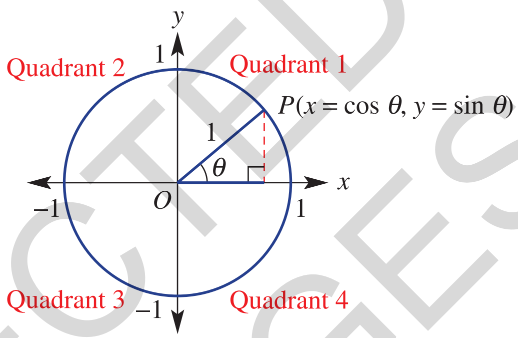
4J The four quadrants 10A

Interactive Widgets HOTsheets Walkthroughs

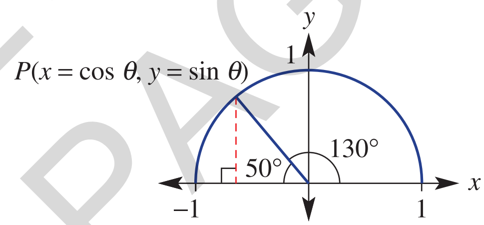
From section 4F we calculated $\sin\theta$, $\cos\theta$ and $\tan\theta\left(=\dfrac{\sin\theta}{\cos\theta}\right)$ using obtuse angles. We will now extend this to include the four quadrants of the unit circle, using $0° < \theta < 360°$.

Recall the following.

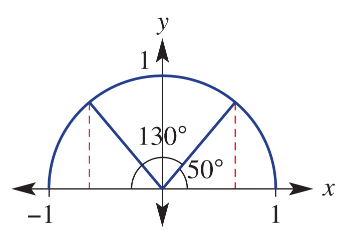
- The unit circle has radius one unit and has centre (0, 0) on a number plane.
- θ is defined anticlockwise from the positive x-axis.
- There are four quadrants, as shown.



- The coordinates of P, a point on the unit circle, are $(x, y) = (\cos\theta,\ \sin\theta)$.
- $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$
- In the second quadrant $\sin\theta$ is positive, $\cos\theta$ is negative and $\tan\theta$ is negative. In this diagram we can see $P(\cos 130°, \sin 130°)$, where $\cos 130°$ is negative, $\sin 130°$ is positive and so $\tan 130°$ will be negative.



In the diagram at right showing 130°, a 50° angle (180° – 130°) drawn in the first quadrant can help relate trigonometric values from the second quadrant to the first quadrant. By symmetry we can see that $\sin 130° = \sin 50°$ and $\cos 130° = -\cos 50°$. This 50° angle is called the **reference angle** (or related angle).



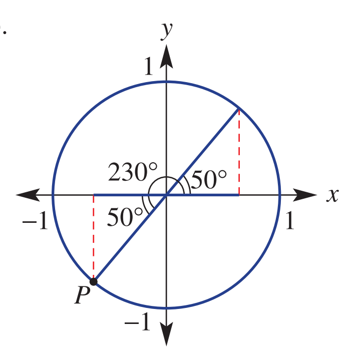
In this section we explore these symmetries and reference angles in the third and fourth quadrants.

Let's start: Positive or negative

For the angle 230°, the reference angle is 50° and $P = (\cos 230°,\ \sin 230°)$.

Since P is in the third quadrant, we can see that:

- $\cos 230° = -\cos 50°$, which is negative.
- $\sin 230° = -\sin 50°$, which is negative.



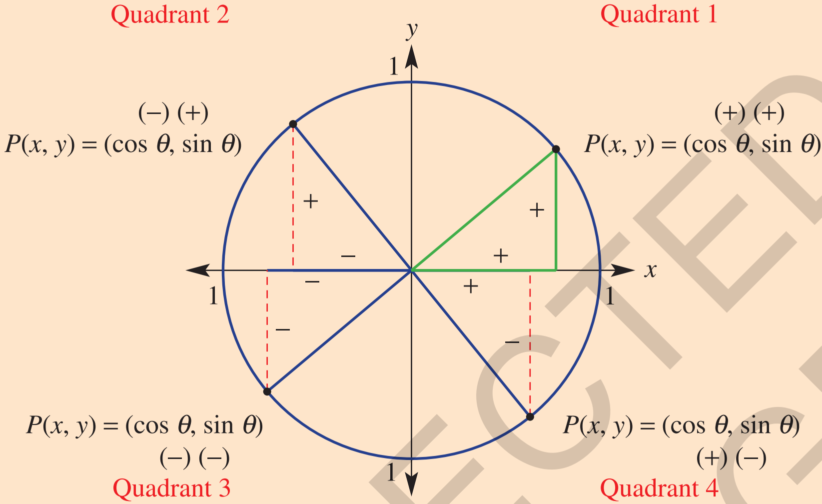
Now determine the following for each value of θ given below. You should draw a unit circle for each.

- What is the reference angle?
- Is $\cos\theta$ positive or negative?
- Is $\sin\theta$ positive or negative?
- Is $\tan\theta$ positive or negative?

a $\theta = 240°$ **b** $\theta = 210°$ **c** $\theta = 335°$ **d** $\theta = 290°$ **e** $\theta = 162°$

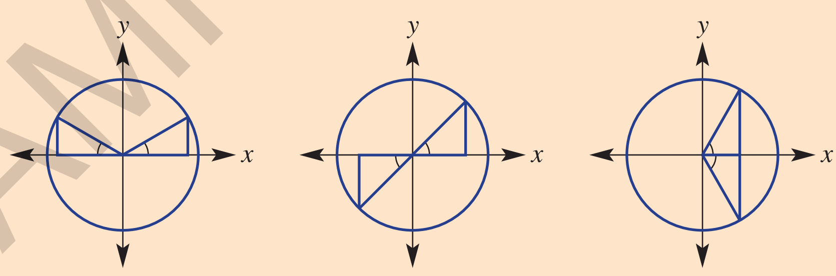
Key ideas

- Every point $P(x, y)$ on the unit circle can be described in terms of the angle θ such that: $x = \cos\theta$ and $y = \sin\theta$, where $-1 \le \sin\theta \le 1$ and $-1 \le \cos\theta \le 1$.



  - For different quadrants, $\cos\theta$ and $\sin\theta$ can be positive or negative.
  - Recall that $\tan\theta = \frac{\sin\theta}{\cos\theta}$.
  - **ASTC** means:
    - Quadrant 1: **A**ll $\sin\theta$, $\cos\theta$ and $\tan\theta$ are positive.
    - Quadrant 2: Only **S**inθ is positive.
    - Quadrant 3: Only **T**anθ is positive.
    - Quadrant 4: Only **C**osθ is positive.
- A **reference angle** (sometimes called a related angle) is an acute angle that helps to relate $\cos\theta$ and $\sin\theta$ to the first quadrant.

Angle θ	90° to 180°	180° to 270°	270° to 360°
Reference angle	$180° - \theta$	$\theta - 180°$	$360° - \theta$



- **Exact values** can be easily used when the reference angles are 30°, 45° or 60°.
- Multiples of 90°.

θ	0°	90°	180°	270°	360°
$\sin\theta$	0	1	0	–1	0
$\cos\theta$	1	0	–1	0	1
$\tan\theta$	0	undefined	0	undefined	0

Example 18 Positioning a point on the unit circle

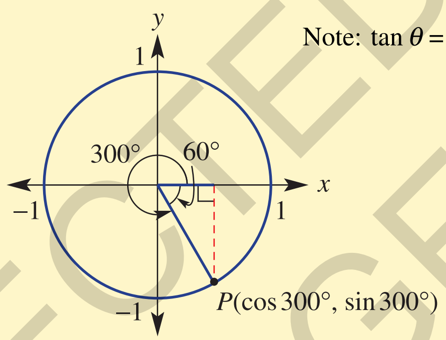
Decide in which quadrant θ lies and state whether $\sin\theta$, $\cos\theta$ and $\tan\theta$ are positive or negative.

a $\theta = 300°$ **b** $\theta = 237°$

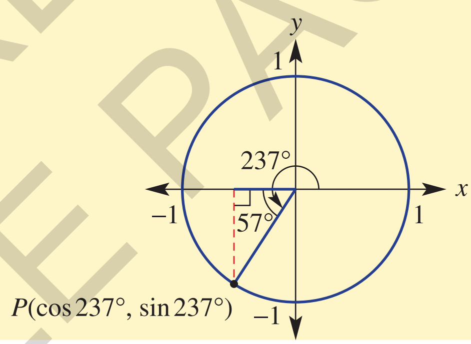
SOLUTION / **EXPLANATION**

a $\theta = 300°$ is in quadrant 4.
$\sin\theta$ is negative
$\cos\theta$ is positive
$\tan\theta$ is negative

Note: $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$



b $\theta = 237°$ is in quadrant 3.
$\sin\theta$ is negative
$\cos\theta$ is negative
$\tan\theta$ is positive



Example 19 Using a reference angle

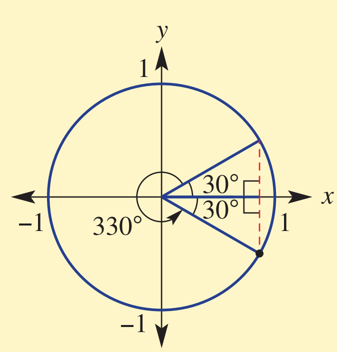
Write the following using their reference angle.

a $\sin 330°$ **b** $\cos 162°$ **c** $\tan 230°$

SOLUTION / **EXPLANATION**

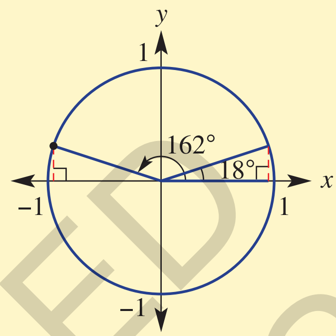
a $\sin 330° = -\sin 30°$

$\sin 330°$ is negative and the reference angle is $360° - 330° = 30°$.



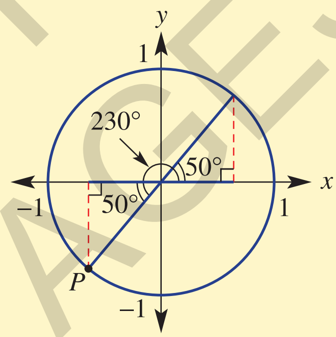
b cos 162° = – cos 18°

cos 162° is negative and the reference angle is 180° – 162° = 18°.



c tan 230° = tan 50°

tan 230° is positive (negative ÷ negative) and the reference angle is 230° – 180° = 50°.



Exercise 4J

1–3, 4(½) | 2, 3 | —

UNDERSTANDING

1 Which quadrant in the unit circle corresponds to these values of θ?

a $0° < \theta < 90°$ **b** $180° < \theta < 270°$ **c** $270° < \theta < 360°$ **d** $90° < \theta < 180°$

2 Decide which quadrants make the following true.

a $\sin\theta$ is positive **b** $\tan\theta$ is negative **c** $\cos\theta$ is negative
d $\cos\theta$ is positive **e** $\tan\theta$ is positive **f** $\sin\theta$ is negative

3 Complete this table.

θ	0°	90°	180°	270°	360°
$\sin\theta$					0
$\cos\theta$			–1		
$\tan\theta$		undefined			

4 Use a calculator to evaluate the following, correct to three decimal places.

a sin 172° **b** sin 84° **c** sin 212° **d** sin 325°
e cos 143° **f** cos 255° **g** cos 321° **h** cos 95°
i tan 222° **j** tan 134° **k** tan 42° **l** tan 337°

4J

5–6(½)	5–7(½)	5–7(½)

FLUENCY

Example 18

5 Decide in which quadrant θ lies and state whether $\sin\theta$, $\cos\theta$ and $\tan\theta$ are positive or negative.

a $\theta = 172°$ b $\theta = 295°$ c $\theta = 252°$ d $\theta = 73°$
e $\theta = 318°$ f $\theta = 154°$ g $\theta = 197°$ h $\theta = 221°$
i $\theta = 210°$ j $\theta = 53°$ k $\theta = 346°$ l $\theta = 147°$

Example 19

6 Write each of the following using its reference angle.

a sin 280° b cos 300° c tan 220° d sin 140°
e cos 125° f tan 315° g sin 345° h cos 238°
i tan 227° j sin 112° k cos 294° l tan 123°

7 If θ is acute, find the value of θ.

a $\sin 150° = \sin\theta$ b $\sin 240° = -\sin\theta$ c $\sin 336° = -\sin\theta$
d $\cos 220° = -\cos\theta$ e $\cos 109° = -\cos\theta$ f $\cos 284° = \cos\theta$
g $\tan 310° = -\tan\theta$ h $\tan 155° = -\tan\theta$ i $\tan 278° = -\tan\theta$

8, 9	8(½), 9	10, 11

PROBLEM-SOLVING

8 Write the reference angle (i.e. related angle) in the first quadrant for these angles.

a 138° b 227° c 326° d 189°
e 213° f 298° g 194° h 302°

9 For what values of θ, in degrees, are the following true?

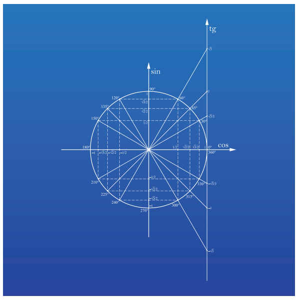
a All of $\sin\theta$, $\cos\theta$ and $\tan\theta$ are positive. b Only $\sin\theta$ is positive.
c Only $\cos\theta$ is positive. d Only $\tan\theta$ is positive.

10 Complete the table by finding a second angle, θ_2, that gives the same value for the trigonometric function as θ_1. Use the unit circle to help in each case and assume $0 \le \theta_2 \le 360°$.

Trigonometric function	$\sin\theta$	$\cos\theta$	$\sin\theta$	$\tan\theta$	$\cos\theta$	$\tan\theta$	$\cos\theta$	$\sin\theta$	$\tan\theta$
θ_1	30°	45°	190°	15°	125°	320°	260°	145°	235°
θ_2									

11 Decide which quadrant suits the given information.

a $\sin\theta < 0$ and $\cos\theta > 0$ b $\tan\theta > 0$ and $\cos\theta > 0$
c $\tan\theta < 0$ and $\cos\theta < 0$ d $\sin\theta > 0$ and $\tan\theta < 0$
e $\sin\theta > 0$ and $\tan\theta > 0$ f $\sin\theta < 0$ and $\cos\theta < 0$



REASONING

12	12, 13(½)	13(½), 14, 15

12 Recall the exact sine, cosine and tangent values for 30°, 45° and 60°.

θ	30°	45°	60°
$\sin\theta$	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$
$\cos\theta$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$
$\tan\theta$	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$

a State the reference angle for 225°.

b Hence, give the exact value of the following.
  i sin 225° **ii** cos 225° **iii** tan 225°

c State the reference angle for 330°.

d Hence, give the exact value of the following.
  i sin 330° **ii** cos 330° **iii** tan 330°

e State the reference angle for 120°.

f Hence, give the exact value of the following.
  i sin 120° **ii** cos 120° **iii** tan 120°

13 Give the exact value of the following.

a sin 135° **b** tan 180° **c** cos 150° **d** sin 240°
e tan 315° **f** cos 210° **g** sin 330° **h** tan 120°
i cos 225° **j** sin 270° **k** tan 330° **l** cos 300°
m sin 180° **n** tan 270° **o** tan 225° **p** cos 180°

14 Explain why $\tan\theta > 0$ when $180° < \theta < 270°$.

15 Explain why tan 90° and tan 270° are undefined.

Exploring truth

ENRICHMENT

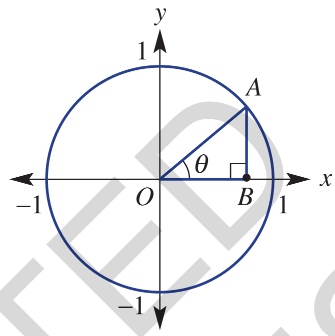
—	—	16, 17

16 By considering a unit circle, state whether the following are true or false.

a sin 10° < cos 10° **b** sin 50° < tan 50°
c cos 80° > sin 80° **d** cos 90° = sin 0°
e tan 180° = sin 180° **f** cos 170° > sin 170°
g sin 120° > tan 120° **h** sin 90° = cos 180°
i tan 230° < cos 230° **j** cos 350° < sin 85°
k sin 260° < cos 110° **l** tan 270° = cos 180°

4J

17 Trigonometric identities are mathematical statements that may involve $\sin\theta$ and/or $\cos\theta$ and/or $\tan\theta$ and hold true for all values of θ. In previous exercises you will have already considered the trigonometric identity $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$.



a Consider the triangle OAB in the unit circle shown.

- **i** Given that $OA = 1$, $OB = \cos\theta$ and $AB = \sin\theta$, use Pythagoras' theorem to prove the trigonometric identity: $\sin^2\theta + \cos^2\theta = 1$. Note: $\sin^2\theta = (\sin\theta)^2$.
- **ii** Check your identity using a calculator to see if it holds true for $\theta = 30°, 145°, 262°$ and $313°$.

b
- **i** Evaluate the given pairs of numbers using a calculator.
  $(\sin 60°, \cos 30°)$, $(\sin 80°, \cos 10°)$, $(\sin 110°, \cos -20°)$, $(\sin 195°, \cos -105°)$
- **ii** What do you notice about the value of each number in the pairs above? Drawing a unit circle illustrating each pair of values may help.
- **iii** What is the relationship between θ in $\sin\theta$ and θ in $\cos\theta$ that is true for all pairs in part **b i**?
- **iv** In terms of θ, complete this trigonometric identity: $\sin\theta = \cos(\_\_\_\_\_)$.
- **v** Check this identity for $\theta = 40°, 155°, 210°$ and $236°$.

c Explore other trigonometric identities by drawing diagrams and checking different angles, such as:

- **i** $\sin\theta = \sin(180° - \theta)$
- **ii** $\cos\theta = \cos(360° - \theta)$
- **iii** $\tan\theta = \tan(180° + \theta)$
- **iv** $\sin 2\theta = 2\sin\theta\cos\theta$
- **v** $\cos 2\theta = \cos^2\theta - \sin^2\theta$
- **vi** $\cos 2\theta = 2\cos^2\theta - 1$
- **vii** $\cos 2\theta = 1 - 2\sin^2\theta$

Medieval Arab astronomers invented this device, a sine quadrant, for measuring the angles of overhead stars.

4K Graphs of trigonometric functions 10A

Interactive
Widgets
HOTsheets
Walkthroughs

As the angle θ increases from 0° to 360°, the values of $\sin\theta$, $\cos\theta$ and $\tan\theta$ increase or decrease depending on the value of θ. Graphing the values of $\sin\theta$, $\cos\theta$ and $\tan\theta$ against θ gives a clear picture of this.

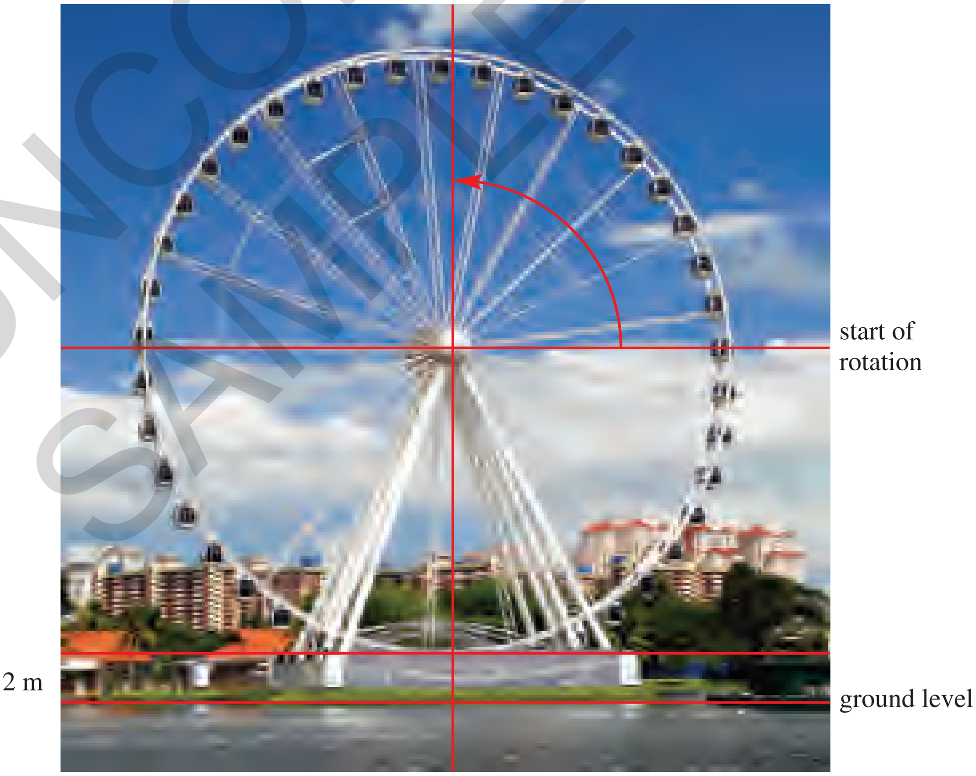
These wave-like graphs based on trigonometric functions are used to model many variables from the height of the tide on a beach to the width of a soundwave giving a high or low pitch sound.

Let's start: Ferris wheel ride

Have you ever had a ride on a Ferris wheel? Imagine yourself riding a Ferris wheel again. The wheel rotates at a constant rate, but on which part of the ride will your vertical upwards movement be fastest? On which part of the ride will you move quite slowly upwards or downwards?

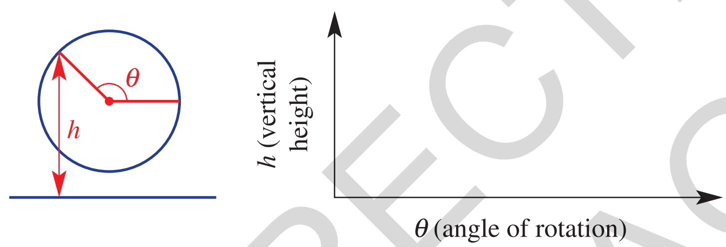
Work in groups to discuss these questions and help each other to complete the table and graph on page 294.

For this example, assume that the bottom of the Ferris wheel is 2 m above the ground and the diameter of the wheel is 18 m. Count the start of a rotation from halfway up on the right, as shown below. The wheel rotates in an anticlockwise direction.



Position	Angle of rotation, θ, from halfway up	Vertical height, h, above ground level (m)
Halfway up	0°	
Top	90°	
Halfway down		
Bottom		2
Halfway up		
Top		
Halfway down		

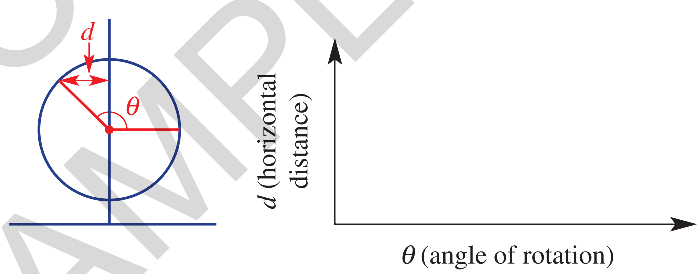
Now draw a graph of the **vertical height** (h) above the ground (vertical axis) versus the **angle** (θ) of anticlockwise rotation for two complete turns of the Ferris wheel.



As a group, discuss some of the key features of the graph.

- What are the maximum and minimum values for the height?
- Discuss any symmetry you see in your graph. How many values of θ (rotation angle) have the same value for height? Give some examples.

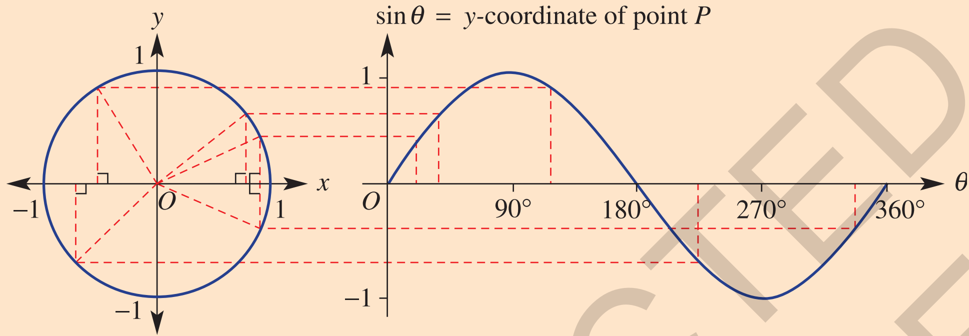
Discuss how the shape would change for a graph of the **horizontal distance** (d) from the circumference (where you sit) to the central **vertical** axis of the Ferris wheel versus the angle (θ) of rotation. Sketch this graph.



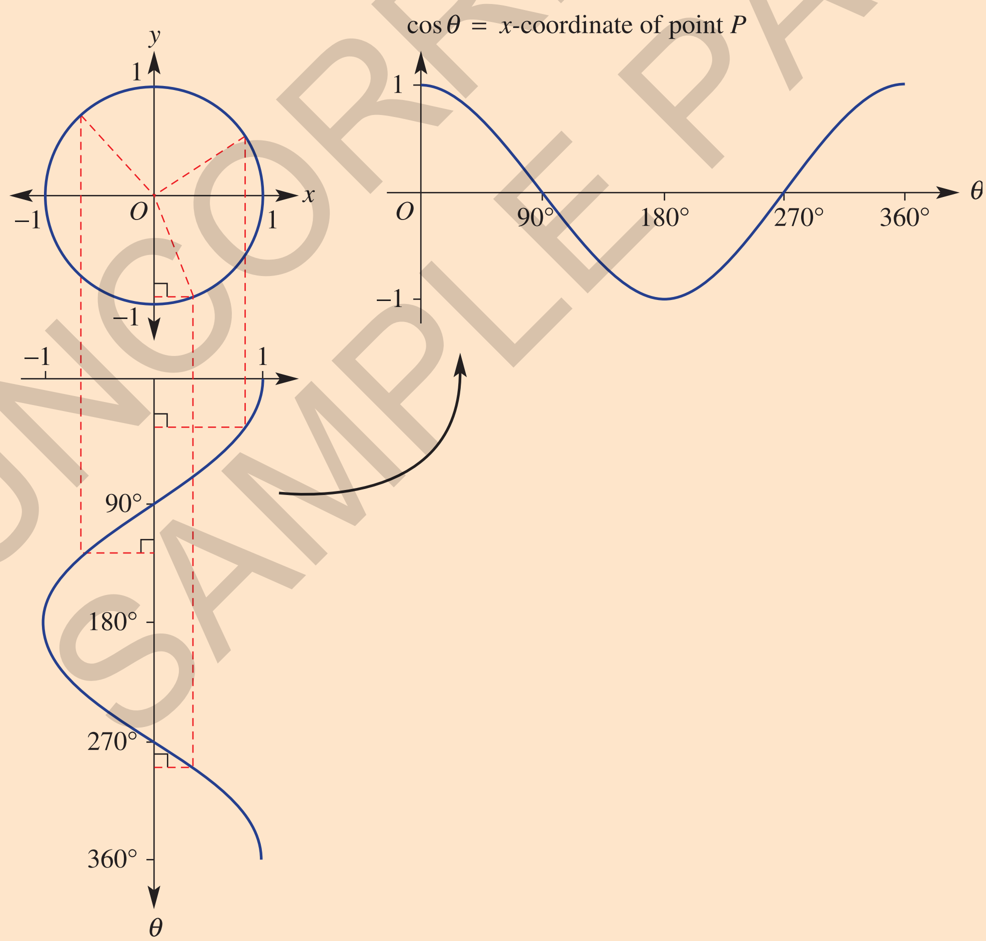
The shapes of the Ferris wheel graphs you have drawn are examples of **periodic functions** because the graph shape continuously repeats one cycle (for each period of 360°) as the wheel rotates. The graph of height above the ground illustrates a sine function ($\sin\theta$). The graph of the distance from a point on the circumference to the central vertical axis of the Ferris wheel illustrates a cosine function ($\cos\theta$).

Key ideas

- By plotting θ on the x-axis and $\sin\theta$ on the y-axis we form the graph of $\sin\theta$.
  $\sin\theta = y$-coordinate of point P on the unit circle.
  - $y = \sin\theta$

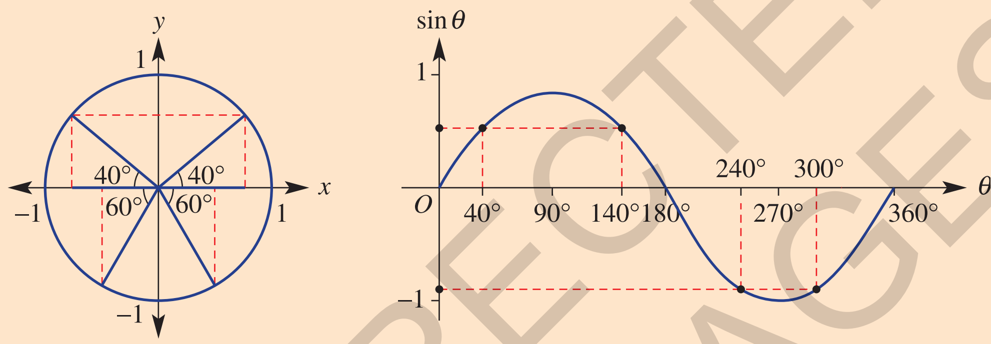


- By plotting θ on the x-axis and $\cos\theta$ on the y-axis we form the graph of $\cos\theta$.
  $\cos\theta = x$-coordinate of point P on the unit circle.
  - When we write $y = \cos\theta$, the y variable is not to be confused with the y-coordinate of the point P on the unit circle.
  - $y = \cos\theta$

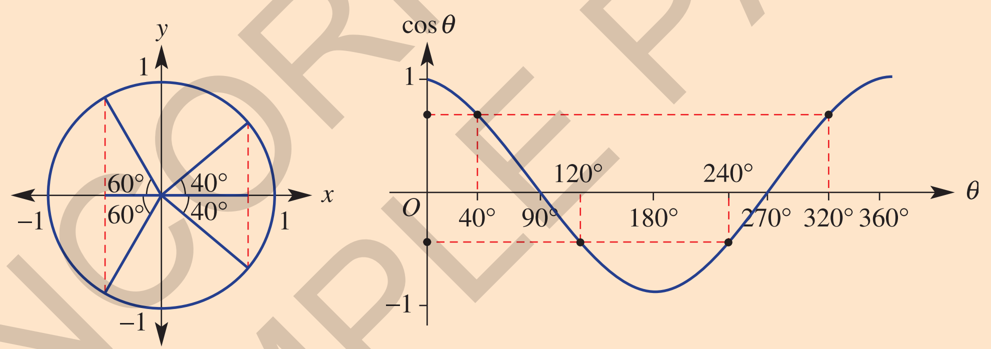


Key ideas

- **Amplitude** is the maximum displacement of the graph from a reference level (here it is the x-axis).
- The **period** of a graph is the time taken (or number of degrees) to make one complete cycle.
- Both $y = \sin\theta$ and $y = \cos\theta$ have Amplitude = 1 and Period = 360°.
- **Symmetry** within the unit circle using reference angles can be illustrated using graphs of trigonometric functions.
  - This shows $\sin 40° = \sin 140°$ (reference angle 40°) and $\sin 240° = \sin 300°$ (reference angle 60°).



  - This shows $\cos 40° = \cos 320°$ (reference angle 40°) and $\cos 120° = \cos 240°$ (reference angle 60°).



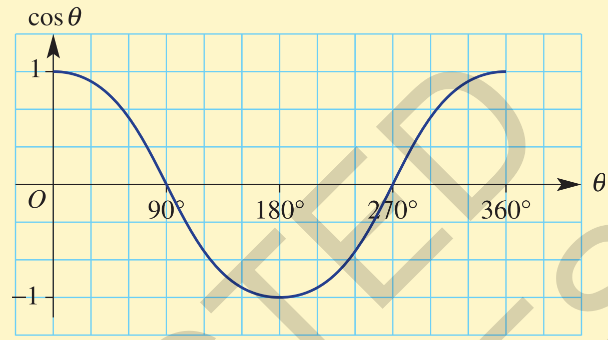
There are many applications of trigonometric functions in physics and engineering. The sine wave displayed on this oscilloscope, for example, could represent an electromagnetic wave or an alternating power source.

Example 20 Reading off a trigonometric graph

Use this graph of $\cos \theta$ to estimate:

a the value of $\cos \theta$ for $\theta = 200°$

b the two values of θ for which $\cos \theta = 0.7$



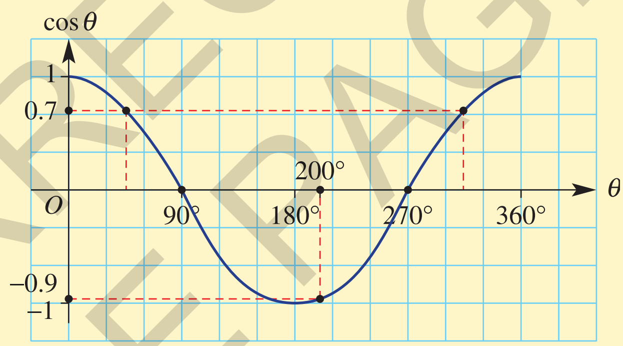
SOLUTION

a $\cos 200° \approx -0.9$

b $\cos \theta = 0.7$

$\theta \approx 46°$ or $314°$

EXPLANATION



Example 21 Comparing the size of the sine of angles

Use the graph of $y = \sin \theta$ to state whether or not the following are true or false.

a $\sin 70° < \sin 130°$

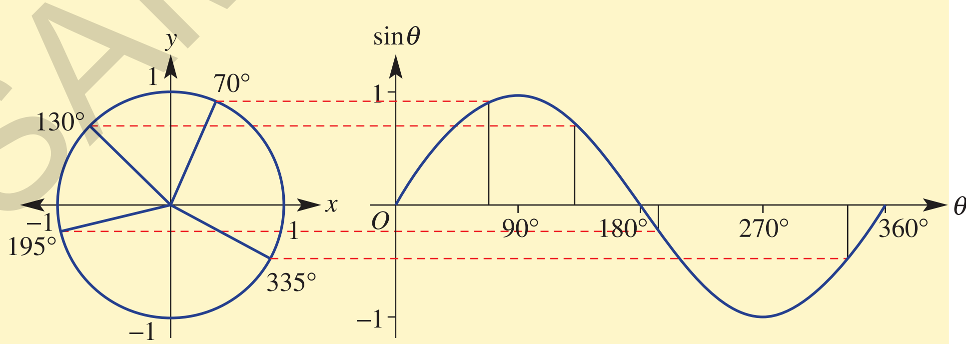
b $\sin 195° > \sin 335°$

SOLUTION

a false

b true

EXPLANATION



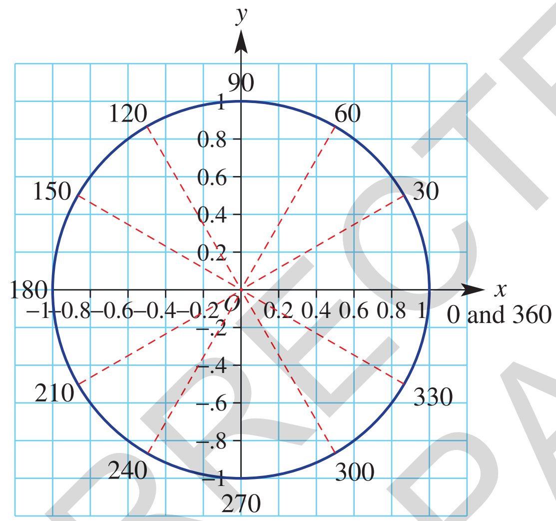
Exercise 4K

1–3	3	—

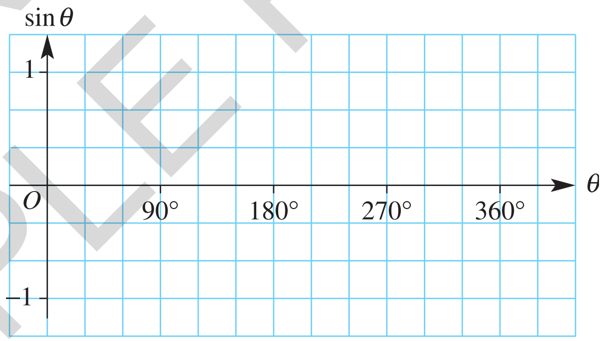
UNDERSTANDING

1 **a** Complete the table below for sin θ, writing the y-coordinate of each point at which the angle intersects the unit circle.

θ	0°	30°	60°	90°	120°	150°	180°	210°	240°	270°	300°	330°	360°
sinθ	0	0.5			0.87			–0.5					



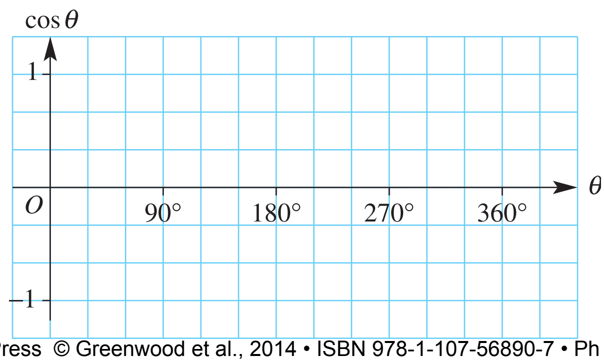
b Graph the points above and join them to make a smooth curve for sin θ.



2 **a** Using the diagram in Question 1, complete the table below for cos θ, writing the x-coordinate of each point at which the angle intersects the unit circle.

θ	0°	30°	60°	90°	120°	150°	180°	210°	240°	270°	300°	330°	360°
cosθ	1	0.87			–0.5			–0.87					

b Graph the points above and join them to make a smooth curve for cos θ.



4K

UNDERSTANDING

3 **a** For the graph of $\sin \theta$ and using $0° \le \theta \le 360°$, state:
  i the maximum and minimum values of $\sin \theta$
  ii the values of θ for which $\sin \theta = 0$.

b For the graph of $\cos \theta$ and using $0° \le \theta \le 360°$, state:
  i the maximum and minimum values of $\cos \theta$
  ii the values of θ for which $\cos \theta = 0$.

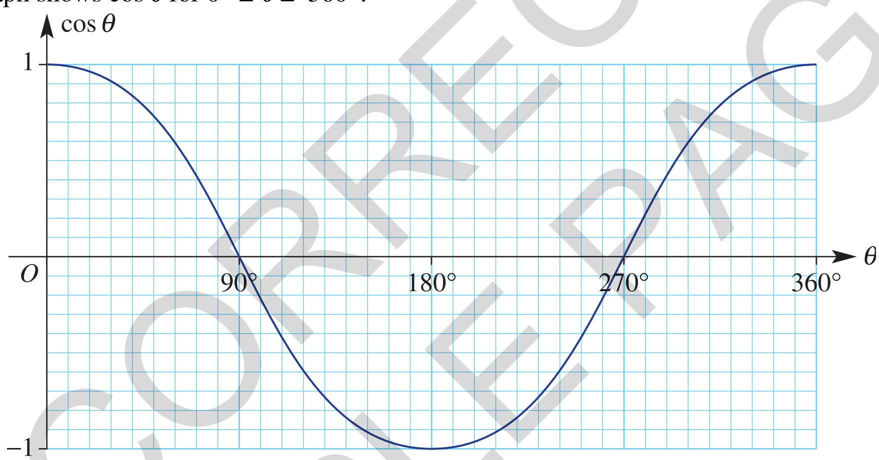
c State the values of θ for which:
  i $\cos \theta < 0$
  ii $\sin \theta < 0$

4–5(½) 4–6(½) 4–6(½)

FLUENCY

Example 20

4 This graph shows $\cos \theta$ for $0° \le \theta \le 360°$.



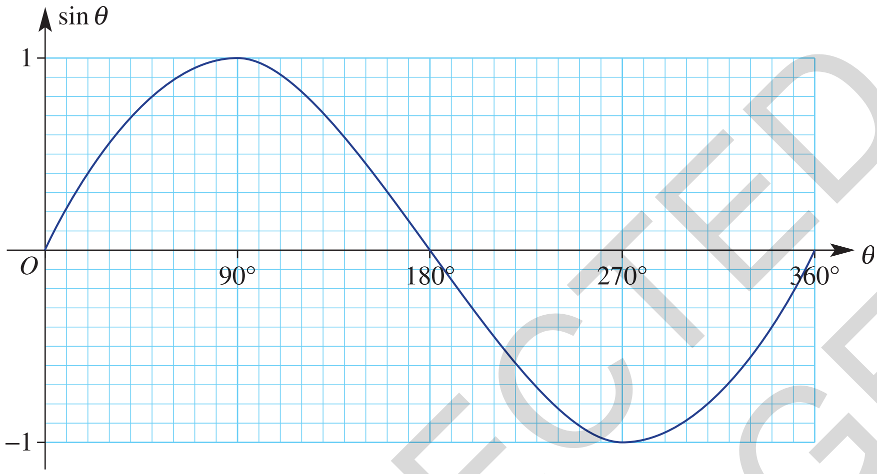
a Use this graph to estimate the value of $\cos \theta$ for the following.
  i $\theta = 35°$ **ii** $\theta = 190°$ **iii** $\theta = 330°$ **iv** $\theta = 140°$
  v $\theta = 260°$ **vi** $\theta = 75°$ **vii** $\theta = 115°$ **viii** $\theta = 305°$

b Use the same graph to estimate the two values of θ for each of the following.
  i $\cos \theta = 0.8$ **ii** $\cos \theta = 0.6$ **iii** $\cos \theta = 0.3$ **iv** $\cos \theta = 0.1$
  v $\cos \theta = -0.4$ **vi** $\cos \theta = -0.2$ **vii** $\cos \theta = -0.8$ **viii** $\cos \theta = -0.6$

4K

FLUENCY

5 This graph shows $\sin\theta$ for $0^\circ \le \theta \le 360^\circ$.



a Use this graph to estimate the value of $\sin\theta$ for the following.

i $\theta = 25^\circ$ **ii** $\theta = 115^\circ$ **iii** $\theta = 220^\circ$ **iv** $\theta = 310^\circ$

v $\theta = 160^\circ$ **vi** $\theta = 235^\circ$ **vii** $\theta = 320^\circ$ **viii** $\theta = 70^\circ$

b Use the same graph to estimate the two values of θ for each of the following.

i $\sin\theta = 0.6$ **ii** $\sin\theta = 0.2$ **iii** $\sin\theta = 0.3$ **iv** $\sin\theta = 0.9$

v $\sin\theta = -0.4$ **vi** $\sin\theta = -0.8$ **vii** $\sin\theta = -0.7$ **viii** $\sin\theta = -0.1$

Example 21

6 By considering the graphs of $y = \sin\theta$ and $y = \cos\theta$, state whether the following are true or false.

a $\sin 60^\circ > \sin 200^\circ$ **b** $\sin 100^\circ < \sin 300^\circ$ **c** $\sin 135^\circ < \sin 10^\circ$

d $\sin 200^\circ = \sin 340^\circ$ **e** $\cos 70^\circ < \cos 125^\circ$ **f** $\cos 315^\circ > \cos 135^\circ$

g $\cos 310^\circ = \cos 50^\circ$ **h** $\cos 95^\circ > \cos 260^\circ$ **i** $\sin 90^\circ = \cos 360^\circ$

j $\cos 180^\circ = \sin 180^\circ$ **k** $\sin 210^\circ > \sin 285^\circ$ **l** $\cos 15^\circ > \cos 115^\circ$

7–8(½)	7–9(½)	9–10(½)

PROBLEM-SOLVING

7 For each of the following angles, state the second angle between 0° and 360° that gives the same value for $\sin\theta$.

a 70° **b** 120° **c** 190° **d** 280°

e 153° **f** 214° **g** 307° **h** 183°

8 For each of the following angles, state the second angle between 0° and 360° that gives the same value for $\cos\theta$.

a 80° **b** 10° **c** 165° **d** 285°

e 224° **f** 147° **g** 336° **h** 199°

9 Give the reference angle in the first quadrant that matches these angles.

a 150° **b** 120° **c** 195° **d** 290°

e 235° **f** 260° **g** 125° **h** 205°

i 324° **j** 252° **k** 117° **l** 346°

4K

PROBLEM-SOLVING

10 Use a calculator to find the two values of θ for $0° \le \theta \le 360°$, correct to one decimal place, for these simple equations.

a $\sin\theta = 0.3$ b $\sin\theta = 0.7$ c $\cos\theta = 0.6$
d $\cos\theta = 0.8$ e $\sin\theta = -0.2$ f $\sin\theta = -0.8$
g $\cos\theta = -0.4$ h $\cos\theta = 0.65$ i $\sin\theta = 0.48$

REASONING 11 | 11, 12 | 12, 13

11 a How many values of θ satisfy $\sin\theta = 2$? Give a reason.
b How many values of θ satisfy $\cos\theta = -4$? Give a reason.

12 Recall the exact values for $\sin\theta$ and $\cos\theta$ for 30°, 45° and 60° in the first quadrant.

a Complete this table.

θ	0°	30°	45°	60°	90°
$\sin\theta$		$\frac{1}{2}$			
$\cos\theta$					0

b Using the reference angles in the table above, state the exact value of the following.

i sin 150° ii cos 120° iii cos 225° iv sin 180°
v cos 300° vi sin 240° vii cos 270° viii sin 135°
ix cos 210° x sin 330° xi sin 315° xii cos 240°
xiii sin 225° xiv sin 120° xv cos 150° xvi cos 330°

13 For θ between 0° and 360°, find the two values of θ that satisfy the following.

a $\cos\theta = \frac{\sqrt{2}}{2}$ b $\sin\theta = \frac{\sqrt{3}}{2}$ c $\sin\theta = \frac{1}{2}$
d $\sin\theta = -\frac{1}{2}$ e $\cos\theta = -\frac{1}{2}$ f $\cos\theta = -\frac{\sqrt{3}}{2}$

ENRICHMENT — | — | 14

Trigonometric functions with technology

14 Use technology to sketch the graph of the following families of curves on the same axes, and then write a sentence describing the effect of the changing constant.

a i $y = \sin x$ ii $y = -\sin x$
b i $y = \cos x$ ii $y = -\cos x$
c i $y = \sin x$ ii $y = 3\sin x$ iii $y = \frac{1}{2}\sin x$
d i $y = \cos x$ ii $y = \cos(2x)$ iii $y = \cos\left(\frac{x}{3}\right)$
e i $y = \sin x$ ii $y = \sin(x) + 2$ iii $y = \sin(x) - 1$
f i $y = \cos x$ ii $y = \cos(x - 45°)$ iii $y = \cos(x + 60°)$

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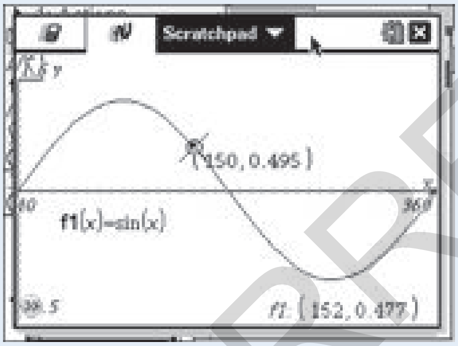
Using calculators to graph trigonometric functions

1 Sketch the graph of $y = \sin(x)$ for $0^\circ \le x \le 360^\circ$ and trace to explore the behaviour of y.

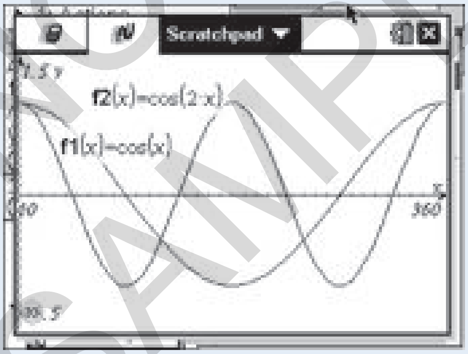
2 Sketch the graph of $y = \cos(x)$ and $y = \cos(2x)$ for $0^\circ \le x \le 360^\circ$ on the same set of axes.

Using the TI-Nspire:

1 In a graphs and geometry page, define $f1(x) = \sin(x)$ and press **enter**. Select **menu**, **Window/Zoom**, **Window Settings**, and set x from 0 to 360 and y from about –1.5 to 1.5. Select **menu**, **Trace**, **Graph Trace** and then scroll along the graph. Ensure you are in degree mode.

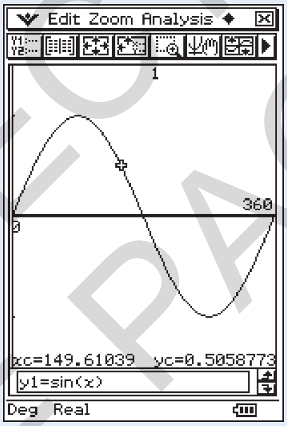


2 In a graphs and geometry page, define $f1(x) = \cos(x)$ and $f2(x) = \cos(2x)$ and press **enter**. Use settings as before.

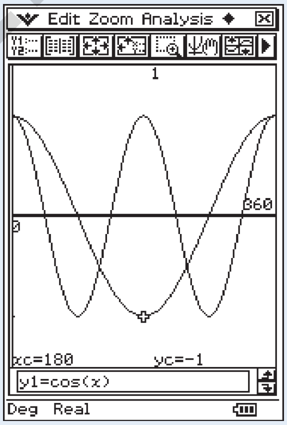


Using the ClassPad:

1 With the calculator in **Degree** mode, go to the **Graph&Table** application. Enter the rule $y1 = \sin(x)$ followed by **EXE**. Tap to see the graph. Tap and set x from 0 to 360 and y from about –1.5 to 1.5. Tap **Analysis**, **Trace** and then scroll along the graph.



2 In the **Graph&Table** application, define $y1 = \cos(x)$ and $y2 = \cos(2x)$. Tap . Use settings as before.



Investigation

Solving trigonometric equations

Trigonometric relations are not necessarily restricted to angles of less than 90°, and this is illustrated by drawing a graph of a trigonometric relation for angles up to 360°. Solving problems using trigonometric relations will therefore result in an equation that can have more than one solution.

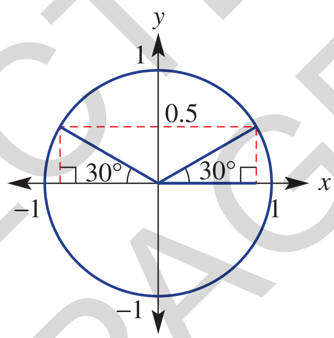
For example, consider the equation $\sin\theta = 0.5$ for $0° \le \theta \le 360°$. Since $\sin\theta$ is the y-coordinate on the unit circle, there are two angles that satisfy $\sin\theta = 0.5$.

Solution 1: $\sin\theta = 0.5$

$$\theta = \sin^{-1}(0.5) = 30°$$

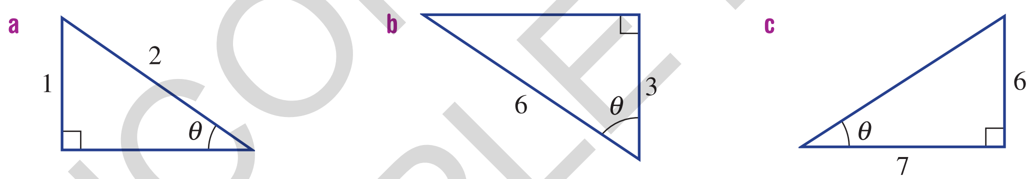
Solution 2:

$$\theta = 180° - 30° = 150°$$



Single solutions (0° ≤ θ ≤ 90°)

For these right-angled triangles, write an equation in terms of θ and then solve the equation to find θ.



Two solutions (0° ≤ θ ≤ 360°)

At point P on the unit circle $x = \cos\theta$ and $y = \sin\theta$.

For each of the following:

i Use a calculator to find a value for θ between 0° and 360°.

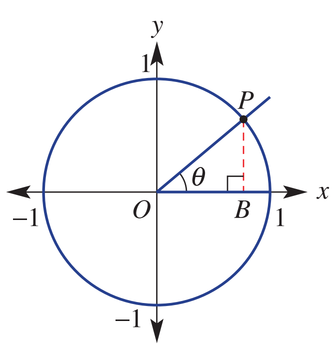
ii Find a second angle between 0° and 360° that also satisfies the given trigonometric equation.

a $\sin\theta = 0.5$

b $\cos\theta = 0.2$

c $\cos\theta = -0.8$

d $\sin\theta = -0.9$



Harder trigonometric equations

a Solve these trigonometric equations for $0° \le \theta \le 360°$.

  i $5 \sin \theta - 1 = 0$ **ii** $2 \cos \theta + 3 = 0$

b Solving an equation such as $\sin(2\theta) = \frac{1}{2}$ for $0° \le \theta \le 360°$ means that $0° \le 2\theta \le 720°$, which includes two rotations of the point P around the unit circle. Solve these equations by first solving for 2θ and then dividing by 2 to solve for θ.

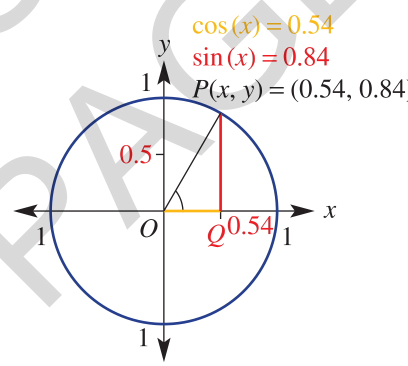
  i $\sin(2\theta) = \frac{1}{2}$ **ii** $\cos(2\theta) = 0.3$

Building a dynamic unit circle

Use dynamic geometry software to construct a unit circle with a point $P(\cos \theta, \sin \theta)$ that can move around the circle.

a Construction steps:

- Show a grid and axes.
- Construct a unit circle with centre (0, 0) and radius 1, as shown.
- Place a point P on the unit circle to construct OP. Show the coordinates of P.
- Construct QP and OQ so that $OQ \perp QP$ and Q is on the x-axis.
- Measure the angle $\theta = \angle QOP$.



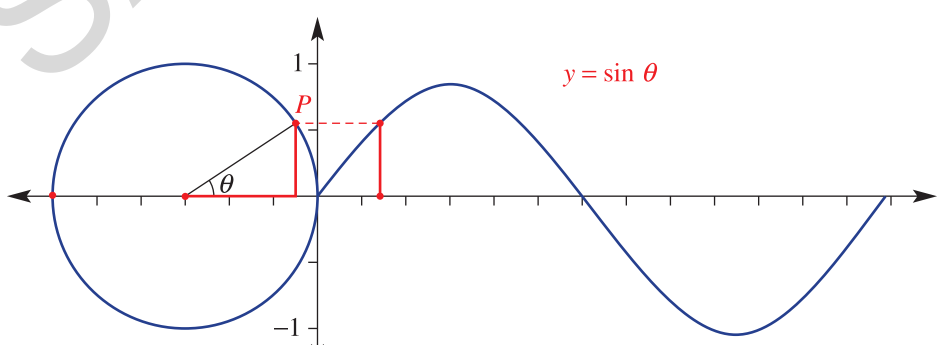
b Now drag point P around the unit circle and observe the coordinates of $P(\cos \theta, \sin \theta)$.

c Describe how $\cos \theta$ and $\sin \theta$ change as θ increases from 0° to 360°.

Extension

a Calculate $\tan \theta$ as $\frac{\sin \theta}{\cos \theta}$ and observe how the value of $\tan \theta$ changes as θ increases.

b Construct the graph of $y = \sin \theta$ by transferring the value of θ and $\sin \theta$ to a set of axes and constructing the set of coordinates $(\theta, \sin \theta)$. Trace $(\theta, \sin \theta)$ as you drag P to form the graph for $\sin \theta$.
Repeat for $\cos \theta$ and $\tan \theta$.



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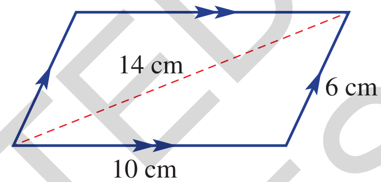
Problems and challenges

Check out the 'Working with unfamiliar problems' poster on the inside cover of your book to help you answer these questions.

1 Two adjacent sides of a parallelogram have lengths of 6 cm and 10 cm. If the length of the longer diagonal is 14 cm, find:

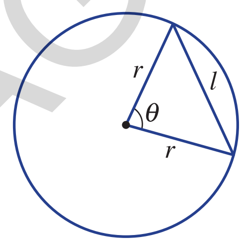
  a the size of the internal angles of the parallelogram

  b the length of the other diagonal, to one decimal place.



2 Two cyclists, Stuart and Cadel, start their ride from the same starting point. Stuart travels 30 km on a bearing of 025°T, while Cadel travels the same distance but in a direction of 245°T. What is Cadel's bearing from Stuart after they have travelled the 30 km?

3 Show that for a circle of radius r the length of a chord l that subtends an angle θ at the centre of the circle is given by $l = \sqrt{2r^2(1 - \cos\theta)}$.



4 Akira measures the angle of elevation to the top of a mountain to be 20°. He walks 800 m horizontally towards the mountain and finds the angle of elevation has doubled. What is the height of the mountain above Akira's position, to the nearest metre?

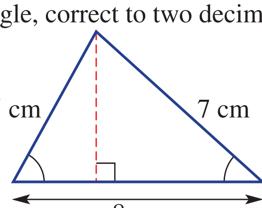
5 A walking group sets out due east from the town hall at 8 km/h. At the same time, another walking group leaves from the town hall along a different road in a direction of 030°T at 5 km/h.

  a How long will it be before the groups are 15 km apart? Give your answer to the nearest minute.

  b What is the true bearing of the second group from the first group, to the nearest degree, at any time?

6 Edwina stands due south of a building 40 m tall to take a photograph of it. The angle of elevation to the top of the building is 23°. What is the angle of elevation, correct to two decimal places, after she walks 80 m due east to take another photo?

7 Calculate the height of the given triangle, correct to two decimal places.

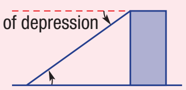


Chapter summary

Trigonometry

2D applications

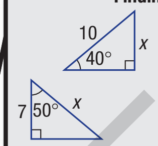
- Draw right-angled triangle with key information.
- Use trigonometry to find unknown.



Finding angles

If $\sin\theta = k$
$\theta = \sin^{-1}(k)$
If $\cos\theta = k$
$\theta = \cos^{-1}(k)$
If $\tan\theta = k$
$\theta = \tan^{-1}(k)$
$-1 \le k \le 1$ for sin and cos.

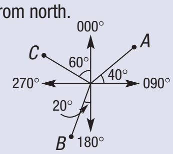
Finding lengths



$\sin 40° = \frac{x}{10}$
$\therefore\ x = 10 \times \sin 40°$
$\cos 50° = \frac{7}{x}$
$x \times \cos 50° = 7$
$x = \frac{7}{\cos 50°}$

Bearings

True bearings are measured clockwise from north.

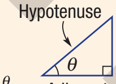


A: 050°T
B: 200°T
C: 300°T

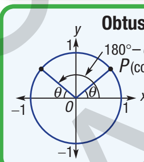
Opposite directions differ by 180°.

Trigonometric ratios

SOHCAHTOA
$\sin\theta = \frac{O}{H}$
$\cos\theta = \frac{A}{H}$
$\tan\theta = \frac{O}{A} = \frac{\sin\theta}{\cos\theta}$



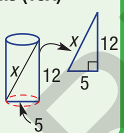
Obtuse angles (10A)



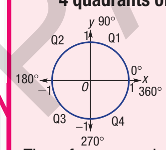
$\sin(180° - \theta) = \sin\theta$
$\cos(180° - \theta) = -\cos\theta$
$\tan(180° - \theta) = -\tan\theta$

3D applications (10A)

- Identify and redraw right-angled triangle.
- Use trigonometric ratios.
- Answer in words.



4 quadrants of unit circle (10A)

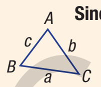


Q1: All are positive
Q2: only sin positive
Q3: only tan positive
Q4: only cos positive

The reference angle relates an angle to its acute angle in quadrant 1.
Symmetry can then be used to find $\sin\theta$ or $\cos\theta$ or $\tan\theta$.
e.g. sin 210° in quadrant 3 is negative
$210° - 180° = 30°$
$\sin 210° = -\sin 30°$
$= -\frac{1}{2}$

To find reference angle of θ in Q1:
Q2: $180° - \theta$
Q3: $\theta - 180°$
Q4: $360° - \theta$

Sine rule (10A)

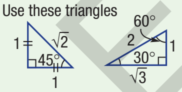


$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

When finding angles, determine whether you require the acute or the obtuse angle.

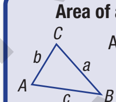
Exact values (10A)

Use these triangles



θ	0°	30°	45°	60°	90°
$\sin\theta$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
$\cos\theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0
$\tan\theta$	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	undefined

Area of a triangle (10A)

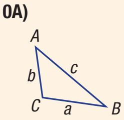


$\text{Area} = \frac{1}{2}ab\sin C$

Cosine rule (10A)

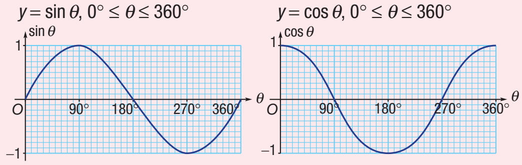
$c^2 = a^2 + b^2 - 2ab\cos C$

Rule is used to find:
- third side, given two sides and included angle
  or
- an angle, given all three sides.



Graphs of sin θ and cos θ (10A)

$y = \sin\theta,\ 0° \le \theta \le 360°$ $\quad$ $y = \cos\theta,\ 0° \le \theta \le 360°$



Symmetry of the unit circle can be observed in graphs.

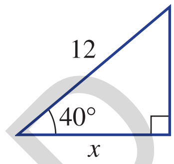
Multiple-choice questions

4A

1 The value of x in the diagram shown is equal to:

A $\dfrac{12}{\cos 40^\circ}$ **B** $12\sin 40^\circ$ **C** $\dfrac{\sin 40^\circ}{12}$

D $12\cos 40^\circ$ **E** $\dfrac{12}{\tan 40^\circ}$

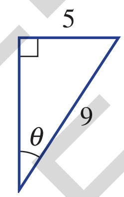


4B

2 The angle θ, correct to one decimal place, is:

A 56.3° **B** 33.7° **C** 29.1°

D 60.9° **E** 42.4°



4C

3 The angle of depression from the top of a communications tower measuring 44 m tall to the top of a communications tower measuring 31 m tall is 18°. The horizontal distance between the two towers is closest to:

A 12 m **B** 4 m **C** 14 m **D** 42 m **E** 40 m

4D

4 A yacht is sailed from A to B on a bearing of 196°T. To sail from B directly back to A the true bearing would be:

A 074°T **B** 096°T **C** 164°T **D** 016°T **E** 286°T

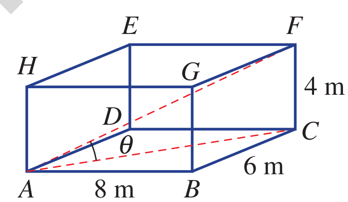
4E

10A

5 The angle θ that AF makes with the base of the rectangular prism is closest to:

A 22° **B** 68° **C** 16°

D 24° **E** 27°



4F

10A

6 The obtuse angle θ such that $\cos\theta^\circ = -\cos 35^\circ$ is:

A $\theta = 125^\circ$ **B** $\theta = 145^\circ$ **C** $\theta = 140^\circ$ **D** $\theta = 175^\circ$ **E** $\theta = 155^\circ$

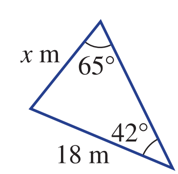
4G

10A

7 The side length x in the triangle shown, correct to one decimal place, is:

A 10.9 **B** 29.7 **C** 13.3

D 12.6 **E** 17.1



4H

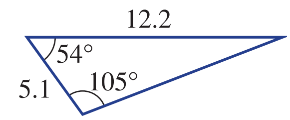
10A

8 The smallest angle in the triangle with side lengths 8 cm, 13 cm and 19 cm, to the nearest degree is:

A 19° **B** 33° **C** 52° **D** 24° **E** 29°

4I 10A

9 The area of the triangle shown can be determined by calculating:



A $\frac{1}{2}\times 5.1\times 12.2\times \cos 54°$ **B** $\frac{1}{2}\times 5.1\times 12.2\times \sin 105°$

C $\frac{1}{2}\times 12.2\times 5.1$ **D** $\frac{1}{2}\times 12.2\times 5.1\times \sin 54°$

E $\frac{1}{2}\times 6.1\times 5.1\times \sin 21°$

4J 10A

10 The incorrect statement below is:

A $\cos 110° = -\cos 70°$ **B** $\cos 246°$ is negative **C** $\tan 130°$ is positive

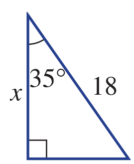
D $\sin 150° = \sin 30°$ **E** $\sin 300°$ is negative and $\cos 300°$ is positive

Short-answer questions

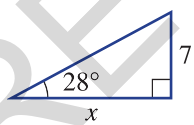
4A

1 Find the value of each pronumeral, rounding your answer to two decimal places.

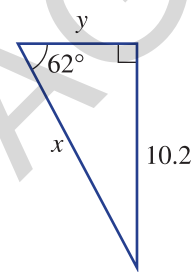
a



b



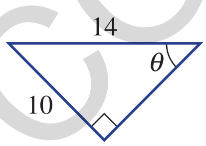
c



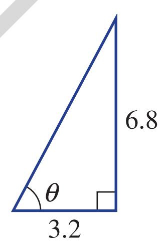
4B

2 Find the value of θ, correct to one decimal place.

a



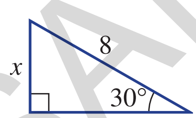
b



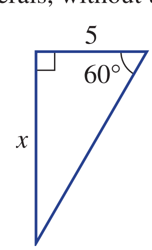
4F 10A

3 Use exact values to find the value of the pronumerals, without using a calculator.

a



b



4C

4 An escalator in a shopping centre from level 1 to level 2 is 22 m in length and has an angle of elevation of 16°. Determine how high level 2 is above level 1, to one decimal place.

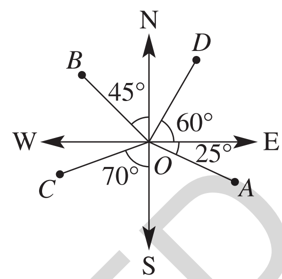
4D

5 **a** Write each bearing A–D as a true bearing.

b Give the true bearing of:

i O from A

ii O from C.



4D

6 A helicopter flies due south for 160 km and then on a bearing of 125°T for 120 km. Answer the following, to one decimal place.

a How far east is the helicopter from its start location?

b How far south is the helicopter from its start location?

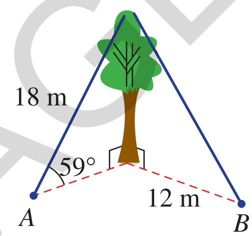
c What bearing must it fly on to return directly to the start location?

4E 10A

7 A tree is being supported by two ropes, as shown. The rope to point A is 18 m long and makes an angle of 59° with the ground. Point B is 12 m from the base of the tree.

a Find the height of the tree, to two decimal places.

b Find the angle the rope to point B makes with the ground, to the nearest degree.



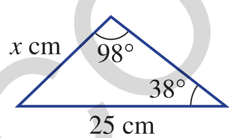
4G/I 10A

8 For these triangles, find the following, correct to one decimal place.

i the value of x

ii the area of the triangle

a



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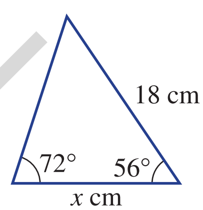
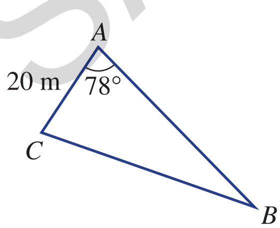
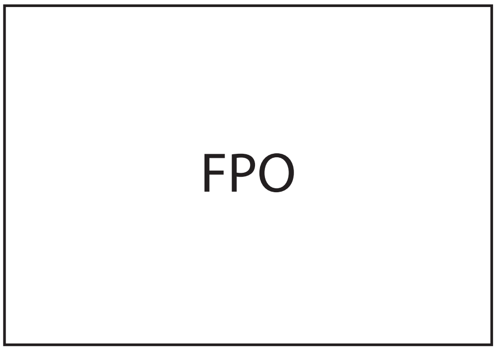


4I 10A

9 Three fences are used to form a triangular pig pen with known dimensions, as shown in the diagram. If the area of the pig pen is 275 m^2, what is the length AB? Round your answer to one decimal place.





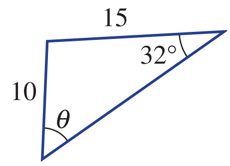
Chapter review

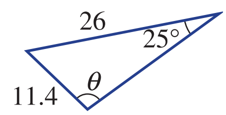
4G 10A

10 Use the sine rule to find the value of θ, correct to one decimal place.

a θ is acute

b θ is obtuse

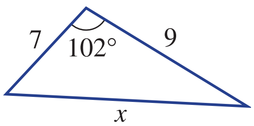




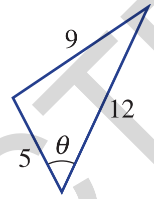
4H 10A

11 Use the cosine rule to find the value of the pronumeral, to one decimal place.

a



b



4J 10A

12 **a** Rewrite the following using their reference angle.

i sin 120° ii cos 210° iii tan 315° iv sin 225°

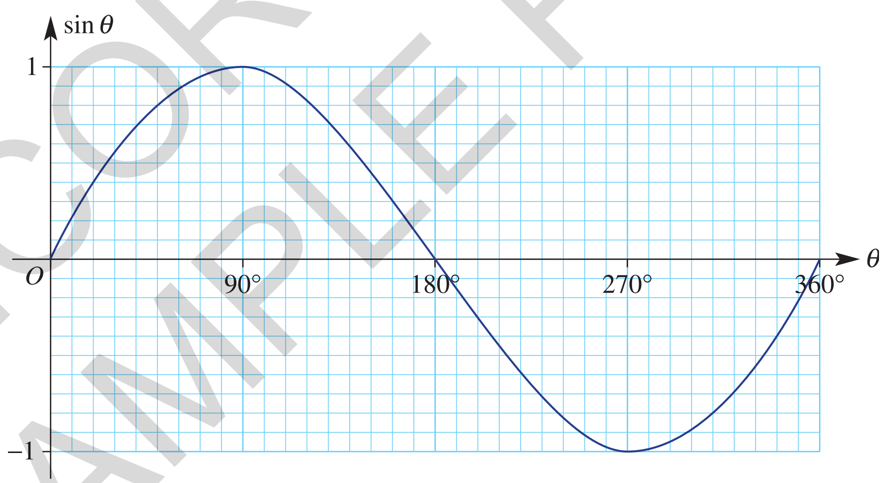
b Hence, give the exact value of each part in **a**.

c State whether the following are positive or negative.

i cos 158° ii tan 231° iii sin 333° iv cos 295°

4K 10A

13 Use the graph of sin θ shown to complete the following.



a Estimate the value of:

i sin 130° ii sin 255°

b Find the values of θ between 0° and 360° such that:

i $\sin\theta = 0.8$ ii $\sin\theta = -0.3$ iii $\sin\theta = 1.5$

c State if the following are true or false.

i sin 90° = 1 ii sin 75° > sin 140° iii sin 220° < sin 250°

Extended-response questions

1 A group of friends set out on a hike to a waterfall in a national park. They are given the following directions to walk from the park's entrance to the waterfall to avoid having to cross a river.

Walk 5 km on a bearing of 325°T and then 3 km due north.

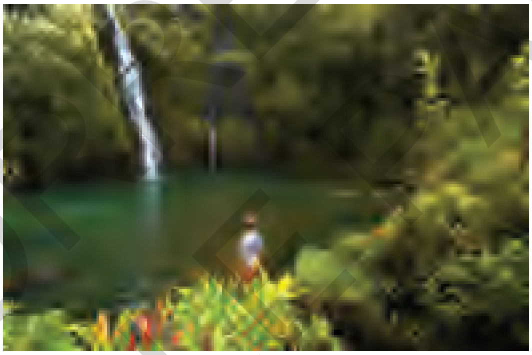
Round each answer to one decimal place.

a Draw and label a diagram to represent this hike.
b Determine how far east or west the waterfall is from the entrance.
c Find the direct distance from the park's entrance to the waterfall.

10A

The friends set up tents on level ground at the base of the waterfall at points *A* (35 m from the base of the waterfall) and *B* (28 m from the base of the waterfall). The angle of elevation from *A* to the top of the waterfall is 32°.

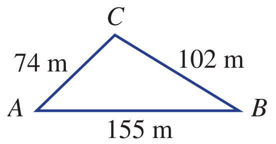
d Determine:
  i the height of the waterfall
  ii the angle of elevation from *B* to the top of the waterfall.



2 A paddock, *ABC*, is fenced off, as shown in the figure.

10A

a Find $\angle A$, to three decimal places.
b Hence, find the area enclosed by the fences. Round your answer to two decimal places.



It is planned to divide the paddock into two triangular paddocks by constructing a fence from point *C* to meet *AB* at right angles at a point *D*.

c Determine how many metres of fencing will be required along *CD*, to the nearest centimetre.
d How far is point *D* from point *A*, to the nearest centimetre?
e The person who constructs the fence *CD* misinterprets the information and builds a fence that does not meet *AB* at right angles. The fence is 45 metres long.
  i Determine, to two decimal places, the two possible angles (i.e. acute and obtuse), this fence line makes with *AB*.
  ii Hence, find the two possible distances of fence post *D* from *A*. Round your answer to one decimal place.