10 Transformations on Curves

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1. [10 marks: 2, 2, 2, 2, 2]

   Describe a sequence of transformations required to convert $y = f(x)$ into $y = g(x)$.

   (a) $f(x) = x^2$ and $g(x) = (x-2)^2 + 4$

   (b) $f(x) = x^3$ and $g(x) = -(2x)^3$

   (c) $f(x) = \dfrac{1}{x}$ and $g(x) = \dfrac{1}{1-x}$

   (d) $f(x) = 3^x$ and $g(x) = -3^{x+1}$

   (e) $f(x) = (2x+1)^2$ and $g(x) = x^2$.

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2. [4 marks: 2, 2]

   Describe a sequence of transformations required to transform:
   (a) $x^2 + y^2 = 100$ into $(x+5)^2 + (y-6)^2 = 100$

   (b) $(x-2)^2 + (y-1)^2 = 64$ into $(x+7)^2 + (y+3)^2 = 64$

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3. [4 marks: 2, 2]

The curve $y = 2^{x+1}$ is transformed into $y = g(x)$.

(a) State the sequence of transformations involved if $g(x) = 2^{0.5x-1}$.

(b) State the sequence of transformations involved if $g(x) = 3(2^x)$.

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4. [4 marks: 2, 2]

The curve $y = 1+\frac{1}{x-2}$ is transformed into $y = g(x)$.

(a) State the sequence of transformations involved if $g(x) = \frac{2}{x-2}$.

(b) State the sequence of transformations involved if $g(x) = -1+\frac{1}{x+2}$.

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5. [10 marks: 2, 2, 2, 2, 2]

Identify the sequence of transformations required to map:

(a) $y = f(x)$ to $y = 2f(2x)$

(b) $y = f(x)$ to $y = f(2x + 1)$

(c) $y = f(x)$ to $y = f(2(x + 1))$

(d) $y = f(x)$ to $y = f(1 - x)$

(e) $y = f(x)$ to $y = 1 - f(x)$

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6. [6 marks: 2; 2, 2]

A parabola has equation $y = x^2 + 2x - 3$. Find the equation of the resulting curve:

(a) if the parabola is dilated by a factor of 2 along the x-axis.

(b) if the parabola is reflected about the x-axis and then translated 2 units along the negative y-axis.

(c) if the parabola is translated 1 unit along the positive x-axis and then reflected about the y-axis.

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7. [6 marks: 2, 2, 2]

The curve $y = 5^x$ is mapped to $y = g(x)$ by the following sequence of transformations. Find $g(x)$.

(a) a translation in the direction of the positive x-axis by 3 units followed by a translation in the direction of the positive y-axis by 2 units

(b) a dilation in the direction of the positive x-axis by a factor of 2 followed by a translation in the direction of the positive x-axis by -2 units

(c) a reflection about the y-axis followed by a dilation in the direction of the positive x-axis by a factor of ½.

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8. [10 marks: 2, 2, 2, 2, 2]

A curve with equation $y = \sqrt{x}$ is transformed into $y = k\sqrt{(ax+b)} + c$ by the following sequences of transformations. State the values of k, a, b and c.

(a) A translation 5 units in the direction of the positive x-axis followed by a dilation parallel to the positive x-axis of factor 2.

(b) A dilation parallel to the positive x-axis of factor 2 followed by a translation 5 units in the direction of the positive x-axis.

(c) A translation 5 units in the direction of the negative y-axis followed by a reflection about the x-axis.

(d) A reflection about the x-axis followed by a translation 5 units in the direction of the negative y-axis

(e) A reflection about the y-axis followed by a dilation of factor 3 parallel to the positive y-axis.

Calculator Free

9. [4 marks: 2, 2]

The circle with equation $(x + 6)^2 + (y - 7)^2 = 81$ is transformed into the circle with equation $(x - a)^2 + (y - b)^2 = r^2$ by the following sequences of transformations. State the values of a, b and r.

(a) A translation 3 units in the direction of the positive x-axis followed by a translation 5 units in the direction of the negative y-axis.

(b) A dilation of factor 2 parallel to the x-axis followed by a dilation of factor 2 parallel to the y-axis.

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10. [4 marks: 2, 2]

The parabola with equation $y^2 = x$ is transformed into the parabola with equation $y^2 = k(x - a)$ by the following sequences of transformations. State the values of a and k.

(a) A reflection about the y-axis followed by a reflection about the x-axis.

(b) A translation 4 units in the direction of the positive x-axis followed by a reflection about the y-axis.

Calculator Free

11. [14 marks: 3, 3, 4, 4]

The curve $y = f(x)$ has a minimum turning point at (–2, –1) and a maximum turning point at (4, 6). Find the minimum and maximum turning points of the following curves. In each case, explain clearly how you obtained your answer.

(a) $y = f(2x)$

(b) $y = 2f(x)$

(c) $y = 1 - f(x)$

(d) $y = f(1 - x)$.

Calculator Assumed

12. [8 marks: 2, 2, 2, 2]

The curve $y = f(x)$ has a maximum point at (1, 5), a minimum point at (–5, 2) and intercepts at (0, 4) and (5, 0). The curve has no other turning points and intercepts.

(a) State the coordinates of the horizontal intercept(s) of the curve $y = f(-x - 1)$.

(b) State the coordinates of a horizontal intercept of the curve $y = f(x + 1) - 2$.

(c) State the coordinates of the vertical intercept(s) of the curve $y = 2f(x + 1)$.

(d) State the coordinates of the maximum and minimum point of $y = -f(-x)$.

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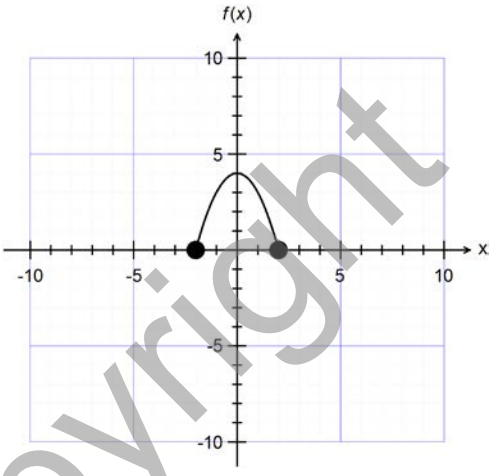
13. [3 marks]

Given that $f(x) = x^2$, solve $f(x) = f(2x + 1)$. Describe clearly how you obtained your answer.

Calculator Assumed

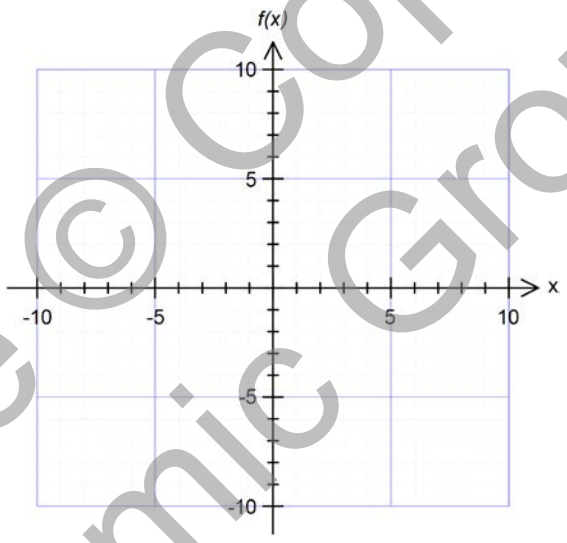
14. [6 marks: 3, 3]

The sketch of $y = f(x)$ is given in the accompanying diagram.

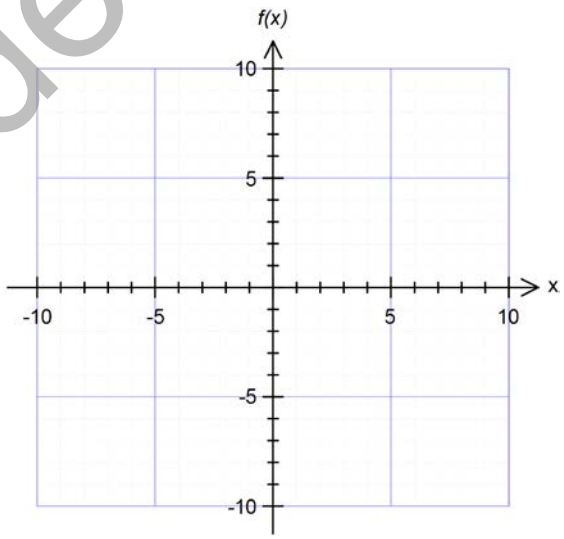


Sketch:

(a) $y = \frac{3}{2} f(x)$



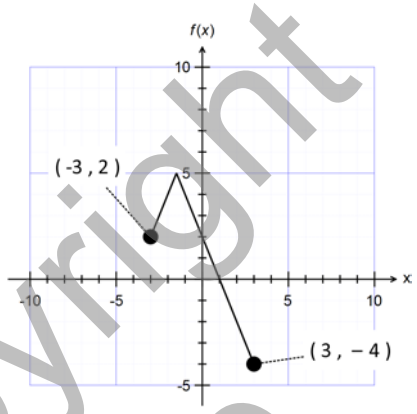
(b) $y = f(\frac{x}{2} + 1)$



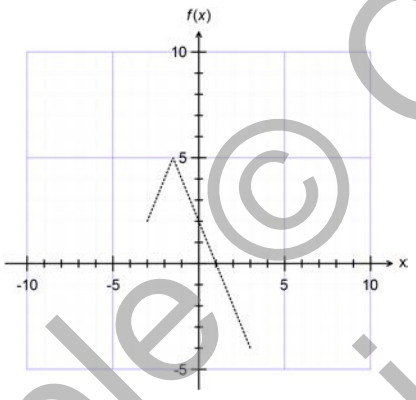
Calculator Assumed

15. [6 marks: 3, 3]

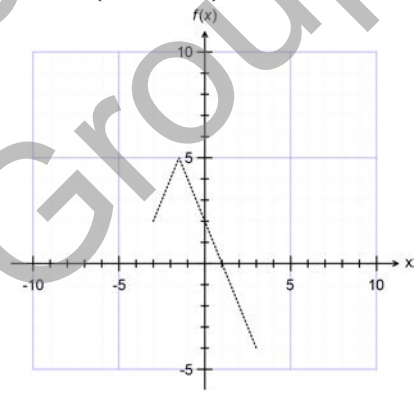
Given the graph of $y = f(x)$, sketch in the axes provided $y = g(x)$.



(a) $g(x) = \frac{1}{2}(f(x)+1)$



(b) $g(x) = f\left(\frac{1}{2}(x+1)\right)$



16. [3 marks]

Given the graph of $y = 5 - f(2x)$, sketch in the axes provided $y = f(x)$.