Macmillan

Australian Curriculum

MathsWorld 9

Australian Curriculum edition

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8

Proportion

Pre-test

Warm-up

Hour glasses were probably invented in the 14th century to measure time. They depend on the fact that the amount of sand that runs through is proportional to the time. If a certain amount of sand runs through from the top to the bottom in one minute, then twice as much sand will run through in two minutes. Egg timers are still used for this purpose to time the cooking of an egg. There are many other instances where one variable is directly proportional to another. For example, the force of gravity acting on an object is proportional to its mass and the pressure exerted by water is proportional to the depth below the surface. There are other cases where the proportion is not direct. In inverse proportion, the value of one variable is halved as the other doubles.

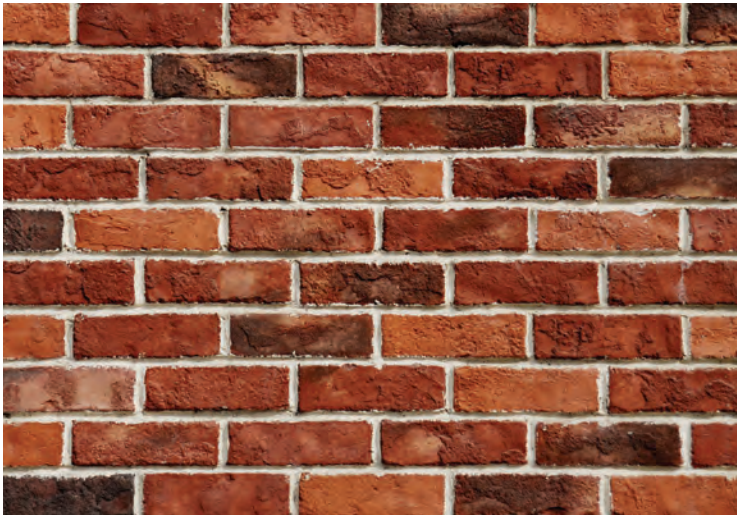
8.1 Direct proportion

BLM
Chapter warm-up

There are many relationships between quantities where one quantity depends directly on the other. For example, the circumference of a circle depends on the diameter, the weight of a load of bricks depends on the number of bricks, or the cost of muffins depends on how many muffins are bought. In each of these examples, there is a linear relationship between the two variables – if the value of one variable increases, the value of the other variable increases in proportion. We say that one variable is **directly proportional** to the other.

Many relationships between physical quantities are examples of direct variation. For example, the pressure in a liquid increases in proportion to the depth below the surface, the electric current flowing through a circuit is proportional to the voltage.

We will start by looking at the simple example of the number of bricks required to build walls of different areas. The photograph shows a section of a brick wall with an area of one square metre. There are 45 whole bricks and 10 half bricks, making up the equivalent of 50 bricks to one square metre.



If we double the area of the wall, we double the number of bricks required. If we multiply the area of the wall by 4, four times as many bricks are required.

× 2 × 4

Area of wall (A m^2)	0	1	2	3	4	5	20
Number of bricks (n)	0	50	100	150	200	250	1000

× 2 × 4

We say that the number of bricks is directly proportional to the area of the wall. If A m^2 is the area of the wall and n is the number of bricks, we can write

$n = kA$ where k is the **constant of proportionality**.

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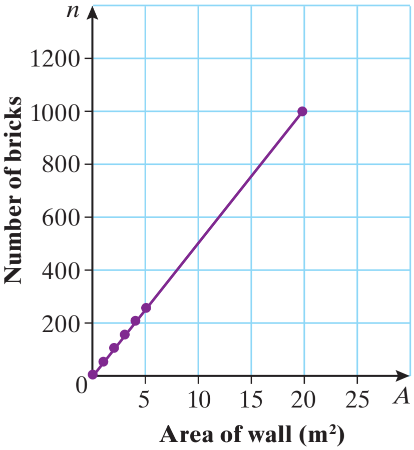
This means that $k = \frac{n}{A}$ so in this example, $k = \frac{50}{1} = \frac{100}{2} = \frac{1000}{20} = 50$. We can see that k is the number of bricks per square metre.

We can also write $n \propto A$ where the mathematical symbol $\propto$ means ‘is proportional to’.

The graph of n against A is a linear graph passing through the origin, with gradient 50.



Direct proportion

In general, direct proportion can be represented as $y \propto x$ or $y = kx$ where k is the constant of proportionality. Direct proportionality can stated in several ways.

- y is directly proportional to x.
- y is proportional to x.
- y varies directly as x.

The graph of $y = kx$ is a straight line passing through the origin, with gradient k. The converse also applies. If the graph of two variables is linear and passes through the origin, we know that one variable is directly proportional to the other.

Example 1

The number of paving stones required is proportional to the area to be paved. If n is the number of paving stones and A m^2 is the area, write the relationship in the following forms

i $y \propto x$

and

ii $y = kx$ where k is the constant of proportionality.

continued

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Example 1 continued

Working

i $n \propto A$

ii $n = kA$

Reasoning

The symbol $\propto$ means 'is directly proportional to'.

Dependent and independent variables

When one variable is directly proportional to another, we usually identify one of the variables as being independent and the other variable as dependent. In the rule $y = kx$, we take x to be the independent variable and y as the dependent variable, that is y depends on x. In the following worked example, Dino's earnings depend on how many matches he referees so n is the independent variable and E is the dependent variable.

- In a table of values the independent variable usually goes in the top row (or in the left hand column if the table is vertical).
- The independent variable goes on the horizontal axis of the graph and the dependent variable on the vertical axis.

Example 2

Dino referees basketball matches and he is paid $15 per match.

a Using n for the number of matches and $\$E$ dollars for Dino's earnings in a season, write this as a rule.

b Complete the table of values to show Dino's seasonal earnings for different numbers of matches refereed.

n	0	1	2	5	10
E					

c Plot a graph to show Dino's seasonal earnings.

d Explain why this is an example of direct variation.

e How many matches would Dino have to referee to earn $135?

Working

a $E = 15n$

b

n	0	1	2	5	10
E	0	15	30	75	150

Reasoning

Dino earns $15 for every match he referees.

Substitute $n = 0$, $n = 1$, and so on, into $E = 15n$.

continued

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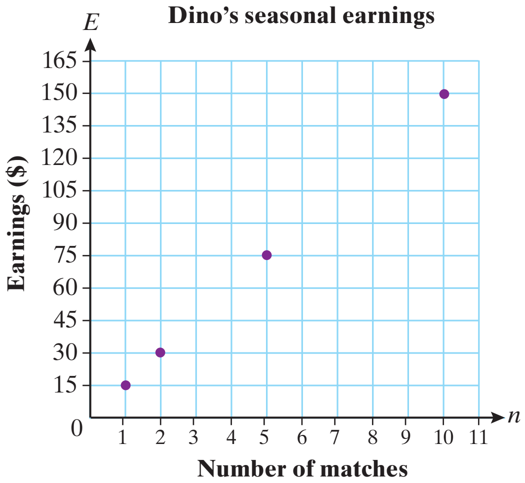
Example 2 continued

Working

Reasoning

c



Dino referees whole matches so it does not make sense to join the points.

d The graph is a straight line passing through zero.

If the number of matches doubles, Dino's earnings double.

e If $E = 135$,

$$135 = 15n$$

$$n = \frac{135}{15}$$

$$n = 9$$

Dino would need to referee 9 matches to earn $135.

Substitute 135 for E to find n.

If we know that one variable is directly proportional to another, we can calculate the constant of proportionality from one pair of values. We can then write the rule linking the two variables and use the rule to calculate other values.

Example 3

T is directly proportional to L.

a Using T and L, write the rule in the forms $y \propto x$ and $y = kx$.

b Find the value of k

c Rewrite the rule with the calculated value for k.

d Complete the table.

L	0	1	2	3	4
T	0			18.75	

Working

a $T \propto L$

$T = kL$

Reasoning

T is directly proportional to L.

continued

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Example 3 continued

Working

b When $L = 3$, $T = 18.75$

$18.75 = 3k$

$$k = \frac{18.75}{3}$$

$$k = 6.25$$

c $T = 6.25L$

d

L	0	1	2	3	4
T	0	6.25	12.50	18.75	25.00

Reasoning

Substitute the given values of T and L to find k.

Substitute 6.25 for k

Multiply each L value by 6.25.

Example 4

The amount of petrol used is directly proportional to the distance travelled. A car uses 27 L of petrol in traveling 360 km.

a Using p for the litres of petrol and d km for the distance travelled, write the rule in the forms $y \propto x$ and $y = kx$.

b Find the value of k.

c Rewrite the rule with the calculated value for k.

d How much petrol (to the nearest litre) would be used for 560 km?

e How far could the car travel with 34 L of petrol?

Working	Reasoning
a $p \propto d$ $p = kd$	The amount of petrol used depends on the distance travelled so d is the independent variable and p is the dependent variable.
b When $d = 360$, $p = 27$. $27 = 360k$ $k = \frac{27}{360}$ $k = 0.075$	Substitute the given values for p and d to find k. (k has been calculated by dividing the amount of petrol in litres by the distance in kilometres, so k represents the litres of petrol per kilometre. Because it is a small number, the petrol consumption of cars is usually stated in litres per 100 km. In this case the petrol consumption would be 7.5 L/100 km.)
c $p = 0.075d$	Substitute the calculated value of k in the rule.
d When $d = 560$, $p = 0.075 \times 560$ $p = 42$ 42 L would be used.	Substitute 560 for d to find p.

continued

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Example 4 continued

Working

e When $p = 34$,

$$34 = 0.075d$$

$$d = \frac{34}{0.075}$$

$$d = 453.\dot{3}$$

The car could travel approximately $453\frac{1}{3}$ km.

Reasoning

Substitute 34 for p to find d.

Example 5

For a given speed, the distance travelled is proportional to the length of time travelling.

a Using d km for the distance and t h for the time, express this in the forms $y \propto x$ and $y = kx$.

b A car travels 102 km in 1 hr 12 minutes. Calculate the value of k.

c What does k represent in this situation?

d Rewrite the rule using the calculated value of k.

e How far would the car travel in 2 hours at the same average speed?

f How long would it take for the car to travel 212 km at the same average speed?

Working	Reasoning
a $d \propto t$ $d = kt$	The distance travelled depends on the travel time. t is the independent variable and d is the dependent variable.
b $102 = 1.2k$ $k = \frac{102}{1.2}$ $k = 85$	$1 \text{ h } 12 \text{ min} = 1\frac{12}{60} \text{ h} = 1.2 \text{ h}$
c k represents the speed of the car in km/h.	k is calculated by dividing the distance in kilometres by the time in hours, so k represents the number of kilometres per hour, that is, the speed.
d $d = 85t$	Substitute 85 for k in the rule.
e When $t = 2$, $d = 85 \times 2$ $d = 190$ The car would travel 190 km.	Substitute 2 for t in the rule.

continued

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Example 5 continued

Working

f When $d = 212$,

$$212 = 85 \times t$$

$$t = \frac{212}{85}$$

$$t = 2.49...$$

It would take approximately $2\frac{1}{2}$ hours to travel 212 km.

Reasoning

Substitute 212 for d in the rule.

Many everyday problems involve direct proportion. Problems of this type can be solved using the following steps.

- Express the relationship in the form $y = kx$.
- Solve for k.
- Write the rule using the value of k.
- Substitute to find the required quantity.

Example 6

A 2.5 L tin of paint covered an area of 35 m^2 of wall. How much paint would be required to paint 64 m^2? Give your answer to one decimal place.

Working

Let $x\,\text{m}^2$ be the area of wall and y L be the amount of paint.

When $x = 35$, $y = 2.5$

$$y = kx$$

$$2.5 = 35k$$

$$k = \frac{2.5}{35}$$

$$k = \frac{1}{4}$$

$$y = 0.07142x$$

When $x = 64$,

$$y = \frac{1}{4} \times 64$$

$$y = 4.57...$$

Approximately 4.6 L of paint is required.

Reasoning

This problem could be solved in other ways:

2.5 L	35 m^2
p L	64 m^2

Writing a proportion equation, we obtain

$$\frac{p}{2.5} = \frac{64}{35}$$

$$p = \frac{64 \times 2.5}{35}$$

$$p \approx 4.6$$

Approximately 4.6 L of paint is required.

The unitary method could also be used:

35 m^2	2.5 L
1 m^2	$\frac{2.5}{35}$ L
64 m^2	$\frac{2.5 \times 64}{35} \approx 4.6$ L

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exercise 8.1

LINKS TO Example 1

1 Rewrite each of the following in the form

i $y \propto x$. **ii** $y = kx$.

a W is directly proportional to m.

b C is directly proportional to n.

c p varies directly as t.

d C is proportional to r.

e V varies directly as h.

f I is proportional to t.

LINKS TO Example 2

2 Each of the following tables of values represents the numbers of tiles, n, of a particular size, needed to pave an area, A m^2. Complete each table.

a

A	0	1	2	3	4	5
n			76			

b

A	0	1	2	3	4	5
n				57		

c

A	0	1	2	3	4	5
n					20	

d

A	0	1	2	3	4	5
n					25	

3 A concreting contractor charges \$55 per square metre to concrete a driveway.

a Using A m^2 for the area of the driveway and \$$C$ for the cost, write this as a rule.

b Complete the table of values to show the cost for different areas.

A	0	10	20	30	40	50	60
C	0						

c Use a spreadsheet to graph the cost against area.

d Explain why this is an example of direct variation.

e How many squares metres could be concreted for \$935?

f What would it cost to have 26 m^2 concreted?

LINKS TO Example 4

4 A fencing contractor charges \$4480 for 56 m of paling fence.

a Using L m for the length of the fence and \$$C$ for the cost, write this as a rule in the form $y \propto x$ and $y = kx$.

b Calculate the value of k.

c Rewrite the rule with the calculated value of k.

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d Copy and complete the table to show the cost for fences of different lengths.

Length of fence (L m)	0	10	20	30	40	50	56
Cost (\$$C$)	0						4480

e Calculate the cost of 68 m of fencing.

5 To build a wall from these blocks, 27 blocks per square metre are required.

a Copy and complete the table to show the number of blocks required for walls with different areas.

Area of wall, A (m^2)	0	1	2	3	4	5	10
Number of blocks, n	0						

b Using A m^2 for the area of the wall and n for the number of blocks, write the rule in the form $y \propto x$ and $y = kx$.

c Write the rule with the calculated value of k.

d What area of wall could be built using 500 blocks?

LINKS TO Example 5

6 A car travelling at constant speed travels 144 km in 1 hour 30 minutes.

a Using d km for the distance travelled and t hr for the time, write the rule in the forms $y \propto x$ and $y = kx$.

b Find the value of k.

c What is the speed of the car?

d Write the rule with the calculated value of k.

e How far would the car travel in 2 hours 30 minutes at the same speed?

f How long would it take for the car to travel 312 km at the same speed?

7 A car travelling at constant speed travels 133 km in 1 hour 24 minutes.

a Using d km for the distance travelled and t hr for the time, write the rule in the forms $y \propto x$ and $y = kx$.

b Find the value of k.

c What is the speed of the car?

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d Write the rule with the calculated value of k.

e How far would the car travel in 2 hours 15 min at the same speed?

f How long would it take for the car to travel 304 km at the same speed?

LINKS TO Example 6

8 Jeremy and Claire are making mini pizzas for a party. The topping for 32 pizzas requires 120 mL tomato paste. How much tomato paste will they require for 200 mini pizzas?

9 For a given voltage, the electrical power W watt is proportional to the current I amp that flows through it.

a Write this as a rule in the form and $y = kx$.

b When $W = 1200$, $I = 5$. Find the value of k.

c Rewrite the rule with the calculated value of k.

d Find I when $W = 60$

10 The circumference, C, of a circle is proportional to the diameter, d.

a Write this as a rule in the form $y \propto x$ and $y = kx$.

b What is the constant of proportionality?

11 A cake recipe includes 90 g of butter and 150 g flour. How much flour would be required if a larger cake is to be made with 120 g butter?

12 A car uses 9.5 L of petrol for 100 km. How much petrol would be required for 230 km in similar driving conditions?

13 The amount of a particular medicinal drug given to children in a hospital is directly proportional to the child's weight. A child weighing 34 kg is given 51 mg of the medicine.

a Using m kg for the child's mass and d mg for the amount of drug, write the rule in the forms $y \propto x$ and $y = kx$.

b Find the value of k.

c Rewrite the rule using the calculated value of k.

d How much of the drug should a child who weighs 42 kg be given?

14 In everyday language we often use the word *weight* when we mean *mass*. The weight of an object is the force of gravity acting on it and is measured in a unit called the Newton (named after Isaac Newton). The weight of an object, W Newtons, is proportional to its mass, m kg.

a Express this in the forms $y \propto x$ and $y = kx$.

b A brick has mass 2.7 kg and on Earth its weight is 26.46 Newton. Find the value of k.

c Rewrite the rule with the calculated value for k.

d What is the weight in Newtons of a person who has a mass of 68 kg?

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exercise 8.1 challenge

⑮ The gas ethane burns in oxygen to produce carbon dioxide and water. If 60 g of ethane and 224 g of oxygen theoretically produce 176 g of carbon dioxide and 108 g of water. If the amount of ethane is changed, the other substances change in the same proportion.

Copy and complete this table.

ethane	60	120	200	500
oxygen	224			
carbon dioxide	176			
water	108			

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8.2 Inverse proportion

We now look at a different type of proportionality. Suppose, for example, a 48 cm length of liquorice is shared between several people. The *greater* the number of people, the *shorter* the length each will receive. We say that the length of liquorice received is **inversely proportional** to the number of people.

In the following table, x is the number of people and y is the length of liquorice in centimetres that each person receives.

x	1	2	3	4	6	8	12	16	24	48
y	48	24	16	12	8	6	4	3	2	1

We can write $y \propto \frac{1}{x}$ or $y = \frac{k}{x}$ where k is the constant of proportionality.

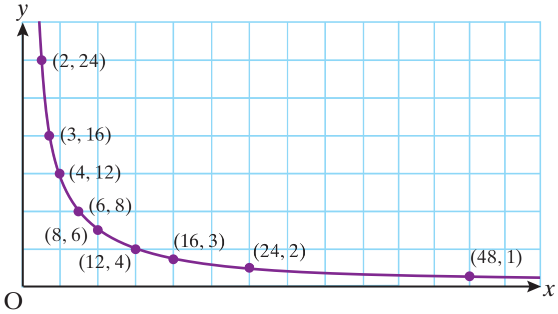
Note that this can also be written in the form $xy = k$, that is, the product of each x value and y value is constant.

In the case of the liquorice, when $x = 3$, for example, $y = 16$. So $16 = \frac{k}{3}$, that is, $k = 48$.

We can then write $y = \frac{48}{x}$.

Multiplying both sides of this rule by x gives $xy = 48$.

The graph of this relationship is called a hyperbola.



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Inverse variation

If two quantities, x and y, vary inversely, then they can be represented by $y \propto \frac{1}{x}$ or $y = k \times \frac{1}{x}$ or $y = \frac{k}{x}$ or $xy = k$. These relationships can also be called **reciprocal relationships** because y is proportional to the reciprocal of x.

The product xy is constant.

We can express inverse proportion as

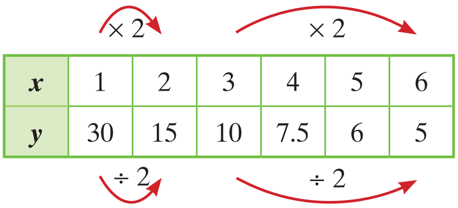
- y is inversely proportional to x.
- y is proportional to the reciprocal of x.
- y varies inversely as x.

Example 7

The table of values represents inverse proportion. Show the doubling and halving of values.

x	1	2	3	4	5	6
y	30	15	10	7.5	6	5

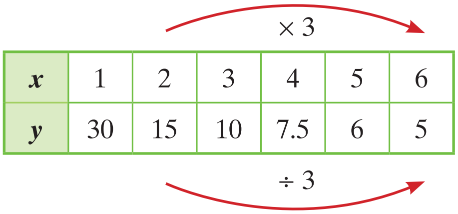
Working



x	1	2	3	4	5	6
y	30	15	10	7.5	6	5

Reasoning

Whatever factor the x-value is multiplied by, the y-value is divided by that factor.



x	1	2	3	4	5	6
y	30	15	10	7.5	6	5

Example 8

v is inversely proportional to m. Write this in the forms $v \propto \frac{1}{x}$ and $y = \frac{k}{x}$.

Working

$$v \propto \frac{1}{m}$$

$$v = \frac{k}{m}$$

Reasoning

Use k as the constant.

Example 9

x and y are inversely proportional. When $x = 3$, $y = 16$.

a Express the relationship using the constant k.

b Find the value of k.

c Write the rule with the calculated value for k.

d Complete the table.

x	1	2	3	4	5	6
y			16			

Working

a $y = \frac{k}{x}$

b When $x = 3$, $y = 16$

$$16 = \frac{k}{3}$$

$$k = 48$$

c $y = \frac{48}{x}$

d

x	1	2	3	4	5	6
y	48	24	16	12	9.6	8

Reasoning

y is inversely proportional to x means that y is proportional to the reciprocal of x.

Substitute the given values for x and y into $y = \frac{k}{x}$ and solve for k.

Substitute 48 for k in $y = \frac{k}{x}$.

Substitute each of the values of x in $y = \frac{48}{x}$ to find the y values.

For example, when $x = 2$.

$$y = \frac{48}{2}$$

$$= 24$$

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Example 10

One bottle of orange juice is shared equally between drinking glasses. The number of glasses of juice that can be poured from the bottle is inversely proportional to the volume of juice in each glass.

a Using n for the number of glasses and V mL for the volume of juice in each glass, write the relationship in the forms $y \propto \frac{1}{x}$ and $y = \frac{k}{x}$ where k is the constant of proportionality.

b If each glass has 150 mL of juice, 8 glasses can be poured. Find the value of k.

c Rewrite the rule using the value of k.

d What does k represent in this situation?

e How much would each glass contain if the juice was distributed between 6 glasses?

Working	Reasoning
a $n \propto \frac{1}{V}$ $n = \frac{k}{V}$	The symbol $\propto$ means 'is directly proportional to'. n is inversely proportional to V.
b $8 = \frac{k}{150}$ $k = 8 \times 150$ $k = 1200$	Substitute 8 for n and 150 for V to find k.
c $n = \frac{1200}{V}$	The rule could be written as $nV = 1200$. For inverse variation there is a constant product of the two variables.
d k is the volume of juice in the bottle before any is poured into the glasses. There must be 1200 mL of juice in the bottle.	8 glasses each with 150 mL means that there was 1200 mL in the bottle.
e $n = \frac{1200}{V}$ When $n = 6$,	Substitute $n = 6$ in the rule and solve for V.
$6 = \frac{1200}{V}$	Multiply both sides by V.
$6V = 1200$ $V = 200$	Divide both sides by 6.
There would be 200 mL in each glass.	

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Example 11

If 16 people work on a particular task it can be finished in 2 days. The task will take twice as long if the number of people is halved.

a Using p for the number of people working and d for the number of days taken, express this relationship in the forms $y \propto \frac{1}{x}$ and $y = \frac{k}{x}$.

b What is the value of k?

c Complete the table of values.

p	1	2	4	8	16	32
d		16				

d Graph the values.

Working

Reasoning

a $d \propto \frac{1}{p}$

d is inversely proportional to p.

$d = \frac{k}{p}$

k is the constant of proportionality.

b $d = \frac{k}{p}$

$2 = \frac{k}{16}$

$k = 32$

Substitute the given values for p and d into $d = \frac{k}{p}$ and solve for k.

c

p	1	2	4	8	16	32
d	32	16	8	4	2	1

Substitute each of the values of p in $d = \frac{48}{p}$ to find the d values.

For example, when $p = 4$,

$$d = \frac{32}{4}$$
$$= 8$$

continued

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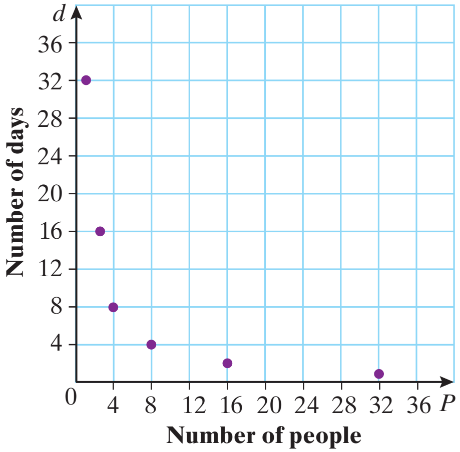
Example 11 continued

d



In this example, it is not appropriate to join the points as the number of people must be a whole number.
The graph approaches each axis but does not touch the axes. If there were zero people it is meaningless to talk about the number of days to complete the task. Similarly, the task could not be completed in zero days. With a large number of people, the task may be completed in a fraction of a day, but never in zero time. So the graph approaches both axes without actually touching them.

exercise 8.2

LINKS TO Example 7

1 For each of these tables of values,

- **i** copy and use arrows to show the doubling and halving.
- **ii** find the value of k.
- **iii** write the rule using the value of k.

a

x	1	2	3	4	5
y	30	15	10	7.5	6

b

x	1	2	4	5	10
y	40	20	10	8	4

c

x	0.5	1	2	4	8
y	32	16	8	4	2

d

x	2	3	4	8	15
y	60	40	30	15	8

LINKS TO Example 8

2 Write each of these in the forms $y \propto \frac{1}{x}$ and $y = \frac{k}{x}$.

a W is inversely proportional to A.

b T is inversely proportional to S.

c I varies inversely as R.

d n is inversely proportional to C.

e n varies inversely as A.

f V is inversely proportional to P.

LINKS TO Example 9

3 These tables of values represent inverse proportion.

- **i** Copy and complete each table.
- **ii** Find the value of k.
- **iii** Write the rule using the value of k.

a

x	1	2	5	8	10	20
y				15		

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b

x	1	2	3	4	5	6
y			12			

c

x	1	2	3	5	6	10
y				30		

d

A	1	2	3	4	5	6
n				36		

4 Tyson had a box of 24 chocolates. If he ate all the chocolates himself, he would get 24 chocolates. If he shared them equally with his sister, he would get 12 chocolates.

a Construct a table of values to show the number, c, of chocolates that each person would receive if the chocolates were divided between n people, where $n = 1, 2, 3, 4, 5$ and 6.

b Explain why this is an example of inverse proportion.

c Write the rule in the forms $y \propto \frac{1}{x}$ and $y = \frac{k}{x}$.

d What is the value of k?

e Rewrite the rule using the value of k.

▶LINKS TO Example 10

5 Maddy has made 720 mL of muffin mixture. She has tins for six different sizes of muffin. The number of muffins Maddy can make from the 720 mL of mixture is inversely proportional to the size of each muffin.

a Complete the table to show the number of muffins that can be made for each different size.

Size (S mL)	30	60	90	120	144	180
Number (n)						

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b Write the rule in the forms $y \propto \frac{1}{x}$ and $y = \frac{k}{x}$.

c What is the value of k?

d Rewrite the rule using the value of k.

LINKS TO Example 11

6 Ahmed has \$60 and plans to buy several CDs. He decides to check out the prices in five different shops before he spends any money.

a At the first shop, CDs are \$15 each. How many could Ahmed buy?

b At the second shop, CDs are \$12 each. How many could Ahmed buy at this shop?

c At the other three shops, Ahmed finds that the price of a CD is \$20, \$10 and \$6 respectively. Make a table of values to show the number of CDs (n) that Ahmed could buy if one CD costs p dollars.

d Draw a graph of the number, n, of CDs that Ahmed could buy against the cost, \$$p$, per CD. Is it appropriate to join the points?

7 Sketch the graph for each of the following rules. Make sure that the axes are labelled correctly and mark the coordinates of two points on each graph.

a $y = \frac{24}{x}$ **b** $t = \frac{20}{s}$ **c** $P = \frac{16}{V}$ **d** $c = \frac{36}{p}$

8 For a given area of 36 cm^2, there are many different rectangles that could be drawn.

a Make a table of ten possible values for the length, L cm, with the corresponding values for the width, W cm.

b Express the proportionality in the forms $y \propto \frac{1}{x}$ and $y = \frac{k}{x}$.

c What is the constant of proportionality?

d Rewrite the rule using the value of k.

e Graph W against L.

f Is it appropriate to join the points? Explain.

9 The exterior angle size of regular polygons is inversely proportional to the number of sides of the regular polygon. The table shows the size of each exterior angle, $e°$, for regular polygons with n sides.

n	3	4	5	6	8	10
e	120	90	72			

a Express the proportionality in the forms $y \propto \frac{1}{x}$ and $y = \frac{k}{x}$.

b What is the constant of proportionality?

c Rewrite the rule using the value of k.

d Copy and complete the table.

e What is the size of each exterior angle of a 12-sided regular polygon?

f The size of each exterior angle of a regular polygon is 15°. How many sides does the polygon have?

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exercise 8.2 challenge

10 The electrical current, I amp, flowing through an appliance is inversely proportional to its resistance, R ohm. $I = \frac{V}{R}$ where the constant of proportionality is the voltage.

The voltage of electricity supplied to homes in Australia is 240 volts. The table shows the resistance of some domestic appliances.

Appliance	**Resistance R ohm**	**Current I amp**
Fan heater	25	
Toaster	50	
Refrigerator	60	
Heated towel rail	120	
TV	300	
Desktop computer	400	
Incandescent light globe	600	
Compact fluorescent light	5000	

a Calculate the current flowing in each of the appliances when operating.

b Use a spreadsheet or graphing calculator to graph current against resistance. Join the points to make a smooth curve.

c Find the resistance if the current is 0.2 amp.

11 In the word game Scrabble™, letters of the alphabet are printed on tiles that are used to build words. The number of tiles for each letter is related to the frequency of that letter in the English language. For example, E is the most commonly occurring letter, so there are more E's than any other letter in a set of Scrabble tiles.

On the other hand, a higher score is obtained for using a less common letter, such as Q. In general, the more commonly used the letter, the lower the score for that letter. Amanda is designing her own version of Scrabble. For each letter of the alphabet, the product of the number of tiles, n, and the score for that letter, S, will be 42. (Different letters may have the same number of tiles and the same score.) Both n and S are whole numbers.

a Copy and complete the table show all the possible values for n and S.

n	1							
S	42							

b Write a rule for the relationship between n and S in the form $y = \frac{k}{x}$.

c Draw a graph of S against n. Is it appropriate to join the points?

d Would it be possible for any letter to have a zero score? Explain.

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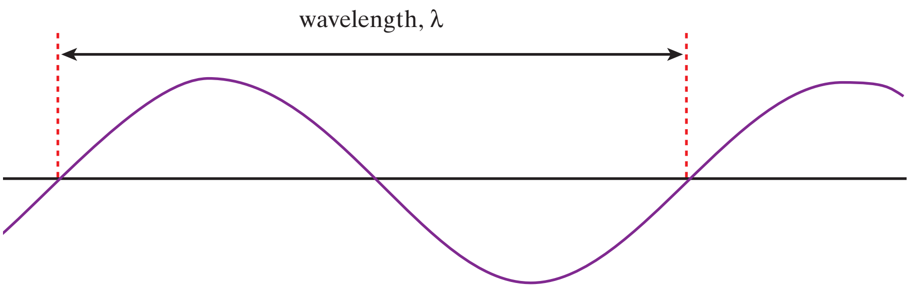
Analysis task

Radio waves

Radio waves represent just one section of the whole electromagnetic spectrum. Electromagnetic radiation also includes light, ultra-violet radiation and X-rays. All these forms of radiation can be represented as wave motion. The wavelength is the length of one complete wave and is represented by the Greek letter, λ, pronounced 'lambda'.



The frequency, *f*, of a wave is how many waves are passing a point in a second. We can see that if the wavelength is doubled, only half as many waves will pass the point in a second. So wavelength is inversely proportional to frequency.

Frequency is measured in a unit called the hertz (Hz) and wavelength is measured in metres.

a Express the inverse relationship between frequency and wavelength in the forms $y \propto \frac{1}{x}$ and $y = \frac{k}{x}$ using the symbols *f* and λ.

b *k* represents the speed of electromagnetic radiation, which is approximately 3×10^8 m/sec. Rewrite the rule using the value of *k*.

Radio stations transmit waves in the radio region of the spectrum and each radio station is assigned a particular frequency.

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The table below shows the frequencies in megahertz of several Melbourne radio stations. (1 MHz = 10^6 Hz)

c Convert each frequency into hertz.

d Calculate the wavelength in metres correct to three decimal places.

Radio station	Frequency (MHz)	Frequency (Hz)	Wavelength (m)
MW 9 3ppS Radio	93.1		
ABC Classic FM	105.9		
Triple J	107.5		
Classic Rock 91.5	91.5		
Nova 100	100.3		
Mix 101.1	101.1		
Fox	101.9		
Triple M	105.1		

e Using the website www.ausradiostations.com, find the frequencies of radio stations in your capital city or country region.

f Calculate the wavelength for each as in parts **c** and **d**.

g Ham radio operators often broadcast on the 6.0 metre band. What is the frequency in MHz of this wavelength?

h In Australia beginning amateur radio operators (with a Foundation Licence) are permitted to use the following wavelengths 40 m, 15 m, 10 m, 2 m and 70 cm. What frequencies correspond to these wavelengths? Give your answers in MHz to one decimal place, where required.

Wavelength	Frequency (MHz)
40 m	
15 m	
10 m	
2 m	
70 cm	

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i Amateur radio operators with an Advanced Licence can use many other wavelengths including the following. Calculate the frequency for each in MHz, current to one decimal place where required.

Wavelength	Frequency (MHz)
6 cm	
3 cm	
1.25 cm	
7.5 mm	
3.7 mm	
2.5 mm	
2 mm	
1.25 mm	

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Review Proportion

Summary

Direct proportion

- $y \propto x$, $y = kx$
- The graph of $y = kx$ is a straight line passing through the origin, with gradient k.

Inverse proportion

- $y \propto \frac{1}{x}$, $y = k \times \frac{1}{x} = \frac{k}{x}$
- The product xy is constant and equal to k.
- The graph of $y = \frac{k}{x}$ is a hyperbola.

Visual map

constant of proportionality
directly proportional
hyperbola
inversely proportional
proportion
variation

Revision

Multiple-choice questions

1 If T is directly proportional to R and $R = 11.6$ when $T = 174$, the relationship between x and y is given by

A $R = 15T$ **B** $T = 15R$ **C** $T = \frac{15}{R}$ **D** $R = \frac{15}{T}$ **E** $y = -\frac{0.4}{x}$

2 If x and y are directly proportional and $x = 15$ when $y = 2$, the value of x when $y = 24$ is

A 1.2 **B** 1.25 **C** 3.2 **D** 112.5 **E** 180

3 If x and y vary inversely and $x = 1.2$ when $y = 3$, the relationship between x and y is given by

A $y = 3.6x$ **B** $y = 0.4x$ **C** $y = \frac{3.6}{x}$ **D** $y = \frac{0.4}{x}$ **E** $y = -\frac{0.4}{x}$

4 If a and b are inversely proportional and $b = 63$ when $a = 2.4$, the value of b when $a = 5.6$ is

A 26.25 **B** 27 **C** 147 **D** 25.875 **E** 151.2

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5 The frequency, f vibrations per second, of a musical note varies inversely as its wavelength, w m. One particular musical note has a frequency of 321 vibrations per second and a wavelength of 3.2 m. The wavelength of another musical note that has a frequency of 374 vibrations per second is closest to

A 2.0 m. **B** 2.3 m. **C** 2.7 m. **D** 3.7 m. **E** 4.0 m

Short-answer questions

6 Rewrite each of these in the form

- **i** $y \propto x$
- **ii** $y = kx$.

a C is directly proportional to n.

b F is directly proportional to m.

c A varies directly as w.

d P is proportional to T.

7 A is directly proportional to n.

n	0	1	2	3	4	5
A			54			

a Complete the table.

b Calculate the value of k.

c Rewrite the rule with the calculated value of k.

8 A painter charges $14 per square metre to paint the walls and ceiling inside a house.

a Using A m^2 for the area of to be painted and $\$C$ for the cost, write this as a rule.

b Complete the table of values to show the cost for different areas.

A	0	100	200	250	400	500	600
C	0						

c Use a spreadsheet to graph the cost against area.

d Explain why this is an example of direct variation.

e How many square metres (to the nearest square metre) could be painted for $5000?

f What would it cost to have 640 m^2 painted?

9 The cost of putting guttering around a house is proportional to the length of guttering.

a Using L m for the length of the guttering and $\$C$ for the cost, write this as a rule in the form $y \propto x$ and $y = kx$.

b A roofing contractor charges $4104 for 108 m of guttering around a roof. Calculate the value of k.

c Rewrite the rule with the calculated value of k.

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d Copy and complete the table to show the cost of different lengths of guttering.

Length of guttering, *L* (m)	0	50	108	120	150	200
Cost, $ (*C*)	0					

10 The number, n, of Frequent Flyer points earned is directly proportional to the distance travelled, d km.

***d* (km)**	1000	2000	3600	8000	12 000
n				12 872	

a Calculate the value of k.

b Complete the table.

c Write the rule with the calculated value of k.

d How many points would be earned for 15 400 km of travel?

e How far would you need to fly to earn 30 000 points?

11 A bank offers rewards for paying by credit card. The points earned can be redeemed for cash or for travel or shopping vouchers. 18 334 points can be redeemed for a $50 voucher.

a How many points would be needed for

  i a $100 voucher?

  ii a $20 voucher?

b 8367 points can be redeemed for two child cinema passes. How much are these two cinema passes worth?

12 Write each of these in the forms $y \propto \frac{1}{x}$ and $y = \frac{k}{x}$.

a L is inversely proportional to W.

b R is inversely proportional to I.

c T varies inversely as n.

d n is inversely proportional to R

Extended-response questions

13 In this table, y is inversely proportional to x.

x	1	2	4	5	8	10
y		2.5		1		

a Copy and complete the table, using arrows to show the doubling and halving.

b Find the value of k.

c Write the rule using the value of k.

d Sketch the graph of y against x, labeling two points on the graph.

e What is the value of y when $x = 25$

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14 An events organizer has \$1200 to spend on food for a conference dinner. She needs to choose between several different menus that cost \$24, \$25, \$30, \$40 and \$48 per head. The number, n, of people who can be invited is inversely proportional to the cost per head, \$$C$.

a Copy and complete the table.

\$C	24	25	30	40	48
n					

b Write the rule in the forms $y \propto \frac{1}{x}$ and $y = \frac{k}{x}$.

c What is the value of k?

d Rewrite the rule using the value of k.

e Sketch a graph of n against C, labelling two points on the graph.

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