Syllabus Checklist

By the end of this chapter, you should be able to:

- ☐ review the concepts and language of outcomes, sample spaces and events as sets of outcomes (ACMMM049)
- ☐ use set language and notation for events, including:
- ☐ $\overline{A}$ (or A') for the complement of an event A,
- ☐ $A \cap B$ and $A \cup B$ for the intersection and union of events A and B respectively,
- ☐ $A \cap B \cap C$ and $A \cup B \cup C$ for the intersection and union of the three events A, B, and C respectively,
- ☐ recognise mutually exclusive events (ACMMM050)
- ☐ use everyday occurrences to illustrate set descriptions and representations of events, and set operations. (ACMMM051)
- ☐ review probability as a measure of 'the likelihood of occurrence' of an event (ACMMM052)
- ☐ review the probability scale: $0 \leq P(A) \leq 1$ for each event A, with $P(A) = 0$ if A is an impossibility and $P(A) = 1$ if A is a certainty (ACMMM053)
- ☐ review the rules: $P(\overline{A}) = 1 - P(A)$ and $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ (ACMMM054)
- ☐ use relative frequencies obtained from data as point estimates of probabilities. (ACMMM055)
- ☐ understand the notion of a conditional probability and recognise and use language that indicates conditionality (ACMMM056)
- ☐ use the notation $P(A|B)$ and the formula $P(A \cap B) = P(A|B)(PB)$ (ACMMM057)
- ☐ understand the notion of independence of an event A from an event B, as defined by $P(A|B) = P(A)$ (ACMMM058)
- ☐ establish and use the formula $P(A \cap B) = P(A)P(B)$ for independent events A and B, and recognise the symmetry of independence (ACMMM059)
- ☐ use relative frequencies obtained from data as point estimates of conditional probabilities and as indications of possible independence of events. (ACMMM060)

5.1 CORE THEORY

i) Set notation

A set is the collective noun used in mathematics to describe the objects found in a group. These objects are called elements.

If set Z contains the letters b, c, d, e, and f, then we can write this set as Z = {b, c, d, e, f}. These brackets are always used to list the elements of a set.

We can also state that this set has five elements. This can be expressed either by writing n(Z) = 5 or | Z | = 5. Also, it is true that

- b is an element of Z, or b $\in$ Z
- r is not an element of Z, or r $\notin$ Z
- If set Y is {b, c, d}, then Y is a subset of Z, or Y $\subset$ Z
- The intersection ($\cap$) of multiple sets refers to the set of elements that are common to both sets. For example, if C = {5, 6, 7, 8} and D = {6, 7, 8, 10} then C$\cap$D = {6, 7, 8}.
- If multiple sets have no intersection, then these sets are said to be disjoint.
- The union ($\cup$) of multiple sets refers to the combination of elements from those sets without repetition. For example, if G = {10, 11, 12} and H = {12, 13} then G$\cup$H = {10,11, 12,13}.
- If a set has no elements, it is referred to as an empty set. This is expressed using the symbol $\emptyset$ or a set of empty brackets {   }.
- The complement of a set consists of all those elements not in the set. The complement of A can be denoted as A' or $\overline{A}$.

ii) Probability

To calculate the theoretical probability of an event happening use the formula:

$$\text{Probability of event happening} = \frac{Number\ of\ favourable\ outcomes}{Total\ number\ of\ outcomes}$$

- The probability, P, of an event happening must lie between the values $0 \leq P \leq 1$.
- 0 represents an event never happening; 1 represents the certainty of an event happening.
- In set notation, $\overline{A}$ is the complement of A. Concerning probability, P($\overline{A}$) is the complement of P(A). The complement can be calculated through: P($\overline{A}$) = 1 − P(A).

Compound Events
There are two different types of compound events:

- P (A and B) or P (A $\cap$ B) refers to both events occurring simultaneously.
- P(A or B) or   P(A $\cup$ B) refers to either one event or both events occurring.

Addition Rule Principles
If n (A $\cup$ B) = n (A) + n (B) − n (A $\cap$ B), then P (A $\cup$ B) = P (A) + P (B) − P (A $\cap$ B).

Conditional Probability
This principle applies to situations where you are asked to find the probability of an event happening **given that** another event has already occurred. To work out a problem involving conditional probability, always determine the given event first (this becomes the total # of possible outcomes).

The notation for finding the probability of Event X happening given that Event Y has happened is : P (X | Y). Another way of reading this could be the probability of X given Y.
The formula for finding conditional probability is:

$$P(X \mid Y) = \frac{P(X \cap Y)}{P(Y)}$$

Independent Events
An event A is independent of event B if the probability that A occurring is not influenced by whether B has or has not occurred.

Two events A and B are independent if:

$$P(A \cap B) = P(A) \times P(B)$$

$$P(A|B) = P(A)$$

Mutually Exclusive Events

Two events are mutually exclusive if both events cannot occur at the same time.

If sets A and B are mutually exclusive then:

- $P(A \cap B) = 0$
- $P(A \cup B) = P(A) \times P(B)$

iii) Tree diagrams

Tree diagrams list the total number of possible outcomes of an event, or the sample space. Consider the tree diagram below which shows the possible gender combinations for a family of three children.

1st child	2nd child	3rd child	OUTCOMES
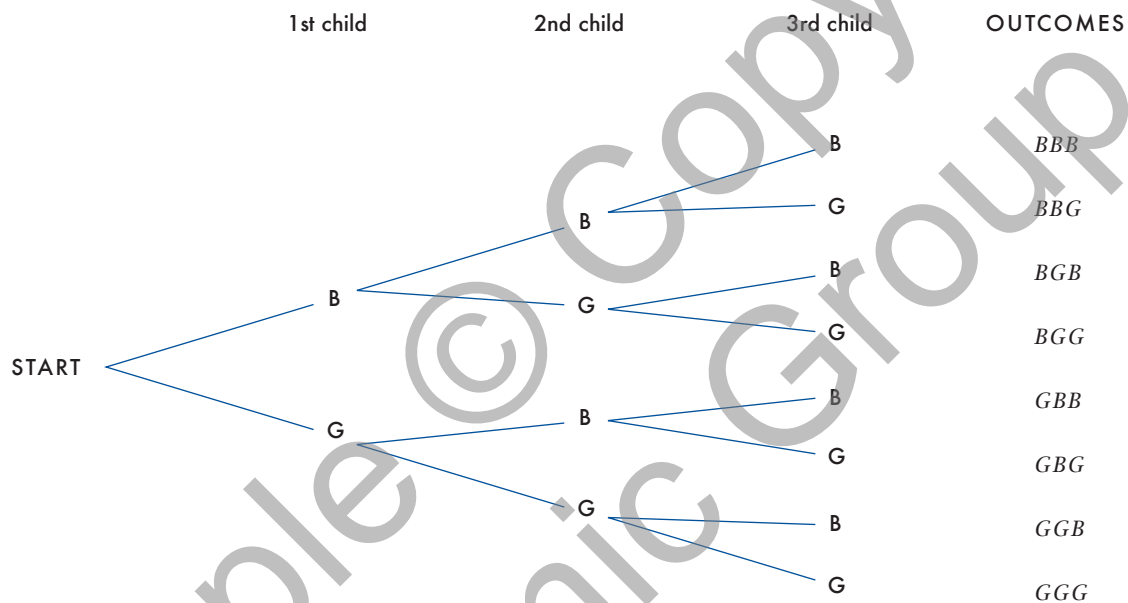
iv) Venn Diagrams

Venn diagrams show the entire universal set within the boundaries of a rectangle. Many sets can be represented simultaneously within the same Venn diagram. If we have Set F comprising { 1, 2, 3, 4}, Set G consisting of { 2, 4, 6, 8, 10}, and the Universal Set including all positive integers from 0 to 20, this information would make the Venn diagram as such:

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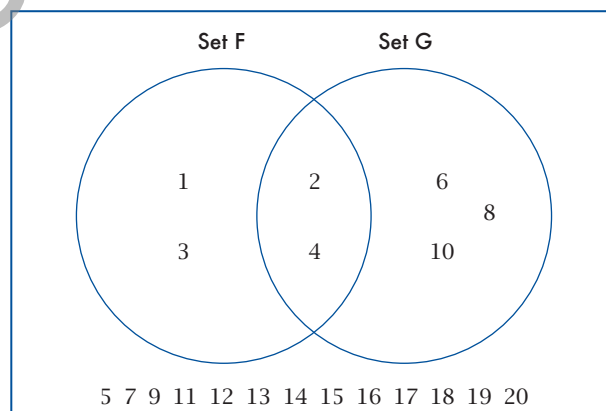
It is helpful to recognise how unions and intersections can be used to 'shade' in parts of a Venn diagram. Some more common examples include those below:

$P\,(A \cup B)$

$P\,(A \cap B)$

$P\,(A \cup B)'$

$P\,(A' \cap B)$
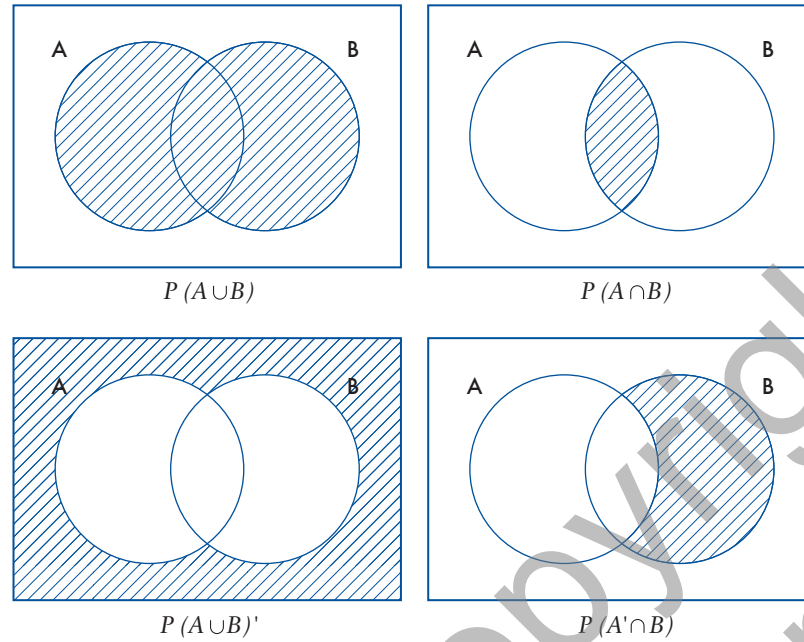
v) Two-Way Tables

Another way of representing data is through the use of a two-way table. Such a table can be generated through the occurrence or preference of two events. To set up a two-way table consider the following steps:

- Write the headings of P (A), P'(A) and Total across the top row (A is the first event)
- Write the headings of P (B), P'(B) and Total down the first column (B is the second event)
- Fill in the given probabilities from the question and determine the missing values.

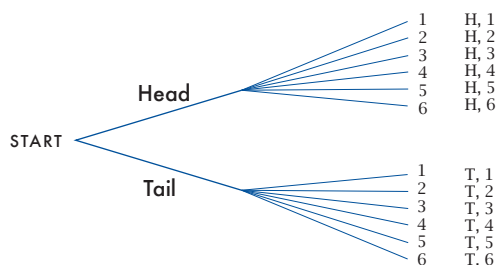In table form, this is represented as:

	P (A)	P' (A)	Total
P (B)			
P'(B)			
Total			

WORKED EXAMPLES

5.1   A fair coin and a fair die are tossed. Draw a tree diagram to represent this sample space, and hence, find the probability that
   a)   a tail and a 6 show
   b)   a tail shows
   **c)   a head and an odd number show**

A tree diagram is required to obtain the sample space.

a) A tail and a six occurs only once in this sample space.
   Thus, the probability associated with this event is $\frac{1}{12}$

b) There are six ways a tail can occur. Therefore, $\frac{6}{12} = \frac{1}{2}$

c) $\frac{3}{12} = \frac{1}{4}$

5.2 **At a certain year group of 60 Year 12 students, the amount of students studying History is 35, the amount of students studying Biology is 45, and the amount of students taking both classes is 30. Use a Venn diagram to determine the probability of**
   a) **the amount of students not studying History**
   b) **the amount of students not studying Biology**
   c) **(Biology U History)**

A Venn diagram is needed to represent the given data.
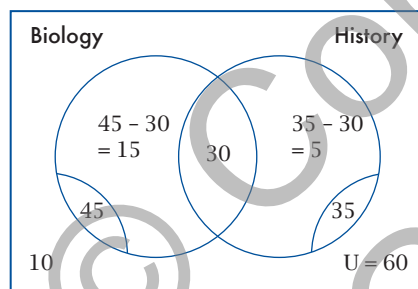
a) The amount of students not studying History is represented by all those students outside of the History circle. This involves $= \frac{25}{60} = \frac{5}{12}$

b) This probability is the reverse of part a), and would be $= \frac{15}{60} = \frac{1}{4}$

c) This probability is the complement of all students who take History or Biology, or the amount of students outside the two circles. Hence, $\frac{10}{60} = \frac{1}{6}$. (The set $(B \cup H) =$, leaving a remainder of 10 students who sat neither subject.)

5.3 **If $P(A \cap B) = 0.4$, $P(A) = 0.5$ and if events A and B are independent determine:**

(a) $P(A|B)$

(b) $P(A'|B)$

If events A and B are independent then:

$$P(A \cap B) = P(A) \times P(B)$$
$$0.4 = 0.5 \times P(B)$$
$$P(B) = \frac{0.4}{0.5}$$
$$P(B) = 0.8$$

Using conditional probability formula

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$
$$P(A|B) = \frac{0.4}{0.8}$$
$$P(A|B) = 0.5$$

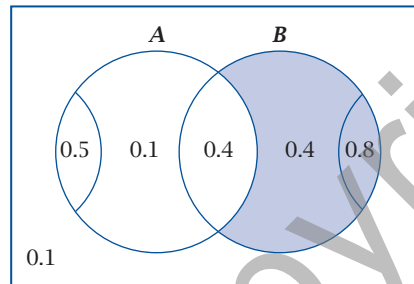5

If events A and B are independent then

$$P(A'|B) = \frac{P(A' \cap B)}{P(B)}$$

$$P(A'|B) = \frac{0.4}{0.8}$$

$$P(A'|B) = 0.5$$

To assist in calculating $P(A' \cap B)$ use a Venn Diagram.

$P(A' \cap B)$ ▢

5.4 **At a local university 500 students were enrolled in a particular course. Within this course 329 students took Chemistry, 186 students took Physics, and 295 students took Calculus. It was also known that the following numbers of students took 2 courses:**

> **Chemistry and Physics – 83 students**
> **Chemistry and Calculus – 217 students**
> **Physics and Calculus – 63 students**

Determine the number of students who took:
(a)   Chemistry, Physics and Calculus
(b)   Only Chemistry
**(c)   Exactly one of the classes: Chemistry, Physics or Calculus**

Step 1. Place all of the known information into a Venn Diagram. Note how there are no 'non-participants'; said another way, all of the 500 students took at least one of the three classes offered, leaving $n(A \cup B \cup C)' = 0$. Denote the middle region as x.
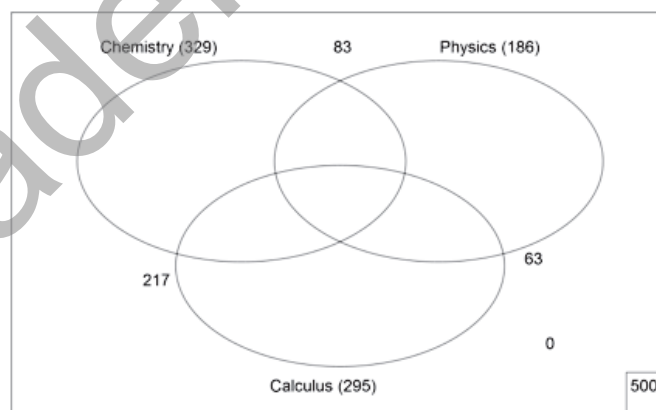
Step 2. Express the 6 unknown regions in terms of x. For instance, the intersection of Chemistry and Physics has 83 students, with x students taking Chemistry, Physics and Calculus. To obtain the number of students taking only Chemistry and Physics (and not Calculus) subtract x from 83 i.e. 83 – x. Repeat this procedure for the remaining 2 intersecting regions.
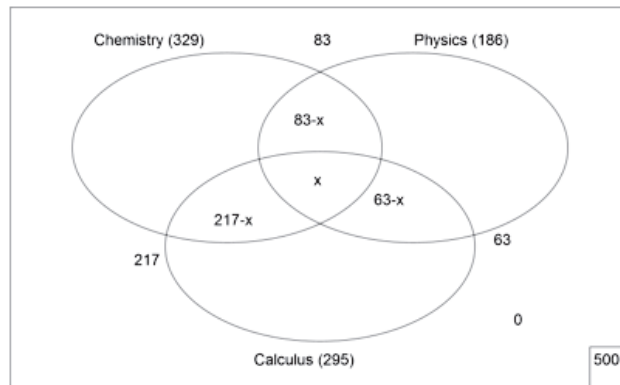
Step 3. Express the number of students taking only one subjects offered in terms of x. For instance, those students taking only Chemistry can be expressed by 329 − [(83 −x) + x + (217- x)] which is equivalent to 29 + x. Repeat this procedure for the remaining 2 'only' regions.
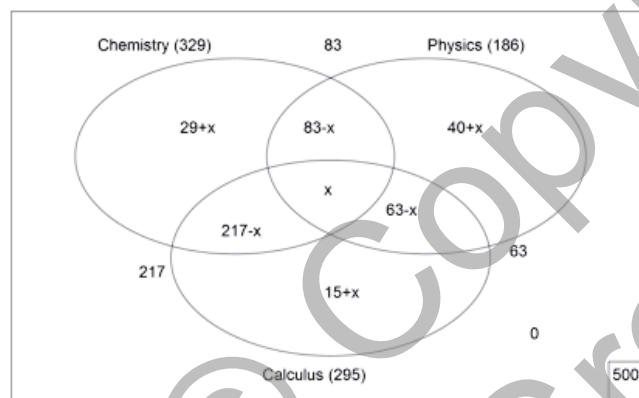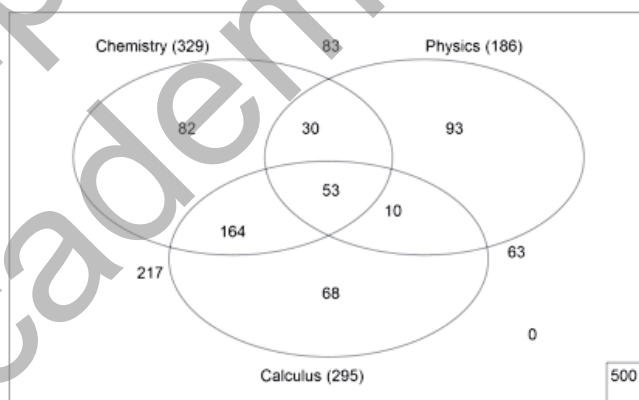

Step 4. Add all of the 8 regions (including the region 'outside' the circles) and sum them to the amount in the Universal Set.
In other words, 29 + x + 83 − x + x + 217 − x + 15 + x + 63 − x + 40 + x + 0 = 500.
Solving gives x = 53

Step 5. Redisplay the Venn Diagram after determining the number of students within each region (use x = 53).

Now the questions can be answered:
(a)   The number of students who took Chemistry, Physics and Calculus is the 'middle' value, 53
(b)   The number of students who only did Chemistry is 82. Note that these students are within the Chemistry set but are not in any region intersecting with Physics nor Calculus.
(c)   Those students who studies exactly one of Chemistry, Physics and Calculus can be found by adding the 3 'only' regions in the Venn Diagram. Hence, 82 + 68 + 93 = 243 students studied exactly one subject.

PROBLEMS TO SOLVE

1. The table below shows the results of 100 votes at a local warehouse to poll opinions about whether to take industrial strike action. There are three times that workers can commence their shift; morning, afternoon and night. The 100 workers polled gave a response of yes, no or undecided concerning whether to strike or not.

|	YES	NO	UNDECIDED	TOTAL
MORNING	24	21	10	
AFTERNOON		3		
NIGHT	11	6	3	
TOTAL				100

a) Complete the table above using the following information. The Afternoon Shift workers who responded 'yes ' represented $\frac{2}{3}$ of the Morning Shift workers who responded 'yes', while the undecided Night Shift workers represented only half of the total of undecided Afternoon Shift workers.

b) For a randomly surveyed person, determine the probability that the person:

   i) worked Afternoon Shift and was in favour of striking;

   ii) worked the Morning Shift:

   iii) was not in favour of going on strike;

   iv) was a Night Shift worker given that the person was in favour of striking;

   v) was not in favour of taking industrial action if the person worked on Morning Shift;

   vi) was not on Night Shift if the person did not want to go on strike;

   vii) was either not in favour or undecided about striking given the person worked afternoons.

2. A famous musical group is playing one concert in the park. There is a choice of two seating packages, the Diamond Package or the Green Package. Patrons selecting the Diamond Package will be provided with chairs and may either buy food at the concert or bring their own. Patrons selecting the Green Package may bring their own chairs or picnic rug to sit on and bring their own food or buy their food at the venue. Programmes are sold at the venue. Patrons who buy their food are given a programme for free. Therefore, patrons need to decide whether to select the Diamond or Green package, whether to bring or buy their food and whether or not to buy a programme.

   a) Draw a clearly labelled tree diagram to show all the possibilities.

   b) If all the possibilities are equally likely, what is the probability that a patron chooses the Green Package?

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c)   Given a patron has a programme, what is the probability that they got it for free?

d)   What is the probability that patrons choose to bring their own food?

3.   In a Human Biology class of 30 students, the teacher discovers that four of the twenty boys and one of the girls are colour blind. Eight of the boys, including one which is colour blind, have blue eyes. Of the six girls with blue eyes, five are not colour blind.

(a)   Use the Venn Diagram below to represent the above information.
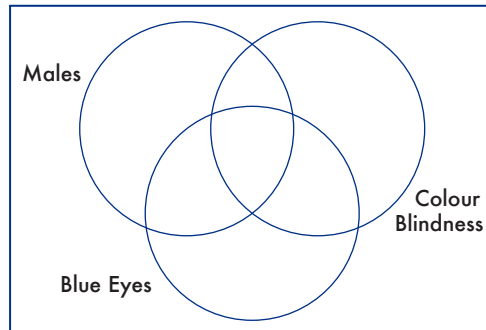
If one of these thirty students is chosen at random, determine the probability of the chosen student being

(b)   a person with blue eyes

(c)   a colour blind person with blue eyes

(d)   a male with blue eyes who is not colour blind

(e)   a male given that they are not colour blind

(f)   a male given that they do not have blue eyes

4.   For a Universal Set of $\{0,1,2,3,4,5,6,7,8,9\}$, with $A = \{1,3,5,7,9\}$, $B = \{2,4,6,8\}$ and $C = \{2,3,5,7\}$, list the elements of :

(a) $A \cup B$

(b) $B \cap C$

(c) $\overline{B}$

(d) $A \cap \overline{C}$

(e) $A \cap B \cap C$

5.   A new pharmacy assistant types three prescriptions for three patients based on an e-mail sent to her by the pharmacist. Unfortunately, during the process, the e-mail was deleted accidently. To avoid looking incompetent, the assistant sent the prescriptions out randomly to the patients.

(a) Draw an appropriate sample space.

(b) Use the sample space to find the probability that:

  (i) all prescriptions end up with the correct patients.

  (ii) no prescriptions end up the correct patients.

6. A statistician for Champion Records questioned teenagers on what Compact Disc they owned. Of 105 people questioned, 78 owned a Hip-Hop CD, 57 owned R & B and 73 owned Pop Rock.   Twenty nine people owned CDs produced by all 3, while 10 had R & B and Hip-Hop but not Pop Rock. Seven only owned Hip-Hop, while 12 had R & B and Pop Rock but not Hip-Hop CDs.

  a) Draw a Venn diagram to represent this information.

  b) How many teenagers did not own any of the CDs?

  c) How many teenagers owned only Pop Rock CDs?

  d) How many teenagers did not own an R & B CD?

7. Papa Joe's Pizza Parlour sells pizza in two styles, thick or thin crust. Customers have the choice of four toppings – cheese, pineapple, ham and tomato. From these four toppings a customer can choose two toppings to make a pizza; however, thick crust is never served with cheese, and thin crust is never served with ham.

  a) Draw a tree diagram to illustrate the possible outcomes.

  b) If every pizza topping combination is equally likely to be chosen, find the probability that a customer buys a pizza

    (i) with ham and pineapple as the toppings

    (ii) with pineapple as one of the choices of topping

    (iii) which does not have tomato as a topping

    (iv) with cheese on it given that the pizza was thin crust.

8. In a survey, 500 males and females were asked at what age they planned to start a family. The information from the survey is displayed in the table below.

	Age "20s"	Age "30s"	Over age 40	Total
Male	21	115		197
Female	20	212		
Total				

  a) Complete the table of values in the spaces provided.

 b) How many females planned to have children by the age of 40?

c) What is the probability that a person will have started a family by 40 years of age given that he is male?

9. A 3A/3B student conducted a survey in the Fremantle area with 150 participants. The survey sought to research the preferred AFL and WAFL teams of people with the following results:

		AFL Team			
		Dockers	Eagles	Other	Total
WAFL Team	East Freo.	40	9	17	66
	South Freo.	8	22	11	41
	Other	18	12	13	43
	Total	66	43	41	150

Determine the probability that a person selected at random from those surveyed:

a) Supported South Fremantle and the Dockers?

b) Supported South Fremantle or supported the Eagles?

c) Did not support South Fremantle?

d) Did not support the Eagles or South Fremantle?

e) Supported the Dockers given they supported South Fremantle?

f) Barracked for East Fremantle or South Fremantle if they did not support the Dockers?

10. A group of 120 Year 10 students were asked whether they were liked playing football or soccer. The survey showed:

60% liked Football
35% liked Soccer
and 32.5% liked both Football and Soccer.

(a) Use the above information to complete the Venn diagram below
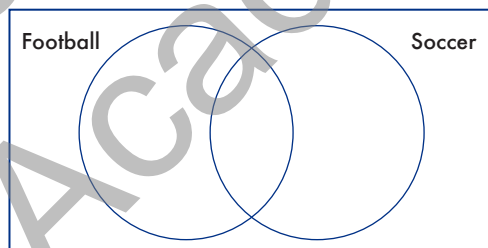
(b) How many students did Soccer but not Football?

(c) What is the probability that a student selected at random from the group liked both sports, given they liked at least one of the sports?

(d) What percentage of students liked neither sport?

11. A wardrobe has 3 shelves – the top, middle and bottom – and contained a mixture of 100 shirts and jumpers. 45% of all the clothing items were shirts. There are 20 jumpers on the middle shelf, and a total of 30 articles of clothing occupy the top shelf. Half of the clothing items on the bottom shelf are shirts, the whole of this shelf having 30 fewer clothing items than the middle shelf.

   a) Use this information to complete the table on the following page.

	Shirts	Jumpers	Total
Top			
Middle			
Bottom			
Total			

   a) What fraction, in simplest form, of all the clothing items were on the bottom shelf?

   b) What percentage of those clothing items on the top shelf were shirts?

12. The universal set, U, and the two sets A and B contained within it are such that

$$|A \cap B| = 35 \qquad |A| = 77 \quad |B| = 51 \qquad |U| = 100$$

   Determine

   a) $\left| \overline{A} \right|$

   b) $\left| \overline{B} \right|$

   c) $\left| \overline{A \cup B} \right|$

   d) $\left| \overline{A \cap B} \right|$

13. 90 Year 10 pupils at a certain school were asked whether they liked hot chips or hash browns for recess. 18 said they liked both, 40 said they liked only hash browns and 4 said they liked neither.

   a) Put this information into a two-way table.

   b) Use the table to find out how many pupils liked hot chips for recess.

14. The numbers in the Venn Diagram below indicate the number of insects in each of the sets A, B and C. If one insect is chosen at random from the universal group, U, determine;

U =
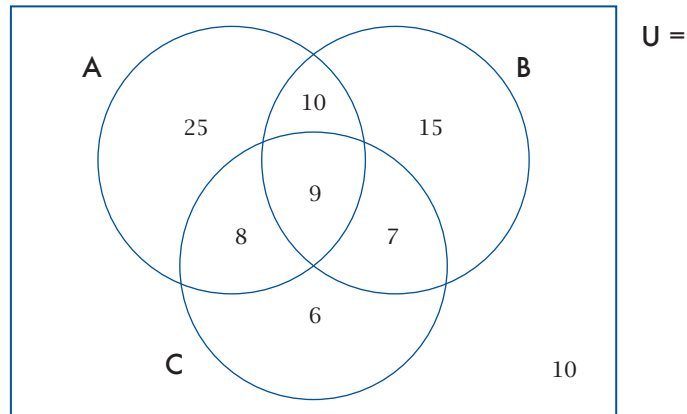
(i)   P(A)

(ii)   P(C)

(iii)   P(A $\cup$ B)

(iv)   P(A $\cap$ B $\cap$ C)

(v)   P(A | (B $\cap$ C))

(vii)   P(A | $\overline{(B \cap C)}$)

15.   Shade the regions, as defined, on the Venn diagrams below:
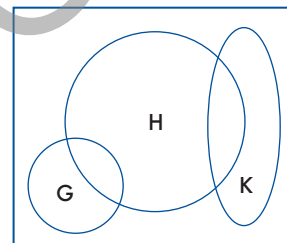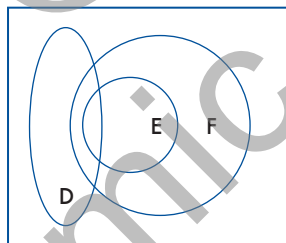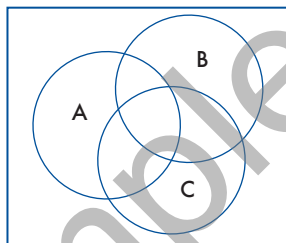
a) (A $\cap$ C) $\cup$ B   b) (D $\cap$ F) $\cap$ E'   c) (H $\cap$ K') $\cup$ G

16.   a)   Write down the set F of Fibonacci numbers less than 35.

      b)   Write down the set P of prime numbers less than 35.

      c)   Hence, verify that n(P $\cup$ F) = n(P) + n(F) – n (P $\cap$ F)

      d)   What is the probability that if:

          (i)   an element is chosen from P, it will also be in F?

          (ii)   an element is chosen from F, it will also be in P?

          (iii)   a whole number is chosen between 2 and 17 inclusive, it will be
                a prime Fibonacci number?

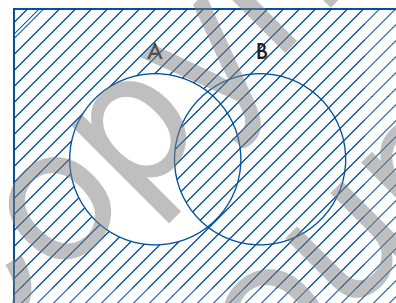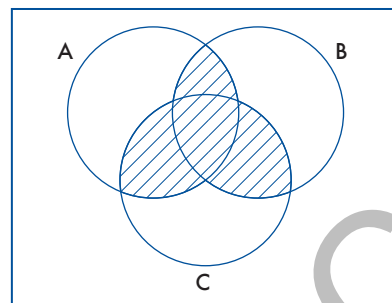17. Shade the following Venn diagrams according to the area indicated.
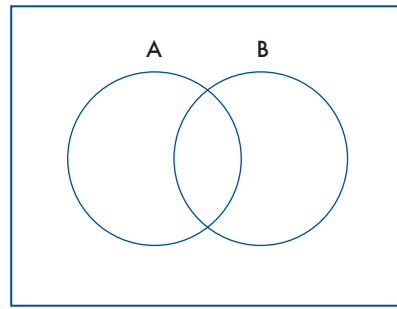
$A' \cap B'$
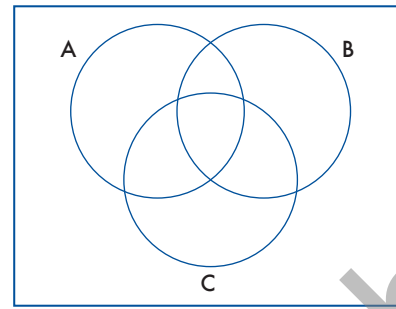
$A' \cap (B \cup C)$

18. Label the shaded areas of the following Venn diagrams.

19. If $P(A \cap B') = x$ $P(A' \cap B) = y$

     $P(A \cap B) = z$ $P(A|B) = 0.6$

     $P(B|A) = 0.4$ $P(A \cup B)' = 0.2$

     Determine the values of x, y and z.

20. Given $P(A) = 0.5$ $P(A \cap B \cap C) = 0.1$

           $P(B) = 0.8$ $P(A \cap B' \cap C') = 0.1$

           $P(C) = 0.6$ $P(A' \cap B \cap C') = 0.2$

                                 $P(A' \cap B' \cap C) = 0.1$

     $P(A \cap B \cap C') = x$

     $P(A \cap B' \cap C) = y$

     $P(A' \cap B \cap C) = z$

     $P(A \cup B \cup C) = 1$

Find:

(a)   x, y and z.

(b) $P(B' \cap A \cup C)$

(c) $P(A|B)$

(d)   Are events A and B independent? Support your answer with reasoning.

21.   The probability of Alex achieving an A grade in Maths, Physics and Chemistry are 0.2, 0.3 and 0.9 respectively. If the grades are independent events determine the probability that Alex receives:

(a)   all A grades

(b)   no A grades

(c)   exactly two A grades.

22.   Mr and Mrs Brown decide to go shopping. The probability that only Mr Brown goes shopping is 0.3 and is identical to the probability of only Mrs Brown going shopping. The probability they both go shopping is 0.09.

(a)   Are the events 'Mr Brown going shopping' and 'Mrs Brown going shopping' independent? Give reasons for your answer.

(b)   What is the probability that neither Mr or Mrs Brown go shopping?

23.   If the probabilities that A and B will die within a year are x and y respectively, determine the probability in terms of x and/or y that only one of them will be alive at the end of the year.

24.   One hundred people were surveyed regarding their relaxation activities. Forty eight watched TV, fifty eight exercised and sixty two read a good book. Twenty eight people read a book and watched TV, forty two read a book and exercised, while eight watched TV and exercised only.

The probability that a person does not get involved in any of the three activities is 0.1. Thirty three people surveyed participated in exactly one activity.

(a)   How many people participated in all three activities?

If a person is selected at random determine the probability that he/she participates in:

(b)   exactly two activities

(c)   all three activities, given they participate in at least one

(d)   the exercise or reading a book activity

25. Given the probabilities in the Venn Diagram below, determine whether or not the events A and B are independent.
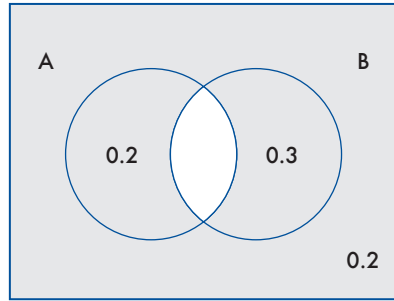
26. Given that events A and B are independent, prove that $P(A / B) = P(A)$ if $P(A / B) = 0.52$ and $P(B) = 0.43$.

$$P(A / B) = \frac{P(A \cap B)}{P(B)}$$

27. Show that the following events are independent:

Event A: A heart is drawn from a pack of playing cards

Event B: A queen is drawn from a pack of playing cards

28. A and B are two mutually exclusive events with $P(A) = \frac{2}{5}$ and $P(A \cup B) = \frac{7}{10}$. Find

(a) $P(B)$

(b) $P(B')$

(c) $P(A' \cap B')$.