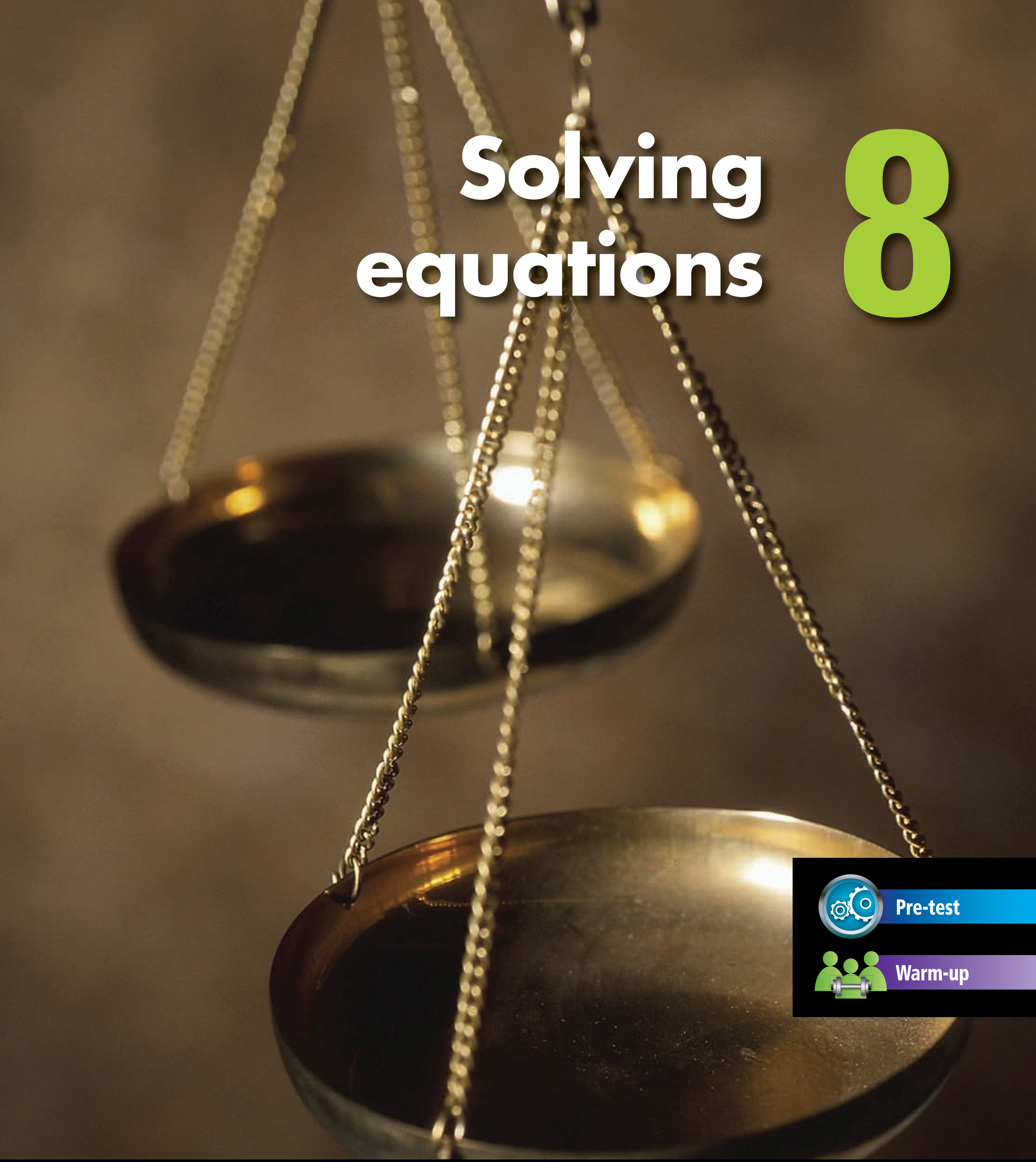
Solving equations 8



Pre-test

Warm-up

We can think of an algebraic equation as being like a set of scales. The two sides of the equation are equal, so the scales are balanced. If we add something to one side of the scales without adding something to the other side, the scales will no longer be balanced. We can solve equations by doing the same to both sides.

8.1 What is an equation?

Comparing the values of expressions

In mathematics, when two expressions have the same value we use an equals symbol ('=') to show this. For example, we know that 3×4 and $6 + 6$ each has the value 12, so we can write $3 \times 4 = 6 + 6$. This type of number statement is called an **equation** because it states that the expressions on each side of the equation are equal in value. We can also describe such a statement as **true**.

Example 1

In each of the following equations, insert a number on the blank line to make the equation true.

a $\_\_\_ + 5 = 17$

b $4 \times \_\_\_ = 32$

Working

a $\_\_\_ + 5 = 17$
$12 + 5 = 17$

b $4 \times \_\_\_ = 32$
$4 \times 8 = 32$

Reasoning

If the missing number is 12, the equation will be true.

If the missing number is 8, the equation will be true.

If two expressions are *not* equal in value, then mathematicians may use the symbol '≠' to show this. The statement $3 \times 7 = 4 \times 5$ is an example of this. Since the two expressions are not equal in value, the statement is **false** (not true) and the '≠' should be used instead of the '=' symbol $3 \times 7 \neq 4 \times 5$.

The following symbols are often used to compare the value of two expressions.

Symbol	Meaning	Example
$=$	is equal to	$3 - 1 = 2 \times 1$
$\neq$	is not equal to	$3 - 1 \neq 4 \times 1$
$>$	is greater than	$3 - 1 > 1 \times 1$
$<$	is less than	$3 - 1 < 4 \times 1$

Example 2

State whether each of the following statements is true or false.

a $5 + 2 = 2 \times 4 + 1$

b $3 \times 4 = 5 \times 2 + 2$

continued

Example 2 continued

Working	Reasoning
a $5 + 2 = 2 \times 4 + 1$ LS $= 5 + 2 = 7$ RS $= 2 \times 4 + 1$ $= 8 + 1$ $= 9$ LS $\neq$ RS so the statement is false.	If the statement is true, the right side and the left side must be equal. Here they are not equal so the statement is not true.
b $3 \times 4 = 5 \times 2 + 2$ LS $= 3 \times 4$ $= 12$ RS $= 5 \times 2 + 2$ $= 10 + 2$ $= 12$ LS $=$ RS so the statement is true.	The right side and the left side are equal so the statement is true.

Example 3

By changing only the expression on the right side of the equation $3 \times 4 = 6 + 6$, find three other expressions which make this statement true.

Working	Reasoning
$3 \times 4 = 9 + 3$	$3 \times 4 = 12$ $9 + 3 = 12$ Both expressions have the same value.
$3 \times 4 = 24 \div 2$	$3 \times 4 = 12$ $24 \div 2 = 12$ Both expressions have the same value.
$3 \times 4 = 18 - 6$	$3 \times 4 = 12$ $18 - 6 = 12$ Both expressions have the same value.

exercise 8.1

LINKS TO Example 1

1 In each of the following equations, insert a number on the blank line to make the equation true.

a $\_\_ + 7 = 12$
b $15 - \_\_ = 9$
c $3 \times \_\_ = 48$
d $9 + \_\_ = 6 + 8$
e $2 \times \_\_ + 3 \times 4 = 8 \times 2$
f $7 \times 9 - \_\_ \times 9 = 4 \times 9$
g $6 \times (3 + \_\_) = 6 \times 5$
h $5 + \_\_ \times 3 = 17$
i $(5 - \_\_) \times 6 = 18$
j $6 \times 5 - 24 \div \_\_ = 22$

LINKS TO Example 2

2 State whether each of the following statements is true or false.

a $3 + 4 = 4 - 2 + 5$

b $4 - 3 = 7 \times 1$

c $2 \times 6 = 4 \times 3 - 2$

d $\frac{12}{2} - 1 = 7 - 2$

e $16 \div 8 + 3 = 5 + 0$

f $2 \times 1 \times 3 = 6 \div 2 + 2$

g $12 - 3 \times 2 = 15 + 3$

h $10 + 14 \div 2 = 30 \div 2 + 2$

i $24 \div 3 + 1 = 72 \div 9 + 1$

j $2 \times 8 \div 4 = 20 \div 5$

k $3 \times (4 + 2) = 10 + 4 \times 2$

l $10 \div (3 - 1) = 42 \div 7$

LINKS TO Example 3

3 The equation $10 \times 4 = 32 + 8$ is true. Which one of the following could replace the expression on the right side of the equation whilst still making it a true statement?

A $6 + 2 \times 5$

B $5 \times 5 + 3$

C $5 \times 10 - 1$

D $5 \times 2 \times 3 + 1$

E $30 + 5 \times 2$

LINKS TO Example 3

4 The statement $4 \times (3 + 2) = 28 - 8$ will become false if the left side is replaced with:

A $3 \times 8 - 4$

B $(10 + 2) \times 5$

C $36 - 4^2$

D $4 + 8 \times 2$

E $25 - 10 \div 2$

5 Rani says that if a statement such as $19 - 2 \times 3 > 10$ is true, then it is also correct that $19 - 2 \times 3 \neq 10$. Is she right? Explain your answer.

exercise 8.1 challenge

LINKS TO Example 3

6 Change a single number in each of the following to turn the statement into an equation. Then replace the $<$, $>$ or $\neq$ with $=$.

a $2 \times 7 \neq 2 \times 5 + 2$

b $9 - 4 + 3 \neq 3 \times 5 + 3$

c $7 \times 3 - 5 \times 4 \neq 3 + 8$

d $18 - 6 \div 2 \neq 4 \times 3$

e $(18 - 9) \neq 20 \div 2 - 4$

f $3 + 3 > 10 - 7 + 1$

g $30 - 5 - 10 \div 2 < 24 - 3$

h $50 \div 10 - 4 < 20 \div 4 - 3$

i $25 - 3 \times 4 > 32 \div 8 + 5$

j $(12 - 8) \div 4 < 16 \div 4 - 1$

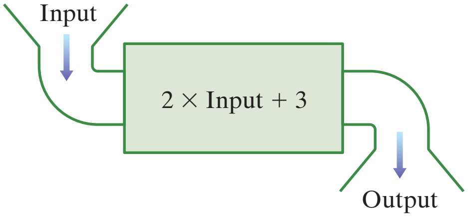
k $(18 - 6) \div 3 \times 5 > (5 + 1) \times 4$

l $16 \div 2 \div 8 \neq 30 \div 5 \div 2$

8.2 Input and output machines

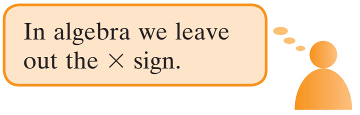
The equations considered so far have only contained numbers, and so you can tell that they are true statements. In an equation involving pronumerals as well as numbers, the statement may be true only for certain values of the pronumerals.

Consider a number machine that takes in numbers, doubles them and then adds 3.



$2 \times \text{Input} + 3 = \text{Output}$

Supposing the output number is 11. If the pronumeral n represents the input number, we can write $2 \times n + 3 = 11$ or simply, $2n + 3 = 11$.



$2n + 3 = 11$ is an equation. This means that the left side must equal the right side. There is only one value of n that makes this equation true. We can see that if the input number was 4, then the output number would be $2 \times 4 + 3$, which equals 11.

Finding a value of the pronumeral that makes an equation true is called **solving** the equation. A value of the pronumeral that makes an equation true is called a **solution** for the equation.

The solution to the equation $2n + 3 = 11$ is $n = 4$.

In example 4 the number machine processes are turned into algebraic expressions and then an equation is written. In later sections of this chapter we will look at ways of solving equations.

When translating a written statement into an algebraic expression there are some key words that indicate the operations involved in the expression. Some examples are provided in the following table. Notice that it does not matter which pronumeral we choose for the unknown number.

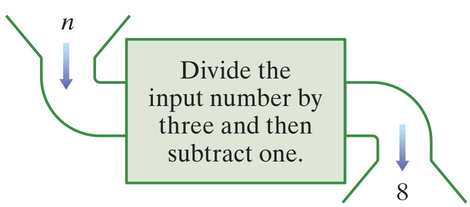
Addition		Subtraction	
Statement	**Expression**	**Statement**	**Expression**
A number plus two	$a + 2$	A number minus four	$d - 4$
The sum of a number and five	$b + 5$	The difference between ten and a number	$10 - e$
Four more than a number	$c + 4$	Six less than a number	$f - 6$

Multiplication		Division	
Statement	**Expression**	**Statement**	**Expression**
Four times a number	$4p$	A number divided by six	$\frac{m}{6}$
The product of three and a number	$3q$	The quotient of a number and eight	$\frac{n}{8}$
A number multiplied by five	$5r$	One fifth of a number	$\frac{t}{5}$

Example 4

Write an equation which represents the following number machines.

a



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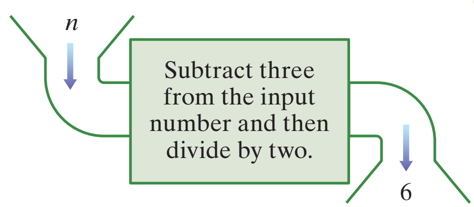


Working

a $\frac{n}{3} - 1 = 8$

b $\frac{n - 3}{2} = 6$

Reasoning

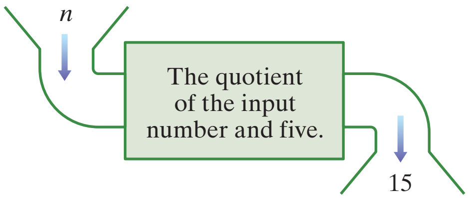
Divide the input number by 3 and then subtract 1. Put this expression equal to the output number.

Subtract 3 from the input number and then divide by 2. Put this expression equal to the output number.

Example 5

Write an equation which represents the following number machines.

a



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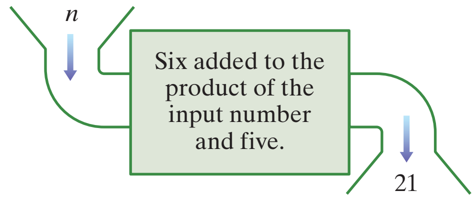


Working

a $\frac{n}{5} = 15$

b $5n + 6 = 21$

Reasoning

Divide the input number by 5. Put this expression equal to the output number.

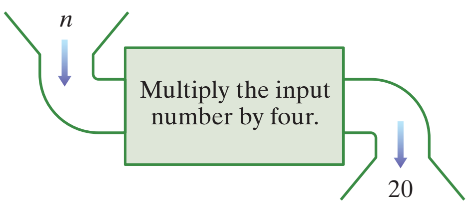
Multiply the input number by 5 and add 6. Put this expression equal to the output number.

exercise 8.2

LINKS TO Example 4

1 Write an equation which represents the following number machines.

a



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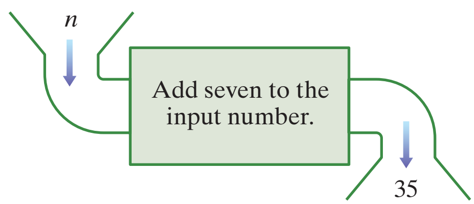

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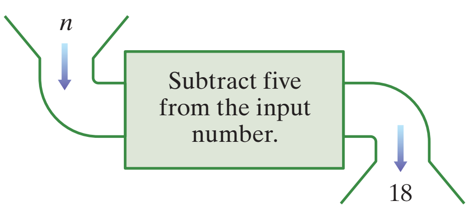

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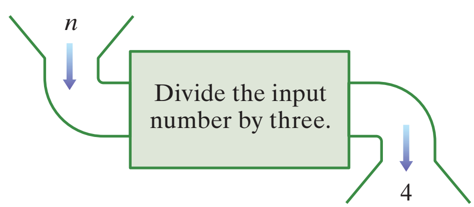

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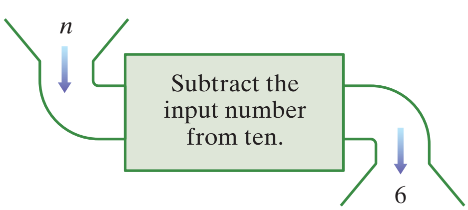

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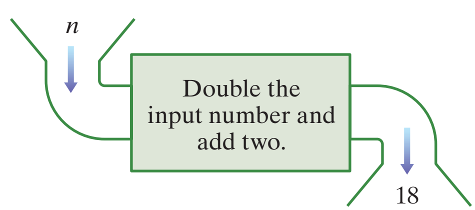

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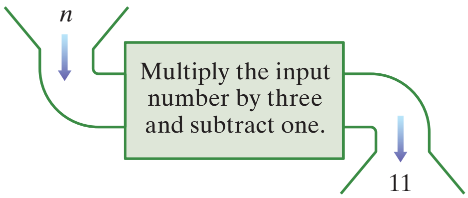

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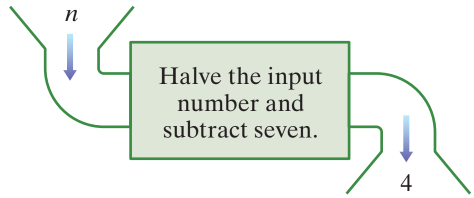

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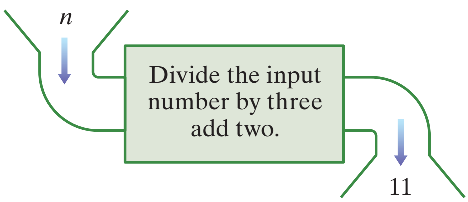

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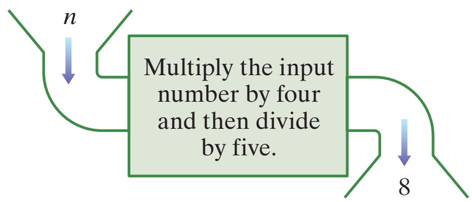

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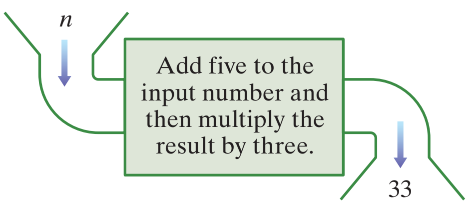

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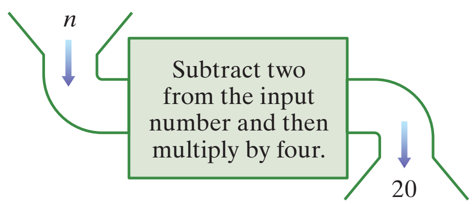
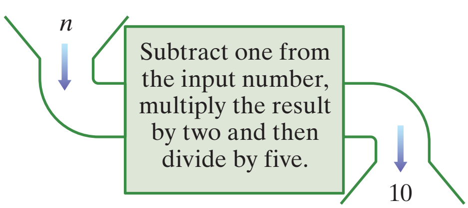


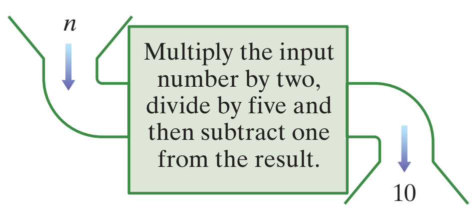
2 Which one of the following number machines is represented by the equation

$$\frac{2x-1}{5} = 10?$$

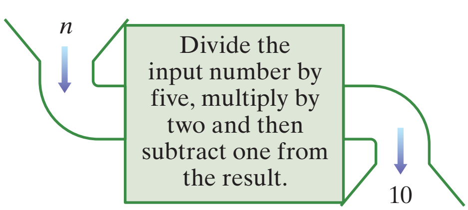
A



B



C



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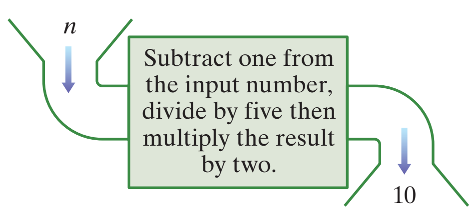

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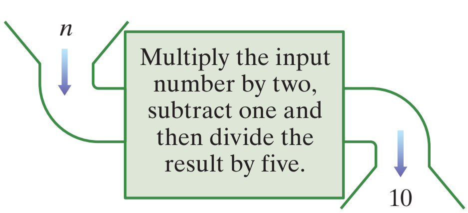
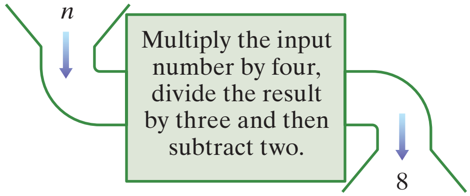


3 Which one of the following equations represents the number machine?



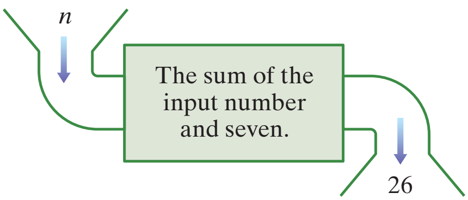
A $\frac{4(n-2)}{3} = 8$ **B** $\frac{4n-2}{3} = 8$ **C** $\frac{4n}{3} - 2 = 8$

D $4n - \frac{2}{3} = 8$ **E** $4\left(n - \frac{2}{3}\right) = 8$

LINKS TO Example 5

4 Write an equation that represents each of the following number machines.

a



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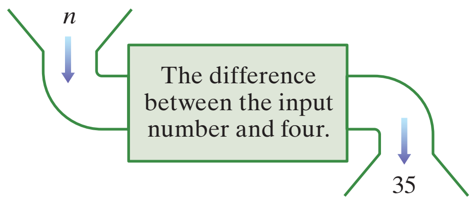

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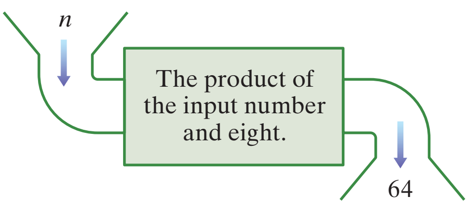

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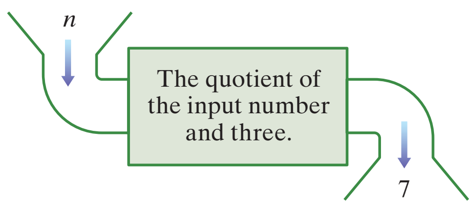

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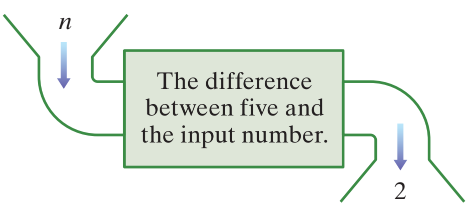

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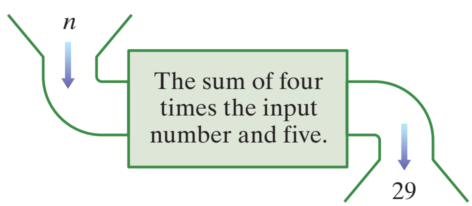

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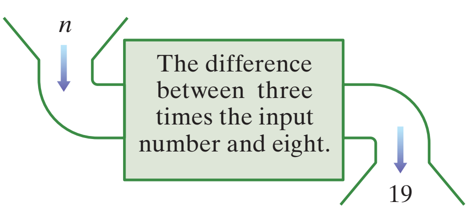

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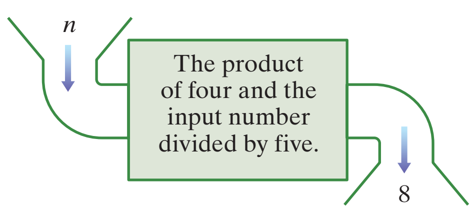

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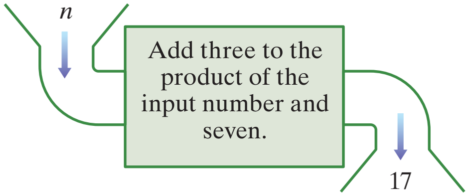

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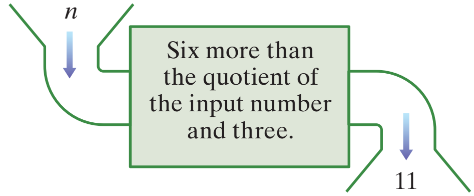

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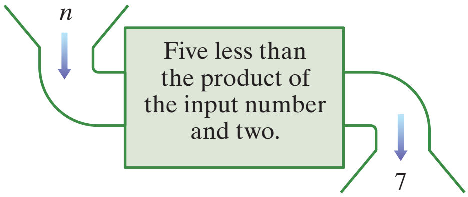

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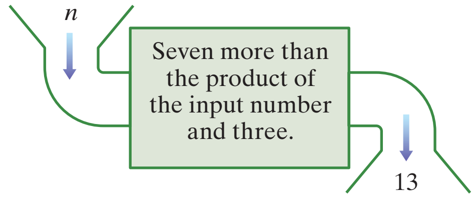
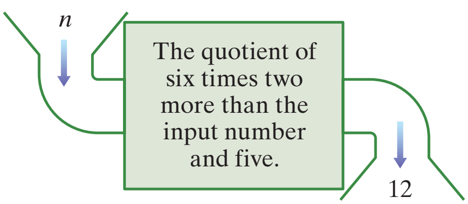


exercise 8.2 challenge

5 Which one of the following equations represents the number machine below?



A $\dfrac{6n + 2}{5} = 12$ **B** $\dfrac{6n}{5} + 2 = 12$ **C** $6n + \dfrac{2}{5} = 12$

D $\dfrac{6(n + 2)}{5} = 12$ **E** $\dfrac{5}{6(n + 2)} = 12$

6 Adrienne uses a two-step number machine. The outputs for three input values are shown in the table below.

Input	4	8	10
Output	10	22	28

a If the input number is 5, what is the output number?

b If the output number is 31, what is the input number?

c If the output number is x, what is the output number?

8.3 Solving equations: arithmetic strategies

Finding the value(s) of the pronumeral that makes an equation true is called solving the equation. A value of the pronumeral that makes an equation true is called a solution for the equation. This section considers some ways of solving equations using arithmetic rather than algebra.

Substitution

In section 8.2 we saw that the equation $2n + 3 = 11$ was true if $n = 4$.

If we substitute other values for n we find that the right side (RS) is not equal to the left side (LS).

n	$2n + 3$	Value of LS	Value of RS	LS = RS?
0	$2 \times 0 + 3$	3	11	False
1	$2 \times 1 + 3$	5	11	False
2	$2 \times 2 + 3$	7	11	False
3	$2 \times 3 + 3$	9	11	False
4	$2 \times 4 + 3$	11	11	True
5	$2 \times 5 + 3$	13	11	False
6	$2 \times 6 + 3$	15	11	False

By substituting a value for the pronumeral in an equation, we can find if that value makes the equation true.

Example 6

Use substitution to determine if the number given in brackets is a solution to the equation.

a $3x - 7 = 11$ (6)

b $\frac{2p}{3} - 1 = 4$ (9)

Working	Reasoning
a $3x - 7$	Work with the LS of the equation.
$= 3 \times 6 - 7$	Substitute the value for x.
$= 18 - 7$ $= 11$	Simplify.
$x = 6$ is a solution to the equation.	LS = RS

continued

Example 6 continued

Working	Reasoning
b $\frac{2p}{3} - 1$	Work with the LS of the equation.
$= \frac{2 \times 9}{3} - 1$	Substitute the value for p.
$= \frac{18}{3} - 1$ $= 6 - 1$ $= 5$	Simplify. LS = 5, RS = 4
$p = 9$ is not a solution to the equation.	LS $\neq$ RS

Observation

The solution to an equation may be obvious just from looking at the equation. For example, we can see that the solution to the equation $x + 3 = 12$ is $x = 9$.

Example 7

Solve the following equations by observation.

a $3x = 15$ **b** $\frac{s}{4} = 6$ **c** $m - 7 = 9$

Working	Reasoning
a $3x = 15$ $3 \times 5 = 15$ $x = 5$	Write down the equation. Think of a number that can be multiplied by 3 to give 15. Try 5. Write down the solution to the equation.
b $\frac{s}{4} = 6$ $\frac{24}{4} = 6$ $s = 24$	Write down the equation. Think of a number that can be divided by 4 to give 6. Try 24. Write down the solution to the equation.
c $m - 7 = 9$ $16 - 7 = 9$ $m = 16$	Write down the equation. Think of a number from which you can subtract 7 and get 9. Try 16. Write down the solution to the equation.

Table of values

We can find the value of the pronumeral that makes an equation true by constructing a table of values for the left side.

Example 8

Use the table of values provided to find the value of x that makes the equation $2x + 7 = 19$ true.

x	$2x + 7$
1	9
2	11
3	13
4	15
5	17
6	19
7	21

Working

$x = 6$

Reasoning

When $x = 6$, $2x + 7$ has the value 19. The right side of the equation is 19.

x	$2x + 7$
6	19

Guess, check and improve

Another useful technique is *guess, check and improve*—try a value for the variable, and then check whether this value makes the equation a true statement. You can repeat this with other values until you get a value which does make the LS = RS.

Example 9

Find the value of x that will make the equation $2x - 3 = 9$ a true statement.

Working	Reasoning
$2x - 3$ $= 2 \times 10 - 3$ $= 20 - 3$ $= 17$	Work with the LS of the equation. Substitute a value for x. Make a guess that $x = 10$ might work. Simplify.
$17 \neq 9$ so the equation is not true for $x = 10$	LS $\neq$ RS
	$x = 10$ was too big so try a smaller value.
$2x - 3$ $= 2 \times 6 - 3$ $= 12 - 3$ $= 9$	Work with the LS of the equation Substitute a value for x. This time, make a guess that $x = 6$ might work. Simplify.
$9 = 9$ so the equation is true for $x = 6$	LS = RS

exercise 8.3

LINKS TO Example 6

1 Use substitution to determine if the number given in brackets is a solution to the equation.

a $5x = 15; (3)$ **b** $\frac{y}{4} = 5; (28)$ **c** $p - 1 = 7; (6)$

d $2x - 5 = 15; (10)$ **e** $5x - 4 = 7; (2.2)$ **f** $\frac{m}{9} + 9 = 12; (15)$

g $3(x - 4) = 6; (6)$ **h** $\frac{2y}{3} - 3 = 2; (6)$ **i** $4(2z + 3) = 20; (2)$

j $\frac{2(m - 5)}{3} = 6; (13)$ **k** $3x - 0.2 = 12.1; (4.1)$ **l** $\frac{2(4m + 3)}{5} = 5; \left(\frac{1}{2}\right)$

2 If $p = 3$, which of the following are true statements?

a $3p = 9$ **b** $p + 4 = 12$ **c** $p - 1 = 2$

d $\frac{4p - 2}{5} = 3$ **e** $3p + 1 = 10$ **f** $\frac{p}{3} - 1 = 1$

LINKS TO Example 7

3 In each of the following, use the table of values provided to find the value of the variable that makes the equation true.

a $3x + 1 = 13$

x	$3x + 1$
1	4
2	7
3	10
4	13
5	16

b $2y - 4 = 12$

y	$2y - 4$
4	4
6	8
8	12
10	16
12	20

c $2x + 1 = 9$

x	$2x + 1$
1	1
2	3
3	5
4	7
5	9

LINKS TO Example 7

4 Complete each of the following tables to find the value of the variable that makes the equation true.

a $3x - 1 = 14$

x	$3x - 1$
1	
2	
3	
4	
5	

b $5x + 7 = 27$

x	$5x + 7$
0	
1	
2	
3	
4	

c $2x - 3 = 9$

x	$2x - 3$
4	
5	
6	
7	
8	

d $4p + 1 = 13$

p	$4p + 1$
0	
1	
2	
3	
4	

e $5x - 1 = 44$

x	$5x - 1$
6	
7	
8	
9	
10	

f $\frac{m}{2} - 5 = 1$

m	$\frac{m}{2} - 5$
10	
12	
14	
16	
18	

LINKS TO Example 8

5 Solve the following equations by observation.

a $a + 2 = 6$ **b** $b + 4 = 7$ **c** $c - 1 = 12$ **d** $d - 4 = 15$

e $e + 8 = 23$ **f** $f - 6 = 21$ **g** $2g = 12$ **h** $5h = 40$

i $7i = 77$ **j** $\frac{j}{3} = 12$ **k** $\frac{k}{5} = 4$ **l** $\frac{l}{7} = 6$

LINKS TO Example 9

6 Use 'guess, check and improve' to find solutions to these equations.

a $2x + 19 = 51$ **b** $4x - 11 = 25$ **c** $3x + 7 = 34$ **d** $5x - 17 = 18$

exercise 8.3 challenge

LINKS TO Example 9

7 Use 'guess, check and improve' to find solutions to these equations.

a $3x + 5 = 71$ **b** $11x - 7 = 202$ **c** $17x + 23 = 159$ **d** $13x - 6 = 137$

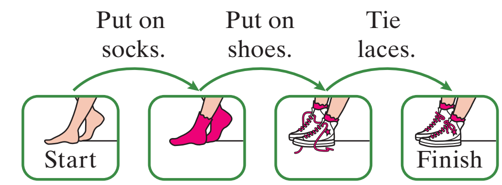
8.4 Forward tracking and backtracking

A flow chart provides a way of visualising a sequence of steps.

For instance, before going out at the start of the day, Sandy

- puts on her socks
- then puts on her shoes
- and finishes by tying her shoelaces.

The flow chart shows the order of the steps.
This is called **forward tracking**.

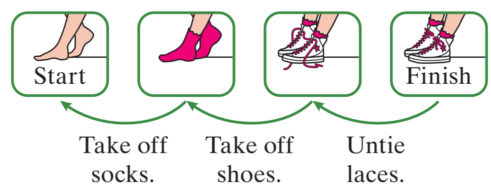


Sandy puts her shoes on—'forward' tracking!

At the end of the day, Sandy reverses these steps and

- unties her shoelaces
- takes off her shoes
- then takes off her socks.

This reverse process is called **backtracking**.



Sandy takes her shoes off—backtracking!

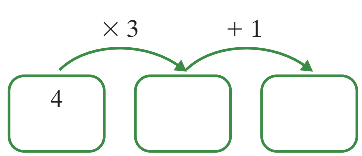
Forward tracking with numbers

In a number flow chart, there is a sequence of operations on the input number. Each operation leads us to working out the next number in the flow chart.

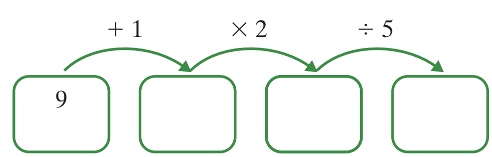
Example 10

Complete the following flow charts and find the output number.

a

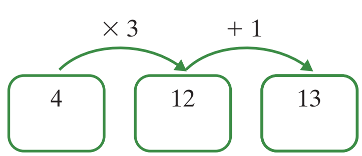


b



Working

a



The output number is 13.

Reasoning

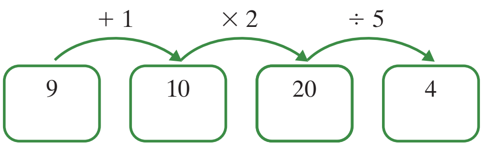
Perform each operation as you move from left to right through the flow chart.

continued

Example 10 continued

Working

b



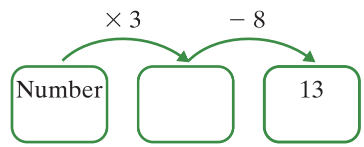
The output number is 4.

Reasoning

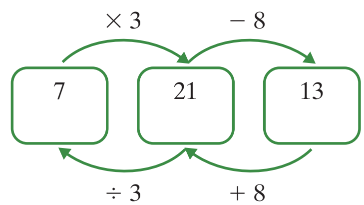
Perform each operation as you move from left to right through the flow chart.

Backtracking with numbers

Consider the following puzzle. Trish thinks of a number, multiplies it by three then subtracts eight and the result is thirteen. The puzzle can be written as a flow chart, shown at right.



To find Trish's number, we work our way backwards through the flow chart doing the opposite. This is **backtracking**.



So Trish's number is 7.

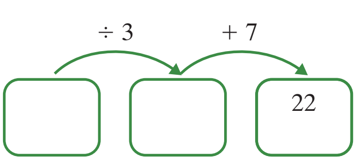
When we backtrack with numbers, we must undo what has been done to the input number. We do this by working backwards in the flow chart, carrying out the opposite or **inverse** number operation; for example, if 8 has been added, then we undo this by subtracting 8.

Operation	Inverse operation
+	−
−	+
×	÷
÷	×

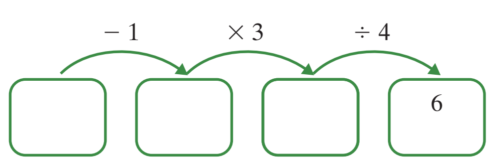
Example 11

Use backtracking to find the input number for each of the following flow charts.

a



b

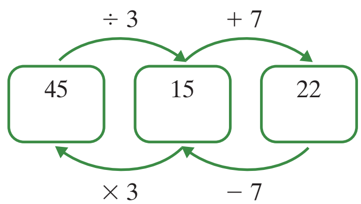


continued

Example 11 continued

Working

a

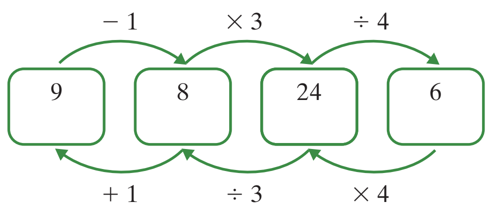


The input number is 45.

Reasoning

Working from right to left, the inverse of +7 is −7 so 22 − 7 is 15.
The inverse of ÷ 3 is × 3 so 15 × 3 is 45.

b



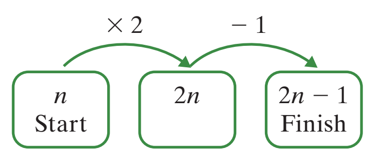
The input number is 9.

Working from right to left, the inverse of ÷ 4 is × 4 so 6 × 4 is 24.
The inverse of × 3 is ÷ 3 so 24 ÷ 3 is 8.
The inverse of −1 is +1 so 8 + 1 is 9.

Using forward tracking to build algebraic expressions

Using flow charts we can build up algebraic expressions. Instead of starting with a known number, we start with a pronumeral. At each step we write the new expression.

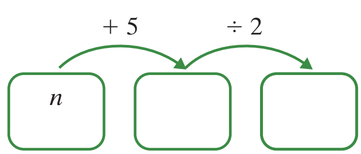
Supposing we start with a number n then multiply it by 2. This gives the expression $2n$. Next we subtract 1, so we now have the expression $2n - 1$.



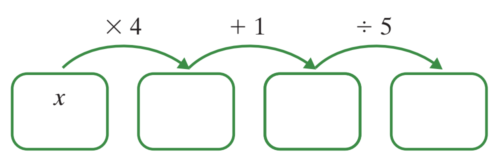
Example 12

Write an algebraic expression that represents each of the following flow charts.

a

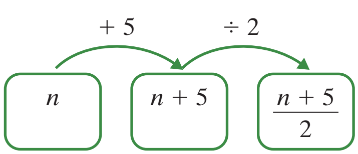


b



Working

a



Reasoning

First add 5 to n. This gives $n + 5$.

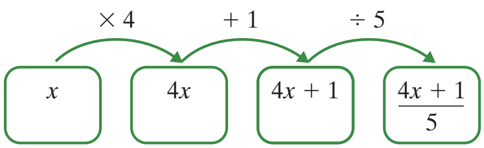
Then divide by 2. This gives $\frac{n + 5}{2}$.

continued

Example 12 continued

Working

b



Reasoning

First multiply x by 4. This gives $4x$.

Then add 1. This gives $4x + 1$.

Finally divide by 5. This gives $\frac{4x + 1}{5}$.

We can work out how an algebraic expression has been built up by thinking about the usual order of operations with numbers. In the expression $3n + 1$ for example, multiplication would be done before addition. So n is multiplied by 3 and then 1 is added.

Example 13

In each of these expressions, what is the first operation that has been carried out on n?

a $2n + 5$ **b** $\frac{n}{3} + 4$ **c** $2(n + 4)$ **d** $\frac{n - 3}{2}$

Working	Reasoning
a n is multiplied by 2	n is multiplied by 2 then 5 is added to the result.
b n is divided by 3	n is divided by 3 then 4 is added to the result.
c 4 is added to n	Brackets are worked out before multiplication. 4 is added to n then the result is multiplied by 2.
d 3 is subtracted from n	We can think of $\frac{n - 3}{2}$ as $\frac{(n - 3)}{2}$. 3 is subtracted from n then the result is divided by 2.

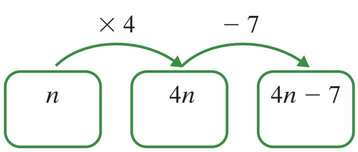
Example 14

Construct a flow chart for each of the following algebraic expressions.

a $4n - 7$ **b** $\frac{2p}{3} + 5$

Working

a



Reasoning

Let the input number be n.
Multiply n by 4. This gives $4n$.
Then subtract 7. This gives $4n - 7$.

continued

Example 14 continued

Construct a flow chart for each of the following algebraic expressions.

a $4n - 7$

b $\frac{2p}{3} + 5$

Working

a n —($\times 4$)→ $4n$ —(-7)→ $4n - 7$

b p —($\times 2$)→ $2p$ —($\div 3$)→ $\frac{2p}{3}$ —($+5$)→ $\frac{2p}{3} + 1$

Reasoning

Let the input number be n.
Multiply n by 4. This gives $4n$.
Then subtract 7. This gives $4n - 7$.

Let the input number be p.
Multiply p by 2. This gives $2p$.
Then divide by 3. This gives $\frac{2p}{3}$.
Finally add 5. This gives $\frac{2p}{3} + 5$.

exercise 8.4

LINKS TO Example 10

1 Copy and complete the following flow charts and find the output number.

a 5 —($\times 3$)→ ☐ —($+ 2$)→ ☐

b 7 —($+ 5$)→ ☐ —($\div 4$)→ ☐

c 20 —($\div 10$)→ ☐ —($\div 2$)→ ☐

d 18 —($- 7$)→ ☐ —($\times 5$)→ ☐

e 12 —($\div 8$)→ ☐ —($\times 1.5$)→ ☐

f 14 —($\times \frac{1}{2}$)→ ☐ —($+ 3$)→ ☐

g 1 —($+ 3.2$)→ ☐ —($\times 5$)→ ☐

h 18 —($\times 0.1$)→ ☐ —($+ 3.4$)→ ☐

i $\frac{1}{4}$ —($+\frac{1}{2}$)→ ☐ —($\div 3$)→ ☐

j 9 —($+ 3$)→ ☐ —($\times 2$)→ ☐ —($\div 3$)→ ☐

k 3 → (+ 5) → □ → (× 2) → □ → (+ 6) → □

l 18 → (÷ 3) → □ → (+ 1) → □ → (× 4) → □

m 30 → (÷ 5) → □ → (÷ 3) → □ → (× 2) → □

n 8 → (× 9) → □ → (÷ 12) → □ → (+ 4) → □

2 For parts a and b, copy and complete the flow chart and use it to find the output number.

a Kristopher starts with the number 12, divides by 3 then adds 9.

□ → □ → □

b Toby starts with the number 12, adds 9 and then divides by 3.

□ → □ → □

c Do Kristopher and Toby obtain the same output number? Why or why not?

LINKS TO Example 11

3 Use backtracking to find the input number for each of the following flow charts.

a □ → (× 2) → □ → (− 1) → 15

b □ → (× 2) → □ → (+ 7) → 25

c □ → (÷ 3) → □ → (× 2) → 16

d □ → (− 5) → □ → (× 4) → 20

e □ → (÷ 4) → □ → (− 3) → 1

f □ → (× 7) → □ → (− 5) → 51

g □ → (× 2) → □ → (+ 3) → 9

h □ → (+ 4) → □ → (× 3) → 60

i $+ 4$, $- 2$, $\div 5$: □ → □ → □ → 2

j $- 3$, $\div 5$, $+ 4$: □ → □ → □ → 7

k $\div 11$, $+ 2$, $\times 5$: □ → □ → □ → 40

l $\times 50$, $+ 8$, $\div 12$: □ → □ → □ → 9

LINKS TO Example 12

4 Copy and complete these flow charts. Write the algebraic expression which represents each flow chart.

a $\times 2$, $+ 4$: x → □ → □

b $\div 3$, $+ 7$: y → □ → □

c $+ 5$, $\div 4$: p → □ → □

d $+ 2$, $\times 9$: y → □ → □

e $- 7$, $\div 2$: k → □ → □

f $\times 5$, $- 1$: z → □ → □

g $\times 4$, $+ 1$, $\div 3$: m → □ → □ → □

h $- 7$, $\div 2$, $+ 5$: c → □ → □ → □

i $\div 4$, $+ 12$, $\times 3$: a → □ → □ → □

j $\times 3$, $\div 5$, $+ 4$: h → □ → □ → □

LINKS TO Examples 13, 14

5 For each of the following

i state the first operation that is carried out on n.

ii copy and complete the flow chart.

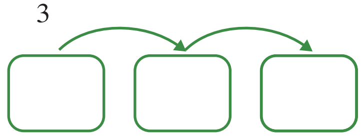
a $2n + 1$

□ → □ → □

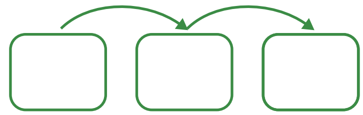
b $4n - 2$

□ → □ → □

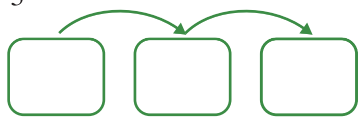
c $\dfrac{n-2}{3}$



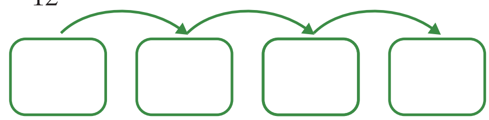
d $2(n-4)$



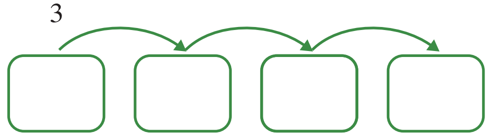
e $\dfrac{n}{3}+4$



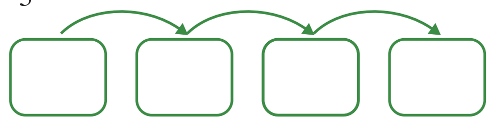
f $\dfrac{n-7}{12}+3$



g $\dfrac{7n-19}{3}$



h $\dfrac{2n}{3}+12$



6 Thuy thinks of a number, then adds 11, doubles the result and finally subtracts 4. If the starting number was n.

a Draw a flow chart.

b Write an expression which represents the final number for this puzzle.

c What would the finishing number be if the starting number was 7?

d What would the starting number be if the finishing number was 22?

7 Hannibal thinks of a number, then adds 1, halves the result and finally adds 4. If the starting number was n.

a Draw a flow chart.

b Write an expression which represents the final number for this puzzle.

c What would the finishing number be if the starting number is 5?

d What would the starting number be if the finishing number is 7?

exercise 8.4 challenge

8 Thom thinks of a number, then multiplies it by 4 and then adds 6. He halves the result, then subtracts the number he first thought of. Finally, Thom subtracts 3.

a Draw a flow chart to represent this number puzzle.

b Write an expression for the final number in terms of a starting number n.

c What is the final number if the starting number is

i 1? **ii** 2? **iii** 3?

d What do you notice about your answers from part c? Explain why this has occurred.

8.5 Solving equations by backtracking

We saw in section 8.4 how flow charts are useful for

- forward tracking to find the finishing number if we know the starting number.
- reversing the steps to find a starting number if we know the finishing number.
- building up algebraic expressions.

In this section we will use backtracking to solve equations. This means finding the starting number that makes the equation true.

Example 15

For each of these number operations

i write an equation.

ii draw a flow chart and use backtracking to find the number.

a 4 is added to a number, n, and the result is 13

b a number, x, is multiplied by 5 and the result is 35

c 7 is subtracted from a number, a, and the result is 11

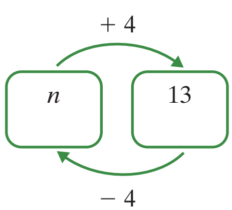
d a number, b, is divided by 8 and the result is 6

Working

a i $n + 4 = 13$

'4 is added to n' means $n + 4$. Put this expression equal to 13.

ii



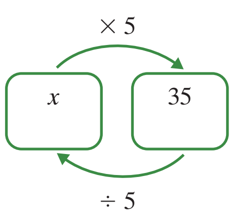
Undo + 4 by subtracting 4. $13 - 4 = 9$

$n = 9$

b i $5x = 35$

'x is multiplied by 5' means $5 \times x$. We write this as $5x$. Put this expression equal to 35.

ii



Undo × 5 by dividing by 5. $35 \div 5 = 7$

$x = 7$

continued

Example 15 continued

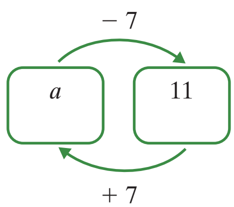
Working

c i $a - 7 = 11$

Reasoning

'7 is subtracted from a number a' means $a - 7$. Put this expression equal to 11.

ii



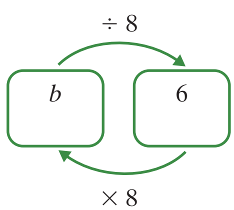
Undo -7 by adding 7.
$11 + 7 = 18$

$a = 18$

d i $\frac{b}{8} = 6$

'b is divided by 8' means $b \div 8$. We write this as $\frac{b}{8}$. Put this expression equal to 6.

ii



Undo $\div 8$ by multiplying by 8.
$6 \times 8 = 48$

$b = 48$

Example 16

Rebus thinks of a number, multiplies it by 2, and then adds 3, giving a finishing number of 15.

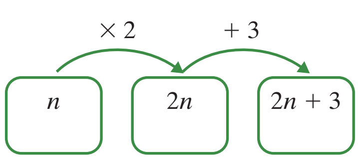
a Using n for Rebus' number, draw a flow chart to represent this 'two-step' process.

b Use the flow chart to write an equation.

c Use backtracking to solve the equation, that is, find the starting number.

Working

a



Reasoning

The first step of the forward tracking process is to multiply by 2, and the second step is to add 3.

b $2n + 3 = 15$

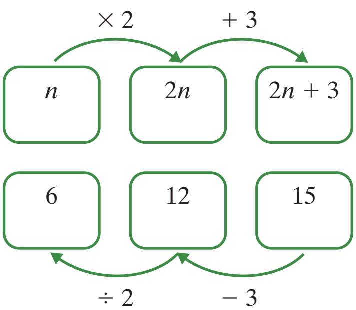
The finishing number is 15.

continued

Example 16 continued

Working

c



The starting number is 6.

$n = 6$

Check:

$\text{LS} = 2 \times 6 + 3$

$\quad = 15$

$\text{RS} = 15$

Reasoning

The first step of the backtracking process is to subtract 3 from 15, and the second step is to divide 12 by 2. Subtracting 3 is the opposite (inverse) of adding 3. Dividing by 2 is the opposite of multiplying by 3.

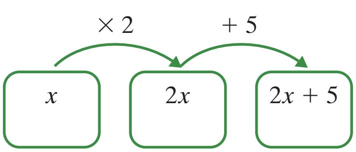
Example 17

Construct a flow chart to represent the left side of each of the following equations and then solve each equation using backtracking.

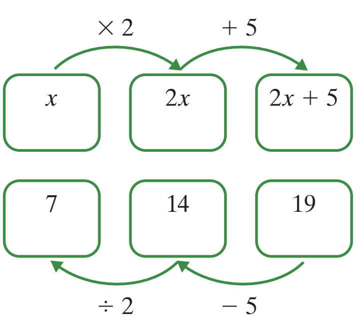
a $2x + 5 = 19$ **b** $\frac{x}{2} - 5 = 7$ **c** $3(x + 4) = 18$

Working

a As a flow chart $2x + 5 = 19$ can be represented as



So backtracking to find the starting number



Reasoning

Start with x, multiply by 2, then add 5.

The result or output is 19. Use backtracking to find x.

The opposite of $+5$ is -5 so subtract 5 from 19, which gives 12.

The opposite of $\times 2$ is $\div 2$ so divide 12 by 2, which gives 6.

Write the solution.

continued

Example 17 continued

Working | **Reasoning**

$x = 6$

Check:

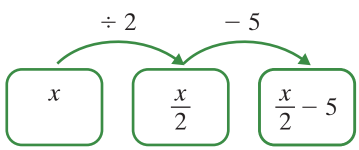
$LS = 2 \times 6 + 5$

$= 19$

$RS = 19$

b As a flow chart $\frac{x}{2} - 5$ can be represented as

Construct a flow chart. Start with x, divide by 2 and then subtract 5.



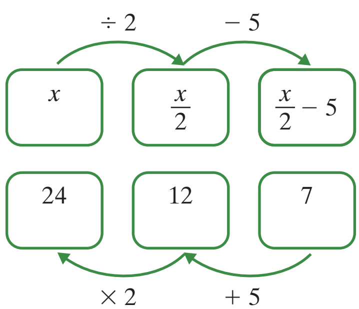
So backtracking to find the starting number

The result or output is 7. Use backtracking to find x.

The opposite to -5 is $+5$ so add 5 to 7 which gives 12.

The opposite to $\div 2$ is $\times 2$ so multiply 12 by 2 which gives 24.

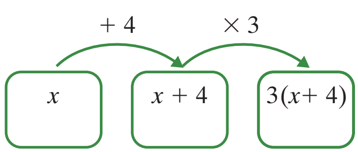
Write the solution.



$x = 24$

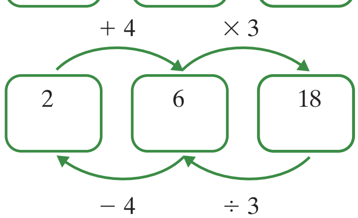
c

Start with x, add 4 then multiply the result by 3. The brackets are necessary to show that the whole expression, $x + 4$, is multiplied by 3.



Backtrack to find the starting number. Dividing by 3 is the opposite of multiplying by 3.

Subtracting 4 is the opposite of adding 4.



$x = 2$

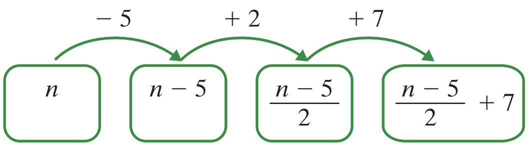
Write the solution.

Example 18

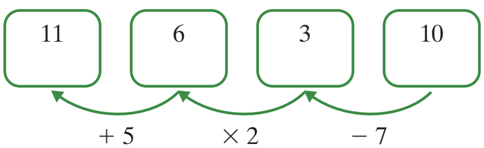
Construct a flow chart for this equation and use it to solve the equation.

$$\frac{n-5}{2} + 7 = 10$$

Working



So backtracking to find the starting number



$n = 11$

Reasoning

This is a ‘three-step’ process.
The first thing that is done to x is that 5 is subtracted from it. The result is then divided by 2. Finally 7 is added.

Use backtracking to find n.

The opposite of $+ 7$ is $- 7$ so subtract 7 from 10, which gives 3.
The opposite of $\div 2$ is $\times 2$ so multiply 3 by 2, which gives 6. The opposite of -5 is $+5$, so add 5 to 6, which gives 11.

exercise 8.5

LINKS TO Example 15

1 For each of the following one-step equations
- **i** draw a flow chart.
- **ii** use backtracking to find the value of n.

a $n + 2 = 18$ **b** $n - 3 = 11$ **c** $5n = 35$

d $\frac{n}{2} = 24$ **e** $n + 4 = 13$ **f** $n - 4 = 12$

g $n + 8 = 17$ **h** $\frac{n}{11} = 5$ **i** $8n = 40$

LINKS TO Example 16

2 For each of the following two-step equations
- **i** draw a flow chart.
- **ii** use backtracking to find the value of n.

a $2n - 1 = 15$ **b** $\frac{n}{2} + 7 = 16$ **c** $2n + 1 = 15$

d $3n - 7 = 8$ **e** $7(n + 3) = 28$ **f** $\frac{4n}{3} = 32$

g $\frac{3n}{2} = 36$ **h** $\frac{n-2}{5} = 1$ **i** $\frac{n+2}{3} = 7$

j $\frac{n}{3} + 1 = 12$ **k** $\frac{n-5}{2} = 4$ **l** $\frac{n}{2} - 5 = 2$

m $4n + 3 = 23$ **n** $7n - 4 = 24$ **o** $\frac{n}{2} + 1 = 5$

p $\frac{n}{4} - 3 = 7$ **q** $\frac{n-7}{2} = 4$ **r** $\frac{n+5}{3} = 6$

LINKS TO Example 17

3 For each of the following three-step equations

i draw a flow chart.

ii use backtracking to find the value of n.

a $\frac{2n+4}{4} = 9$ **b** $3(2n - 7) = 27$ **c** $2(n + 3) - 5 = 19$

d $\frac{n-3}{3} - 3 = 1$ **e** $3(n + 9) - 7 = 20$ **f** $\frac{n-2}{5} - 7 = 1$

g $4(2n - 1) = 12$ **h** $4(n + 1) - 5 = 27$ **i** $3\left(\frac{n}{4} - 1\right) = 15$

LINKS TO Example 18

4 In each of the following, complete a flow chart and use backtracking to find the starting number.

a Cara thinks of a number, divides by 2 and adds 5 to the result. She gets 30 as her answer.

b Viki thinks of a number, adds 5, multiplies the result by 4 and then subtracts 2. She gets 42 as her answer.

c Garth thinks of a number, subtracts 8, divides the result by 3 and then adds 2. His answer is 5.

5 Use a flow chart and backtracking to solve each of the following equations.

a $3(a + 1) = 12$ **b** $2(b - 5) = 20$ **c** $\frac{2a}{3} + 1 = 5$

d $\frac{3s}{4} - 1 = 11$ **e** $\frac{2a+1}{5} = 7$ **f** $2(3m + 1) = 14$

g $4(5k - 2) = 52$ **h** $3(n - 2) + 2 = 11$ **i** $\frac{3n-5}{5} - 3 = 2$

exercise 8.5 challenge

6 For each of the following four-step equations

i draw a flow chart.

ii use backtracking to find the value of n.

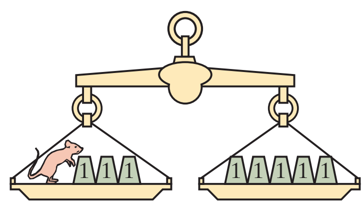
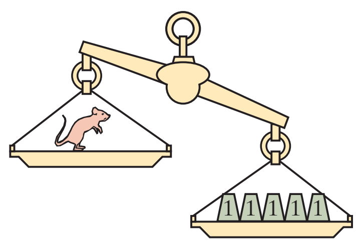
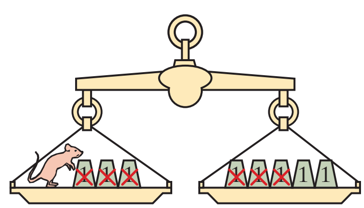
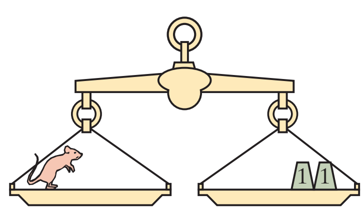
a $\frac{4(n+2)}{3} + 2 = 10$ **b** $4\left(\frac{n-1}{3} + 5\right) = 36$ **c** $5\left(\frac{n}{3} + 1\right) - 4 = 11$

d $\frac{2n+4}{2} + 3 = 7$ **e** $\frac{5(n+1)}{2} + 3 = 18$ **f** $4\left(\frac{n-1}{3} + 5\right) = 32$

8.6 Solving equations: balance scales

One way to think about the solving process is to consider an equation as a balance scale where the two sides of the balance must be kept level.

Consider the equation $x + 3 = 5$ where x kg is the mass of a toy mouse. By writing a series of equivalent equations we can solve the equation and find that $x = 2$.

Equivalent equations	Balance scales diagram
$(x + 3 = 5)$ The scale is balanced because the two sides are equal.	(Balanced) 
$(x + 3 - 3 \neq 5)$ By taking 3 kg from the left side, the scale is no longer balanced.	(Unbalanced) 
$(x + 3 - 3 = 5 - 3)$ By taking 3 kg from the right side as well, the scale is balanced again.	(Balanced) 
$(x = 2)$ The mass of the toy mouse is 2 kg.	(Balanced) 

We can write the solution process that we used with the scales above as follows.

$x + 3 = 5$

$x + 3 - 3 = 5 - 3$

$x = 2$ (doing the same to both sides).

Example 19

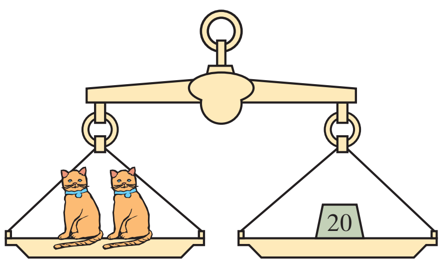
Consider the equation $2x = 20$. Where x kg represents the mass of a cat.

a Construct a balance scales diagram to represent the equation $2x = 20$, and then use diagrams and equivalent statements to solve this equation for x.

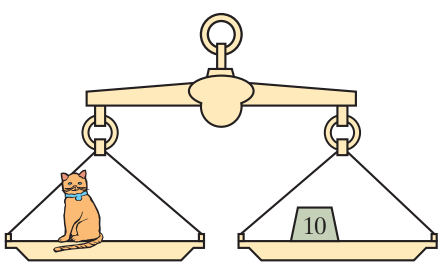
b Check your answer by substituting the answer in to the left side (LS) of the original equation to see whether it equals the right side (RS) of the equation.

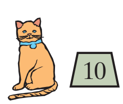
Working

a



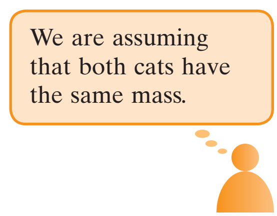
$2x = 20$





$$\frac{2x}{2} = \frac{20}{2}$$

Reasoning

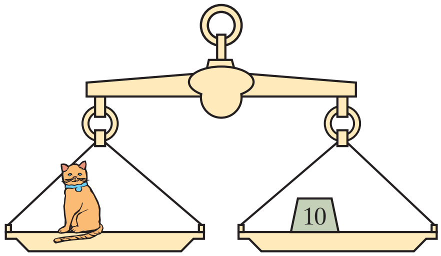


Divide each side by two. This is the same as halving each side.

continued

Example 19 continued

Working



$x = 10$

Reasoning

The cat on the left side balances the 10 kg mass on the right side.

b Check mentally:

$$\begin{aligned} LS &= 2x \\ &= 2 \times 10 \\ &= 20 \\ &= RS \end{aligned}$$

Substituting $x = 10$ into the LS of the original equation shows that it is equal to the RS. If the solution is correct then you should get the same result on the LS and RS of the equation.

To make equivalent equations easier to read, it is common practice in algebra to line up the equals signs vertically, as in example 19. This makes it easier to identify the left side (LS) and right side (RS) of each equation.

The balance scales model can be used to solve more difficult equations; as long as we **do the same thing to both sides**, the balance scales will remain level. Each time you do the 'same to both sides' you are finding an equivalent equation to the step before.

Example 20

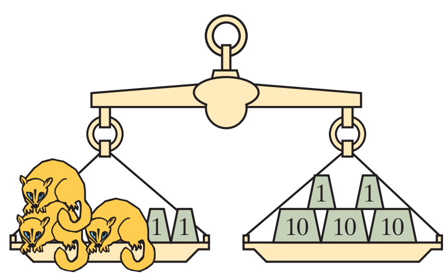
y kg represents the mass of a possum. Solve for y if $3y + 2 = 32$. Check your answer by substituting your solution into the original equation.

Working

$3y + 2 = 32$

Reasoning

In the example y kg represents the mass of a possum.



Working

$$\begin{aligned} 3y + 2 - 2 &= 32 - 2 \\ 3y &= 30 \end{aligned}$$

Reasoning

Subtracting 2 from both sides keeps the equation balanced.

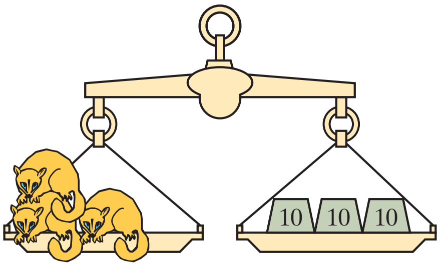
continued

Example 20 continued

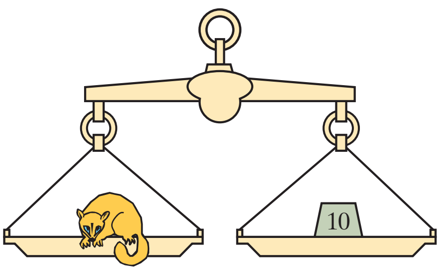
Working

$$\frac{3y}{3} = \frac{30}{3}$$

Reasoning



Divide both sides by 3. This is the same as sharing the 30 kg equally between the three possums.



$y = 10$

Substituting $y = 10$ into the original equation

$$\begin{aligned} LS &= 3y + 2 \\ &= 3(10) + 2 \\ &= 32 \\ &= RS \end{aligned}$$

One possum has a mass of 10 kg, so $y = 10$.

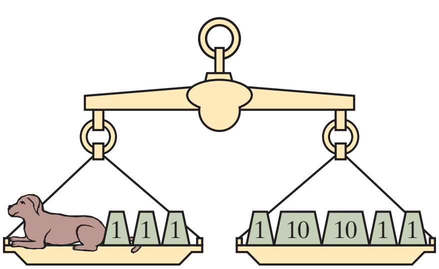
Substitute $y = 10$ into the left side of the equation.

If LS = RS, the solution is correct.

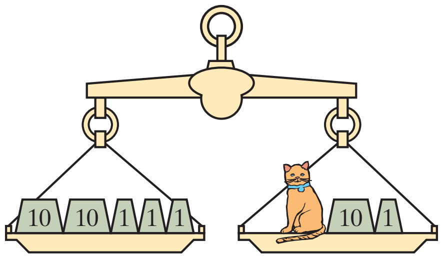
exercise 8.6

LINKS TO Examples 19, 20

1 The diagram below shows Ruffy the dog on a set of balance scales. Draw an equivalent scale balance diagram which shows Ruffy's mass.



2 The diagram below shows Mojo the cat on a set of balance scales. Draw an equivalent balance scales diagram which shows Mojo's mass.

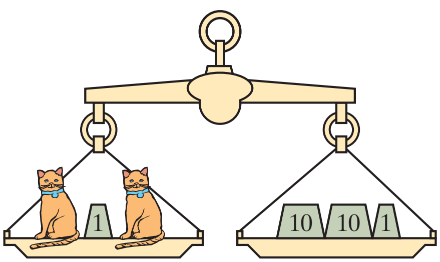


LINKS TO Example 20

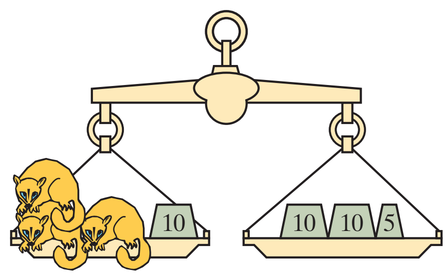
3 For each of the following balance scales

- **i** write an equation that represents the situation shown, using x kg to represent the mass of each toy animal.
- **ii** use equivalent equations to find the mass of the toy animals.

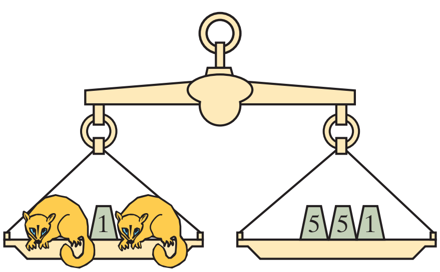
a



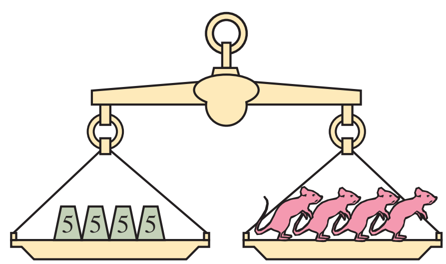
b



c



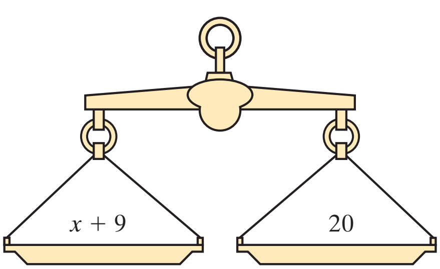
d



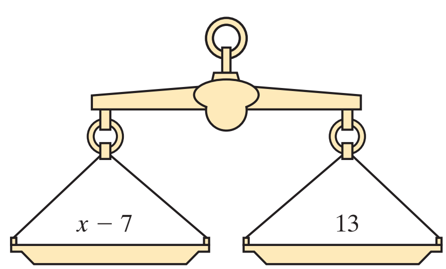
4 For each of the following balance scales

- **i** write the equation represented.
- **ii** describe in words how to get x on its own whilst keeping the scales balanced.
- **iii** find the solution to the equation.

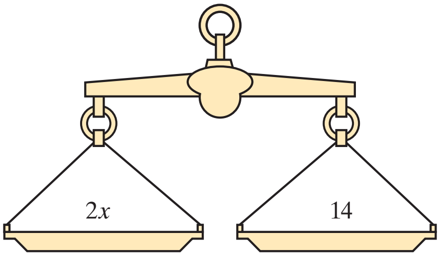
a



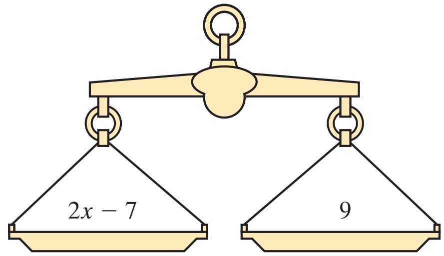
b



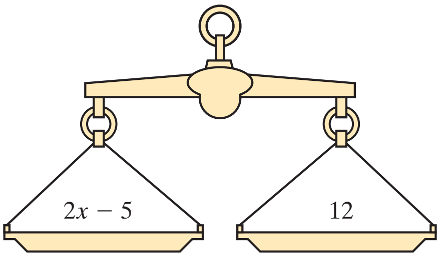
c



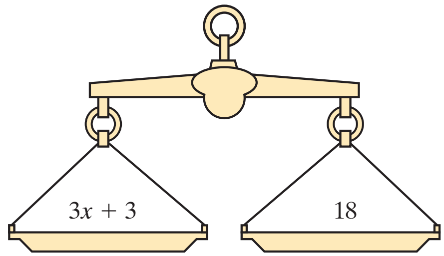
d



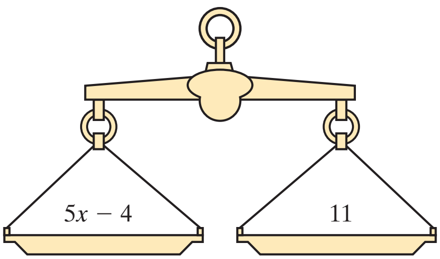
e



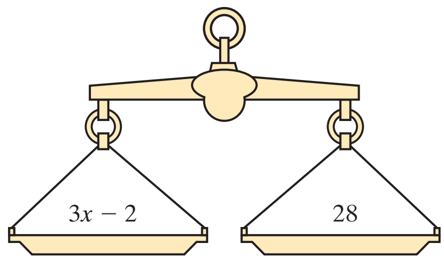
f



g



h

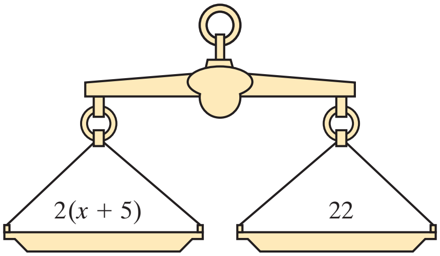


exercise 8.6 challenge

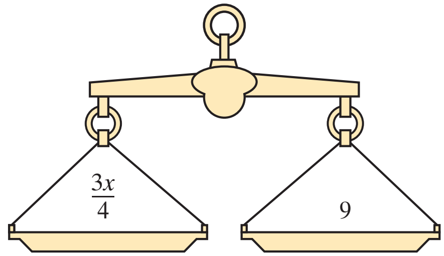
5 For each of the following balance scales

- **i** write the equation represented.
- **ii** describe in words how to get x on its own whilst keeping the scales balanced.
- **iii** find the solution to the equation.

a



b



8.7 Solving equations: doing the same to both sides

So far in this chapter we have looked at several ways of solving equations.

- In section 8.3 we used arithmetic strategies: tables of values, inspection and guess, check and improve. When dealing with more complicated equations, it may not be possible to solve by inspection. It may not be efficient to use guess, check and improve.
- In section 8.5 we used backtracking in flow charts. Like arithmetic strategies, backtracking may not be efficient or useful when dealing with more complicated equations.
- In section 8.6 we used the scale balance approach of keeping the two sides balanced by doing the same to both sides. This ensured that we were always making equivalent equations. We obviously don't want to draw a scale balance every time we solve an equation, but the method of doing the same to both sides is an efficient method for solving equations.

In this section we will focus on algebraic solving of equations by doing the same to both sides to make equivalent equations. We will pay particular attention to careful setting out, even with very simple equations that you would be able to solve mentally.

Example 21

Solve each of the following equations. In each case, check your solution by substitution.

a $m - 7 = 12$　　**b** $n + 6 = 14$

Working	Reasoning
a $m - 7 = 12$ $m - 7 + 7 = 12 + 7$ $m = 19$	m has had 7 subtracted from it. Add 7 to both sides.
Check: LS $= m - 7$ $= 19 - 7$ $= 12$ $=$ RS	Substituting $m = 19$ into the LS of the equation gives the same value as the RS. So, $m = 19$ is correct.
b $n + 6 = 14$ $n + 6 - 6 = 14 - 6$ $n = 8$	n has had 6 added to it. Subtract 6 from both sides.
Check: LS $= n + 6$ $= 8 + 6$ $= 14$ $=$ RS	Substituting $n = 8$ into the LS of the equation gives the same value as the RS. So, $n = 8$ is correct.

Example 22

Solve each of the following equations. In each case, check your solution by substitution.

a $7a = 42$ **b** $\frac{x}{5} = 3$

Working	**Reasoning**
a $7a = 42$	a has been multiplied by 7.
$7a \div 7 = 42 \div 7$	Divide both sides by 7.
$a = 6$	
Check:	Substitute $a = 6$ in the LS of the equation.
LS $= 7a$ $= 7 \times 6$ $= 42$ $=$ RS	
b $\frac{x}{5} = 3$	x has been divided by 5.
$\frac{x}{5} \times 5 = 3 \times 5$	Multiply both sides by 5.
$x = 15$	
Check:	
LS $= \frac{x}{5}$ $= \frac{15}{5}$ $= 3$ $=$ RS	Substitute $x = 15$ in the LS of the equation.

Example 23

Solve each of the following equations. In each case, check your solution by substitution.

a $10 = 2d + 4$ **b** $3b - 3 = 13$

Working	**Reasoning**
a $10 = 2d + 4$ $10 - 4 = 2d + 4 - 4$ $6 = 2d$ $\frac{6}{2} = \frac{2d}{2}$ $3 = d$	We want to get d on one side of the equals sign by itself. In the expression $2d + 4$, d is first multiplied by 2 and then 4 is added. We need to undo the operations in the reverse order. First subtract 4 from both sides. Then divide both sides by 2.
Check:	
RS $= 2d + 4$ $= 2 \times 3 + 4$ $= 6 + 4$ $= 10$ $=$ LS	Substitute $d = 3$ in the RS of the equation.

continued

Example 23 continued

Working

b

$$3b - 3 = 13$$
$$3b - 3 + 3 = 13 + 3$$
$$3b = 16$$
$$\frac{3b}{3} = \frac{16}{3}$$
$$b = \frac{16}{3}$$
$$b = 5\frac{1}{3}$$

Check:

$$\begin{aligned} LS &= 3b - 3 \\ &= 3 \times \frac{16}{3} - 3 \\ &= 16 - 3 \\ &= 13 \\ &= RS \end{aligned}$$

Reasoning

We want to get b on one side of the equals sign on its own. In the expression $3b - 3$, b is first multiplied by 3 and then 3 is subtracted. First add 3 to both sides. Then divide both sides by 3.

Substitute $b = 5\frac{1}{3}$ in the LS of the equation.

Example 24

Solve the equation $\frac{t}{5} + 4 = 7$. Check your solution by substitution.

Working

d

$$\frac{t}{5} + 4 = 7$$
$$\frac{t}{5} + 4 - 4 = 7 - 4$$
$$\frac{t}{5} = 3$$
$$\frac{t}{5} \times 5 = 3 \times 5$$
$$t = 15$$

Check:

$$\begin{aligned} LS &= \frac{t}{5} + 4 \\ &= \frac{15}{5} + 4 \\ &= 3 + 4 \\ &= 7 \\ &= RS \end{aligned}$$

Reasoning

We want to get t on one side of the equals sign on its own. In the expression $\frac{t}{5} + 4$, t is first divided by 5 and then 4 is added.
First subtract 4 from both sides. Then multiply both sides by 5.

Substitute $t = 15$ in the LS of the equation.

8.7

exercise 8.7

LINKS TO Example 21

1 Solve the following equations by doing the same to both sides.

a $x + 7 = 11$	**b** $x - 7 = 11$	**c** $a + 6 = 9$
d $6 + a = 9$	**e** $b + 13 = 20$	**f** $b - 13 = 20$
g $a - 5 = 16$	**h** $d + 14 = 23$	**i** $d - 8 = 32$
j $m - 15 = 8$	**k** $m - 2.7 = 3.1$	**l** $p - 13.5 = 21.2$
m $x + 6.5 = 14.8$	**n** $x - 1.3 = 1.2$	**o** $7 + t = 35$

LINKS TO Example 22a

2 Solve the following equations by doing the same to both sides.

a $7m = 49$	**b** $12b = 60$	**c** $8m = 56$
d $5x = 65$	**e** $9m = 63$	**f** $2.4h = 24$
g $1.5k = 6$	**h** $23a = 115$	**i** $2x = 6.4$
j $5a = 12$	**k** $4b = 14$	**l** $8h = 84$

LINKS TO Example 22b

3 Solve the following equations by doing the same to both sides.

a $\frac{x}{7} = 2$	**b** $\frac{b}{9} = 7$	**c** $\frac{k}{5} = 20$
d $\frac{n}{2} = 25$	**e** $\frac{a}{8} = 12$	**f** $\frac{d}{4} = 2.5$
g $\frac{x}{9} = 15$	**h** $\frac{m}{11} = 6$	**i** $\frac{m}{4} = 1.2$
j $\frac{y}{3} = 2.5$	**k** $\frac{b}{1.5} = 3$	**l** $\frac{a}{2.2} = 7$

4 Matthew wants to solve the equation $4x + 5 = 21$ for x. To do this he needs to

A add 5 to both sides and then multiply both sides by 4.
B subtract 5 from both sides and then multiply both sides by 4.
C add 5 to both sides and then divide both sides by 4.
D subtract 5 from both sides and then divide both sides by 4.
E divide both sides by 4 and then subtract 5 from the result.

LINKS TO Example 23

5 Solve each of these equations for a by doing the same to both sides.

a $3a + 4 = 34$	**b** $2a - 4 = 12$	**c** $3a - 7 = 23$
d $3a - 14 = 10$	**e** $6a + 7 = 31$	**f** $11a + 6 = 83$
g $2a + 6 = 9$	**h** $7a + 13 = 55$	**i** $3a + 14 = 59$
j $4a + 13 = 45$	**k** $4a - 13 = 19$	**l** $8a + 5 = 61$
m $3.2a + 1 = 7.4$	**n** $3a - 4.2 = 1.8$	**o** $2a - 5.6 = 1.2$
p $7a + 4.1 = 25.1$	**q** $2a + 1.3 = 5.7$	**r** $10a - 6.4 = 2.6$

6 The solution to the equation $\frac{m}{2} + 6 = 24$ is

A $y = 9$ **B** $y = 18$ **C** $y = 30$ **D** $y = 36$ **E** $y = 60$

LINKS TO Example 24

7 Solve each of these equations for n by doing the same to both sides.

a $\frac{n}{4} + 5 = 7$ **b** $\frac{n}{4} - 5 = 7$ **c** $\frac{n}{3} - 4 = 8$

d $\frac{n}{3} + 4 = 8$ **e** $\frac{n}{6} + 8 = 15$ **f** $\frac{n}{4} - 3 = 9$

g $\frac{n}{8} - 1 = 7$ **h** $\frac{n}{9} + 7 = 15$ **i** $\frac{n}{6} - 11 = 9$

j $\frac{n}{8} + 3 = 10$ **k** $\frac{n}{6} - 11 = 29$ **l** $\frac{n}{10} - 5.6 = 4.2$

m $\frac{n}{11} + 2 = 11$ **n** $\frac{n}{1.2} + 8 = 10$ **o** $\frac{n}{7} + 3.4 = 3.9$

p $\frac{n}{3} + 1.4 = 2.9$ **q** $\frac{n}{8} + 0.9 = 1.3$ **r** $\frac{n}{7} - 8 = 15$

8 Solve each of these equations.

a $d - 11 = 43$ **b** $\frac{x}{3} - 9 = 14$ **c** $5y = 18$

d $\frac{x}{12} = 9$ **e** $\frac{b}{7} - 4 = 7$ **f** $5a + 18 = 43$

g $m - 11.2 = 38.4$ **h** $\frac{k}{6} + 1 = 13$ **i** $4x = 8.4$

j $n + 7.3 = 15.9$ **k** $\frac{b}{5} - 2.4 = 1.6$ **l** $1.6a + 2.1 = 5.3$

m $\frac{b}{8} = 2.2$ **n** $a - 16.8 = 13.7$ **o** $3a + 12 = 69$

p $4a - 8.2 = 1.8$ **q** $\frac{h}{7} + 1.2 = 2.1$ **r** $13n = 143$

9 Hana thinks of a number, triples it and subtracts 4, which leaves her with 5. Write an equation which describes this number puzzle. Then solve this equation to find what the starting number must have been.

exercise 8.7 challenge

10 Solve for p. Give your answer correct to two decimal places.

a $31 = 3p - 16$ **b** $3.56p + 2.42 = 7.38$ **c** $7.645 = \frac{p}{4} + 7$

d $14 = 3.4p - 1$ **e** $\frac{p}{1.3} + 3 = 7$ **f** $10.41 = 2.23(p - 2)$

11 Solve for the unknown in the following equations.

a $3 + \frac{v}{5} = \frac{17}{5}$ **b** $5(a - 4.5) - 2 = 0.5$ **c** $\frac{q}{4} + \frac{3}{2} = \frac{7}{4}$

d $4a - \frac{3}{10} = \frac{1}{2}$ **e** $\frac{4}{3} = \frac{a}{3} + \frac{1}{6}$ **f** $\frac{k}{4} - \frac{3}{11} = \frac{5}{11}$

8.8 Further equations to solve by doing the same to both sides

Equations with brackets

When an equation includes an expression in brackets, we may be able to simplify the left side by dividing both sides of the equation by a common factor.

Example 25

Solve the equation $2(a + 3) = 12$, checking your solution by substitution.

Working	Reasoning
$2(a + 3) = 12$	$(a + 3)$ has been multiplied by 2.
$\frac{2(a + 3)}{2} = \frac{12}{2}$	There is a common factor of 2 on each side of the equation.
$a + 3 = 6$	Divide both sides by 2
$a + 3 - 3 = 6 - 3$	a has 3 added to it. Subtract 3 from both sides.
$a = 3$	
Check:	
$\text{LS} = 2(a + 3)$	Substitute $a = 3$ in the LS of the equation.
$= 2(3 + 3)$	
$= 2 \times 6$	
$= 12$	
$= \text{RS}$	

If there is no common factor on the two sides, we normally expand the brackets using the distributive law. For example,

$$2(a + 5) = 2 \times a + 2 \times 5$$

$$= 2a + 10$$

Example 26

Solve the equation $2(a + 3) = 11$, checking your solution by substitution.

continued

Example 26 continued

Working

$$2(a + 3) = 11$$
$$2 \times a + 2 \times 3 = 11$$
$$2a + 6 = 11$$
$$2a + 6 - 6 = 11 - 6$$
$$2a = 5$$
$$2a \div 2 = 5 \div 2$$
$$a = 2\frac{1}{2}$$

Check:

$$\begin{aligned} LS &= 2(a + 3) \\ &= 2\left(2\frac{1}{2} + 3\right) \\ &= 2 \times 5\frac{1}{2} \\ &= 11 \\ &= RS \end{aligned}$$

Reasoning

$(a + 3)$ has been multiplied by 2.
Multiply each term inside the brackets by 2.
$2a$ has 6 added to it. Subtract 6 from both sides.
a is multiplied by 2.
Divide both sides by 2.

Substitute $a = 2\frac{1}{2}$ in the LS of the equation.

Example 27

Solve $5(3 + a) + 2 = 20$ for a. Check your answer using substitution.

Working

$$5(3 + a) + 2 = 20$$
$$5(3 + a) + 2 - 2 = 20 - 2$$
$$5(3 + a) = 18$$
$$\frac{5(3 + a)}{5} = \frac{18}{5}$$
$$3 + a = 3.6$$
$$3 + a - 3 = 3.6 - 3$$
$$a = 0.6$$

Check:

$$\begin{aligned} LS &= 5(3 + a) + 2 \\ &= 5(3 + 0.6) + 2 \\ &= 5 \times 3.6 + 2 \\ &= 18 + 2 \\ &= 20 \\ &= RS \end{aligned}$$

Reasoning

Subtract 2 from both sides of the equation.
Divide both sides of the equation by 5.
Subtract 3 from both sides of the equation.
Substitute $a = 0.6$ into the LS of the equation.

An algebraic fraction with an expression such as $x + 7$ in the numerator should be treated as if it were in brackets.

After expanding brackets, it may be possible to simplify the left side before doing the same to both sides.

Example 28

Solve the $\frac{x + 7}{3} = 8$. Check your solution by substitution.

Working	Reasoning
b $\frac{x+7}{3} = 8$	$(x + 7)$ has been divided by 3. Undo by multiplying both sides by 3.
$\frac{(x+7)}{3} \times 3 = 8 \times 3$	
$x + 7 = 24$	
$x + 7 - 7 = 24 - 7$	x has 7 added to it. Subtract 7 from both sides.
$x = 17$	
Check:	
$\text{LS} = \frac{x+7}{3}$	Substitute $x = 17$ in the LS of the equation.
$= \frac{17+7}{3}$	
$= \frac{24}{3}$	
$= 8$	
$= \text{RS}$	

Equations with negative numbers

During the solving of some equations, the pronumeral may end up on its own on the left side with a negative coefficient. Multiplying both sides of the equation by -1 will change the coefficient of the pronumeral to a positive number.

Example 29

Solve the equation $12 - x = 3$. Check your solution by substitution.

Working	Reasoning
Method 1	
$12 - x = 3$	Subtract 12 from both sides.
$12 - 12 - x = 3 - 12$	$3 - 12 = -9$
$-x = -9$	The coefficient of x is negative so multiply both
$-x \times (-1) = -9 \times (-1)$	sides by -1.
$x = 9$	$-x \times (-1) = +x$ and $-9 \times (-1) = +9$

continued

Example 29 continued

Working

Method 2

$$12 - x = 3$$
$$12 - x + x = 3 + x$$
$$12 = 3 + x$$
$$12 - 3 = 3 - 3 + x$$
$$9 = x$$

Check:

$$\begin{aligned} LS &= 12 - x \\ &= 12 - 9 \\ &= 3 \\ &= RS \end{aligned}$$

Reasoning

x has been subtracted from 12.
Add x to both sides.

Subtract 3 from both sides.

Substituting $n = 8$ into the LS of the equation gives the same value as the RS.

Example 30

Solve these equations, checking the solution by substitution.

a $a + 11 = 4$ **b** $7b = -63$

Working

a

$$a + 11 = 4$$
$$a + 11 - 11 = 4 - 11$$
$$a = -7$$

Check:

$$\begin{aligned} LS &= a + 11 \\ &= -7 + 11 \\ &= 4 \\ &= RS \end{aligned}$$

Reasoning

Subtract 11 from both sides.
$4 - 11 = -7$

Substitute $a = -7$ in the LS.

Working

b

$$7b = -63$$
$$\frac{7b}{7} = -\frac{63}{7}$$
$$b = -9$$

Check:

$$\begin{aligned} LS &= 7b \\ &= 7 \times (-9) \\ &= -63 \\ &= RS \end{aligned}$$

Reasoning

Divide both sides by 7.
A negative number divided by a positive number is negative.

Substitute $b = -9$ in the LS.

Example 31

Solve the equation $3x + 2.7 = 0.3$, checking the solution by substitution.

Working

$$3x + 2.7 = 0.3$$
$$3x + 2.7 - 2.7 = 0.3 - 2.7$$
$$3x = -2.4$$
$$\frac{3x}{3} = -\frac{2.4}{3}$$
$$x = -0.8$$

Check:

$$\begin{aligned} LS &= 3x + 2.7 \\ &= 3 \times -0.8 + 2.7 \\ &= -2.4 + 2.7 \\ &= 0.3 \\ &= RS \end{aligned}$$

Reasoning

Subtract 2.7 from both sides.
$0.3 - 2.7 = -2.4$

Divide both sides by 3.
A negative number divided by a positive number is negative.

Substitute $x = -0.8$ in the LS.

Equations with the pronumeral on both sides

In some equations the pronumeral occurs on both sides of the equation. We subtract the term containing the pronumeral from one side of the equation.

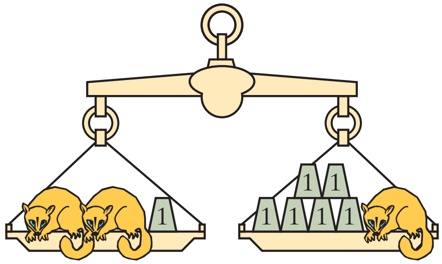
Example 32

If x represents the mass in kilograms of one toy possum in this balance scales diagram,

a write an equation to represent these balance scales.

b solve the equation to find the value of x.

c check the solution.



Working

a $2x + 1 = x + 6$

Reasoning

Mass in kilograms of LS $= 2 \times x + 1$
Mass in kilograms of RS $= x + 6$

continued

Example 32 continued

Working	Reasoning
b $2x + 1 = x + 6$	Take one possum from each side.
$2x + 1 - x = x + 6 - x$	There is now one possum on the left side and none on the right side.
$x + 1 = 6$	Subtract x from both sides so that x is removed from the right side.
$x + 1 - 1 = 6 - 1$	Subtract 1 from both sides.
$x = 5$	
Check:	
$\text{LS} = 2x + 1$	Substitute $x = 5$ into the LS and evaluate.
$= 2 \times 5 + 1$	Substitute $x = 5$ into the RS and evaluate.
$= 11$	
$\text{RS} = x + 6$	
$= 5 + 6$	
$= 11$	
$\text{LS} = \text{RS}$	

exercise 8.8

LINKS TO Example 25

1 Solve the following equations.

a $5(p + 3) = 25$ **b** $24 = 3(c + 1)$ **c** $2(m - 2) = 16$

d $2(x - 5) = 20$ **e** $21 = 3(m + 7)$ **f** $36 = 4(k + 1)$

g $3(z - 2) = 12$ **h** $14 = 7(x - 5)$ **i** $9(4x + 3) = 63$

LINKS TO Example 26

2 Solve the following equations, giving the values of m in fraction form.

a $2(m + 4) = 11$ **b** $4(m + 4) = 30$ **c** $2(m - 3) = 17$

d $5(m - 7) = 2$ **e** $3(m + 1) = 10$ **f** $5(m + 8) = 56$

g $2(m - 13) = 5$ **h** $4(m - 2) = 7$ **i** $3(m + 3) = 11$

LINKS TO Example 27

3 Solve the following equations, giving the value of a in decimal form where appropriate.

a $2(a + 3) + 7 = 25$ **b** $3(a + 7) - 13 = 32$ **c** $2(a + 4) + 11 = 25$

d $4(a + 5) - 7 = 19$ **e** $5(a + 2) - 13 = 9$ **f** $2(a - 3) + 7 = 25$

g $3(a + 4) + 2 = 14$ **h** $2(a + 7) - 9 = 5$ **i** $10(a + 3) - 17 = 41$

LINKS TO Example 28

4 Solve the following equations.

a $\frac{x + 4}{3} = 2$ **b** $\frac{x + 11}{5} = 7$ **c** $\frac{x - 3}{6} = 5$

d $\frac{x - 12}{5} = 4$ **e** $\frac{x + 1}{8} = 2$ **f** $\frac{x + 16}{4} = 4$

g $\frac{x - 13}{2} = 11$ **h** $\frac{x + 4.6}{3} = 7.2$ **i** $\frac{x - 3.8}{5} = 1.8$

LINKS TO Example 29

5 Solve the following equations.

a $6 - b = 4$ **b** $13 - b = 8$ **c** $21 - b = 16$

d $9 - b = 1$ **e** $17 - 2b = 9$ **f** $25 - 3b = 10$

g $26 - 5b = 11$ **h** $41 - 8b = 9$ **i** $17 - 5b = 6$

LINKS TO Example 30a

6 Solve the following equations.

a $x + 11 = 6$ **b** $x + 14 = 3$ **c** $x + 20 = 8$

d $x - 5 = -7$ **e** $x - 8 = -10$ **f** $x + 11 = -2$

g $x - 5 = -5$ **h** $x + 7 = -13$ **i** $x - 8 = -12$

LINKS TO Example 30b

7 Solve the following equations.

a $3m = -18$ **b** $2m = -22$ **c** $5m = -35$

d $-3m = 12$ **e** $-2m = 8$ **f** $-3m = -21$

g $\frac{m}{2} = -9$ **h** $\frac{m}{6} = -11$ **i** $\frac{m}{4} = -7$

j $-\frac{m}{7} = 5$ **k** $-\frac{m}{4} = -3$ **l** $-\frac{m}{3} = -5$

LINKS TO Example 31

8 Solve the following equations.

a $2n - 19 = -37$ **b** $5n + 6 = 31$ **c** $2n + 21 = 15$

d $5 - 2n = 13$ **e** $8 - 3n = -16$ **f** $-5n + 3 = 13$

g $7n + 17 = -39$ **h** $-3n - 8 = 4$ **i** $3n + 14 = -22$

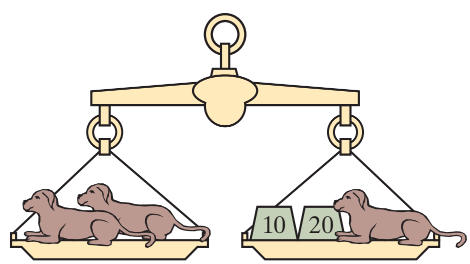
9 For each of these balance scales use x to represent the mass in kilograms of one toy animal.

i Write an equation to represent these balance scales.

ii Solve the equation to find the value of x.

iii Check the solution.

a



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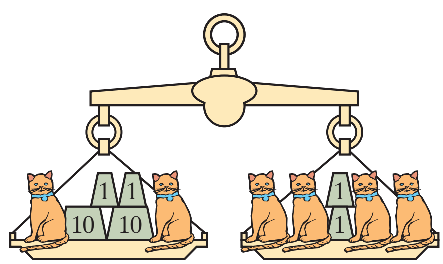

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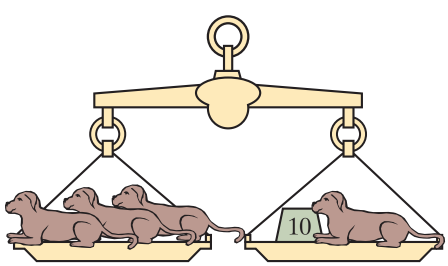

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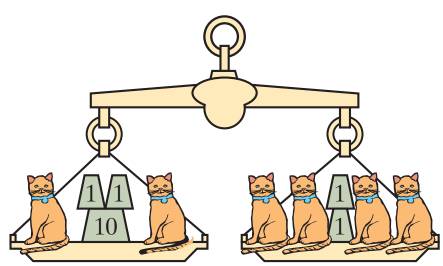

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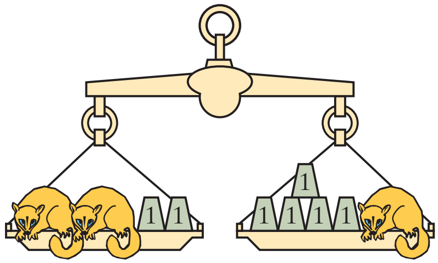
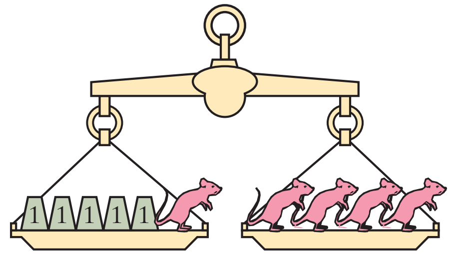


f



LINKS TO Example 32

10 Solve these equations for x.

a $4x + 7 = 3x + 8$

b $7x - 11 = 2x + 9$

c $3x - 13 = x + 15$

d $6x - 11 = -3x + 7$

e $8x - 15 = 4x - 7$

f $3.4x + 7.1 = 2.5x + 8$

g $2x + 22 = -3x + 7$

h $13x - 5 = 11x - 8$

11 Gemma starts with a number and then divides by 3, and subtracts 2 from the result. If she then doubles this, she ends up with the number 8.

a Write an equation which represents this number puzzle.

b Solve this equation to find what the starting number must have been.

12 Radha starts with a number and then subtracts 3, and divides this answer by 5 then adds 9 and the result is 11.

a Write an equation which represents this number puzzle.

b Solve this equation to find what the starting number must have been.

13 Grant thinks of a number, adds 3, multiplies the result by 4, then subtracts 2. His final number is 38. What is the original number?

exercise 8.8 challenge

14 Find the unknown value in each case.

a $\frac{2(p-1)}{5} - 2 = 2$

b $2\left(\frac{m}{4} + 1\right) + 3 = 15$

c $3\left(\frac{2m}{5} - 1\right) = 9$

d $4\left(\frac{k}{6} - 2\right) - 3 = 1$

e $5\left(\frac{a}{3} - 1\right) - 2 = 3$

f $4 + 2(a - 5) = 7$

g $\frac{4(x-1) + 2}{3} = 6$

h $3\left(\frac{m}{4} - 2\right) - 1 = 5$

i $\frac{3(x-4)}{5} + 2 = 11$

8.9 Solving problems with equations

Many worded problems, including those with diagrams, can be solved using algebra. The process of translating the 'words' of a problem into algebra is referred to as **formulation**.

As a starting point, it is important to read the question carefully and identify the unknown, and then try to construct an equation that can be solved.

A four-step strategy exists for setting up and solving worded problems with algebra. It can be summarised as follows.

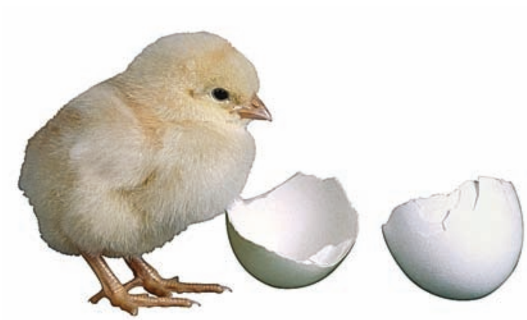
1. **Translate the words into algebra.** Decide on the unknown and give it a pronumeral, then formulate an equation using this pronumeral.
2. **Solve the equation.** Solve by 'doing the same to both sides'.
3. **Check the solution.** Substitute your solution back into the original equation to check that the LS = RS.
4. **Translate the algebra back into words.** Express your solution in terms of the problem wording.

Example 33

Katy has brought home some baby chicks from school. Her sister Rachel brings home another four, so they have a total of 11 chicks.

a Formulate an equation that represents the number of chicks in the home.

b Solve your equation to find out how many chicks Katy brought home.



Working	Reasoning
a Let c be the number of chicks that Katy brought home. $c + 4 = 11$	Total chicks = number of chicks Katy has brought home + number of chicks Rachel brought home So total chicks = $c + 4$ If the total number of chicks is 11, this means that $c + 4 = 11$ $c + 4 = 11$ is an equation that expresses the number of chicks.
b $c = 7$, so Katy brought home seven chicks.	By inspection, it can be seen that $c = 7$.

Equations can be used to solve problems relating to **consecutive numbers**. Consecutive numbers are whole numbers that follow one another. For example, 8, 9, 10 are consecutive numbers. If we let n represent a whole number, then the next consecutive number is one more than n, that is, $n + 1$. The next consecutive number is $n + 2$.

Example 34

The sum of two consecutive whole numbers is 11. Find the numbers.

Working

Let n be the first number, and $n + 1$ be the next number.

$$n + (n + 1) = 11$$
$$2n + 1 = 11$$
$$2n + 1 - 1 = 11 - 1$$
$$2n = 10$$
$$\frac{2n}{2} = \frac{10}{2}$$
$$n = 5$$

Substitute $n = 5$ into the original equation.

$$\begin{aligned} LS &= n + (n + 1) \\ &= 5 + (5 + 1) \\ &= 5 + 6 \\ &= 11 = RS \end{aligned}$$

The two consecutive numbers are 5 and 6.

Reasoning

Step 1: Words into algebra
Deciding on the variable is the first step, and then writing an equation that uses this variable is the next step.

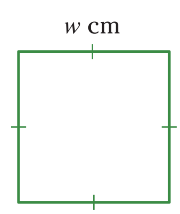
Step 2: Solve the equation
Subtract 1 from both sides.
Divide both sides by 2.

Step 3: Check the solution
Substituting the solution $n = 5$ into the LS.

Step 4: Algebra into words
The first number is n, which is 5. The second number is $n + 1$, which is 6.
Express your answer in terms of the problem.

Example 35

Formulate an equation which shows the perimeter of this shape in terms of the length of each side, w cm, given that the perimeter is known to be 15 cm. Then solve it for w.



continued

Example 35 continued

Working

If w is the side length, and the shape is a square, then an equation representing this is

$$w + w + w + w = 15$$
$$4w = 15$$
$$\frac{4w}{4} = \frac{15}{4}$$
$$w = 3\frac{3}{4} \text{ or } 3.75$$

Check:
If $w = 3.75$,

$$\begin{aligned} LS &= 4w \\ &= 4 \times 3.75 \\ &= 15 \\ &= RS \end{aligned}$$

Reasoning

The unknown variable w (side length) is given.
The sides are of equal length, and the total perimeter is 15 cm. The expression $4w$ has the same value as 15.

Divide both sides by 4.

Example 36

Josef works in a supermarket on the weekend. He earns $8.50 an hour. If Josef earns $105.75 one week but that amount includes a $25 bonus for working an extra shift, use the four-step approach above to find out how many hours he worked in that week.

Working

Let h = the number of hours Josef works.
The amount of money Josef will earn is $8.5h + 25$

$$8.5h + 25 = 105.75$$
$$8.5h + 25 = 105.75$$
$$8.5h + 25 - 25 = 105.75 - 25$$
$$8.5h = 80.75$$
$$\frac{8.5h}{8.5} = \frac{80.75}{8.5}$$
$$h = 9.5$$

If $h = 9.5$, substitute this into the original equation $8.5h + 25 = 105.75$

$$\begin{aligned} LS &= 8.5h + 25 \\ &= 8.5 \times 9.5 + 25 \\ &= 80.75 + 25 \\ &= 105.75 \\ &= RS \end{aligned}$$

Reasoning

Step 1: Words into algebra
Choose a pronumeral for the unknown.
Write an equation.

Step 2: Solve the equation
Subtract 25 from both sides.
Divide both sides by 8.5. h is a number so we write $h = 9.5$, not $h = 9.5$ hours.

Step 3: Check the solution
Substitute the solution $h = 9.5$ into the RS.

continued

Example 36 continued

Working	Reasoning
Josef works for $9\frac{1}{2}$ hours in that week.	**Step 4: Algebra into words** Express your answer in terms of the problem.

exercise 8.9

In this exercise, use the four steps for solving algebra word problems.

LINKS TO Example 33

1 Pearlie has been given a handful of jelly beans, and is then given seven more.

- **a** Let b represents the number of jelly beans that Pearlie started with. Write an expression for the number of jelly beans that Pearlie now has.
- **b** If she ends up with 23 jelly beans, write an equation which represents this situation.
- **c** Solve this equation to find the value of b.
- **d** How many jelly beans did Pearlie have in her hand to start with?

LINKS TO Example 33

2 There are 71 chocolate buttons in two piles. The second pile has 13 more than the first pile.

- **a** If there are c chocolate buttons in the first pile, write an expression for the number of chocolate buttons in the second pile.
- **b** Write an equation to describe this situation.
- **c** Solve the equation to find the value of c.
- **d** How many chocolate buttons are in each pile?

LINKS TO Example 33

3 Aniela has a bag of banana lollies that she shares with three friends. Each person receives k lollies and there are three left over.

- **a** Write an expression for the total number of lollies in the bag.
- **b** If there are 23 lollies in the bag, write an equation to find how many lollies each person receives.
- **c** Solve the equation to find the value of k.
- **d** How many lollies does each person receive?

LINKS TO Example 33

4 Graph paper is sold in packets containing x sheets. Narelle's folder holds six packets of graph paper. She has used five sheets from the folder.

- **a** Write an expression for the number of sheets of graph paper in Narelle's folder,
- **b** If there are 175 sheets of graph paper in Narelle's folder, write an equation to find how many sheets come in a packet?
- **c** Solve the equation to find the value of x.
- **d** How many sheets come in a packet?

5 Two children are trying to work out the age of their two grandparents, Sarah and William. Here is what their grandmother told them.

'William is 10 years older than I am. If you add together our two ages you get 154 years.'

a Write an equation to represent this situation using x to represent Sarah's age in years.

b How old are the grandparents?

6 A teacher asked her class to write down an equation for the following sentence:

'If you add 6 to an unknown number and then divide the result by 3, the answer is 5.'

The responses of two students are shown below. Which student do you think is correct? For the student who is incorrect, what is the error?

Denise	Michael
$n + \frac{6}{3} = 5$	$\frac{n + 6}{3} = 5$

LINKS TO Example 34

7 The sum of two consecutive whole numbers is 29.

a Let n be the smallest of the two numbers. Write an equation to represent this.

b Solve the equation to find the value of n.

c What are the two numbers?

LINKS TO Example 34

8 The sum of three consecutive numbers is 27.

a Let n be the smallest of the three numbers. Write an equation.

b Solve the equation to find the value of n.

c What are the three numbers?

9 A square garden bed has side length x metres.

a Write down an expression for the perimeter of the garden bed in terms of x.

b If the perimeter of the garden bed is 76 metres, write an equation to find the value of x.

c Solve the equation to find the value of x.

LINKS TO Example 35

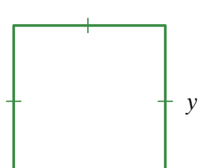
10 For each of these shapes, the perimeter is known, but there are unknown side lengths. For each shape

i write an equation using the given information. Do not include 'cm' in your equation.

ii solve the equation to find the value of the pronumeral.

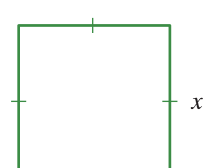
iii list all the side lengths.

a



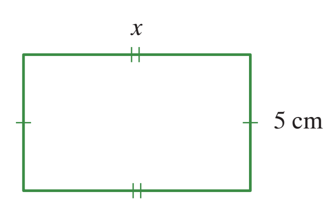
Perimeter = 64 cm

b



Perimeter = 24.8 cm

c



Perimeter = 26 cm

11 If you multiply a decimal number by 6 and then subtract 4, the result is 4.1.

a Using n to stand for the unknown number, write an equation.

b Solve the equation to find the unknown number.

12 Ali plants petunias in 12 rows. There are p petunias in each row and 132 petunias in total.

a Write this information as an equation.

b How many petunias are in each row?

LINKS TO Example 36

13 Alex works in a supermarket on the weekend. He earns $7.50 an hour.

a If Alex works for n hours, write an expression for the amount of money he earns. Do not include the $ sign in your expression.

b If Alex earns $37.50 one week, write an equation to work out how many hours he worked.

c Solve the equation and write down how many hours Alex worked.

LINKS TO Example 36

14 Madeleine works as a waitress at weekends and earns $16.50 per hour. One weekend she earned $265 which included $34 in tips.

a Let n be the number of hours Madeleine worked. Write an equation using the given information. Do not include the $ sign in your equation.

b Solve your equation.

c How many hours did Madeleine work?

15 A nurse worked a total of 48 hours in a week. She worked four normal shifts and 12 hours of overtime.

a Let the length of her normal shift be n hours. Write an equation using the given information. Do not include 'hours' in your equation.

b Solve your equation.

c How long is her normal shift?

exercise 8.9 challenge

16 Lucy is three years older than Peter and she is six years older than Dominic. If you add together the ages of the three children you get 30 years. Let d stand for Dominic's age.

a Write an expression for Lucy's age in terms of d.

b Write an expression for Peter's age in terms of d.

c Write an equation to represent the sum of the three ages.

d Solve the equation for d.

e List the ages of the three children.

Analysis task

Kath and Kim

Kath has \$70 and saves \$20 per week. Kim has \$510, but spends \$35 per week.

a Write an expression for how much money Kath will have after n weeks.

b How much money will Kath have after three weeks?

c After how many weeks will Kath have \$150?

d Write an expression for how much money Kim will have after n weeks.

e How much will Kim have after three weeks?

f How much will Kim have after 14 weeks?

g Use a table of values to find the value of n when Kath and Kim will have the same amount of money.

h Write an equation which can be used to find out when Kath and Kim will have the same amount of money.

i Solve the equation for n. Do you get the same value for n as in part g?

Review Solving equations

Summary

- An equation is a statement that two expressions have the same value.
- Solving is the process of finding which values of the variable will make the equation a true statement.
- Some equations can be solved by using mental strategies, or a guess, check and improve strategy.
- Forward tracking through a flow chart can be used to build up expressions.
- Backtracking involves using inverse operations to move backwards through a flow chart from the finishing number to find what the starting number must have been.
- Equations can be solved by using inverse operations and doing the same to both sides.
- A four-step process of solving algebraic word problems is below.

1. **Translate the words into algebra.** Decide on the unknown and give it a pronumeral, then formulate an equation using this pronumeral.
2. **Solve the equation.** Solve by 'doing the same to both sides'.
3. **Check the solution.** Substitute your solution back into the original equation to check that the LS = RS.
4. **Translate the algebra back into words.** Express your solution in terms of the problem wording.

Visual map

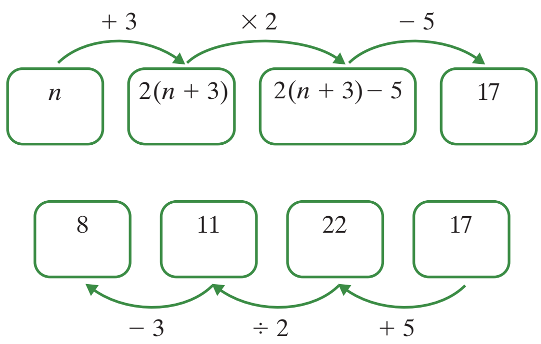
Using the following terms (and others if you wish), construct a visual map that illustrates your understanding of the key ideas covered in this chapter.

backtracking	equivalent equation	solve
consecutive	expression	true statement
doing the same to both sides	product	unknown value
equation	solution	variable

Revision

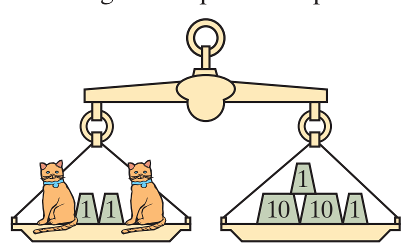
Multiple choice questions

1 The flow chart below represents the solution of which equation?



A $2n + 3 - 5 = 17$
B $n + 3 \times 2 - 5 = 17$
C $3n \times 2 - 5 = 17$
D $(n + 3) \times 2 - 5 = 17$
E $n + (3 \times 2 - 5) = 17$

2 In this balance scales diagram, the mass of each cat is x kg. The equation represented in the diagram is $2x + 2 = 22$. The value of x is



A 10
B 11
C 12
D 20
E 24

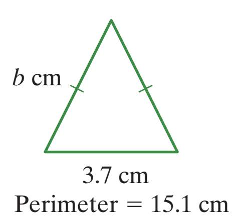
3 The solution to the equation $4x - 13 = 27$ is

A $x = 1$ **B** $x = 3$ **C** $x = 10$ **D** $x = 12$ **E** $x = 13$

4 The solution to the equation $\frac{m}{5} - 3 = 11$ is

A $m = 40$ **B** $m = 14$ **C** $m = 70$ **D** $m = 8$ **E** $m = 26$

5 In the diagram, the value of b is



A 5.7
B 6
C 5
D 11.4
E 18.1

Short answer questions

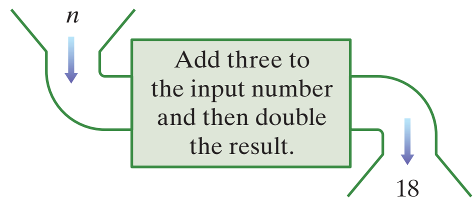
6 Insert the symbol >, < or = in each of the following to make a true statement.

a $2 + 3 \times 5 \;\_\_\_\; 5 \times 3 - 2$ **b** $3 + 3 \times 1 \;\_\_\_\; 4 \times 3 - 6$

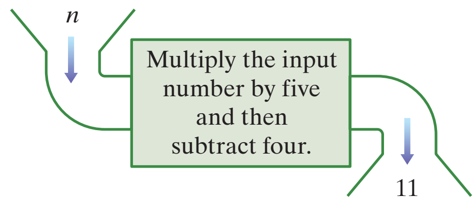
c $\dfrac{3 \times 6}{2} \;\_\_\_\; 3 \times 3 - \dfrac{0}{2}$ **d** $\dfrac{7}{2} + \dfrac{4}{2} \;\_\_\_\; 11$

7 Write an equation which represents the following number machines.

a



b

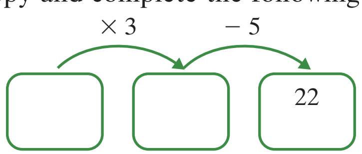


8 Use an arithmetic strategy to find the value of the unknown.

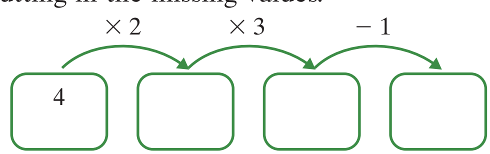
a $1 + 5h = 16$ **b** $7k - 12 = 16$ **c** $\dfrac{x}{3} + 1 = 10$

9 Copy and complete the following flow charts, putting in the missing values.

a



b



10 Use a flow chart and backtracking to work out the starting number for each of the following.

a $3n + 5 = 23$ **b** $3(n - 2) + 2 = 23$

11 Solve each of the following equations by doing the same to both sides.

a $m - 8 = 9$ **b** $b + 13 = 24$ **c** $7x = 84$ **d** $\dfrac{y}{2.4} = 5$

e $3d + 5 = 32$ **f** $\dfrac{n - 3}{5} = 6$ **g** $\dfrac{k}{5} + 1 = 6$ **h** $4n - 20 = 10$

i $\dfrac{p + 2}{3} = 4$ **j** $\dfrac{3y}{2} = 12$ **k** $1.2a + 1.1 = 4.7$

Extended response questions

12 There are 103 corn chips in two piles. The second pile has 11 more than the first pile.

a Write an expression for the number, n, of corn chips in the first pile.

b Write an expression for the number of corn chips in the other pile.

c Write an equation to describe the situation.

d Solve the equation to find out the value of n.

e How many corn chips are in each pile?

13 The sum of two consecutive numbers is 121. Let n be the smaller number.

a Write an equation to represent the sum of the two numbers.

b Solve the equation to find n.

c Find the values of the consecutive numbers.

Practice quiz
Chapter 8