Congruency and quadrilateral properties

7

Pre-test

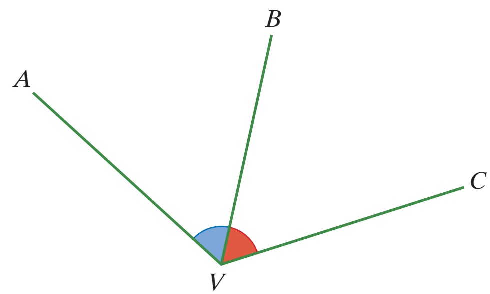
Warm-up

Lines, angles, triangles and quadrilaterals play an important part in our everyday lives in art, architecture, tool design and sport. Triangles are used in the structure of bridges and roof trusses because they are rigid. Quadrilaterals have many uses because they are *not* rigid shapes. Parallelograms and rhombuses formed by hinged bars, such as in this scissor lift, are found in many tools and everyday objects.

7.1 Reviewing angles and parallel lines

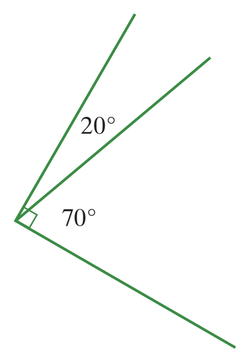
Adjacent angles

The point where two straight lines or segments intersect is called a **vertex**. **Adjacent angles** are angles that are next to each other. They share a common vertex and a common arm. In the diagram, $\angle AVB$ and $\angle BVC$ are adjacent angles. They share the common vertex V and the common arm VB.



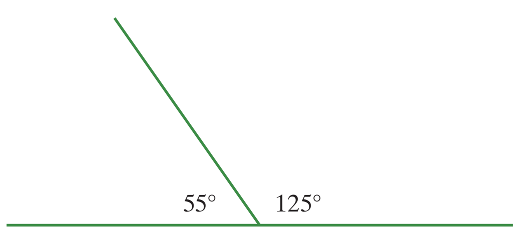
Complementary angles

Angles that add to 90° are called complementary angles . The two angles 70° and 20° are complementary angles. We say that 70° is the **complement** of 20°. Two adjacent angles that are complementary form a right angle.



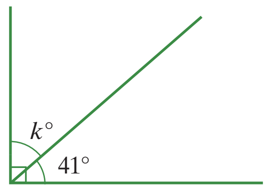
Supplementary angles

Supplementary angles are angles that add to 180°. The two angles 55° and 125° are supplementary angles. We say that 55° is the **supplement** of 125°. Two adjacent angles that are supplementary form a straight line.

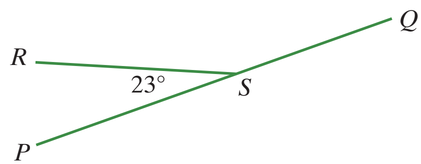


Example 1

a Find the value of k.



b PSQ is a straight line. Find the size of $\angle RSQ$.



Working	Reasoning
a $k = 90 - 41$ $\quad = 49$	P is the common vertex and PB is the common arm. $k + 41 = 90$
b $\angle RSQ = 180 - 23$ $\quad = 157$	$\angle RSQ$ and $\angle RSP$ are supplementary angles because PSQ is a straight line.

Example 2

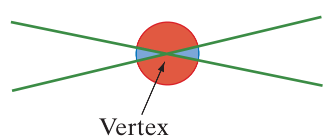
a What is the complement of 43°?

b What is the supplement of 122°?

Working	Reasoning
a Complement of 43° = 47°	90° – 43° = 47°
b Supplement of 122° = 58°	180° – 122° = 58°

Vertically opposite angles

Angles that are on the opposite side of a vertex are called **vertically opposite angles**.

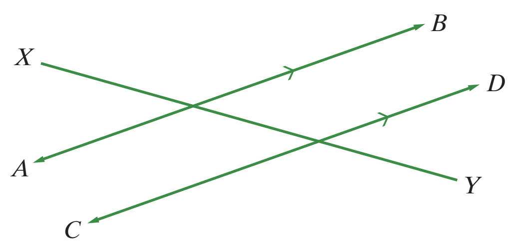


The two angles marked in red are vertically opposite angles.

The two angles marked in blue are vertically opposite angles.

Parallel lines and transversals

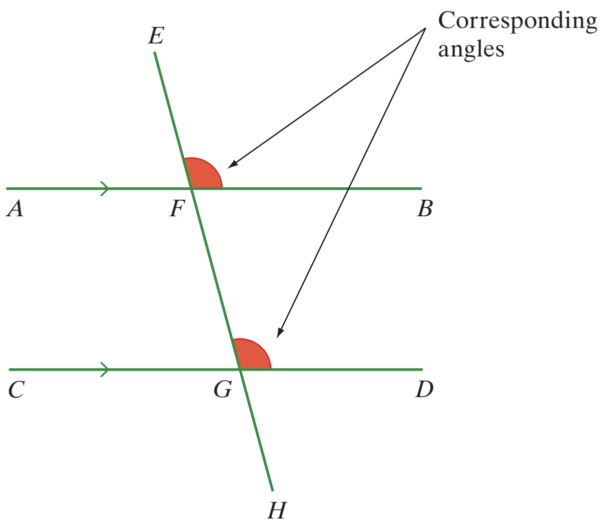
A line or line segment that cuts across a set of parallel lines is called a **transversal**. In this figure, the line segment XY is a transversal, cutting across the parallel lines AB and CD.



Corresponding angles

When parallel lines are crossed by a transversal, **corresponding angles** are angles that are in corresponding positions on the same side of the transversal.

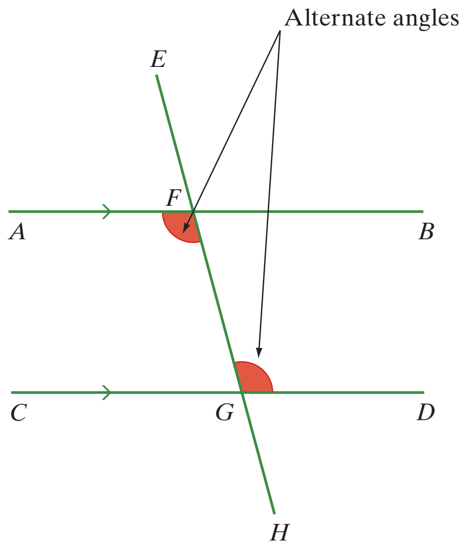
Corresponding angles are equal.



Alternate angles

Alternate angles are angles that are between a pair of parallel lines, but on opposite sides of the transversal.

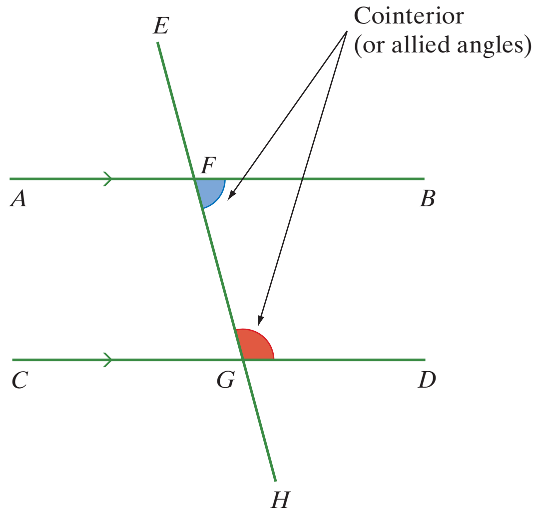
Alternate angles are equal.



Cointerior angles (or allied angles)

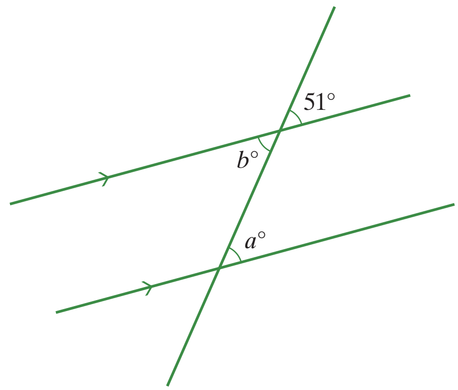
Cointerior angles are sometimes called allied angles. They are angles that are between a pair of parallel lines, but on the same side of the transversal.

Cointerior angles are supplementary.



Example 3

Find the values of the pronumerals.



Working

$a = 51$ (Corresponding angles)

Reasoning

The angles marked $a°$ and $51°$ are corresponding angles. As the lines are parallel, the corresponding angles are equal.

Working

$b = 51$ (Alternate angles)

Reasoning

The angles marked $a°$ and $b°$ are alternate angles. As the lines are parallel, the alternate angles are equal.

Working

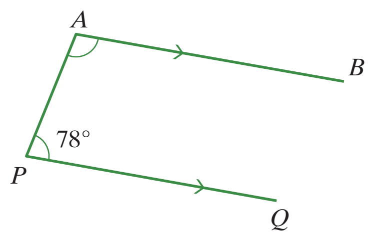
Or
$b = 51$ (Vertically opposite angles)

Reasoning

The angles marked $b°$ and $51°$ are vertically opposite angles, so they are equal.

Example 4

Find the size of $\angle PAB$.



Working

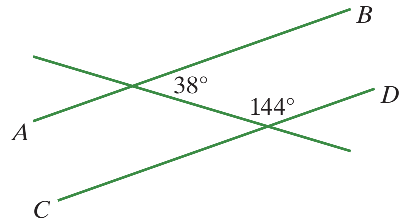
$$\begin{aligned}\angle PAB &= 180° - 78° \\ &= 102° \text{ (Cointerior angles)}\end{aligned}$$

Reasoning

$\angle PAB$ and $\angle APQ$ are cointerior angles. As AB and PQ are parallel, the marked angles add to $180°$.

Example 5

Are line segments AB and CD parallel?



Working

38° and 144° are allied angles.
The allied angles do not add to 180°.

Line segments AB and CD are not parallel.

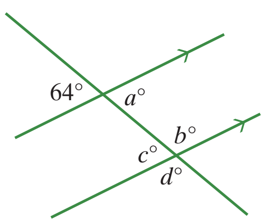
Reasoning

38° and 144° are allied angles.

Allied angles formed by a transversal cutting across two line segments add to 180° if the line segments are parallel.

Example 6

Find the value of a, b, c and d.

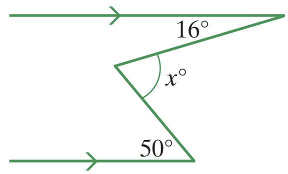


Working	Reasoning
$a = 64$	The angles marked $a°$ and 64° are vertically opposite angles so they are equal.
$b = 116$	The angles marked $a°$ and $b°$ are cointerior angles so they are supplementary. $180° - 64° = 116°$
$c = 64$	The angles marked 64° and $c°$ are corresponding angles so they are equal.
$d = 116$	The angles marked $b°$ and $d°$ are vertically opposite angles so they are equal.

Sometimes we need to draw extra lines to help us find an unknown angle. In the diagram in the next example, drawing a line through the vertex of the unknown angle parallel to the pair of parallel lines allows us to make use of alternate angles. The required unknown angle is then the sum of the two alternate angles.

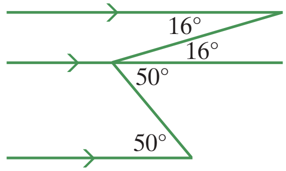
Example 7

Find the value of the pronumeral.



Working

$x = 16 + 50$
$x = 66$

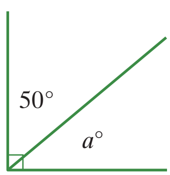


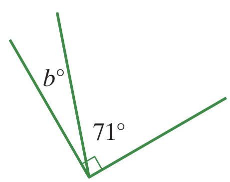
Reasoning

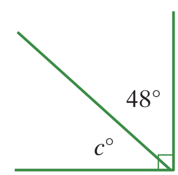
Draw a line through the vertex of the unknown angle parallel to the pair of parallel lines. Label the alternate angles. Add the sizes of the two alternate angles to find the unknown angle.

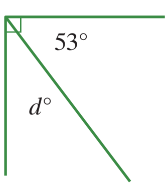
exercise 7.1

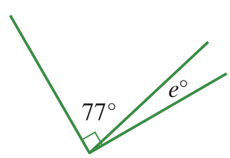
1 Find the unknown angles in each of the following diagrams. Each letter stands for a number of degrees, so your answer is just a number. Do not write a degrees sign after your answer.

a


b


c


d


e


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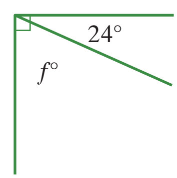
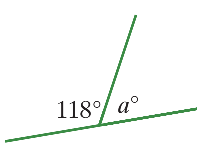


2 Find the unknown angles in each of the following diagrams. Each letter stands for a number of degrees, so your answer is just a number. Do not write a degrees sign after your answer.

a


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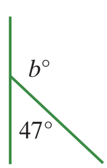

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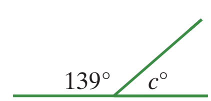

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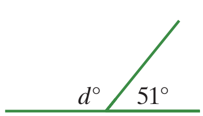

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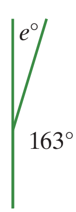

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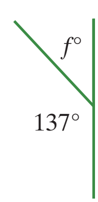
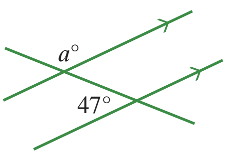


3 Copy and complete the following table where possible. There are two angles for which you will not be able to write the complement.

Angle	Complement	Supplement
13°		
88°		
129°		
54°		
142°		

4 Find the unknown angles in each of the following diagrams. Each letter stands for a number of degrees, so your answer is just a number. Do not write a degrees sign after your answer.

a


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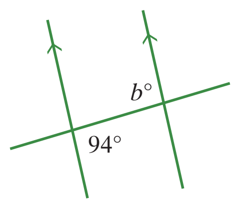

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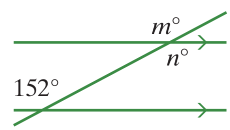

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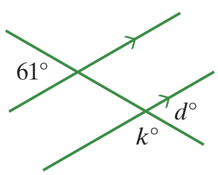

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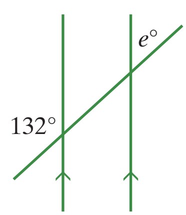

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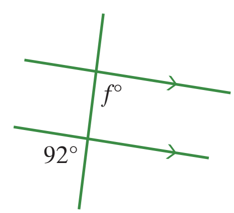

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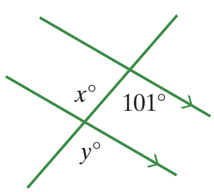

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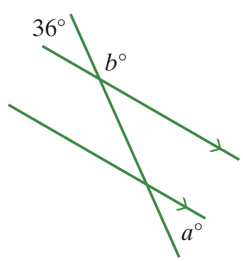

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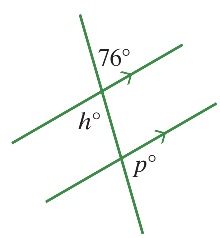

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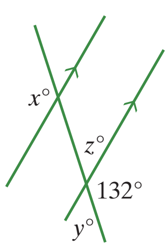

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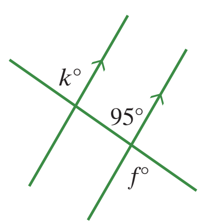

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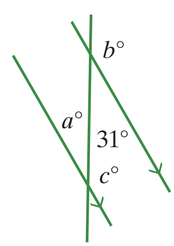
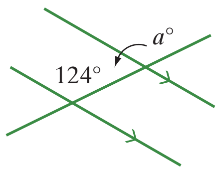


5 Find the unknown angles in each of the following diagrams. Each letter stands for a number of degrees, so your answer is just a number. Do not write a degrees sign after your answer.

a


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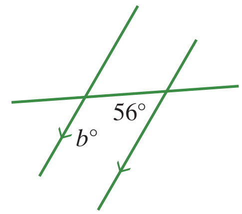

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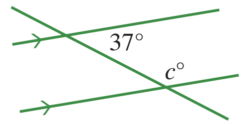

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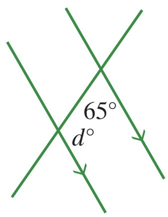

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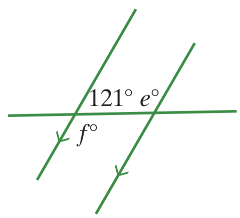

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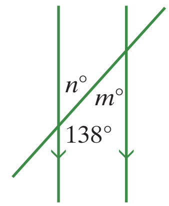

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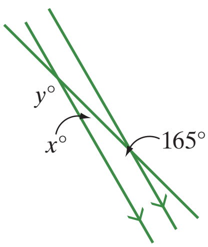

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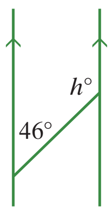

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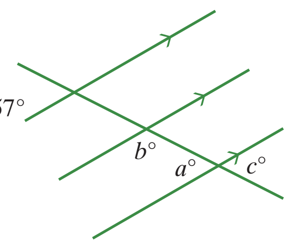

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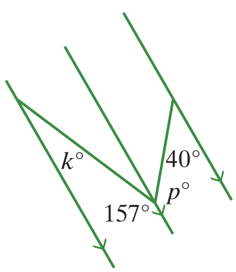

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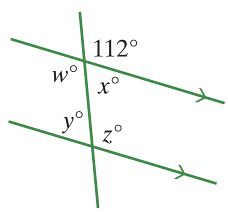

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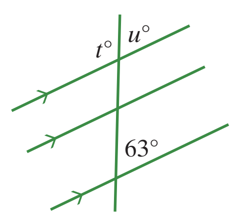
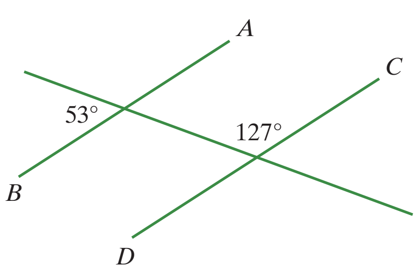


6 State if the line segments AB and CD are parallel in each of the following diagrams. Justify each answer.

a


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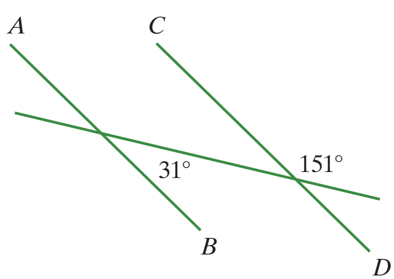

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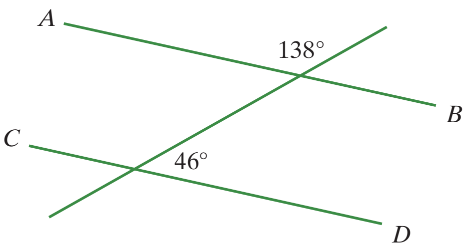

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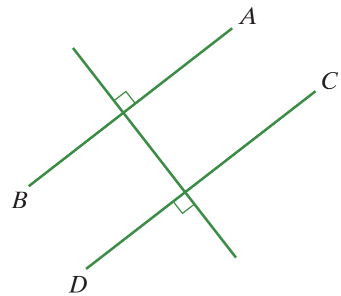
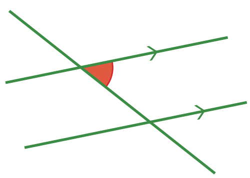
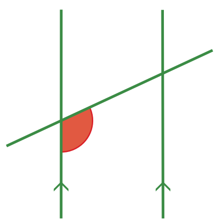
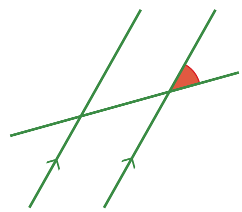


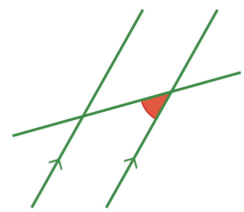
7 Each of the following diagrams has one marked angle. Copy each diagram and mark with a cross the angle indicated, for example in part a, use a cross to mark the alternate angle to the marked angle.

a alternate angle **b** cointerior angle **c** vertically opposite angle **d** alternate angle

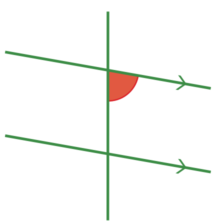


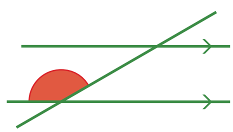


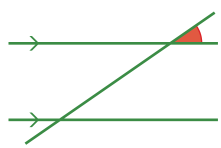


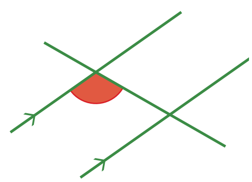


e corresponding angle **f** alternate angle **g** corresponding angle **h** cointerior angle









8 Name each of the following angles:

a the corresponding angle to $\angle ABN$

b the allied angle to $\angle MBC$

c the alternate angle to $\angle NBC$

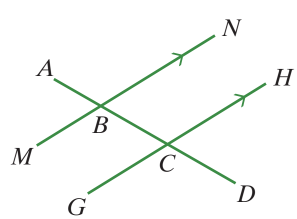
d an angle that is adjacent to $\angle DCH$ and is the supplement of $\angle DCH$

e the corresponding angle to $\angle GCB$

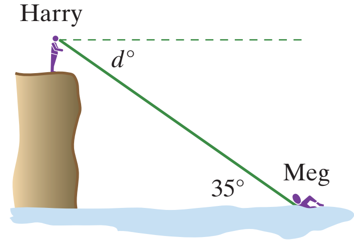
f the allied angle to $\angle HCB$

g the angle vertically opposite $\angle GCD$

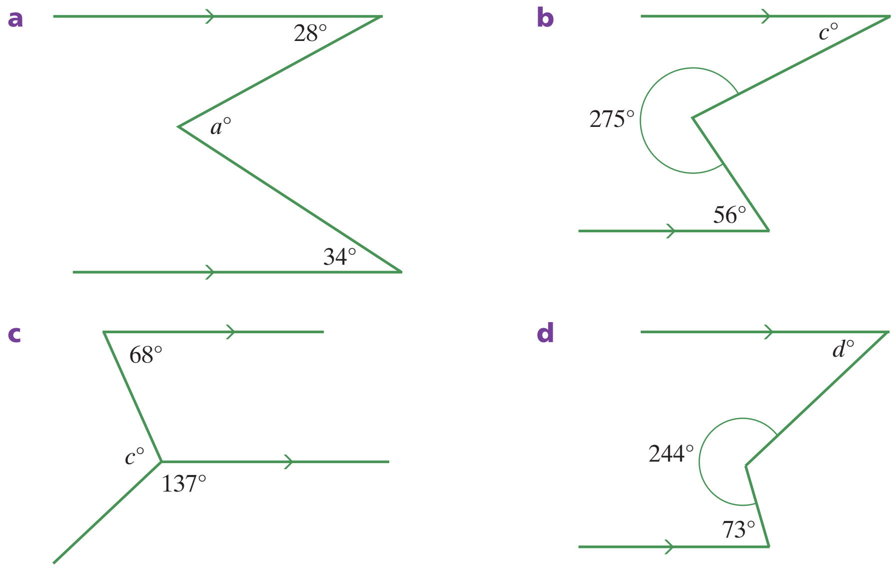
h all the angles that are the supplement of $\angle NBC$



9 Harry is standing on top of a cliff, looking downwards downwards at Meg who is swimming in the sea below.Meg looks up at Harry through an angle of 35°. Through what angle, $d°$, does Harry look down from the horizontal as he watches Meg? Justify your answer.

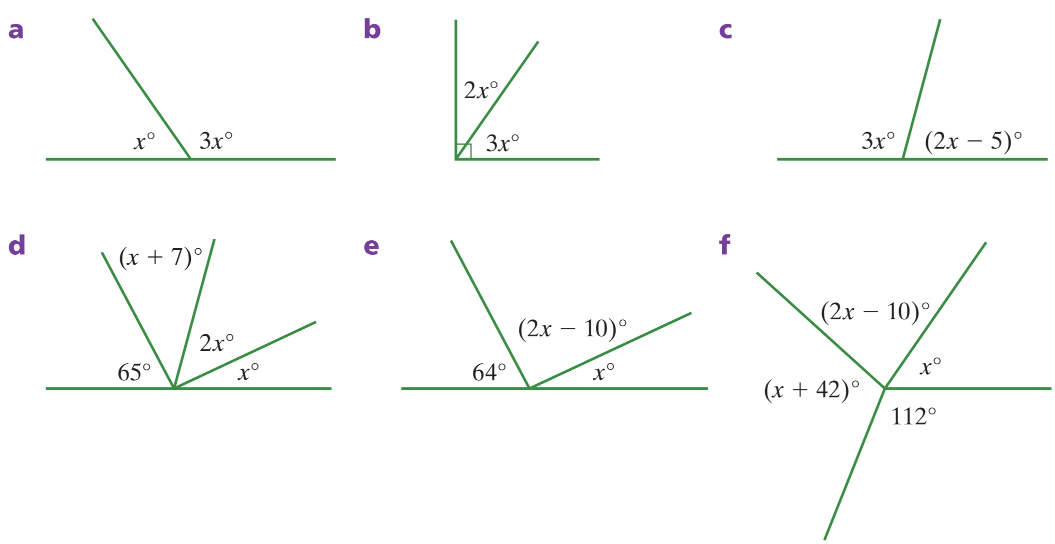


10 Find the value of each pronumeral.



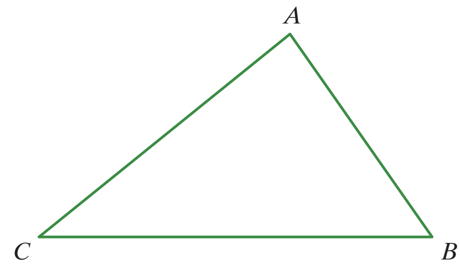
exercise 7.1 challenge

11 Write an equation, then find the value of x in each of the following diagrams.

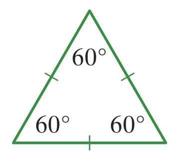
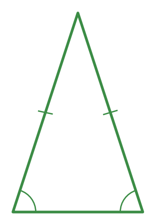
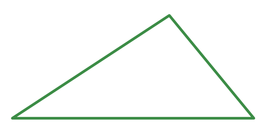
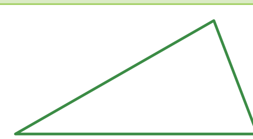
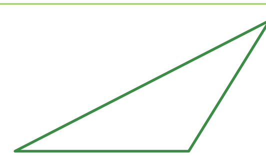
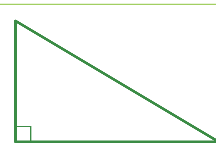


7.2 Reviewing triangles

A triangle is a plane shape with three straight sides. Each 'corner' of the triangle is called a vertex. Triangles are named according to the letters at the vertices. We can start at any vertex. e.g., $\triangle ABC$ or $\triangle CAB$.



Types of triangles

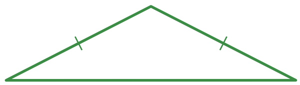
According to sides	
Equilateral triangle Three sides are equal in length. (Each angle is 60°.)	
Isosceles triangle Two sides are equal. (The angles opposite the equal sides are equal.)	
Scalene triangle The sides are all of different lengths.	
According to angles	
Acute-angled triangle All three angles are acute.	
Obtuse-angled triangle One angle is obtuse.	
Right-angled triangle One angle is a right angle.	

Example 7

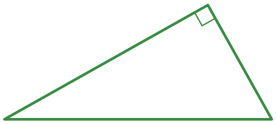
Identify each of these triangles according to

i the sides **ii** the angles

a



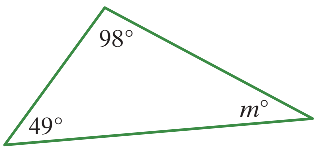
b



Working	Reasoning
a **i** isosceles triangle	The triangle has two equal sides.
ii obtuse-angled triangle	The triangle has one obtuse angle.
b **i** scalene triangle	The three sides have different lengths.
ii right-angled triangle	The triangle has a right-angle.

Example 8

Find the value of m.



Working

$$\begin{aligned} 98 + 49 + m &= 180 \\ m &= 180 - 98 - 49 \\ m &= 33 \end{aligned}$$

Reasoning

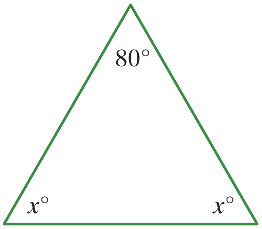
The measures of the three interior angles of a triangle add to 180°.

Example 9

a Write an equation to show the three angles of the triangle adding to 180°.

b Solve the equation to find the value of x.

c State the sizes of each of the angles in the triangle.

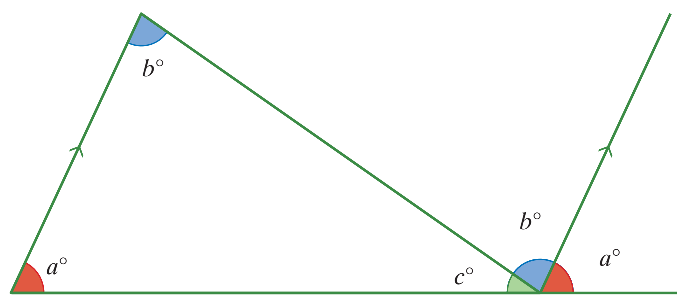


Working	Reasoning
a $x + x + 80 = 180$	The three angles add to 180°.
$2x + 80 = 180$	$x + x = 2x$
b $2x + 80 = 180$	Subtract 80 from both sides.
$2x + 80 - 80 = 180 - 80$	Divide both sides by 2.
$2x = 100$	
$x = 50$	
c The three angle sizes are 50°, 50°, 80°.	One angle is 80°. The other two angles are x° and x°, that is 50° and 50°.

Interior angles of triangles

The three interior angles of a triangle add to 180°.

We can prove this by extending one side of the triangle then drawing a line at this vertex of the triangle so that it is parallel to the opposite side.



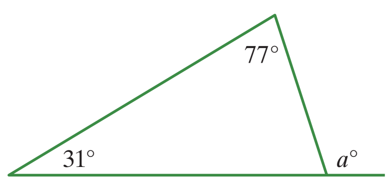
The two red angles (marked $a°$) are equal because they are corresponding angles.

The two blue angles (marked $b°$) are equal because they are alternate angles between parallel lines.

Together the red, blue and green angles make a straight line, that is $a° + b° + c° = 180°$. So the three angles of the triangle must also make 180°.

Example 10

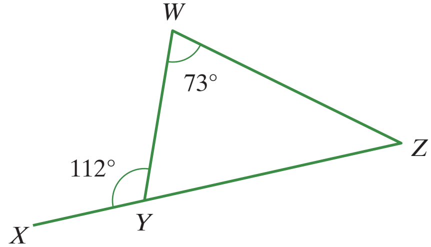
Find the value of a.



Working	Reasoning
Size of missing interior angle $= 180° - 77° - 31°$ $= 72°$	The three angles of the triangle add to 180°.
$a + 72 = 180$ $a = 108$	The exterior angle and the adjacent interior angle are supplementary (add to 180°).
or	
$a = 31 + 77$ $= 108$	An exterior angle of a triangle is equal to the sum of the two interior opposite angles.

Example 11

Find the size of $\angle WZY$.



Working

$$\angle WZY + 73° = 112°$$
$$\angle WZY = 112° - 73°$$
$$\angle WZY = 39°$$

Reasoning

An exterior angle of a triangle is equal to the sum of the two interior opposite angles.

Finding angles

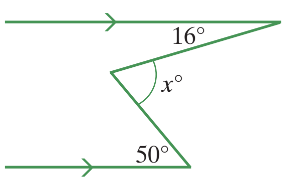
When working out the size of unknown angles in a diagram, we sometimes need to find the size of other angles in the diagram before we get to the angle we want. Questions such as the following enable us to identify clues that might help us in this angle chase.

- Are there any parallel lines cut by transversals?
- Are there any angles that together make up a straight line?
- Are there any exterior angles?
- Is there an isosceles triangle?
- Do we already know two of the three angles of a triangle?

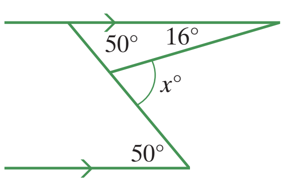
It is often possible to find more than one way of solving an angle problem. In example 12, we return to the angle problem in example 7 and use a different method to find the unknown angle.

Example 12

Use the fact that the exterior angle of a triangle is equal to the sum of the interior opposite angles to find the value of x.



Working



$$x = 16 + 50$$
$$= 66$$

Reasoning

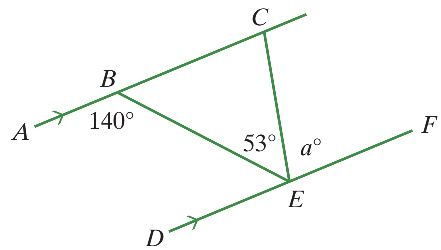
Continue the line segment as shown to make a triangle.

Using alternate angles between the parallel lines we know that the angle in the triangle is 50°.

The angle labelled $x°$ is an exterior angle to the triangle. The exterior angle is equal to the sum of the two interior opposite angles.

Example 13

Find the value of a.



Working

$\angle BED = 40°$

Reasoning

$\angle ABE$ and $\angle BED$ are allied angles so they are supplementary.

Remember that supplementary angles add to 180°.

Working

$\angle BED + \angle BEC + \angle CEF = 180°$

$$\begin{aligned} 40 + 53 + a &= 180 \\ 93 + a &= 180 \\ a &= 180 - 93 \\ a &= 87 \end{aligned}$$

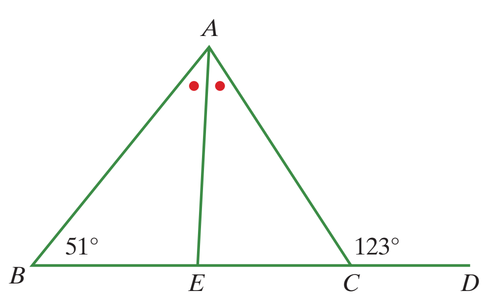
Reasoning

$\angle BED$, $\angle BEC$ and $\angle CEF$ form a straight line.

We don't write $a°$ because a represents a number.

Example 14

In the following diagram, AE bisects $\angle BAC$. Find $\angle AEC$.



Bisect means to cut exactly in half.

continued

Example 14 continued

Working

$\angle BAC + 51° = 123°$

Another way would be to find $\angle ACE$ first ($\angle ACE$ and $\angle ACD$ form a straight line). Then $\angle BAC$ can be found.

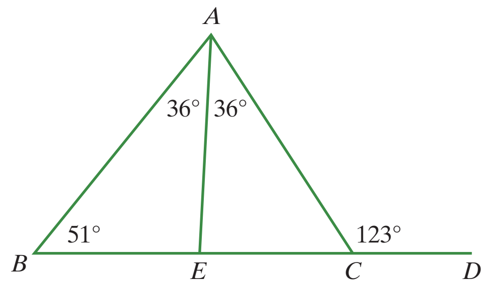
Reasoning

The exterior angle of a triangle is equal to the sum of the two interior opposite angles.

$\angle ACD$ is an exterior angle to $\triangle ABC$, so $\angle BAC + \angle ABC = \angle ACD$.

$\angle BAC = 72°$
$\angle EAC = 36°$

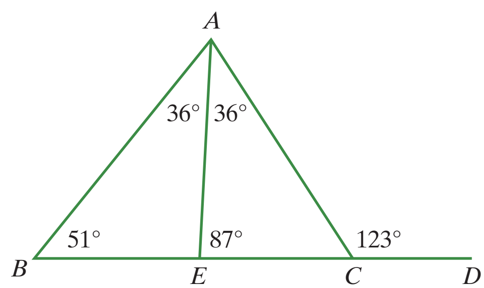
$123° - 51° = 72°$
AE bisects $\angle BAC$ so $\angle EAC = 72° \div 2$.



$\angle AEC + 36° = 123°$
$\angle AEC = 87°$

The exterior angle of a triangle is equal to the sum of the two interior opposite angles.

$\angle ACD$ is an exterior angle to $\triangle AEC$, so $\angle AEC + \angle EAC = \angle ACD$.
$123° - 36° = 87°$



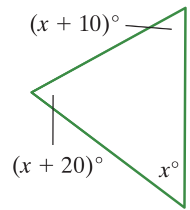
Example 15

For the triangle shown

i write an equation

ii solve the equation to find the value of x

ii find the size of the three angles in the triangle

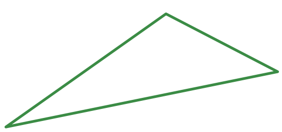


Working	Reasoning
i $x + x + 10 + x + 20 = 180$	The three angles of the triangle add to 180°.
ii $3x + 30 = 180$ $3x = 150$ $x = 50$	Simplify the left side then solve the equation to find the value of x.
iii $x + 10 = 50 + 10$ $= 60$ $x + 20 = 50 + 20$ $= 70$ The three angles are 50°, 60° and 70°.	Substitute $x = 50$ into the expressions for the angles.

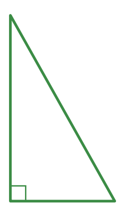
exercise 7.2

1 Classify each of the following triangles according to its sides (i.e. isosceles, equilateral or scalene) and according to its angles (i.e. acute-angled, obtuse-angled or right-angled).

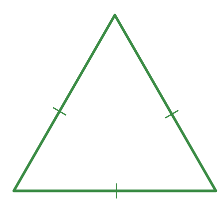
a



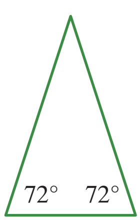
b



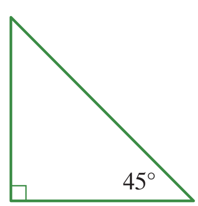
c



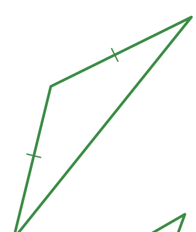
d



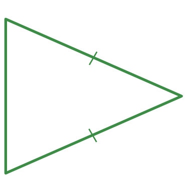
e



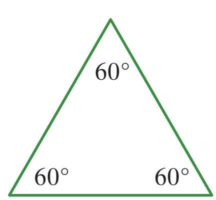
f



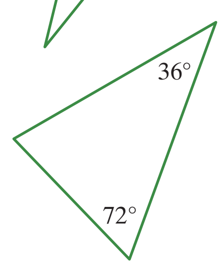
g



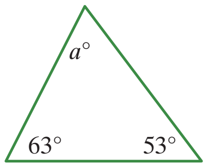
h



i



2 Find the value of the pronumerals in each of the following.

a


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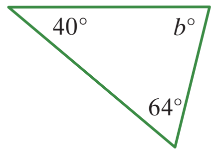
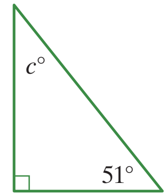


c


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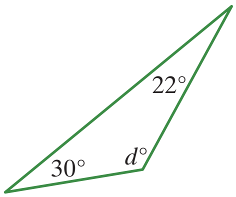

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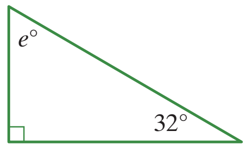

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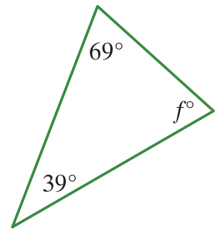
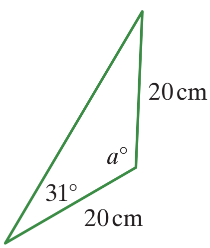
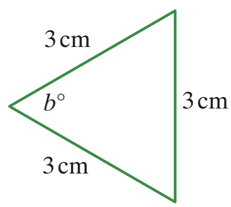


3 If two angles of a triangle are complementary, what do we know about the third angle?

4 Can a triangle have two obtuse angles? Explain.

5 Find the value of the pronumerals in each of the following.

a


b


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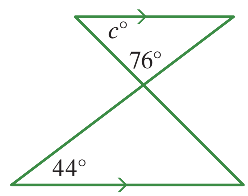

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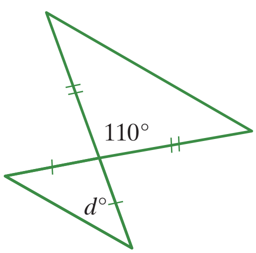

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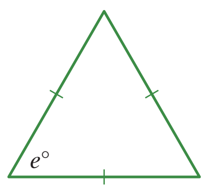

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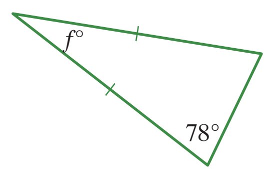

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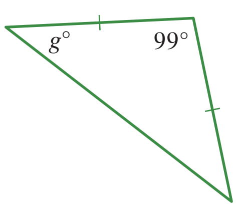

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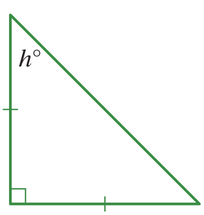

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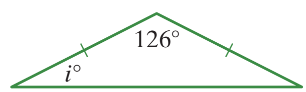

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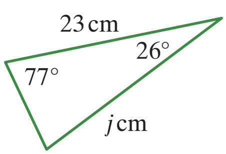

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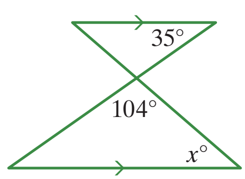
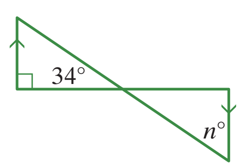
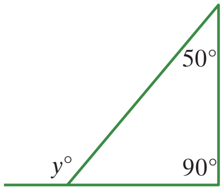
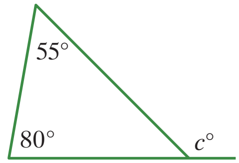
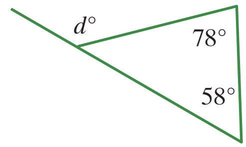


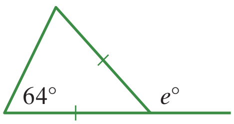
l


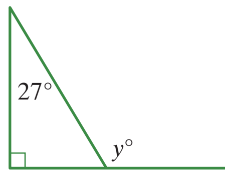
6 Find the value of the pronumeral in each of the following.

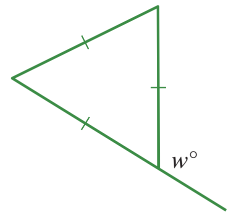
a 

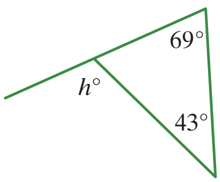
b 

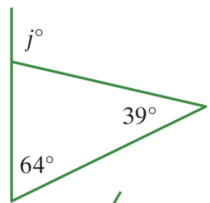
c 

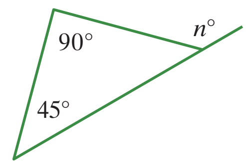
d 

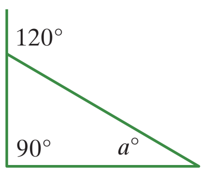
e 

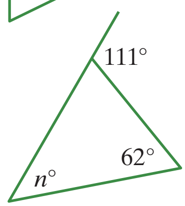
f 

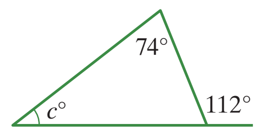
g 

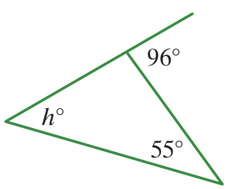
h 

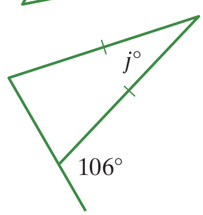
i 

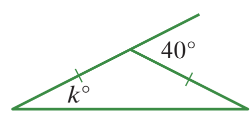
j 

k 

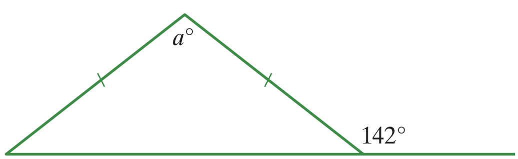
l 

m 

n 

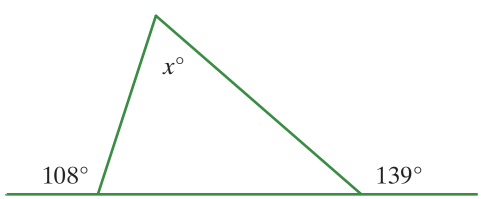
o 

7 The value of a is



A 38° **B** 66° **C** 76° **D** 104° **E** 142°

8 The value of x is



A 27° **B** 41° **C** 67° **D** 72° **E** 113°

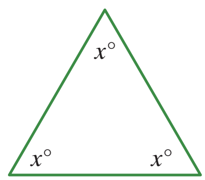
9 For each of the following triangles

i Write an equation showing that the three angles add to 180°. Do not use degrees signs in your equation.

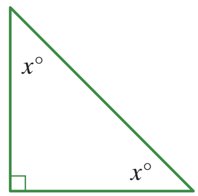
ii Solve the equation to find the value of the pronumeral.

iii State the sizes of the three angles of the triangle. (Check that the angles add to 180°.)

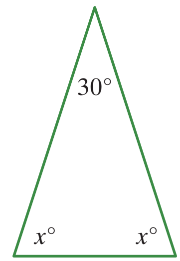
a



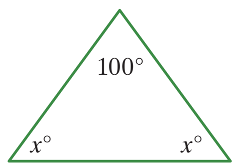
b



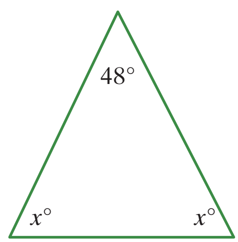
c



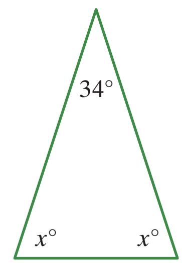
d



e



f



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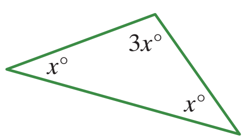
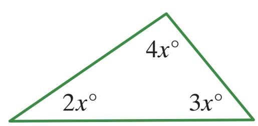
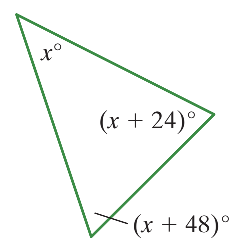


h



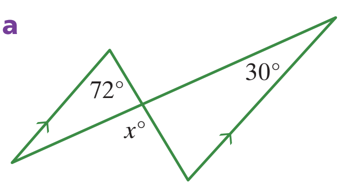
i



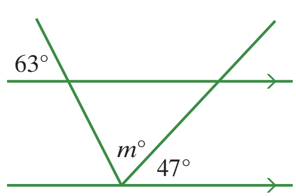
10 Find the unknown angles in question 10 from exercise 7.1 by using the angles in a triangle.

11 Copy each of the following diagrams. Label on your drawings the sizes of all the angles that you had to find in order to calculate the value of the pronumeral, then write the value of the pronumeral.

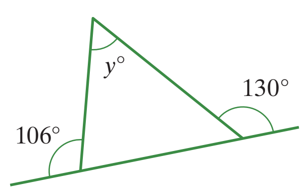
a



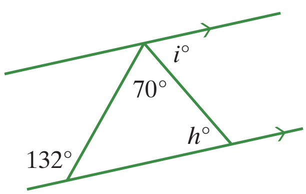
b



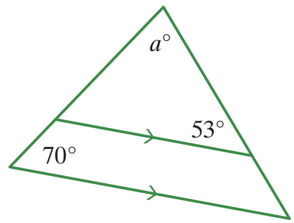
c



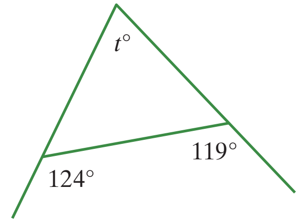
d



e



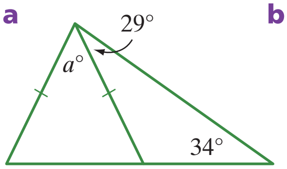
f



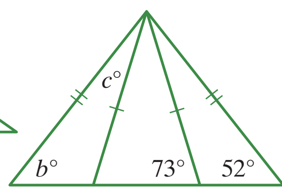
exercise 7.2 challenge

12 Copy each of the following diagrams and find the value of each pronumeral. Hint: you may need to find the size of other angles in the figure before you find the value of the pronumeral. If so, label the size of each angle on your diagram.

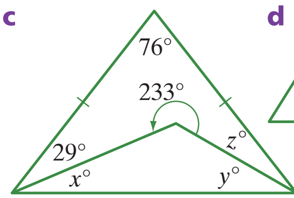
a



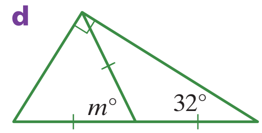
b



c



d



13 Before the development of satellite communication and global positioning systems, sailors used angle measurements to locate their position. One technique used to find the distance from a landmark is called 'doubling the angle'. This involves measuring the angle between the ship's path and an object such as a lighthouse, and then sailing towards the object until the angle is double the previous angle measurement. These angle measurements are referred to as 'the angle on the bow'.

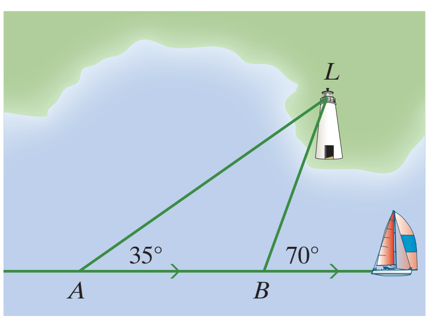
In the diagram, the angle on the bow was 35° at A and 70° at B.

a Explain why $\triangle ABL$ must be an isosceles triangle.

b The boat is travelling at a speed of 8 knots and takes 30 minutes to travel from A to B. Calculate the distance in nautical miles from A to B.

c How far is the boat from the lighthouse when it is at B?

d If 1 nautical mile is equivalent to 1.852 km, what is the distance from the lighthouse in kilometres?

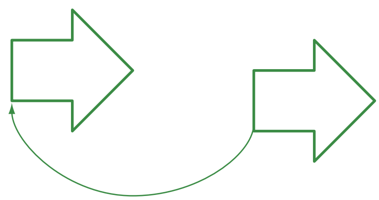


1 knot is a speed of 1 nautical mile per hour.

7.3 Congruency and isometric transformations

If two shapes are **congruent**, one can be placed exactly over the other.

These two shapes are congruent because we can place one exactly over the other.



> Congruent comes from a Latin word meaning *to come together* or *agree*.

If two or more shapes are congruent, both of the following conditions must be true:

- matching angles must be equal
- matching side lengths must be equal

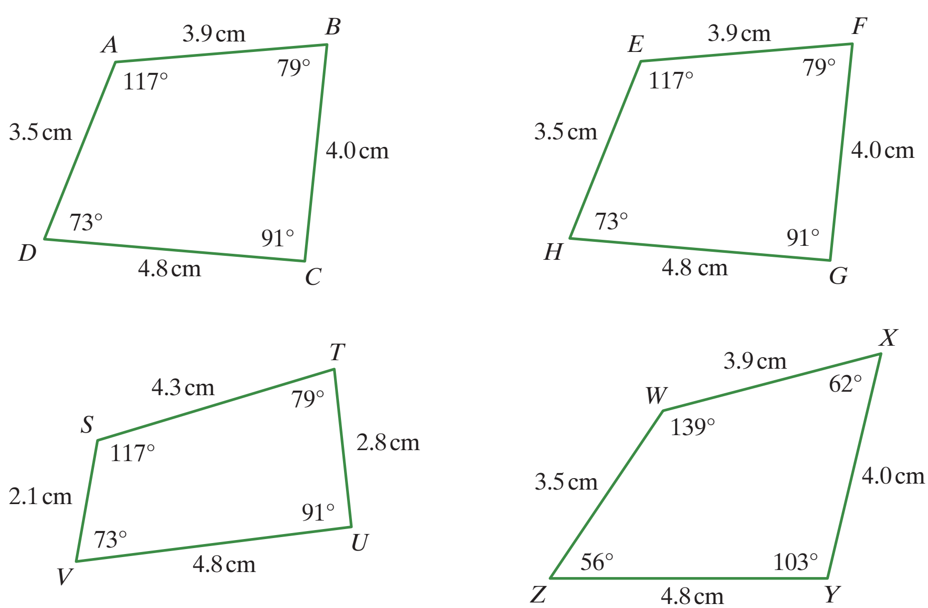
We use the symbol ≡ to indicate that two shapes are congruent.

Example 16

a Are quadrilaterals *ABCD* and *EFGH* congruent? Explain.

b Are quadrilaterals *ABCD* and *STUV* congruent? Explain.

c Are quadrilaterals *ABCD* and *WXYZ* congruent? Explain.



continued

Example 16 continued

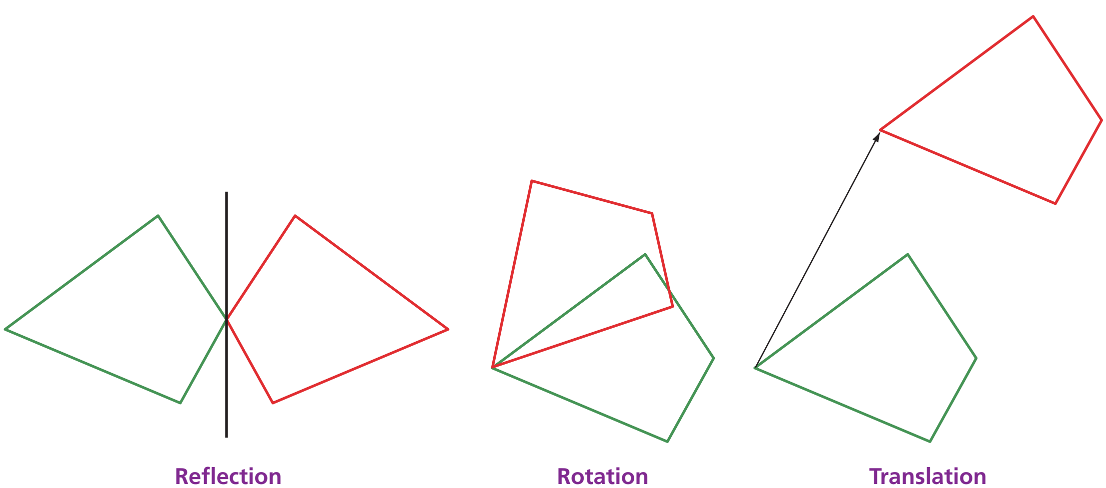
Working	Reasoning
a The angles of quadrilateral *ABCD* are the same as the angles of quadrilateral *EFGH*. The matching sides have the same length. So $ABCD \equiv EFGH$	For shapes to be congruent the matching angles of one must be the same as the angles of the other *and* the matching sides must be the same length. The quadrilaterals must be named according to the matching sides. To match the sides of quadrilateral *ABCD*, the second quadrilateral must be named *EFGH*, not, for example, *GHEF*.
b Quadrilaterals *ABCD* and *STUV* are not congruent. Although their angles are the same, the sides are different.	The angles must be the same *and* the sides must be the same.
c Quadrilaterals *ABCD* and *WXYZ* are not congruent. Although their sides are the same, the angles are different.	The angles must be the same *and* the sides must be the same.

You will recall from Year 7 that a change in the size or position of a figure is called a **transformation**. An **isometric transformation** is where the position of a geometric figure is changed but its shape and size stay the same, that is, a congruent figure is produced.

Three different types of isometric transformation are:

- reflection
- rotation
- translation.

Isometric means *the same measure*.

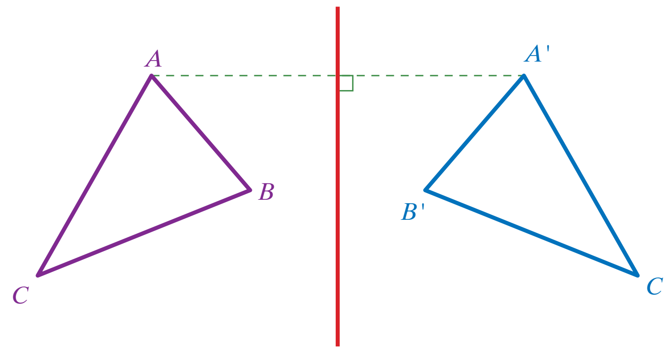


Isometric transformations produce congruent shapes

A shape that is produced by a transformation is called the **image** of the original shape. If we use the letters A, B, C, etc. to label the vertices of the original shape, we normally use A', B', C', etc. to label the vertices of its image. For example, if a triangle is labelled ABC, its image would be labelled A', B', C'. We write A'. We say 'A dash'.

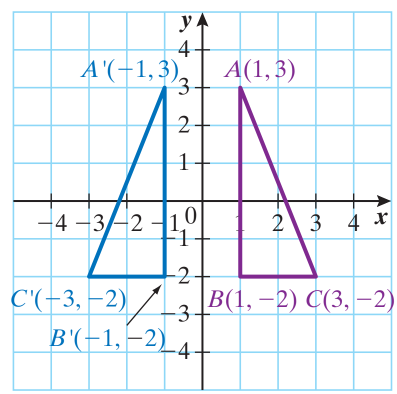
Reflection

Reflection of a shape produces a mirror image. Each point and its image can be joined by a line that is perpendicular to the mirror line. Each point and its image are the same distance from the mirror line. Reflection is an isometric transformation so $\triangle A'B'C' \equiv \triangle ABC$.



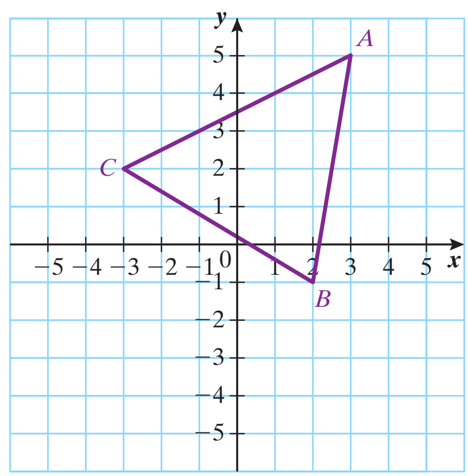
When a shape is reflected on the Cartesian plane:

- reflection in the y-axis changes the sign of the x-coordinate of each vertex but the y-coordinates remain unchanged.
- reflection in the x-axis changes the sign of the y-coordinate of each vertex but the x-coordinates remain unchanged.



Example 17

Reflect triangle ABC in the x-axis then complete the table to show the coordinates of each vertex of the triangle and its image. Each grid square is 1 unit.

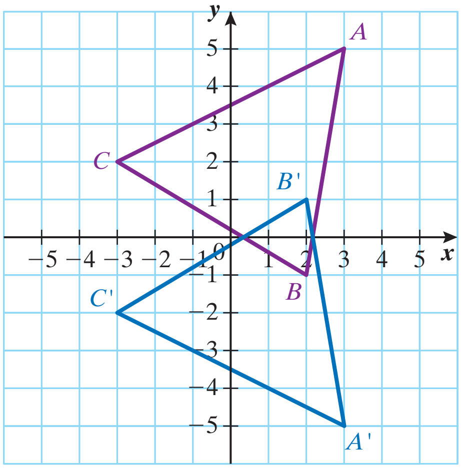


A: (,)	A': (,)
B: (,)	B': (,)
C: (,)	C': (,)

continued

Example 17 continued

Working



A: (3, 5)	A': (3, −5)
B: (2, −1)	B': (2, 1)
C: (−3, 2)	C': (−3, −2)

Reasoning

The sign of each x-coordinate stays the same. The sign of each y-coordinate changes. Reflection is an isometric transformation so $\triangle A'B'C' \equiv \triangle ABC$.

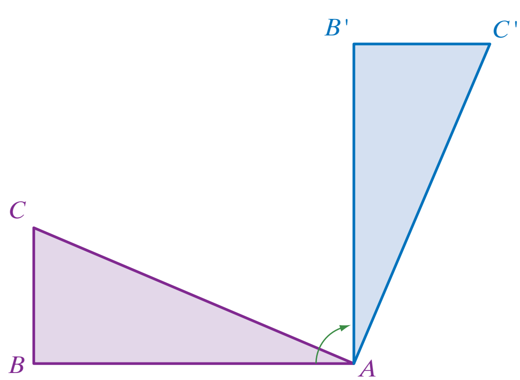
Rotation

Rotation occurs when a shape is turned about a point. In describing a rotation we must state

- the angle of rotation,
- the direction of rotation (clockwise or anticlockwise) and
- the point about which the shape is to be rotated.

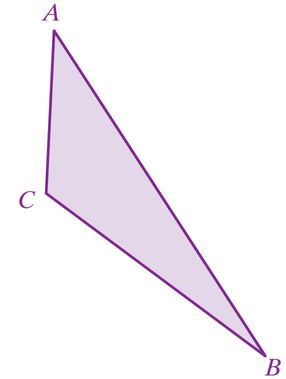
If a shape is rotated through 360° it will end up back in its original position.

Triangle ABC has been rotated through 40° about point A in a clockwise direction to produce the image $A'B'C'$. Rotation is an isometric transformation so $\triangle A'B'C' \equiv \triangle ABC$.

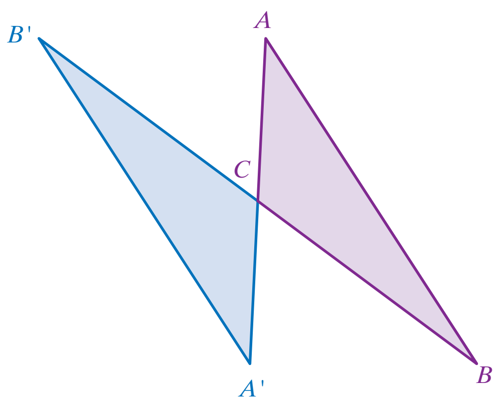


Example 18

Rotate triangle ABC through 180° about point C.



Working

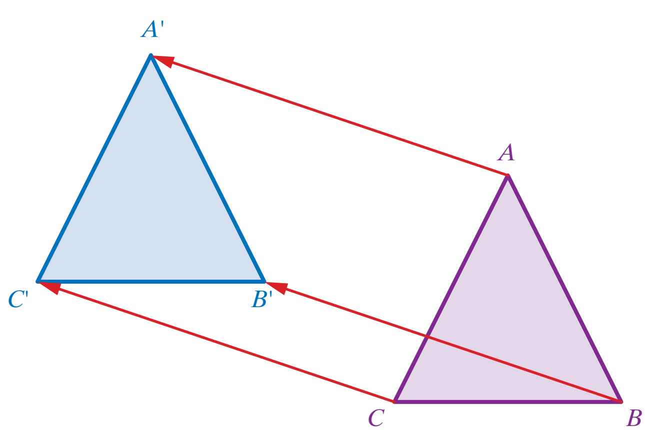


Reasoning

Each side of the triangle is rotated through 180°.
Rotation is an isometric transformation so $\triangle A'B'C' \equiv \triangle ABC$.

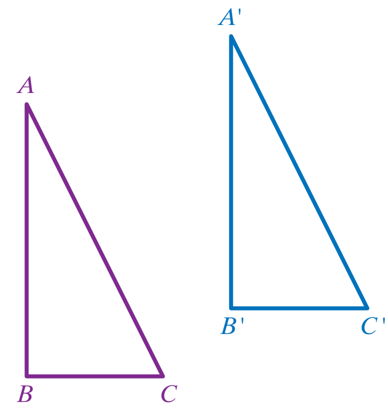
Translation

Translation occurs when a shape slides into a different position without being turned. Each point on the object is translated the same distance. Translation is an isometric transformation so $\triangle A'B'C' \equiv \triangle ABC$.

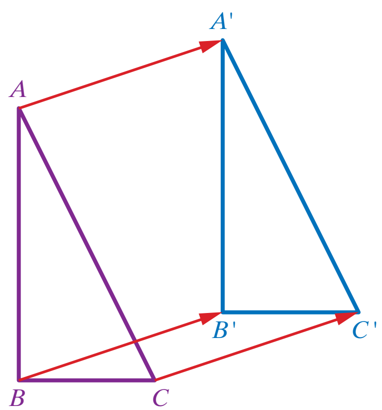


Example 19

Draw arrows at A, B and C to show how each point has been translated.



Working

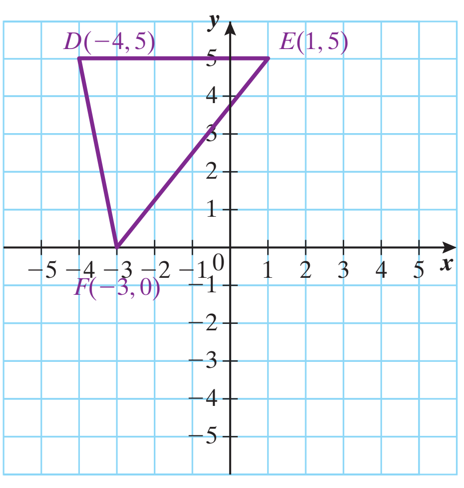


Reasoning

Each point is translated the same distance in the same direction. $\triangle A'B'C'$ is exactly the same shape and size as $\triangle ABC$.
Translation is an isometric transformation so $\triangle A'B'C' \equiv \triangle ABC$.

Example 20

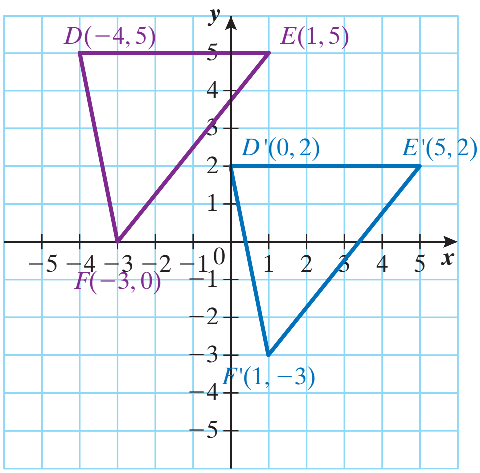
Translate $\triangle DEF$ 4 units to the right and 3 units down.



continued

Example 20 continued

Working



Reasoning

Translating 4 units to the right and 3 units down adds 4 to each x-coordinate and subtracts 3 from each y-coordinate. Translation is an isometric transformation so $\triangle D'E'F' \equiv \triangle DEF$.

If two triangles are congruent we can use one or more isometric transformations to place one exactly over the other.

Example 21

For each of the following pairs of congruent triangles

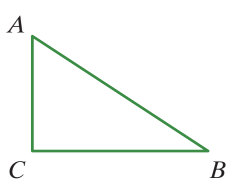
i state the transformations that would be needed to place triangle 2 exactly over triangle 1.

ii List the matching vertices.

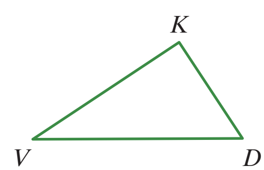
iii List the matching sides.

iv Name triangle 2 to match triangle 1.

a

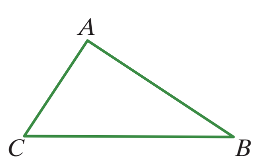


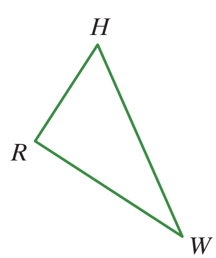
Triangle 1



Triangle 2

b





Triangle 1

Triangle 2

continued

Example 21 continued

Working

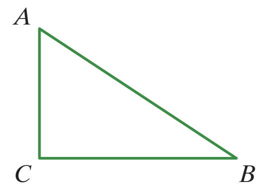
a **i** Rotation and translation

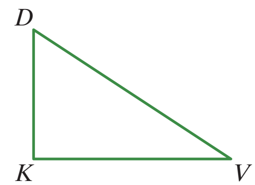
ii $A \leftrightarrow D$
$B \leftrightarrow V$
$C \leftrightarrow K$

iii $AB \leftrightarrow DV$
$BC \leftrightarrow VK$
$CA \leftrightarrow KD$

iv $\triangle DVK$

Reasoning



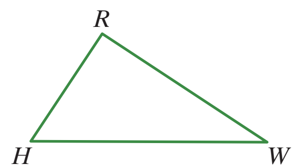


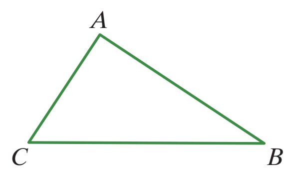
b **i** Rotation, reflection and translation

ii $A \leftrightarrow R$
$B \leftrightarrow W$
$C \leftrightarrow H$

iii $AB \leftrightarrow RW$
$BC \leftrightarrow WH$
$CA \leftrightarrow HR$

iv $\triangle RWH$





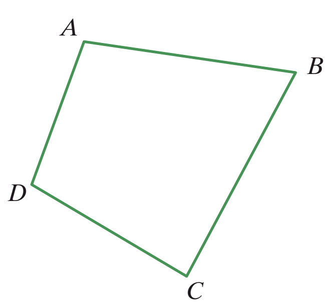
exercise 7.3

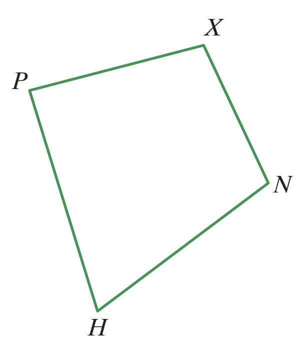
1 These two quadrilaterals are congruent. Name

a the matching vertices

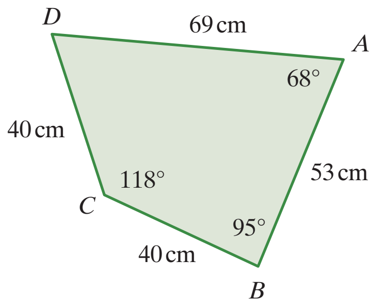
b the matching sides

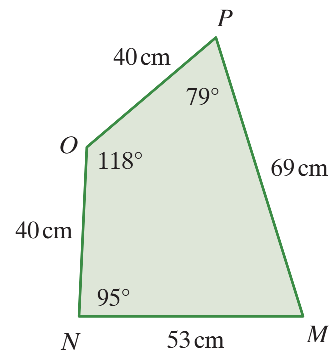
c the quadrilateral on the right to match the name of quadrilateral $ABCD$



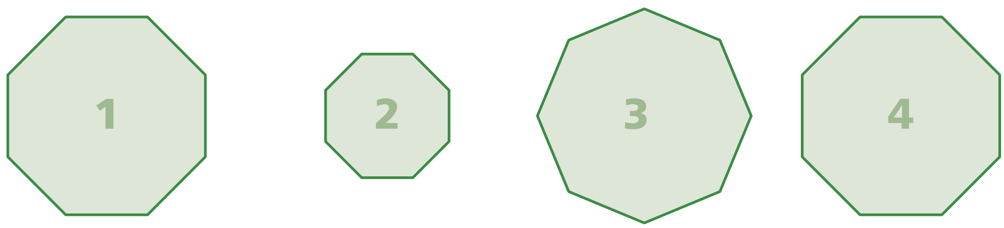


2 Is quadrilateral *ABCD* congruent to quadrilateral *MNOP*? Explain.





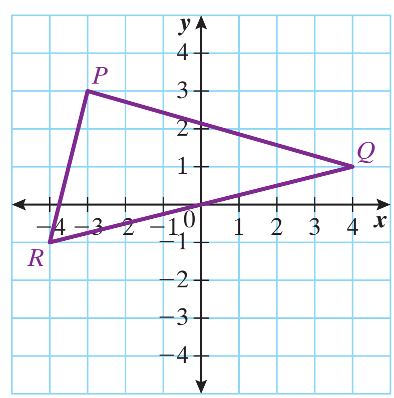
3



Which of the following shapes are congruent?

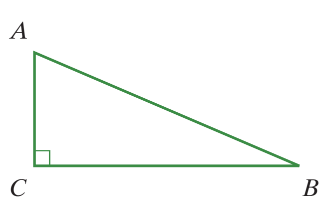
A 1 and 2 **B** 1 and 3 **C** 1 and 4 **D** 1, 2 and 4 **E** 1, 3 and 4

4 Reflect $\triangle PQR$ in the x-axis. Each grid square is 1 unit. Make a table to show the coordinates of P, Q and R and their images.

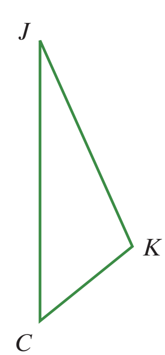


5 Copy each of these triangles and rotate it through the given angle.

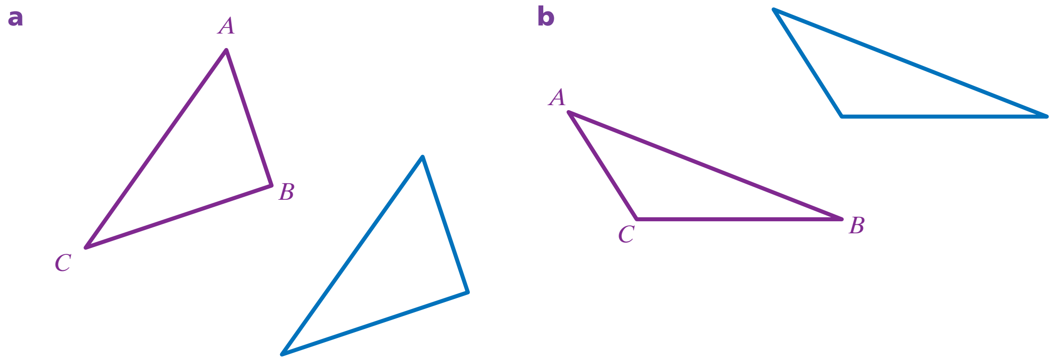
a Rotate $\triangle ABC$ through 90° anticlockwise about point A.



b Rotate $\triangle JKL$ through 180° about point L.



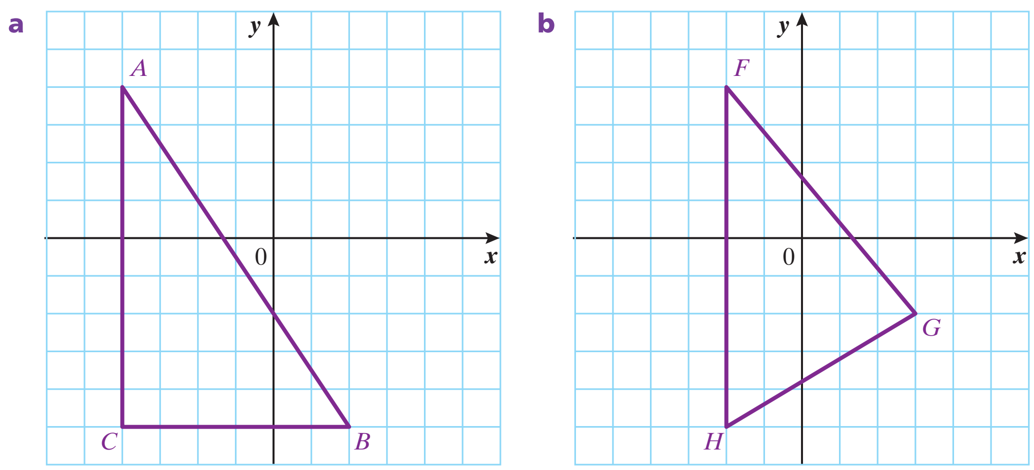
6 Draw translation arrows to show the direction of translation and label the vertices of the translated triangle as A', B' and C'.



7 Translate each of these triangles in the direction indicated. Each grid square is 1 unit. In each case make table to show the coordinates of each vertex and its image.

a Translate $\triangle ABC$ 3 units to the right and 1 unit up.

b Translate $\triangle FGH$ 2 units to the left and 1 unit down.

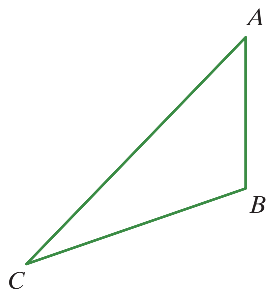


8 $\triangle ABC$ has vertices $A(-3,-5)$, $B(-1,6)$ and $C(4,3)$. On graph paper, carefully draw $\triangle ABC$ and then translate it 4 units to the left and 3 units up. Label the vertices A', B' and C' and show their coordinates.

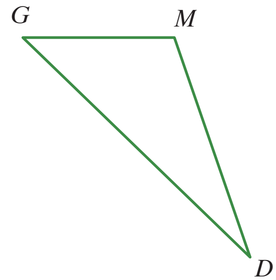
9 For each of the following pairs of congruent triangles

- **i** state the transformations that would be needed to place triangle 2 exactly over triangle 1.
- **ii** List the matching vertices.
- **iii** List the matching sides.
- **iv** Name triangle 2 to match triangle 1.

a

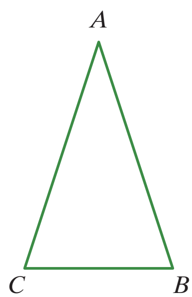


Triangle 1

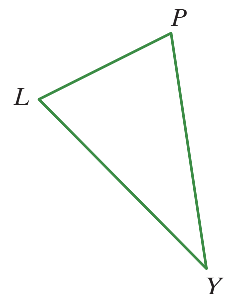


Triangle 2

b

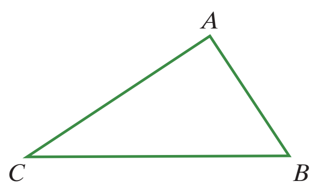


Triangle 1

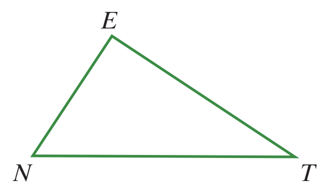


Triangle 2

c

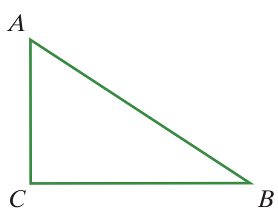


Triangle 1

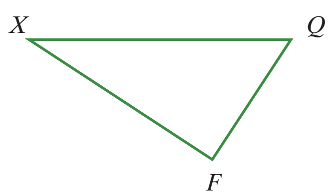


Triangle 2

d



Triangle 1



Triangle 2

7.4 Triangle construction and congruency

The sum of the two shortest sides of a triangle must be greater than the third side. For example, a triangle cannot be constructed if the side lengths are 3 cm, 4 cm and 9 cm as the two shortest sides will not meet.

Example 22

For which of these sets of lengths is it possible to construct a triangle?

a 3 cm, 5 cm, 11 cm **b** 4 cm, 6 cm, 7 cm **c** 2.5 cm, 3 cm, 5.5 cm

Working	Reasoning
a $3\text{ cm} + 5\text{ cm} < 11\text{ cm}$ A triangle cannot be constructed.	The sum of the two shortest sides is less than the third side, so the sides will not meet.
b $4\text{ cm} + 6\text{ cm} > 7\text{ cm}$ A triangle can be constructed.	The sum of the two shortest sides is greater than the third side, so the sides will meet.
c $2.5\text{ cm} + 3\text{ cm} = 5.5\text{ cm}$ A triangle cannot be constructed.	The sum of the two shortest sides is equal to the third side. Although they meet to make the length of the third side, they cannot make a triangle.

Every triangle has three sides and three angles so there are six pieces of information about a triangle that we could be given: the lengths of the three sides and the sizes of the three angles. The question is: do we actually need all six pieces of information or is there a smaller number of pieces of information that will allow us to accurately identify the triangle? In the following examples, four different sets of information are considered.

Side-Side-Side (SSS)

In Year 7 you saw how a compass and ruler could be used to accurately draw a triangle if you were given the three sides.

Given the three side lengths of a triangle, the size and shape of the triangle is determined.

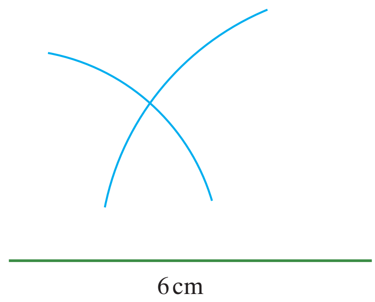
Example 23

Construct a triangle with sides 6 cm, 4.5 cm and 3.5 cm.

a using pencil, compass and ruler **b** using GeoGebra

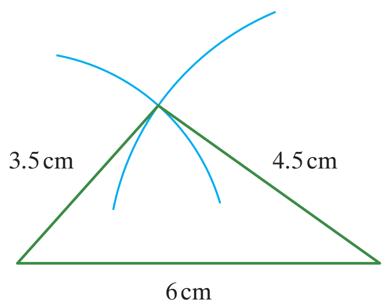
Working

a



Reasoning

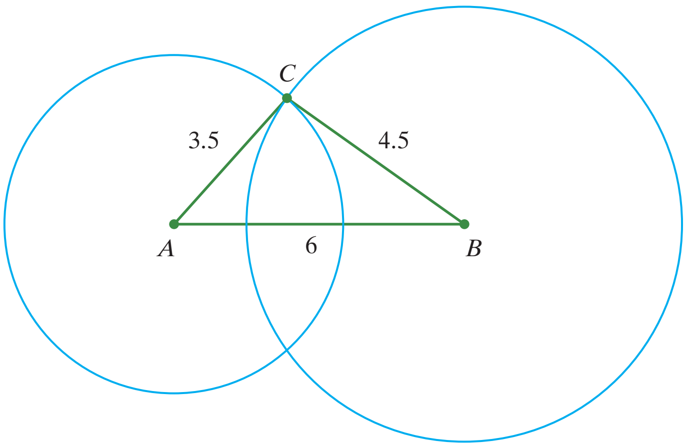
Draw a line segment 6 cm long.



With a compass opened to exactly 4.5 cm, place the compass point at one end of the segment and draw an arc as shown. Then with the compass opened at exactly 3.5 cm, place the compass point on the other end of the 6 cm line segment and draw another arc.

Where the two arcs intersect represents the third vertex of the triangle.

b

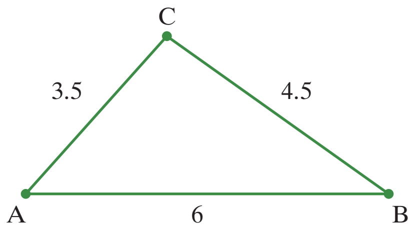


Using GeoGebra, we start by constructing a line interval (line segment) of length 6 cm. Construct circles of radius 3.5 and 4.5 at points A and B respectively. Construct an intersection point C at the intersection of the two circles, then construct line intervals AC and BC.

continued

Example 23 continued

Working

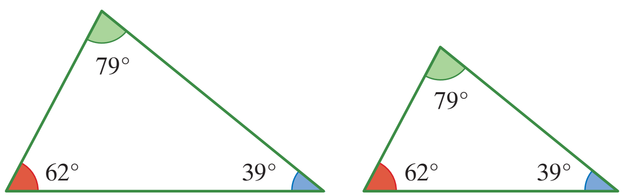


Reasoning

Finally, *hide* the circles by using the Show/hide tool. Note that we do not *delete* the circles because this would cause all those parts of the construction that depended on the two circles to be deleted.

Angle-Angle-Angle (AAA)

The two triangles below both have angles of 62°, 79° and 39°. Although they are the same shape, they are different sizes. Note that we actually need to know only two of the angle sizes because we can calculate the size of the third angle.

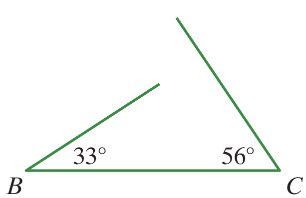


Given the three angles of a triangle, the *shape* of the triangle is determined, but not its *size*.

Example 24

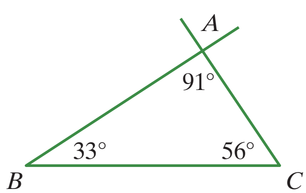
Construct a triangle ***ABC*** with angles 33°, 56° and 91° at ***B***, ***C*** and ***A*** respectively.

Working



Reasoning

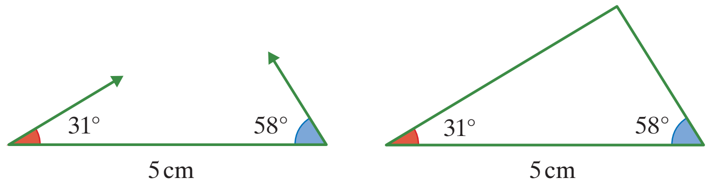
Draw a line segment any length and label it *BC*. Use a protractor to construct an angle of 33° at *B* and an angle of 56° at *C*.



Extend the arms of the angles to meet at *A*. The third angle will be 91°.
The shape of the triangle is fixed by the angle sizes, but the size will depend on how long we make the first line segment.

Angle-Side-Angle (ASA)

If we know the sizes of two angles and the length of the side joining them, there is only one possible triangle we can draw.

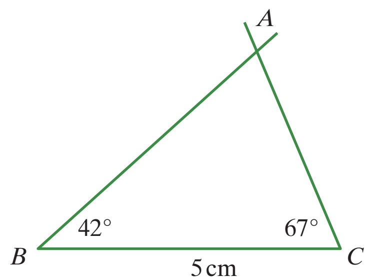


Given two angles of a triangle and the length of the side between them, the shape and size of the triangle is determined.

Example 25

Construct a triangle ***ABC*** in which ***BC*** = 5 cm, ∠***ABC*** = 42° and ∠***ACB*** = 67°.

Working

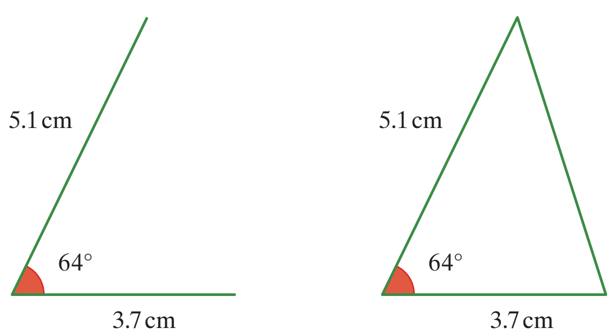


Reasoning

Draw a line segment, *BC*, 5 cm long. Use a protractor to construct an angle of 42° at *B* and an angle of 67° at *C*. Extend the arms of the angles to meet at *A*.

Side-Angle-Side (SAS)

If we know the lengths of two sides and the size of the angle between them, there is only one possible triangle we can draw.

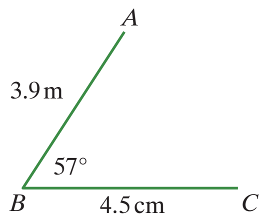


Given two sides of a triangle and the size of the angle between them, the shape and size of the triangle is determined.

Example 26

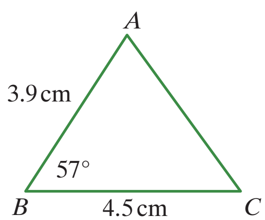
Construct a triangle ***ABC*** in which ***BC*** = 4.5 cm, ***AB*** = 3.9 cm and ∠***ABC*** = 57°.

Working



Reasoning

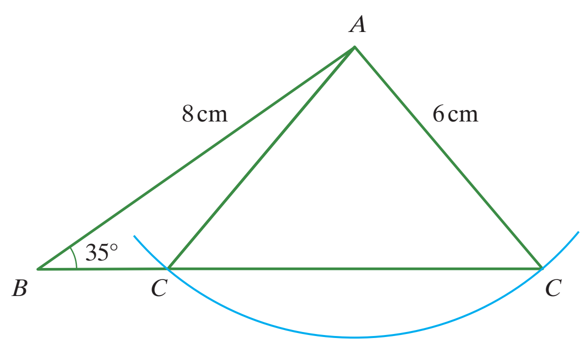
Draw a line segment, *BC*, 4.5 cm long. Use a protractor to construct an angle of 57° at *B*, with *BA* = 3.9 cm.

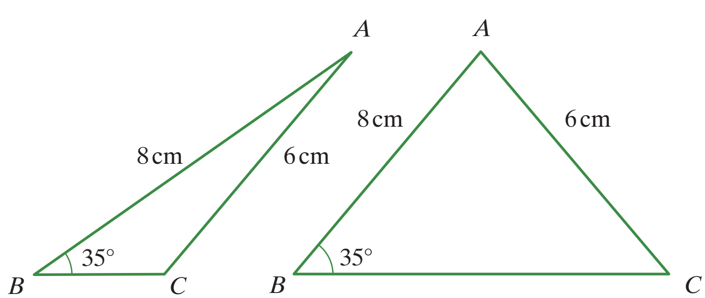


Use a ruler to join *A* and *C*.

Side-Side-Angle (SSA)

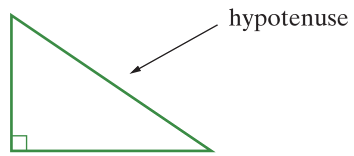
If we are given the lengths of two sides and an angle, but the angle is not between the two given sides, then two different triangles are possible, as shown below. So knowing two sides and an angle that is not the included angle is not sufficient to fix the shape of the triangle.





Right angle-Hypotenuse-Side (RHS)

In any right-angled triangle the side that is opposite the right angle is always the longest side of the triangle. This side of a right-angled triangle has a special name – it is called the **hypotenuse**.

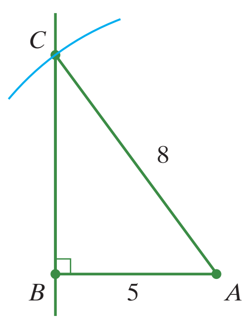


Knowing the length of the hypotenuse and one other side of a right-angled triangle is sufficient to fix the shape of the triangle.

Example 27

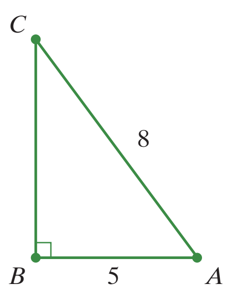
Use interactive geometry software such as GeoGebra to construct a triangle ABC in which $\angle CAB = 90°$, $AB = 5$ cm and $BC = 8$ cm.

Working



Reasoning

Start by constructing a line interval (line segment) AB with length 5. Use the perpendicular line tool to construct a line through point A perpendicular to AB. Then construct a circle with centre B and radius 8. Construct an intersection point C where the circle crosses the perpendicular line through A. Join BC and join AC with line intervals.



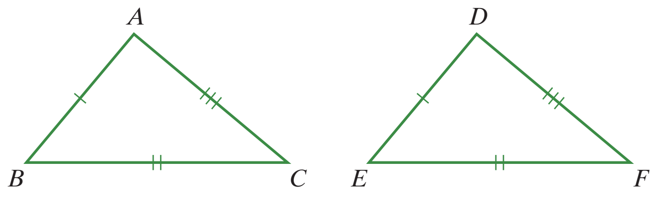
Hide the circle and the perpendicular line.

Conditions for triangles to be congruent

We have seen that certain sets of information about a triangle ensure that only one shape is possible for the triangle. These are also the sets of information that allow us to decide if two triangles are congruent.

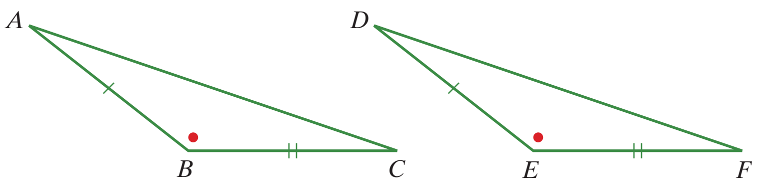
Side–Side–Side (SSS)

The three sides of one triangle are equal in length to the three sides of the other triangle.



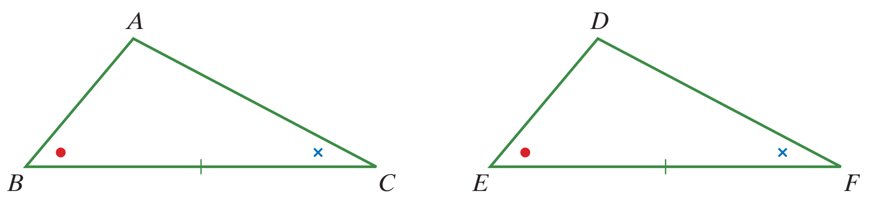
Side–Angle–Side (SAS)

Two sides of one triangle are equal to the corresponding two sides of the other triangle, and the angles in between these two sides are equal.



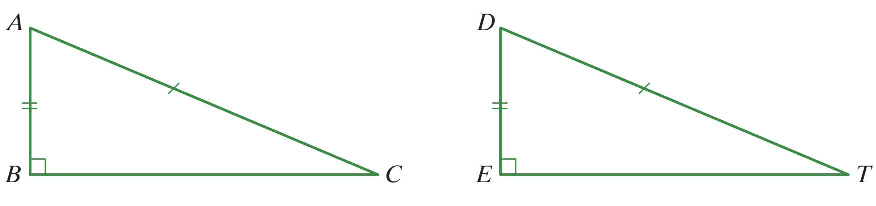
Angle–Side–Angle (ASA)

Two angles and a side of one triangle are equal to two angles and the matching side in the other triangle.



Right angle–Hypotenuse–Side (RHS)

Both triangles are right-angled and the hypotenuse and another side of one triangle are equal to the hypotenuse and the matching side of the other triangle.

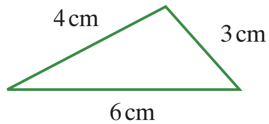


The sets of conditions that do **not** guarantee that two triangles are congruent are AAA and ASS.

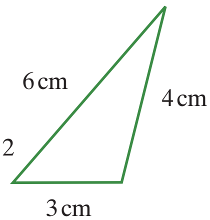
Example 28

Are these pairs of triangles congruent?

a

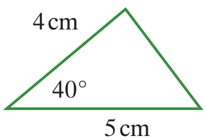


Triangle 1

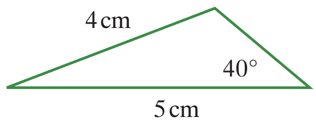


Triangle 2

b



Triangle 3



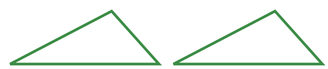
Triangle 4

Working

a Triangles 1 and 2 are congruent because the sides of triangle 1 are the same length as the sides of triangle 2.

Reasoning

Rotate Triangle 2 so it is in the same orientation as Triangle 1.



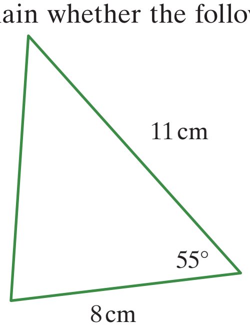
Triangles are congruent if the three sides of one are equal in length to the three sides of the other.

b Triangles 3 and 4 are not congruent because in Triangle 4 the angle of 40° is not between the matching sides of 5 cm and 4 cm.

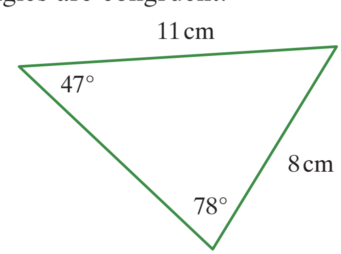
Triangles are congruent if two sides of one triangle and the size of the angle between them are the same as the matching two sides of the other triangle and the side between them.

Example 29

Explain whether the following triangles are congruent.



E



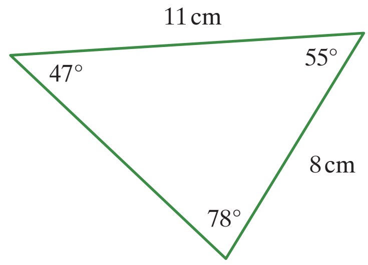
F

continued

Example 29 continued

Working

In triangle F, the angle between the two given sides is 55°.



So two sides and the angle between them of triangle E are equal to two sides and the angle between them of triangle F.

The triangles are congruent (SAS).

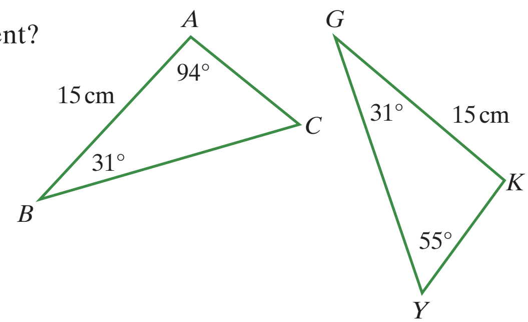
Reasoning

The third angle of triangle F
$= 180° - (47° + 78°)$
$= 55°$

If two sides and the angle between them of one triangle are equal to two sides and the angle between them of the other triangle, the triangles are congruent.

Example 30

Are these triangles congruent?



Working

$\angle GKY = 180° - 55° - 31°$

$\angle GKY = 94°$

Both triangles have angles of 31° and 94° joined by a side of length 15 cm.

$\triangle ABC \equiv \triangle KGY$ (AAS)

Reasoning

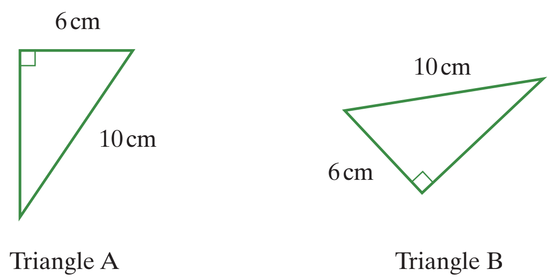
The sum of the angles of a triangle is 180°.

Two triangles are congruent if two angles and a side of one triangle are equal to two angles and the matching side of the other triangle.

We name the congruent triangles by matching the vertices. If we rotate the second triangle so that it is exactly on top of $\triangle ABC$, K corresponds to A, G to B and Y to C.
So we say that $\triangle ABC \equiv \triangle KGY$.

Example 31

Are these triangles congruent?



Working

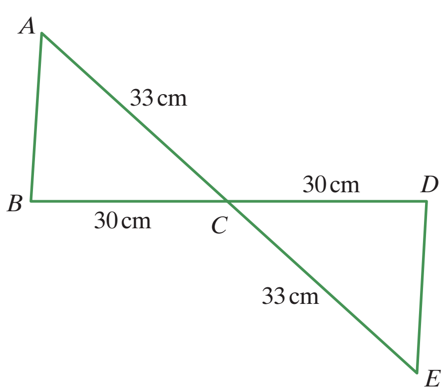
Both triangles are right-angled.
The hypotenuse is the same length in each triangle.
Both triangles have another pair of matching sides equal.
The triangles are congruent (RHS)

Reasoning

Two right-angled triangles are congruent if the hypotenuse and another side of one triangle are equal to the hypotenuse and another side of the other triangle.

Example 32

Show that these triangles are congruent.



Working

$BC = DC$
$AC = EC$

$\angle ACB = \angle ECD$ (vertically opposite angles)
Two sides and the included angle of $\triangle ABC$ are equal to the matching two sides and the included angle of $\triangle EDC$.
$\triangle ABC \equiv \triangle EDC$ (SAS)

Reasoning

Two matching sides of each triangle are given. The included angles are equal because they are vertically opposite angles.

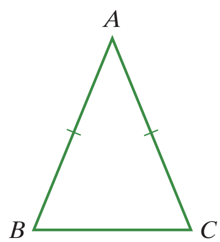
When we name the congruent triangles, we put the letters in matching order.

Showing that two triangles are congruent enables us to show many properties in geometry to be true.

Example 33

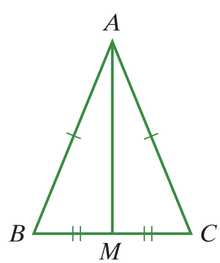
Show that the base angles of an isosceles triangle are equal.

Working



Prove that $\angle ABC = \angle ACB$.

$AB = AC$ (definition of isosceles triangle)
M is the midpoint of BC (construction).



In $\triangle AMB$ and $\triangle AMC$
$AB = BC$ (definition of isosceles triangle)
$BM = MC$ (M is midpoint of BC)
AM is common to both triangles.
$\triangle AMB \equiv \triangle AMC$ (SSS)

So $\angle ABC = \angle ACB$ (corresponding angles in congruent triangles)

Reasoning

Draw a diagram and state what is to be proved.

In constructing a proof, we need to have a plan of where we are going and how we are going to get there. It may be necessary to add other lines to the diagram.

By joining A to M, the midpoint of BC, we divide the isosceles triangle into two triangles. If we can prove that these triangles are congruent, then the base angles must be equal.

Two triangles are congruent if three sides of one triangle are equal to three sides of the other triangle.

If two triangles are congruent, the angles of one triangle are equal to the corresponding (matching) angles of the other triangle.

exercise 7.4

1 For which of these sets of lengths is it possible to construct a triangle? Explain.

- **a** 8 cm, 4 cm, 7 cm
- **b** 3 cm, 8 cm, 12 cm
- **c** 5 cm, 8 cm, 6 cm
- **d** 6 cm, 7 cm, 13 cm
- **e** 5 cm, 6 cm, 7 cm
- **f** 6.5 cm, 3.4 cm, 2.9 cm
- **g** 5.6 cm, 2.9 cm, 4.3 cm
- **h** 4.8 cm, 2.4 cm, 2.4 cm

In each of the questions 2–6, compare your triangles with those of other students in the class. Are your triangles the same shape and size?

2 Use ruler, pencil and compass to construct each of these triangles. Label the vertices A, B and C and label the side lengths.

a $AB = 5\,\text{cm}$, $AC = 3\,\text{cm}$, $BC = 4\,\text{cm}$

b $AB = 7\,\text{cm}$, $AC = 6\,\text{cm}$, $BC = 3\,\text{cm}$

c $AB = 8\,\text{cm}$, $AC = 6\,\text{cm}$, $BC = 4\,\text{cm}$

d $AB = 50\,\text{mm}$, $AC = 50\,\text{mm}$, $BC = 50\,\text{mm}$

3 Use ruler, pencil and protractor to construct each of these triangles. Label the vertices A, B and C and label the angle sizes.

a $40°$ at vertex A, $60°$ at vertex B, $80°$ at vertex C

b $30°$ at vertex A, $75°$ at vertex B, $75°$ at vertex C

c $55°$ at vertex A, $35°$ at vertex B, $90°$ at vertex C

d $85°$ at vertex A, $30°$ at vertex B, $65°$ at vertex C

4 Use ruler, pencil and protractor to construct each of these triangles. Label the vertices A, B and C and label the length of AB and the angle sizes.

a $AB = 7\,\text{cm}$, $40°$ at vertex A, $30°$ at vertex B

b $AB = 11\,\text{cm}$, $20°$ at vertex A, $15°$ at vertex B

c $AB = 6\,\text{cm}$, $25°$ at vertex A, $45°$ at vertex B

d $AB = 85\,\text{mm}$, $40°$ at vertex A, $25°$ at vertex B

5 Use interactive geometry software such as GeoGebra to construct each of these triangles.

a $\triangle ABC$ in which, $AB = 6$ cm, $BC = 7$ cm $\angle ABC = 60°$

b $\triangle ABC$ in which, $AB = 8$ cm, $BC = 6$ cm $\angle ABC = 45°$

c $\triangle ABC$ in which, $AB = 4$ cm, $BC = 7$ cm $\angle ABC = 70°$

d $\triangle ABC$ in which, $AB = 6$ cm, $BC = 8$ cm $\angle ABC = 35°$

6 Use interactive geometry software such as GeoGebra to construct each of these triangles.

a $\triangle ABC$ in which $\angle CAB = 90°$, $AB = 6$ cm and $BC = 10$ cm

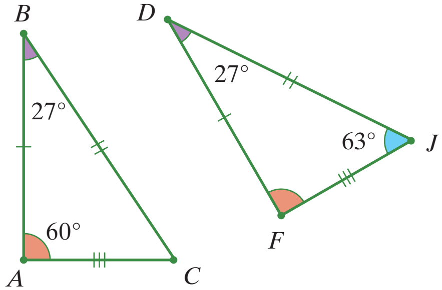
b $\triangle ABC$ in which $\angle CAB = 90°$, $AB = 4$ cm and $BC = 6$ cm

c $\triangle ABC$ in which $\angle CAB = 90°$, $AB = 5$ cm and $BC = 7$ cm

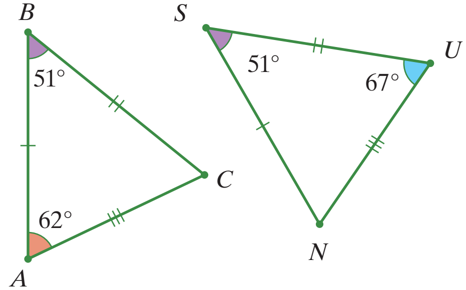
d $\triangle ABC$ in which $\angle CAB = 90°$, $AB = 6$ cm and $BC = 9$ cm

7 Each of these pairs of triangles are congruent. Name the triangle that is congruent to $\triangle ABC$, making sure the order of letters is correct.

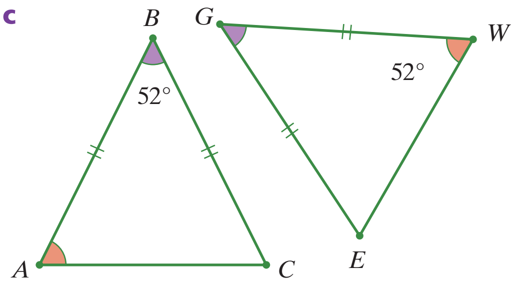
a



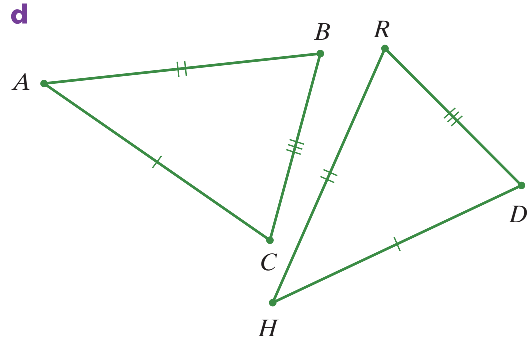
b



c

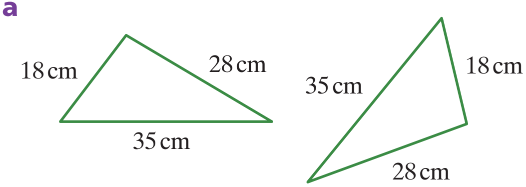


d

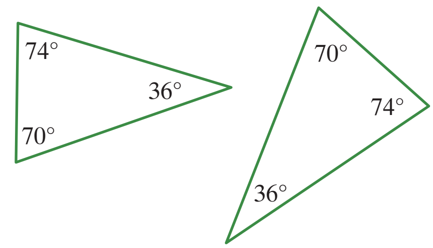


8 State whether the following pairs of triangles are congruent, giving your reasons.

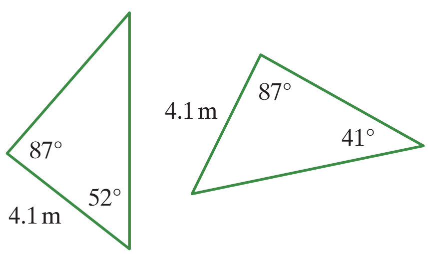
a



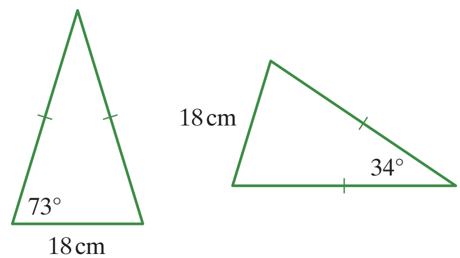
b



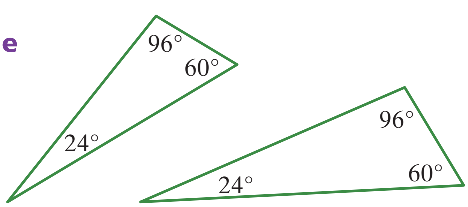
c



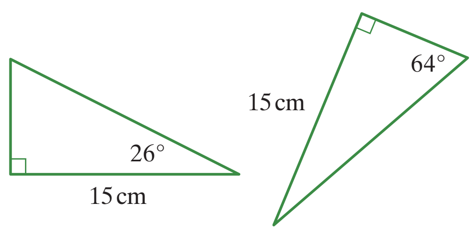
d



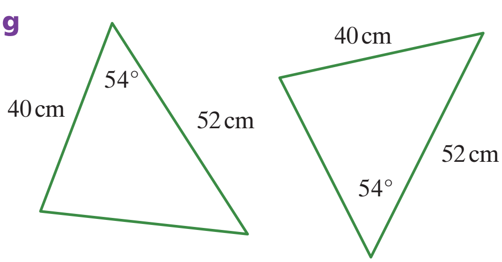
e



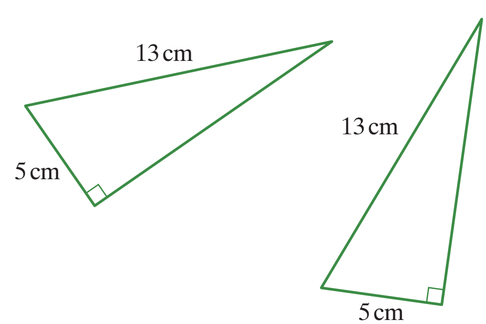
f



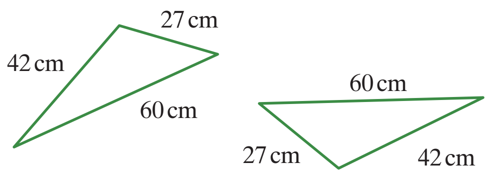
g



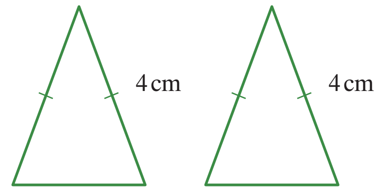
h



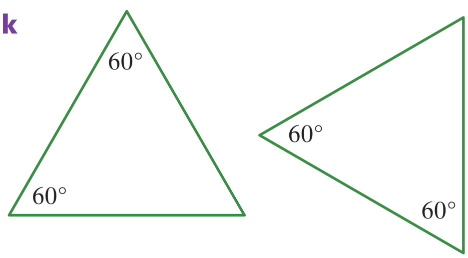
i



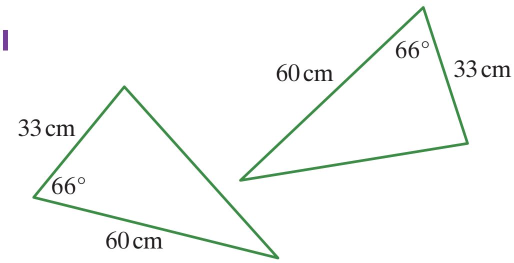
j



k

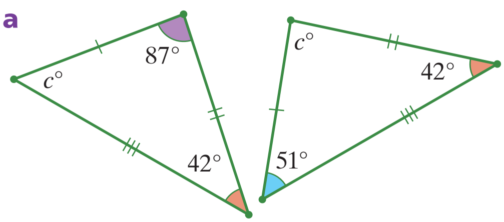


l

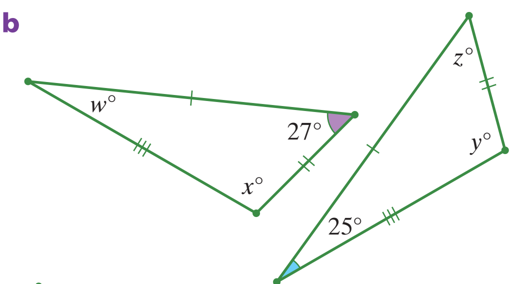


9 Find the values of the pronumerals in these pairs of congruent triangles.

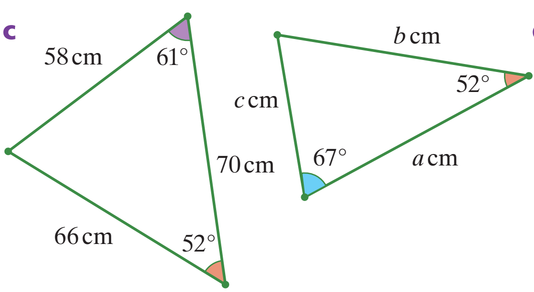
a



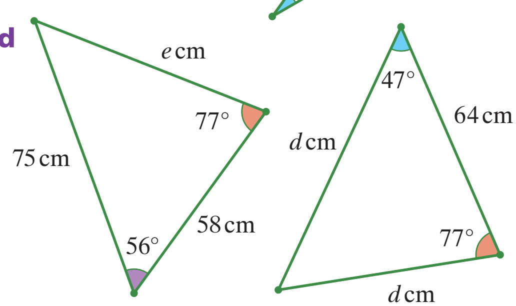
b



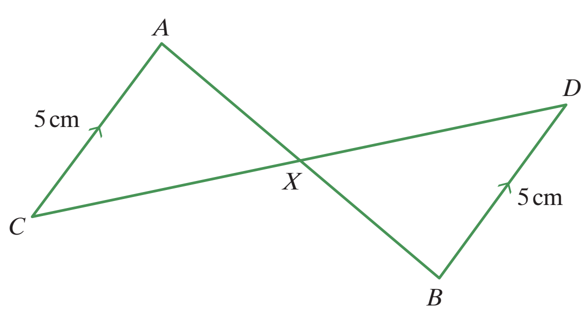
c



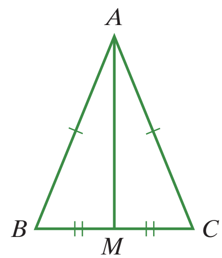
d



10 Show that triangles $\triangle AXC$ and $\triangle BXD$ are congruent.



11 In example 33, it was shown that $\triangle AMB \equiv \triangle AMC$. Show that the line segment from the vertex A of an isosceles triangle to the midpoint M of the base is perpendicular to the base.



12 **a** Construct $\triangle PQR$ where $\angle PQR$ is 30°, $PR = 3$ cm and $QR = 7$ cm.
b Construct $\triangle KLM$ where $\angle KLM$ is 30°, $KL = 3$ cm and $LM = 7$ cm.
c Are triangles PQR and KLM congruent?

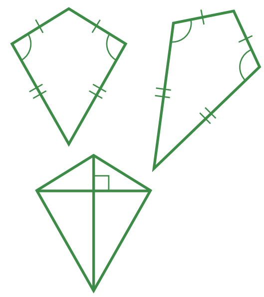
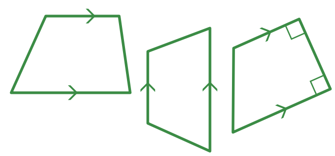
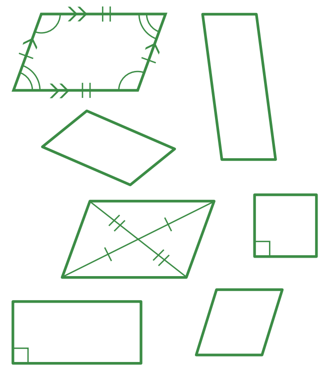
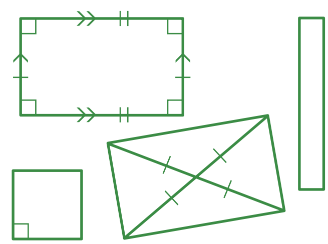
exercise 7.4 challenge

13 Andy and Josh are working with a set of plastic geometry strips with the following lengths: 7.5 cm, 11 cm, 15 cm and 22 cm. There are three strips of each length. Triangles can be made by joining the strips together with paper fasteners. For example, a triangle can be made by joining 11 cm, 11 cm and 15 cm strips. What is the total number of possible triangles that Andy and Josh could make? Assume that they pull each triangle apart and reuse the strips. Hint: make a systematic list.

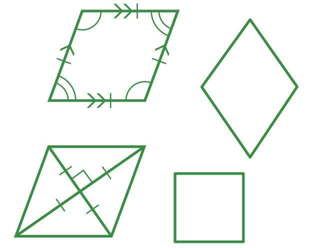
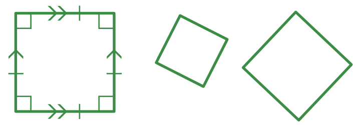
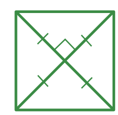
7.5 Quadrilaterals and their properties

A quadrilateral is a plane figure with four straight sides. Some quadrilaterals have special properties.

Properties of special quadrilaterals

Name	Properties	Examples
Kite	■ Two pairs of adjacent (next to each other) sides are equal. ■ One pair of opposite angles are equal. ■ Diagonals are perpendicular. ■ One diagonal bisects the unequal angles.	
Trapezium	■ One pair of opposite sides are parallel. ■ Two pairs of angles are supplementary.	
Parallelogram	■ Both pairs of opposite sides are parallel. ■ Both pairs of opposite sides are equal. ■ Opposite angles are equal. ■ Adjacent angles are supplementary. ■ Diagonals bisect each other.	
Rectangle	■ Both pairs of opposite sides are parallel and equal. ■ All angles are right angles. ■ Diagonals are equal in length. ■ Diagonals bisect each other.	

continued

Name	Properties	Examples
Rhombus	■ Both pairs of opposite sides are parallel. ■ All four sides are equal. ■ Opposite angles are equal. ■ Adjacent angles are supplementary. ■ Diagonals bisect each other. ■ Diagonals are perpendicular. ■ Diagonals bisect the angles of the rhombus.	
Square	■ Both pairs of opposite sides are parallel. ■ All four sides are equal. ■ All angles are right angles. ■ Diagonals are equal in length. ■ Diagonals are perpendicular. ■ Diagonals bisect each other. ■ Diagonals bisect the right angles of the square.	 A square is a special rectangle and a special rhombus. 

Example 34

Identify each of these quadrilaterals. Draw a diagram for each to show the given information.

a Both pairs of opposite sides of the quadrilateral are parallel. The four angles of the quadrilateral are 72°, 108°, 72°, 108°. The diagonals intersect at right angles.

b A quadrilateral has one pair of adjacent sides with length 8 cm and the other pair with length 5 cm. There are no parallel sides.

Working	Reasoning
a The quadrilateral is a rhombus.	The first sentence tells us that the quadrilateral belongs to the family of parallelograms. The second sentence tells us that it is not a square or a rectangle. The diagonals of squares, rhombuses and kites intersect at right angles. The quadrilateral must be a rhombus as we know it is not a kite or a square.
b The quadrilateral is a kite.	The quadrilateral is not a parallelogram or a trapezium. Kites have two pairs of adjacent sides equal.

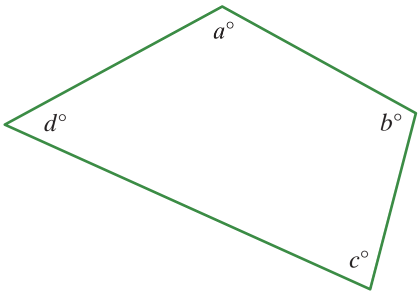
Squares, rhombuses and rectangles are all special parallelograms because they all share the properties of parallelograms, that is, both pairs of opposite sides are parallel.

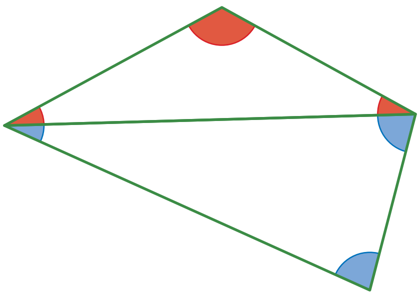
A square is a special rhombus that has right angles.

A square is also a special rectangle that has four equal sides.

Interior angles of quadrilaterals

The sum of the four angles of any quadrilateral is 360°.





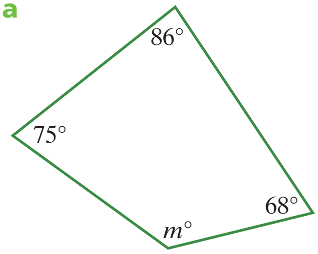
$\boldsymbol{a + b + c + d = 360}$

We can easily prove this by dividing the quadrilateral into two triangles. The three angles of each triangle have a sum of 180°. (These three angles shown in red add to 180° and the three angles shown in blue add to 180°.) This makes a total of 360° for the angles of the quadrilateral.

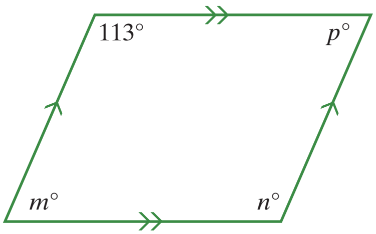
Example 35

Find the value of the pronumerals in these quadrilaterals.

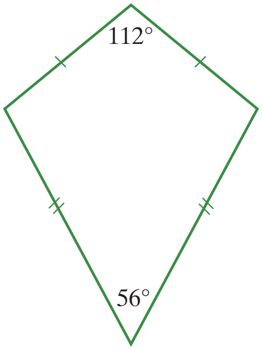
a



b



c



Working	Reasoning
a $a + 75 + 86 + 68 = 360$ $a + 229 = 360$ $a = 360 - 229$ $a = 131$	The four angles of a quadrilateral add to 360°.

continued

Example 35 continued

Working

b $m = 180 - 113$

$m = 67$

$n = 113$

$p = 67$

Reasoning

Adjacent angles of a parallelogram are supplementary.

Opposite angles of a parallelogram are equal.

Opposite angles of a parallelogram are equal.

Working

c $360 - (112 + 56)$

$= 360 - 168$

$= 192$

$x = \frac{1}{2}$ of 192

$x = 96$

Reasoning

The quadrilateral is a kite. The other unknown angle is also $x°$.
Add the two given angles and subtract from 360°.
Halve the result.

Example 36

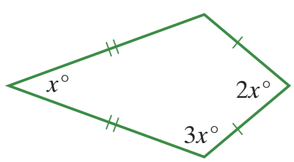
For each of the following figures

i write an equation

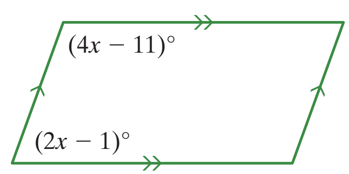
ii find the value of x

iii find the sizes of the four angles.

a



b



Working

a

$$3x + 3x + 2x + x = 360$$
$$9x = 360$$
$$x = 40$$

The four angles are 40°, 80°, 120°, 120°.

Reasoning

The figure is a kite, so the angle opposite the angle marked $3x°$ is also $3x°$. The four angles add to 360°.

Substitute $x = 40$ into the expression for each angle.

Working

b

$$2x - 1 + 4x - 11 = 180$$
$$6x - 12 = 180$$
$$6x = 192$$
$$x = 32$$

The four angles are 63°, 63°, 117°, 117°.

Reasoning

Adjacent angles in a parallelogram are supplementary.

Substitute $x = 32$ into the expressions $2x - 1$ and $4x - 11$.

exercise 7.5

1 **a** Copy and complete the following table. For each property, list all the special quadrilaterals with that property.

	Special quadrilaterals
Both pairs of opposite sides equal	
All sides equal	
Two pairs of adjacent sides equal but opposite sides not equal	
Two pairs of parallel sides	
One pair of parallel sides	

b Which special quadrilaterals fit the definition of a parallelogram and can therefore be regarded as belonging to the family of parallelograms?

2 List the special quadrilaterals that match the following descriptions.

a Both pairs of opposite angles are equal.

b One pair of opposite angles is equal.

c All angles are right angles.

3 Investigate the diagonals of the six special quadrilaterals shown in the diagrams on the next page. Record your answers to the following questions (Yes or No) in a table as shown.

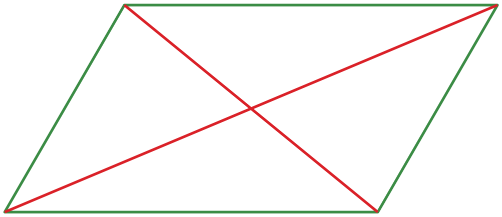
a For which quadrilaterals are the two diagonals equal in length?

b For which quadrilaterals do the two diagonals bisect each other?

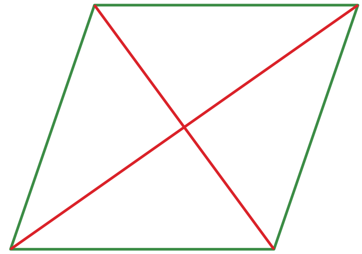
c For which quadrilaterals do the two diagonals intersect at right angles?

Special quadrilateral	**Diagonals equal in length?**	**Diagonals bisect each other?**	**Diagonals intersect at right angles?**
Parallelogram			
Rhombus			
Rectangle			
Square			
Trapezium			
Kite			

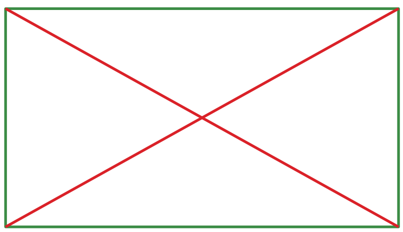
<note that this page is the (missing) next page referred to in line 1 of Q3>



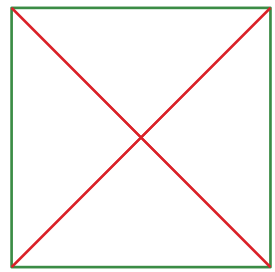
Parallelogram



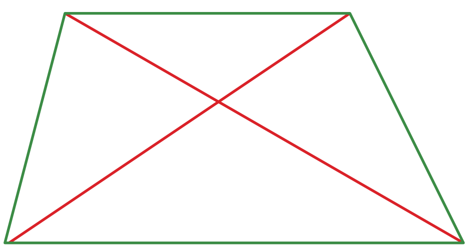
Rhombus



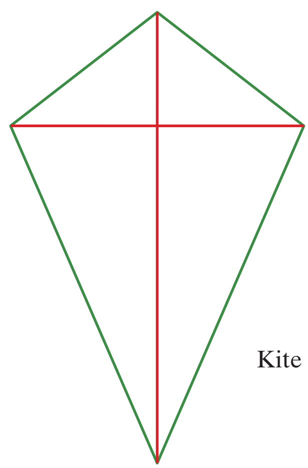
Rectangle



Square



Trapezium



Kite

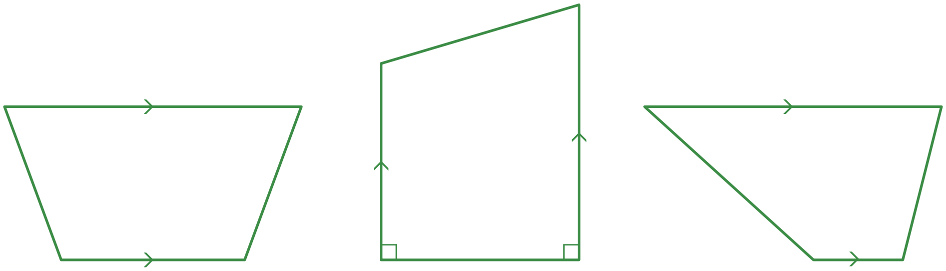
4 A quadrilateral has both pairs of opposite sides are parallel. The diagonals are equal in length and intersect at right angles. The quadrilateral is a

A rhombus **B** square **C** kite **D** rectangle **E** parallelogram

5 A quadrilateral has two pairs of equal sides. The diagonals are different lengths and intersect at right angles. One of the diagonals is bisected by the other. The quadrilateral is a

A trapezium **B** rectangle **C** parallelogram **D** kite **E** rhombus

6 **a** To which special group of quadrilaterals do each of these quadrilaterals belong?



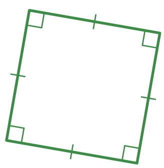
b What do they all have in common?

7 For each of the following figures:

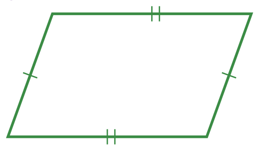
i list the properties marked on the figure

ii identify the shape.

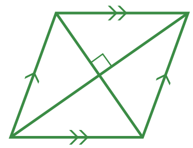
a



b



c



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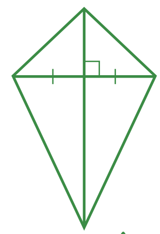
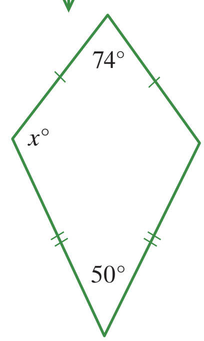




8 The value of x is

A 56 **B** 112 **C** 118 **D** 124 **E** 236

9 Identify each of the following quadrilaterals and make a drawing of each to show the properties described.

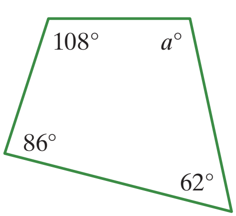
a All sides are equal and the diagonals are of different lengths but intersect at right angles.

b Both pairs of opposite sides are parallel and the diagonals are equal in length but do not intersect at right angles.

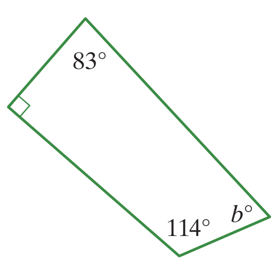
c The sides are all of different lengths and two pairs of adjacent angles are supplementary.

10 Find the value of each of the pronumerals in the following quadrilaterals.

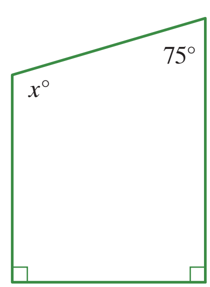
a



b



c



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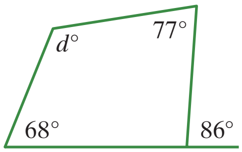

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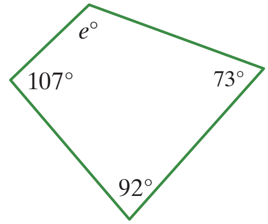

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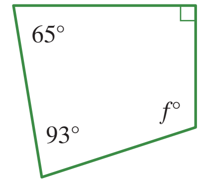

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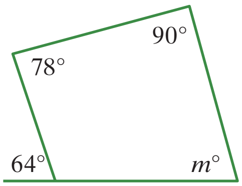
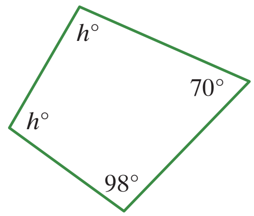
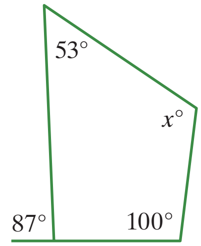


h



i



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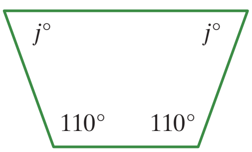
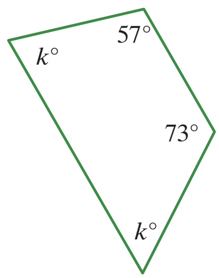
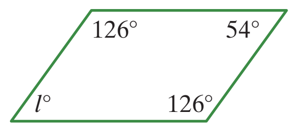


k



l



11 In each of the following, three angles of a quadrilateral are given. Calculate the size of the fourth angle for each.

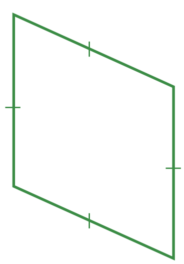
a 73°, 115°, 100° **b** 127°, 53°, 90° **c** 29°, 143°, 97° **d** 156°, 37°, 19°
e 48°, 67°, 140° **f** 110°, 38°, 105° **g** 150°, 75°, 90° **h** 108°, 72°, 72°

12 **a** A quadrilateral has angles 115° and 47° with the other two angles equal to each other. What is the size of each of these angles? Explain how you worked out your answer.

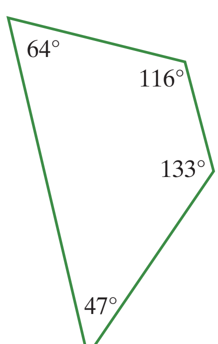
b In a quadrilateral one angle is 57° and the other three angles are equal to each other. What is the size of each of these angles? Explain how you worked out your answer.

13 Identify each of these quadrilaterals, justifying your answer.

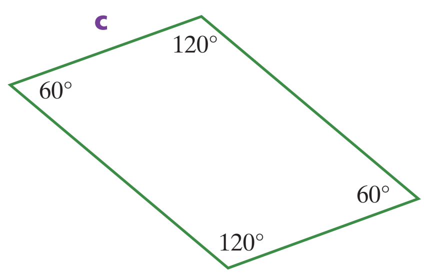
a



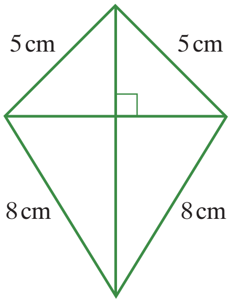
b



c



d



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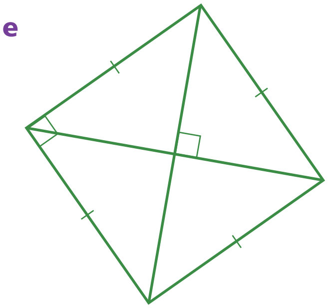

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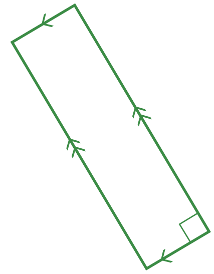
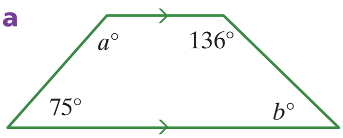


14 Find the value of each of the pronumerals. You may find it easier to copy each diagram and label the size of each angle as you find it.

a



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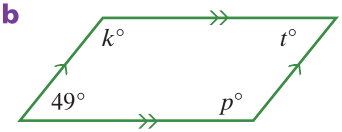

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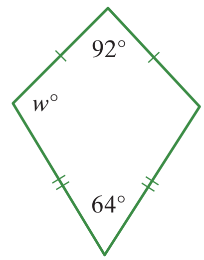

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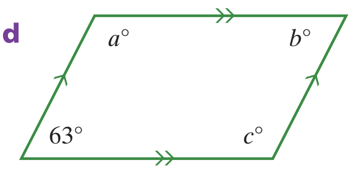

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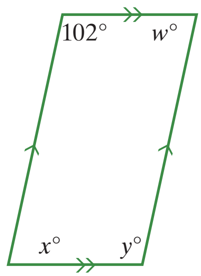

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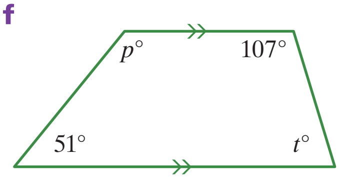

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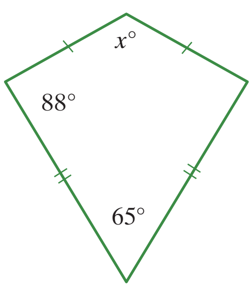

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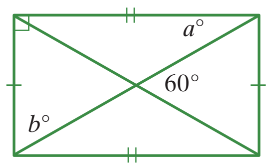
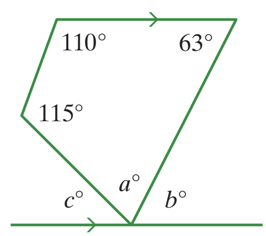


i



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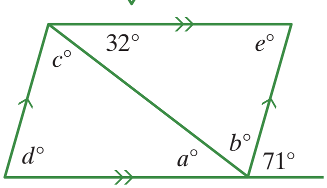

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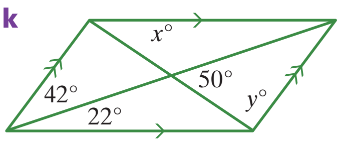

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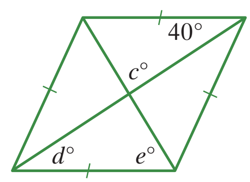


15 A quadrilateral has both pairs of opposite sides parallel and has four right angles. The quadrilateral is

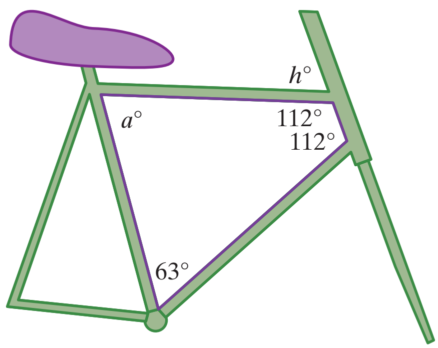
A a rectangle **B** a square **C** a rhombus

D a rectangle or a square **E** a rectangle, a square or a rhombus

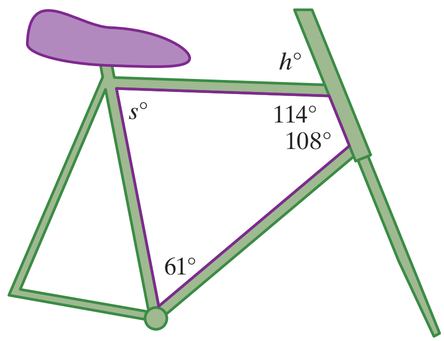
16 The diagonals of a quadrilateral are equal in length and intersect at right angles. Which of the special quadrilaterals (square, rectangle, rhombus, kite or trapezium) could the quadrilateral be? Is there more than one possibility? Justify your answer.

17 Part of the frame of the bicycle shown below forms a quadrilateral. The angle, $h°$, which the head tube makes with the horizontal is called the *head angle*. The angle, $s°$, which the seat tube makes with the horizontal is called the *seat angle*. Slight changes in these angles are important. Triathlon bikes generally have steeper seat angles than road bikes.

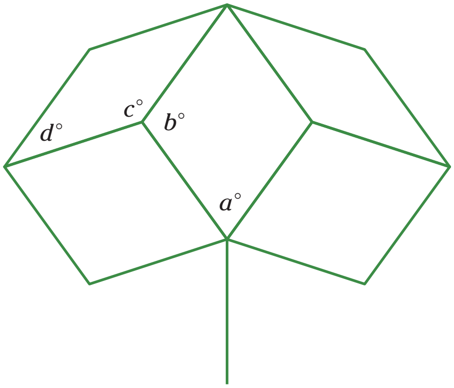
a Road bicycle



b Triathlon bicycle



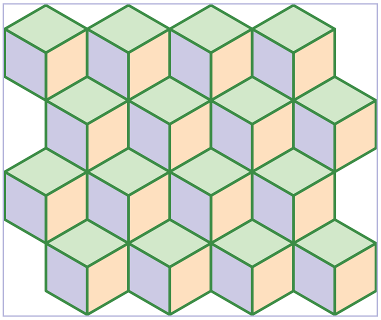
18 There are two different shaped rhombuses in the window below, one with angles labelled $a°$ and $b°$ and the other with angles $c°$ and $d°$. By looking at the way the rhombuses fit together at the centre, calculate the sizes of the angles in each of the two rhombuses.



19 In the patchwork quilt design shown below, three rhombuses combine to form each hexagon.

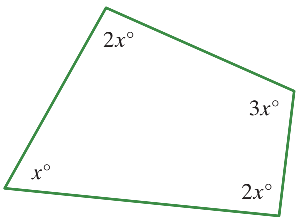
a By looking at the way the rhombuses fit together, calculate the sizes of the angles in each rhombus.

b Make a careful, accurate drawing of one of the rhombuses. Label the angle sizes.

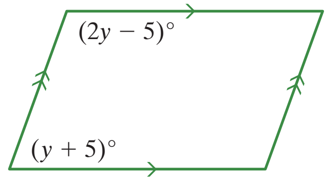


20 For each of the following
- **i** write an equation
- **ii** solve the equation for x
- **iii** find the size of each of the angles

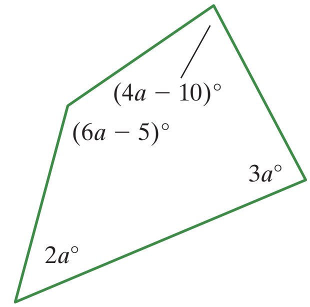
a



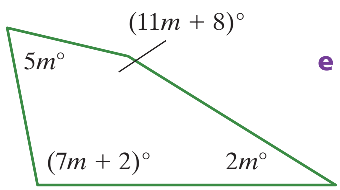
b



c



d



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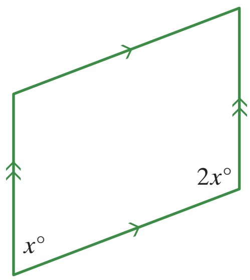

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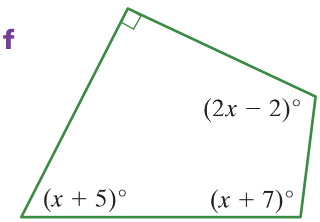

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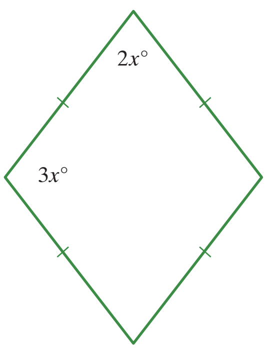

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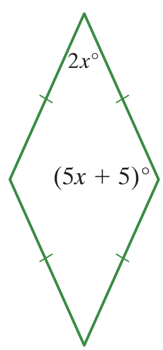
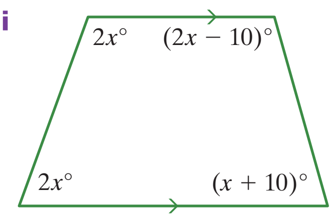


i



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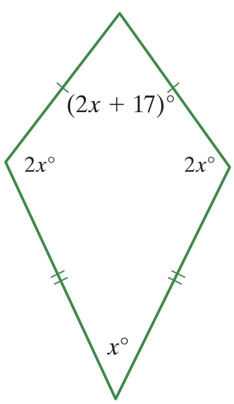

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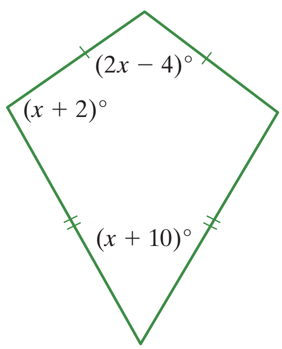

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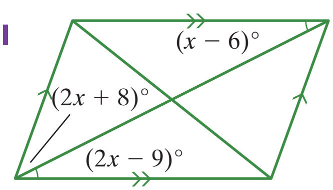
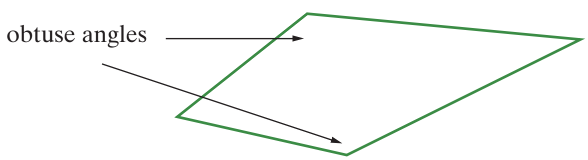


7.6 Using congruent triangles to explain quadrilateral properties

How do we know if a mathematical statement is true or false?

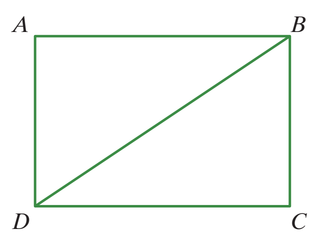
- **Definitions**
  Some statements are true 'by definition'. For example, both pairs of opposite sides of a parallelogram are parallel. This is true because this is the definition of a parallelogram.
- **Counterexamples**
  If we can find one example of a statement which is not true, then the statement is false. The example is called a counterexample. If a student claimed that a quadrilateral cannot have two obtuse angles, we could draw an example of a quadrilateral with two obtuse angles to show that the student's claim was wrong.



  Although finding a counterexample can show that a statement is false, we cannot claim that a statement is true just because we haven't found any counterexamples.
- **Deductive reasoning**
  We start with something we already know is true. We then develop a logical sequence of statements, using evidence to support our reasoning. The evidence we use must be based on definitions or statements that have already been shown to be true—either by mathematicians in the past or by ourselves. This type of reasoning is called deductive reasoning.

Example 37

Show that the diagonal BD of rectangle $ABCD$ divides the rectangle into two congruent triangles.



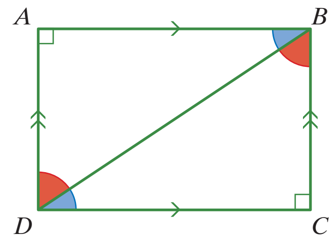
continued

Example 37 continued

Working

In $\triangle ABD$ and $\triangle CDB$
$\angle ABD = \angle CDB$ (alternate angles between parallel sides)
$\angle ADB = \angle CBD$ (alternate angles between parallel sides)
BD is shared by both triangles.
So $\triangle ABD \equiv \triangle CDB$ (ASA)

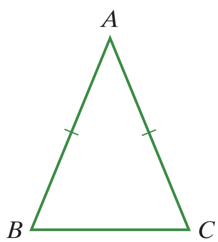
Reasoning



Example 38

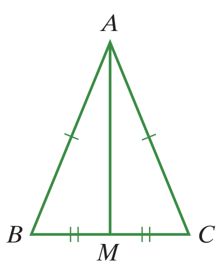
Show that the base angles of an isosceles triangle are equal.

Working



Show that $\angle ABC = \angle ACB$.

$AB = AC$ (definition of isosceles triangle)
M is the midpoint of BC (construction).



In $\triangle AMB$ and $\triangle AMC$
$AB = BC$ (definition of isosceles triangle)
$BM = MC$ (M is midpoint of BC)
AM is common to both triangles.
$\triangle AMB \equiv \triangle AMC$ (SSS)

So $\angle ABC = \angle ACB$ (matching angles in congruent triangles)

Reasoning

Draw a diagram and state what is to be shown.

When asked to show that a property is true, we need to have a plan of where we are going and how we are going to get there. It may be necessary to add other lines to the diagram.

By joining A to M, the midpoint of BC, we divide the isosceles triangle into two triangles. If we can show that these triangles are congruent, then the base angles must be equal.

Two triangles are congruent if three sides of one triangle are equal to three sides of the other triangle.

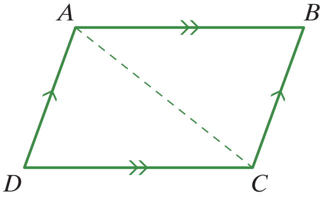
If two triangles are congruent, the angles of one triangle are equal to the corresponding (matching) angles of the other triangle.

We define a parallelogram is as 'a quadrilateral with opposite sides parallel'.

We can use this definition to show other properties of a parallelogram.

Example 39

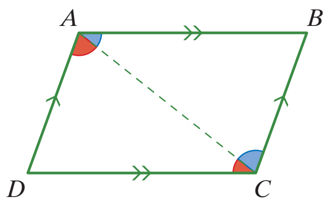
In this figure, $ABCD$ is a parallelogram and AC is a diagonal. From the definition of a parallelogram, we know that $AB \parallel DC$ and $AD \parallel BC$. Using this information and your knowledge of angles associated with parallel lines and transversals, show that $\triangle ABC$ is congruent to $\triangle CDA$. Justify each statement that you make.



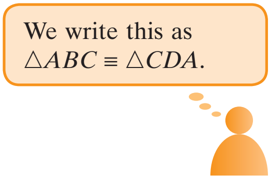
Working

In $\triangle ABC$ and $\triangle CDA$, side AC is common to both triangles.

$\angle DAC = \angle BCA$ (alternate angles, $AD \parallel BC$, AC is a transversal)
$\angle ACD = \angle CAB$ (alternate angles, $AB \parallel DC$, AC is a transversal)



$\triangle ABC \equiv \triangle CDA$ (ASA)



Reasoning

Look for clues: $\triangle ABC$ and $\triangle CDA$ share the side AC.

Diagonal AC is a transversal cutting across the parallel sides. This suggests that we could find equal angles in $\triangle ABC$ and $\triangle CDA$.

Two triangles are congruent if two angles of one triangle are equal to two angles of the other triangles and the sides joining the two equal angles are equal.

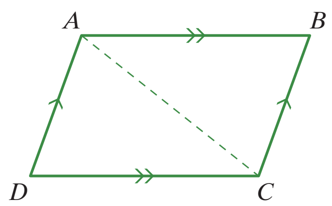
Example 40

Using what was shown in example 39, show that both pairs of opposite sides of a parallelogram are equal.

continued

Example 40 continued

Working



$\triangle ABC \equiv \triangle CDA$ (shown in example 39)
So
$AB = DC$
$AD = BC$

Reasoning

Matching sides of congruent triangles are equal.

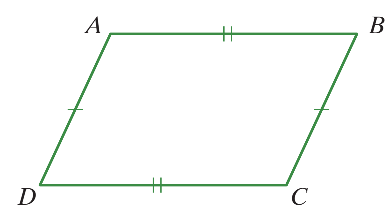
Is the converse (opposite) true?

We know that a quadrilateral with both pairs of opposite sides parallel is a parallelogram because this is the definition of a parallelogram. We have also shown that the opposite sides of a parallelogram are equal. However, can we assume that the converse is true? In other words, is a quadrilateral with both pairs of opposite sides equal necessarily a parallelogram?

Example 39 shows that a quadrilateral with both pairs of opposite sides equal is a parallelogram.

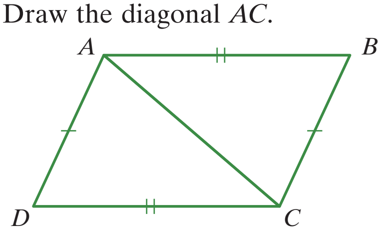
Example 41

Show that quadrilateral $ABCD$ is a parallelogram.



Working

Draw the diagonal AC.



$\triangle ABC \equiv \triangle CDA$ (ASA)
In $\triangle ABC$ and $\triangle CDA$
$AD = BC$ (given)
$AB = DC$ (given)
AC is shared by both triangles.
So $\triangle ABC \equiv \triangle CDA$ (SSS)

Reasoning

We could have drawn the diagonal BD instead and worked with $\triangle AB$ and $\triangle CDB$

Example 41

Working

So $\angle BCA = \angle DAC$

But $\angle BCA$ and $\angle DAC$ are alternate angles

So $AD \parallel BC$

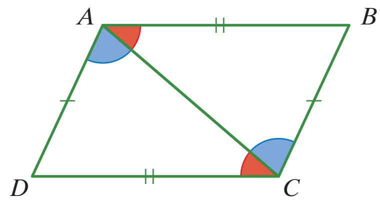
Similarly $\angle BAC = \angle DCA$.

But $\angle BAC$ and $\angle DCA$ are alternate angles

So $AB \parallel DC$.

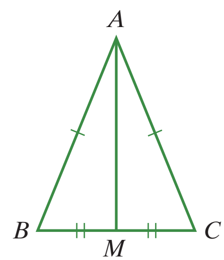
So quadrilateral $ABCD$ is a parallelogram.

Reasoning



exercise 7.6

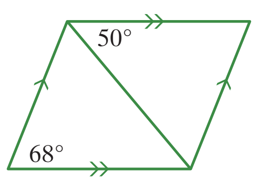
1 In example 38 it was shown that $\triangle AMB \equiv \triangle AMC$. Use this to show that $AM \perp BC$.



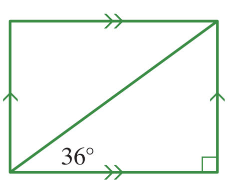
2 These special quadrilaterals have all been divided into two triangles by a diagonal.

- **i** Use your knowledge of parallel lines and special quadrilaterals to write the angle sizes in each triangle. Use symbols to mark the equal sides.
- **ii** In which of the quadrilaterals are two congruent triangles formed?

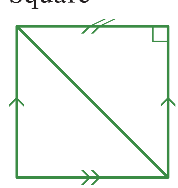
a Parallelogram



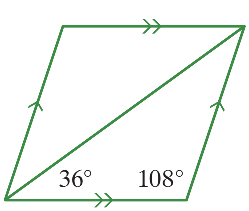
b Rectangle



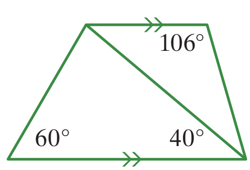
c Square



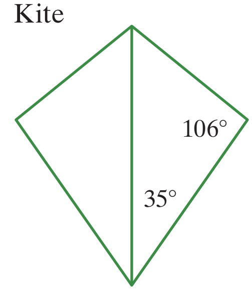
d Rhombus



e Trapezium



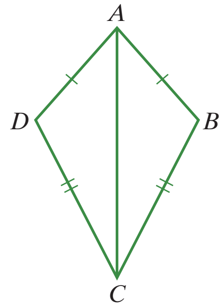
f Kite



3 $ABCD$ is a kite.

a Show that $\triangle ADC \equiv \triangle ABC$

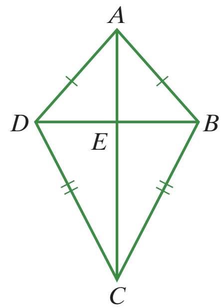
b Hence show that $\angle ADC = \angle ABC$



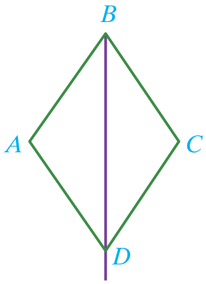
4 Now that we have shown that $\triangle ADC \equiv \triangle ABC$, we can show that other properties of the kite are true.

a Show that the diagonal AC bisects $\angle ADC$ and $\angle ABC$.

b Show that the diagonals of a kite intersect at right angles.



5 The tool shown below is an angle bisector. It consists of a rhombus hinged at the four vertices. One vertex of the rhombus can slide along the vertical slotted bar. As it does so, the length of BD changes.



a What name do we give to BD in the rhombus?

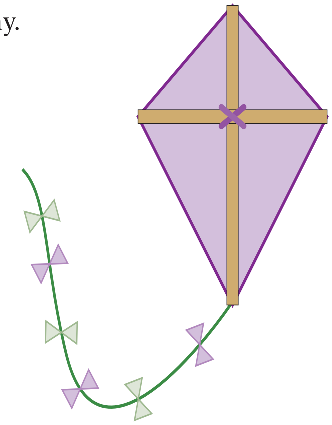
b What can you say about $\triangle ABD$ and $\triangle CBD$? Explain why.

c What does this tell you about $\angle ABD$ and $\angle CBD$?

d Explain why the tool is called an angle bisector.

e Is there any other special quadrilateral that would work in the same way?

6 Kenji made the frame of a kite by tying two sticks together as shown below. What properties of the mathematical kite was Kenji making use of in constructing the frame for his kite from the two sticks?

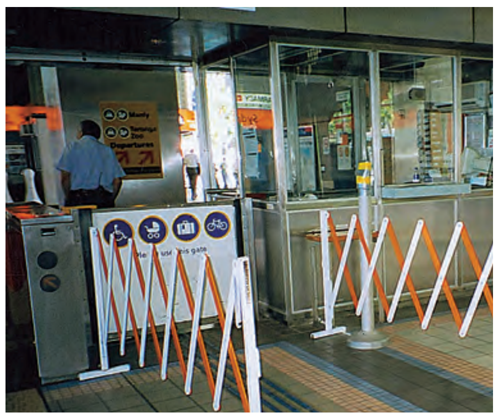


7 The photograph below shows a common car jack for lifting a car to change a wheel. The top of the jack is placed underneath the wheel axle and the horizontal screw is turned to raise or lower the jack.

a On which special quadrilateral shape is the jack based?

b Which property of this quadrilateral shape ensures that the car goes vertically upwards as the screw is turned?

8 The photograph shows temporary barriers at a railway station.

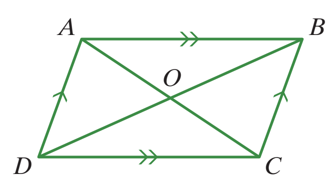


a On which special quadrilateral is the design of these barriers based?

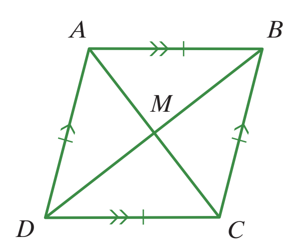
b Which properties of this special quadrilateral make it particularly suitable for using in the barriers?

exercise 7.6 challenge

9 Show that the diagonals of a parallelogram bisect each other. Hint: which triangles will you use?



10 Using the property you shown in question 6, show that the diagonals of a rhombus intersect at right angles.

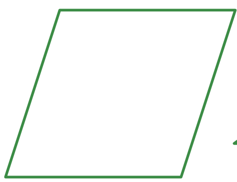


Analysis tasks

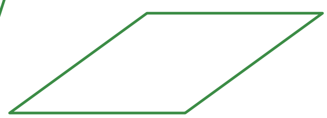
Penrose tiles

Most tessellations form a repeating pattern. This means it is possible to copy part of the tessellation and slide (translate) the copy (without rotation) to match up exactly with another region of the tessellation.

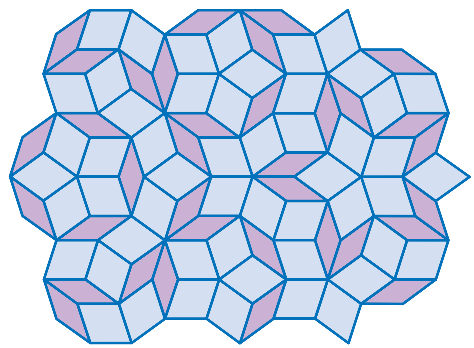
Certain tile shapes, however, can tessellate in such a way that there is no repeating pattern. Sir Roger Penrose, a mathematician at the University of Oxford in England, was investigating shapes to see if he could find shapes that would form non-repeating patterns. In1974 he realised that two simple rhombuses were able to tessellate for ever without ever repeating exactly the same arrangement. The two rhombuses became known as the fat and thin rhombuses and are also known now as Penrose tiles.



Fat rhombus



Thin rhombus



a By looking at the way the rhombuses fit together, work out the angle sizes of the fat and thin rhombuses.

b Using ruler and protractor or computer drawing tools, carefully construct a fat rhombus and a thin rhombus with sides 4 cm long and the angle sizes you have calculated.

Penrose tiles have been used in several paving and building designs in different places in the world. The photograph on the left below shows a pavement of Penrose rhombuses at the University of Oxford in England. The arcs on each rhombus serve the same purpose as the bumps and dents on the pieces of a jigsaw puzzle and must be matched when the rhombuses are fitted together. The fat and thin rhombuses can then fit together in only one way, resulting in the non-periodic tessellation. Without these bands, the rhombuses could, of course, be arranged to make a periodic tessellation.

c Make a drawing to show how the fat and thin rhombuses could make a repeating pattern.

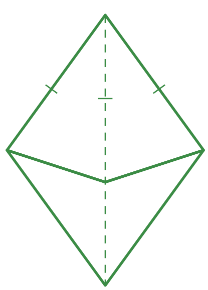
In Australia, Penrose tiles have been used in the design of Storey Hall at RMIT University in Melbourne, and in the floor of the Chemistry building at the University of Western Australia in Perth. Sir Roger Penrose is shown in the photograph on the right when he visited Storey Hall in 2000. The green bands on the rhombuses serve the same purpose as the arcs on the paving stones: to ensure that a non-repeating tessellation is produced.

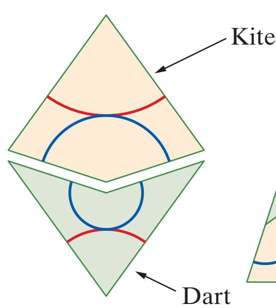
d It has been found that the ratio of fat rhombuses to thin rhombuses is equal to the golden ratio. This was the ratio of length to width of a rectangle that the ancient Greeks thought was most pleasing to the eye. It is equal to $\frac{1 + \sqrt{5}}{2}$:1.
Use your calculator to find the approximate value of this correct to 1 decimal place.

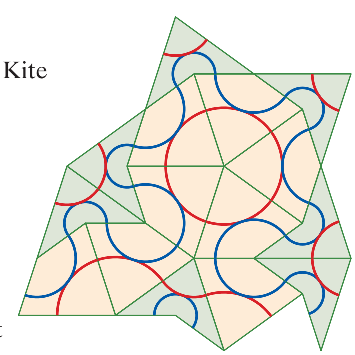
e Using the Penrose tile templates provided in the student e-book, cut out about 32 fat rhombuses and 20 thin rhombuses. Explain why these numbers of each tile have been suggested.

f Fit the tiles together, making sure that the bands match as shown above. Paste your tessellation onto a sheet of A3 paper.

Sir Roger Penrose found that he could create another pair of tiles that formed a non-repeating tessellation by first dividing the fat rhombus into two pairs of isosceles triangles. He did this by dividing the long diagonal of the fat rhombus in the golden ratio. Each pair of congruent isosceles triangles is then combined to form two shapes that are referred to as kites and darts. As for the rhombuses, the tiles are marked so that they can fit together in only one way. Blue arcs must join with blue arcs and red arcs must join with red arcs.







g Carefully copy the kite and dart. Calculate the size of each of the angles in each tile.

h Sir Roger Penrose found that in an infinite tessellation of kites and darts, the number of kites for each dart is equal to the golden ratio. Using the kite and dart templates provided in the student e-book, cut out about 32 kites and 20 darts. Fit them together, making sure that the bands match as shown above. Paste your tessellation onto a sheet of A3 paper.

Review Congruency and quadrilateral properties

Summary

Angle properties for parallel lines cut by a transversal

- Alternate angles are equal.
- Corresponding angles are equal.
- Cointerior (allied) angles are supplementary.

Triangle properties

- The three angles of any triangle add to 180°.
- An exterior angle of a triangle is equal to the sum of the two interior opposite angles.
- The base angles of an isosceles triangle are equal.
- The line segment joining the midpoint of the base of an isosceles triangle to the opposite vertex is perpendicular to the base.

Congruency

Congruent figures must satisfy *both* of these conditions:

- matching angles must be equal
- matching sides must be equal in length.

Congruent triangles

- SSS (three sides of one triangle equal to three sides of the other)
- SAS (two sides of one triangle equal to two sides of the other triangle and the angle between them the same)
- AAS (two angles of one triangle equal to the two angles of the other triangle and a matching side of the two triangles equal)
- RHS (both triangles right-angled, hypotenuse and another side of one triangle equal to the hypotenuse and another side of the other triangle)

Quadrilateral properties

The four angles of a quadrilateral add to 360°.

Parallelograms

- Both pairs of opposite sides are parallel and equal.
- Diagonals bisect each other.
- Both pairs of opposite angles are equal.

Special parallelograms

- Rectangles
  - Four right angles, diagonals are equal in length
- Rhombus
  - Four equal sides, diagonals are perpendicular
- Squares
  - Four right angles, four equal sides, diagonals are perpendicular and equal in length (squares are special rectangles and special rhombuses)

Trapeziums

- One pair of sides are parallel.

Kites

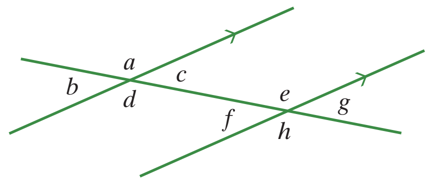
- Two pairs of adjacent sides are equal.
- One pair of opposite angles are equal.
- Diagonals are perpendicular.
- One diagonal bisects the unequal angles.

Visual map

alternate angles
cointerior angles
complementary angles
congruent
corresponding angles
diagonal
equilateral triangle
isometric transformation
kite
parallel
parallelogram
perpendicular isosceles triangle
quadrilateral
rectangle
reflection
rhombus
rotation
scalene triangle
square
supplementary angles
transformation
translation
transversal
trapezium
vertically opposite angles

Revision

Multiple-choice questions



Use the following diagram for questions 1 and 2.

1 The angle that is alternate to angle c is:

A angle a **B** angle b **C** angle d **D** angle e **E** angle f

2 The angle that is corresponding to angle h is

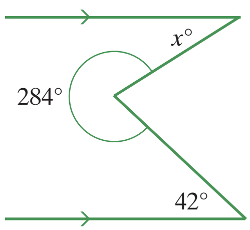
A angle b **B** angle d **C** angle e **D** angle f **E** angle g

3 If the angle at the vertex of an isosceles triangle is 46°, then each of the other two angles is:

A 22° **B** 67° **C** 72° **D** 134° **E** 144°

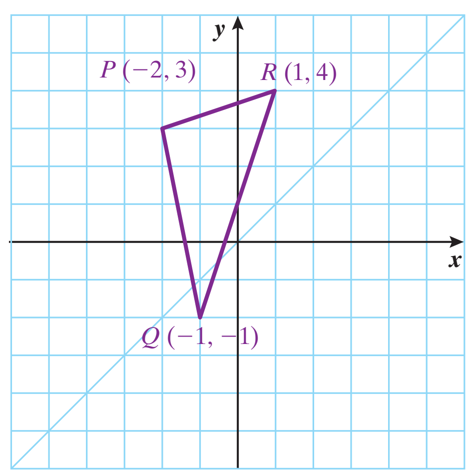
4 The value of x is

A 34
B 42
C 76
D 16
E 118



5 If triangle PQR is translated 4 units to the right and 3 units down, the coordinates of the image $P'Q'R'$ will be:

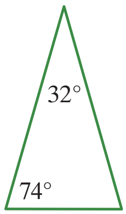
A $P'(-6, 0)$, $Q'(-5, -5)$, $R'(-3, 1)$
B $P'(-5, 7)$, $Q'(-4, 2)$, $R'(-2, 8)$
C $P'(2, 0)$, $Q'(3, -5)$, $R'(5, 1)$
D $P'(2, 6)$, $Q'(3, 1)$, $R'(5, 7)$
E $P'(-3, 2)$, $Q'(-2, 2)$, $R'(0, 3)$



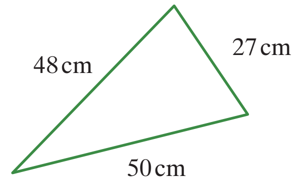
Short-answer questions

6 Identify each of these triangles according to their sides.

a

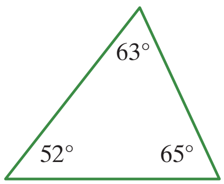


b

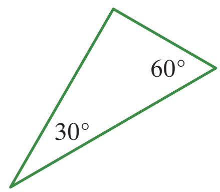


7 Identify each of these triangles according to their angles.

a

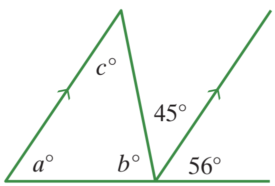


b

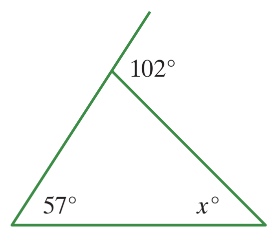


8 Find the value of the pronumerals.

a



b



9 Construct the following triangles, in each case labeling the given side lengths and angle sizes.

a two sides are 4.5 cm and 6.0 cm and the angle between them is 60°.

b the three sides are 5 cm, 8 cm and 9 cm.

c two angles are 40° and 70° and the side joining them is 5 cm.

10 Is it possible to construct triangles with the following side lengths? Explain.

a 5 cm, 8 cm, 13 cm **b** 7 cm, 12 cm, 13 cm **c** 8 cm, 9 cm, 20 cm

11 State whether each of the following statements is true or false.

a If the three angles of one triangle are exactly the same sizes as the three angles of another triangle, then the triangles must be congruent.

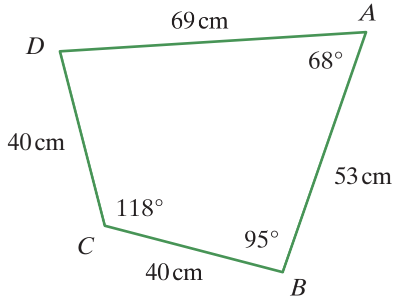
b If the three sides of one triangle are exactly the same lengths as the three sides of another triangle, then the triangles must be congruent.

12 **a** Name the matching pairs of sides in these two quadrilaterals.

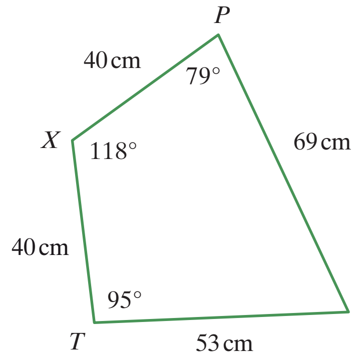
b Calculate the size of $\angle ADC$ and $\angle PFT$

c Are the quadrilaterals congruent?

d If Quadrilateral 1 is called $ABCD$, how do we name Quadrilateral 2?



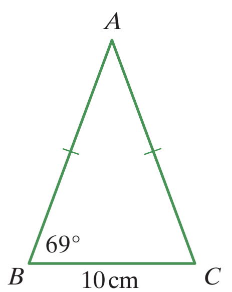
Quadrilateral 1

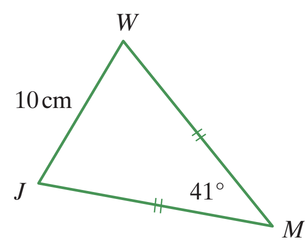


Quadrilateral 2

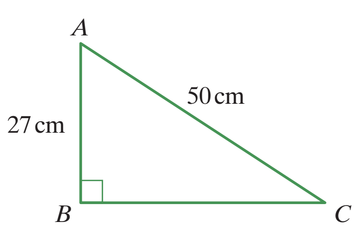
13 For each of these pairs of triangles, decide if the two triangles are congruent, justifying your answer in each case. For those pairs that you decide are congruent, name the triangles according to their matching vertices.

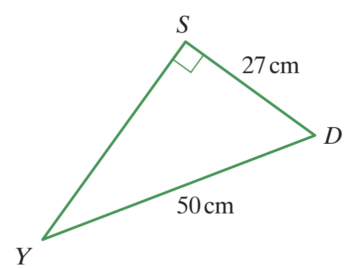
a



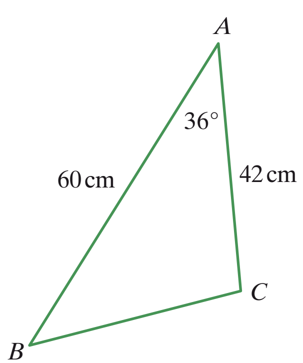


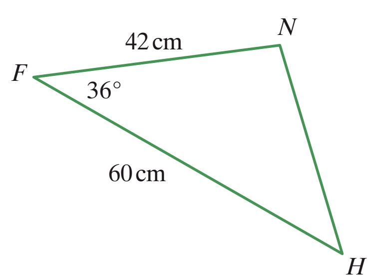
b





c





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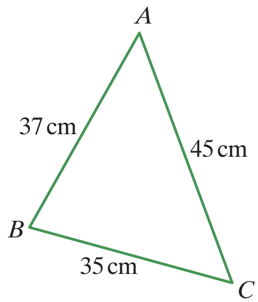
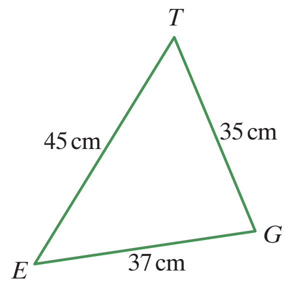
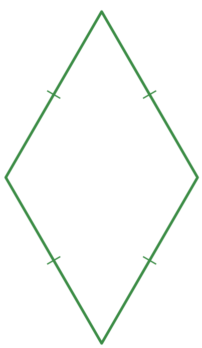




14 Identify these quadrilaterals.

a



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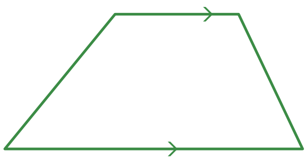
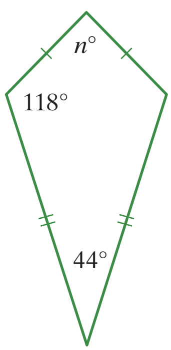


15 Find the value of the pronumerals.

a



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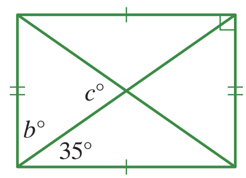

c