CHALLENGING MATHEMATICAL TASKS

UNLOCKING THE POTENTIAL OF ALL STUDENTS

PETER SULLIVAN

OXFORD

CONTENTS

CHALLENGING MATHEMATICAL TASKS – INTRODUCTION

This book complements the title*, Open-Ended Maths Activities: Using 'good' questions to enhance learning in mathematics*. However, this book differs in a couple of ways. Firstly, it argues that learning is enhanced when students work on mathematics problems that they don't yet know how to solve. Secondly, it suggests that these problems are best supported by a particular lesson structure.

THE RATIONALE FOR CHALLENGE

This book focuses on tasks that are challenging. The lessons are designed on the assumption that students will work on the learning tasks prior to instruction from the teacher. The rationale for this approach to planning and teaching is that it takes concentration and effort over an extended period of time to build the connections between concepts, to understand the coherence of mathematical ideas and to be able to transfer learning to practical contexts and new topics.

To this end, students need encouragement to *persist*, which includes them concentrating, applying themselves, believing that they can succeed and making an effort to learn. The lessons in this book are likely to foster such actions in that they allow for the possibility of sustained thinking, decision making and some risk taking by the students.

WHY IS CHALLENGE IMPORTANT TO MATHS LEARNING?

On one hand, the mathematics curriculum might be a linear hierarchy of concepts that students learn by following carefully sequenced and well-supported micro steps. On the other hand, the curriculum might be a network of interconnected ideas in which students can engage through experiences that are challenging and that require some degree of risk taking. To build these networks of ideas it's necessary to process different concepts simultaneously, comparing and contrasting ideas and considering their use in different contexts.

Learning is robust if students connect ideas together for themselves and determine their own strategies for solving problems, rather than following instructions they have been given. Both connecting ideas together and formulating their own strategies is more complex than other approaches, and is therefore more challenging.

Essentially, the notion is for teachers to pose problems that students don't yet know how to solve and to support them in coming to find solutions. This is the essence of inquiry.

WHAT DOES 'CHALLENGING' LOOK LIKE?

The challenging tasks in this book are intended to give students opportunities to inquire into mathematics for themselves. In general, challenging tasks require students to:

- plan their approach, especially sequencing more than one step
- process multiple pieces of information, with an expectation that they make connections between those pieces, and see concepts in new ways
- choose their own strategies, goals and level of accessing the task
- spend time on the task and record their thinking.

At least part of the challenge is the expectation that students:

- record the steps in their solutions
- explain their strategies
- justify their thinking to the teacher and other students
- listen attentively to each other.

Teachers can establish these as expectations for ways of working. The suggestions in this book address important mathematical ideas that are developmentally appropriate, and in which there is a reasonable expectation that students can engage with minimal instruction. It's worth taking the time needed for students to engage with the tasks and the concepts.

WHY IS PERSISTENCE IMPORTANT?

Important mathematical ideas are complex and to engage with those ideas sustained thinking is essential. Students gain satisfaction from the act of overcoming challenge, and this satisfaction leads to improved self-concept.

Teachers are encouraged to communicate with students about the benefits of persisting and to affirm persistence when they identify it.

There are three key actions for teachers:

1. Affirm positive behaviours such as effort, persistence, cooperation, learning from others and flexible thinking.
2. Reflect on ways to affirm, such as: *You did not give up even though you were stuck*; *You tried something different*; *You tried to find more than one answer*.
3. Pose tasks that students find challenging so that they can learn to persist.

THE LESSONS

The lessons in this book include the following features.

LEARNING TASK

This task is posed to students first (unless there is an introductory task – see 'Introduction' below). Write the problem on the board. If concrete materials are required, make sure these are prepared prior to the lesson. Students are expected to engage with the tasks *prior* to instruction from the teacher.

LEARNING FOCUS

This explains the mathematical purpose of the tasks, and includes a suggestion of what might be communicated to students about the intentions of the learning.

INTRODUCTION

In some lessons, there is an introductory activity to check students' familiarity with the language and the representations of the learning task.

KEY MATHEMATICAL LANGUAGE

Some prerequisite and new terminology is suggested in this section.

PEDAGOGICAL CONSIDERATIONS

In most cases, the tasks should be introduced with minimal explanation. Any aspects of the tasks that require clarification are listed in this section.

In general, teachers can adapt the numbers used in the tasks to suit their students, which ensures that the tasks are appropriately challenging.

ENABLING PROMPT

This can be posed to students who haven't been able to make progress on the learning task. The intention is that the students can complete the enabling prompt and then proceed with the learning task. Enabling prompts vary an aspect of the learning task, such as the form of representation, the size of the numbers or the number of steps.

EXTENDING PROMPT

This is for students who finish the learning task quickly. Extending prompts aim to extend their thinking on an aspect of the main task, perhaps inviting generalisation or abstraction.

SUPPLEMENTARY TASKS

It's intended that, having engaged with the learning task and listened to the successful strategies of others, students can then engage productively with the supplementary tasks. The goal is for student learning to develop over the sequence of the lesson – it's not essential that all students have success with the learning task, but the hope is that they can complete the supplementary tasks.

BLMs

All BLMs are available online on www.oxfordowl.com.au. BLMs for the learning tasks are also printed in this book (pp. 170 to 179). BLMs for the supplementary tasks are only available online. It's assumed teachers will create their own prompts and worksheets.

POSSIBLE STUDENT SOLUTIONS TO THE LEARNING TASK

This section presents some answers and strategies. It's important to try to anticipate possible student solutions when choosing students to report on their thinking. This ensures that a range of key strategies is presented. It can also help teachers to connect the different student strategies in productive ways.

THE TASKS

It's recommended that teachers follow this three-phase approach for each task within the lessons. These three phases can occur a number of times within a single lesson.

INTRODUCING THE TASK

- Have students read the task quietly, allowing them time to think about what the question is asking. Check in with any students who can't read at this level after they start working.
- Invite questions to ensure the task is clear, but don't show students how to do it. For example, explain the structure of the BLM or clarify the mathematical language used in the task.

- Set expectations for student working. While it varies from task to task, allow students to work individually on the task for at least five minutes before working with one or more peers. This is to give individuals time to think.
- Advise students if there is more than one possible answer to a task.

WHILE STUDENTS ARE WORKING

- Be prepared to allow some confusion. If the majority of students appear stuck, encourage a class discussion about what has been found so far, or how they are approaching the task.
- Use the enabling prompts for students experiencing difficulty and the extending prompts for students who complete the task quickly.
- Watch what students are doing and select some who represent a range of approaches to explain their thinking later.

REVIEWING STUDENT STRATEGIES

- Use students' solutions, ideas and strategies to highlight the important mathematical ideas in the task.
- Have selected students explain their solution strategy and other insights to the class. Students may need assistance with this initially, but they will improve rapidly over time.
- Provide engaging resources for students to present their work to the class, such as a document camera or iPad.
- Invite questions from other students, ask them to confirm the strategy, and ask them to compare student methods.
- Ask students to describe the successful strategies of other students.
- Connect students' strategies with the formal processes that were the intention of the task in the first place (Smith & Stein, 2011).

REFERENCES

Smith, M. S. & Stein, M. K. (2011). *5 practices for orchestrating productive mathematics discussions*. Reston VA: National Council of Teachers of Mathematics.

ACKNOWLEDGEMENTS

Thank you to the following people who made suggestions for the learning tasks:
Mike Askew (*Multiplicative comparisons*), Alan Bell (*Decimal word problems*), Maria Bartolini Bussi (*Additive word problems*), Jill Cheeseman (*Sharing counters*) and James Russo (*Using the hundreds chart for additive thinking*).

ADDITIVE THINKING

Addition and subtraction continue to be a central focus of primary mathematics instruction. However, recent years have seen the focus shift from pen and paper algorithms and error diagnosis to building upon students' intuitive knowledge and strategies. The emphasis is not only on exploring different ways to solve addition and subtraction problems, but also on creating awareness in students of their ability to work out ways to solve the problems themselves.

Students need to build or visualise models of the problem, and use mental images of number relationships to devise their own ways to solve it. Teachers need to provide students with problems and experiences that offer opportunities for them to progress to the next phase in their learning, with the appropriate level of support. This can be achieved by:

- allowing students to create their own methods to solve problems. This will encourage conceptual understanding of the addition and subtraction processes, rather than simply learning the algorithm.
- ensuring there are clear connections between the problem, its material or illustrative representation and the symbols used. Again, this encourages students to search for connections, instead of just learning the routine for finding the answer.

The table on p. 3 shows the Australian Mathematics Curriculum content descriptions by year level. The matching lessons from this book are listed for reference.

GAMES

Games with dice and counters can be used to develop fluency with addition and subtraction. Some examples of flexible games are included here.

Race to 10
Starting at 0, each player adds 1 or 2, saying the progressive total. The player who says 10 is the winner. There is a winning strategy that students can search for. The target and calculations can be adapted to whatever you are working on – for example:

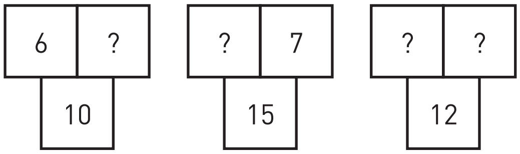
- race to 100 – start at 0, add any number from 1 to 8
- race to 0 – start at 200, take away 5, 10 or 20
- race to $10 – start at 0, add any Australian coin.

Directions
A small group stands in a circle and says a number from a counting sequence in turn. As they say their number, they also indicate a direction (the person on their left or the person on their right). That person then continues the counting sequence.

What's my card?
In groups of four, one person deals a card to each of the other three players. Each player puts the card on their forehead, without looking at it. The dealer says the total value of the cards and the players have to work out the value of their card.

Adding squares

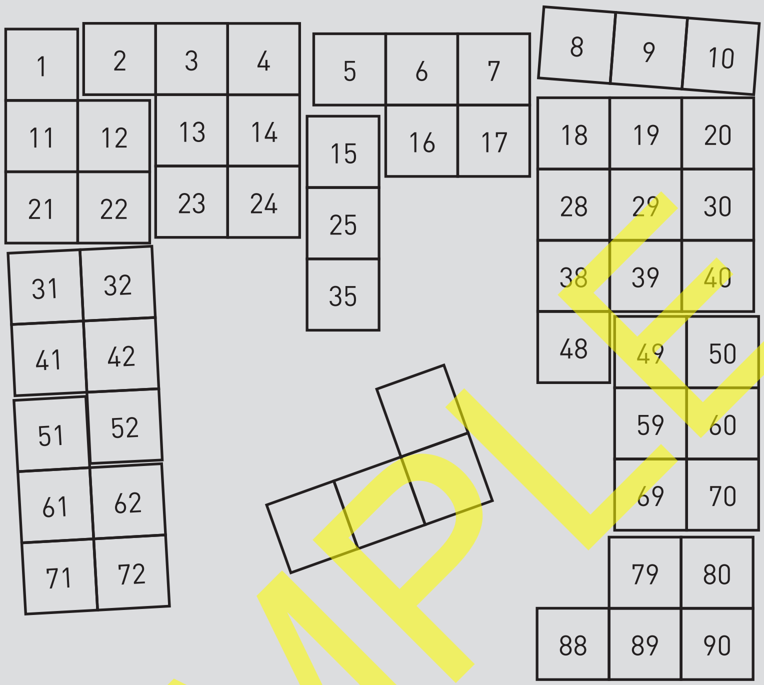
In this game, students have to work out the missing number. Tell students that the numbers in the top squares add up to the number below. This can be played competitively, with the fastest to finish the winner. Below are some examples of questions you can create for any number combinations.



Year	Content description	Lesson
1	Represent and solve simple addition and subtraction problems using a range of strategies including counting on, partitioning and rearranging parts (ACMNA015)	Patterns on the hundreds chart (p. 4)
2	Group, partition and rearrange collections up to 1000 in hundreds, tens and ones to facilitate more efficient counting (ACMNA028) Explore the connection between addition and subtraction (ACMNA029) Solve simple addition and subtraction problems using a range of efficient mental and written strategies (ACMNA030)	Ordering numbers (p. 6) Bridging to simplify (p. 8)
3	Apply place value to partition, rearrange and regroup numbers to at least 10 000 to assist calculations and solve problems (ACMNA053) Recognise and explain the connection between addition and subtraction (ACMNA054) Recall addition facts for single-digit numbers and related subtraction facts to develop increasingly efficient mental strategies for computation (ACMNA055)	Using the hundreds chart for additive thinking (p. 10) Sharing counters (p. 12) Additive word problems (p. 14) Adding numbers efficiently (p. 16)
4	Apply place value to partition, rearrange and regroup numbers to at least tens of thousands to assist calculations and solve problems (ACMNA073)	Pen and paper methods for addition and subtraction (p. 18)
5	Use efficient mental and written strategies and apply appropriate digital technologies to solve problems (ACMNA291)	Thinking flexibly about addition and subtraction (p. 20) Using two pieces of information to solve addition problems (p. 22)
6	Select and apply efficient mental and written strategies and appropriate digital technologies to solve problems involving all four operations with whole numbers (ACMNA123)	Making both sides equal (p. 24) Managing your change (p. 26)

PATTERNS ON THE HUNDREDS CHART

LEARNING TASK



I know that one of the numbers on the L-shaped piece is 65. What might the other numbers on the L-shaped piece be?

Give as many possibilities as you can.

LEARNING FOCUS

Part of understanding place value to 100 is appreciation of the patterns that exist in the 100s chart. There are both horizontal patterns and vertical patterns. For example, the number that appears one row below another number is always 10 more than that number (if there are 10 numbers in each row).

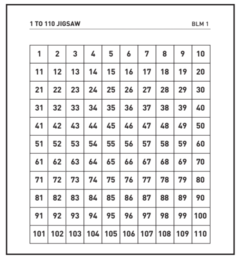
To indicate the intended learning, you might write: ‘There are patterns in the hundreds chart. The patterns can help us answer questions such as: Which number is 10 more or 10 less than another number?’

KEY MATHEMATICAL LANGUAGE

Introduce or revise the key language, such as the number words *more*, *less*, *10 more*, *10 less*.

INTRODUCTION

To provide all students with a common experience, have them complete **BLM 1: 1 to 110 jigsaw** (p. 170). Prior to the lesson, print out the BLM, laminate it and cut it into a jigsaw.



1 TO 110 JIGSAW BLM 1

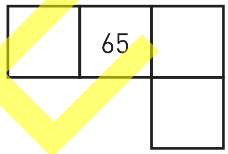
1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100
101	102	103	104	105	106	107	108	109	110

PEDAGOGICAL CONSIDERATIONS

It is assumed that students have had experience with saying number sequences up to and beyond 100, including adding 10 starting from any number, and reading and writing numbers to 100. Note that the hundreds chart in the BLMs goes to 110 so that students experience the numbers just above 100.

While not part of the specific tasks, prompt students to think about adding 10, 10 more, etc.

ENABLING PROMPT



Ask: *What are the missing numbers on this jigsaw piece?*

EXTENDING PROMPT

Have students convince you that they have all of the possible combinations.

SUPPLEMENTARY TASKS

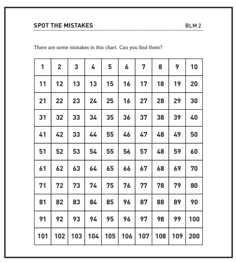
SHAPED LIKE A LETTER

Advise students that the numbers 62 and 84 are on a jigsaw piece shaped like a letter of the alphabet.

Ask them to draw what that piece might look like. Have students suggest at least three possibilities.

SPOT THE MISTAKES

Provide each student with **BLM 2: Spot the mistakes**. Ask: *What are the mistakes?* Have students explain how they found them.



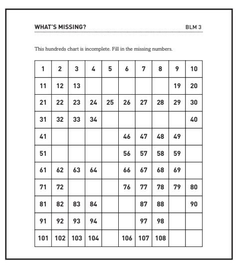
SPOT THE MISTAKES BLM 2

There are some mistakes in this chart. Can you find them?

1	2	3	4	5	6	7	8	9	10
11	12	13	13	15	16	17	18	19	20
21	22	23	24	25	16	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	33	44	55	46	47	48	49	50
51	52	53	54	55	56	57	48	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	96	87	88	89	90
91	92	93	94	95	96	97	98	99	100
101	102	103	104	105	106	107	108	109	200

WHAT IS MISSING?

Have students fill in the missing numbers on the hundreds chart in **BLM 3: What's missing?**



WHAT'S MISSING? BLM 3

This hundreds chart is incomplete. Fill in the missing numbers.

1	2	3	4	5	6	7	8	9	10
11	12	13						19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34						40
41					46	47	48	49	
51					56	57	58	59	
61	62	63	64		66	67	68	69	
71	72				76	77	78	79	80
81	82	83	84			87	88		90
91	92	93	94			97	98		
101	102	103	104		106	107	108		

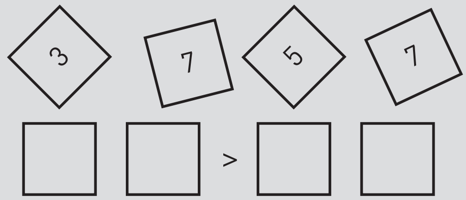
POSSIBLE STUDENT SOLUTIONS TO THE LEARNING TASK

There are 16 different possible solutions to the learning task. The piece can be facing four different ways, and the 65 can go in four different places on each one. If you allow the piece to be flipped, there are a further 16 possibilities.

ORDERING NUMBERS

LEARNING TASK

Place each of these digits onto one of the empty squares to make the sentence true. Find as many different ways of doing this as you can. Record your solutions.



LEARNING FOCUS

One of the key applications of place value thinking is when ordering numbers. The particular numbers used in the learning task help to focus students' attention on the order – for example, 75 and 73 (where the judgment is based on the units), 73 and 37 (where the order of the digits is important), or 73 and 57 (where the tens digits determine the decision).

To indicate the intended learning, you might write: 'Comparing numbers to decide which is larger and which is smaller.'

KEY MATHEMATICAL LANGUAGE

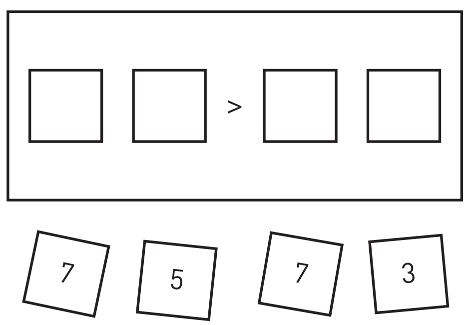
Introduce or revise the necessary language, such as *more than*, *greater than*, *less than*.

PEDAGOGICAL CONSIDERATIONS

To introduce the task, explain the meaning of the greater-than symbol. It's ideal if students can read the symbol readily. (Avoid suggesting an intermediate step, such as thinking about a fish with its mouth open.) Write some sentences on the board and ask them to tell you which is correct – for example, 9 > 3 and 5 > 7.

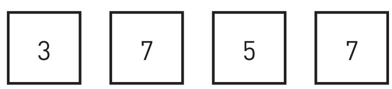
It may help students understand the nature of the learning task if they have pre-prepared cards and a template on which to put them.

Encourage students to record their solutions.



ENABLING PROMPT

Draw these cards on the board and say: *Using just these cards, make some numbers that are both larger than 10 and smaller than 100.*

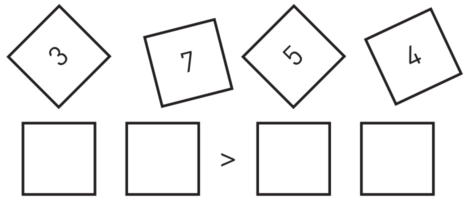


EXTENDING PROMPT

Ask students to convince you that they have all the possible answers.

SUPPLEMENTARY TASKS

MORE ORDERING



Ask students to place each of these numbers onto one of the empty squares to make the sentence true. Have them find as many different ways of doing this as they can and record their solutions.

THREE TEACHERS

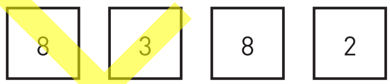
Pose this problem to students: *Ms Smith is 56 years old. Ms Chu is 49 years old. I know that Mr Taylor is older than Ms Chu. Ms Smith is older than Mr Taylor. How old might Mr Taylor be?*

Have students give as many possibilities as they can.

THINKING OF A NUMBER

Pose this problem to students: *I am thinking of a number. 72 is more than my number. My number is more than 60. What might my number be?*

LARGEST AND SMALLEST NUMBERS



Say: *Using just these cards, make as many different 2-digit numbers as you can.*

Ask: *What is the largest 2-digit number you can make? What is the smallest?*

NEWSPAPER NUMBERS

Ask students: *What is the largest 2-digit number that you can find in today's newspaper?*

POSSIBLE STUDENT SOLUTIONS TO THE LEARNING TASK

There are six different possibilities: 77 > 53, 77 > 35, 75 > 73, 75 > 37, 73 > 57 and 57 > 37.

BRIDGING TO SIMPLIFY

LEARNING TASK

Without writing anything, work out the missing number in these equations.

$28 + 5 = 30 + ?$
$27 + 5 = 30 + ?$

What advice would you give to someone on how to work out answers to questions like these in their head?

LEARNING FOCUS

A useful strategy when adding numbers mentally is bridging to the nearest multiple of 10 or the nearest multiple of 100, etc. While some students develop this naturally, others benefit from specific experiences. The learning task focuses students' attention onto partitioning (such as the 5 into $2 + 3$), equivalence ($=$ does not mean 'find the answer') and bridging (the multiples of 10 are helpful).

To indicate the intended learning, you might write: 'Bridging to numbers like 10, 20, 30, etc. can help you to add numbers in your head.'

KEY MATHEMATICAL LANGUAGE

Introduce or revise the necessary language, such as *bridging*, *partitioning*, *equivalence*, *multiples of 10*.

PEDAGOGICAL CONSIDERATIONS

The idea is that students do the tasks mentally but that they explain how they did it either verbally or in writing. While students will initially want to calculate all the steps, encourage them to see that there is a shortcut – bridging. Students will propose a variety of methods, but the focus in this case is on partitioning and bridging.

A useful strategy is for students to show their methods using a number line that is not marked with the numbers. If the numbers are marked, it can encourage students to count on one by one, which is not the point of the task. Likewise, discourage the use of counters.

Focus on the meaning of the equals symbol. Rather than 'find the answer to', it's better to think of it as 'this is the same as that'.

ENABLING PROMPT

Ask: *Find the missing number in this equation:* $8 + 3 = 10 + ?$

EXTENDING PROMPT

Ask: *What is the hardest question like that one that you can do in your head*?

SUPPLEMENTARY TASKS

MAKING THE SIDES EQUAL

Without writing anything, have students work out the missing number in these equations.

27 + 4 = 30 + ?
27 + 6 = 30 + ?
27 + 8 = 30 + ?

SAME AND DIFFERENT

Without writing anything, have students work out the missing number in these equations.

57 + 6 = 60 + ?
57 + ? = 60 + 3
? + 4 = 60 + 3

Ask: *What is the same and what is different about those equations?*

WHAT ARE THE POSSIBILITIES?

Ask: *What might be the missing numbers in these equations?* Have students give at least 10 different answers.

15 + ? = 20 + ?
? + 36 = 42 + ?

Ask students to describe any patterns that they see.

HOW MANY TEAMS OF 10?

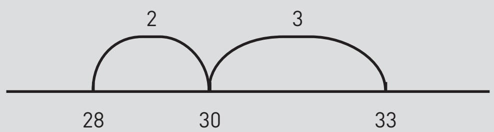
Ask students to solve the following problems.

- Our school is making teams of 10 for a mixed basketball tournament. We have 37 girls and 5 boys. How many full teams can we make?
- Our school is making teams of 10 for a mixed basketball tournament. We have 58 girls and 7 boys. How many full teams can we make?
- Our school is making teams of 10 for a mixed basketball tournament. We have 76 girls and 15 boys. How many full teams can we make?

POSSIBLE STUDENT SOLUTIONS TO THE LEARNING TASK

The intention is for students to see that, for example, 28 + 5 = 28 + 2 + 3 = 30 + 3 = 33. So it is possible to do this in your head.

On a number line, this might look like:



USING THE HUNDREDS CHART FOR ADDITIVE THINKING

LEARNING TASK

I am thinking of 2 numbers on the hundreds chart. One number is 15 more than the other. One of my numbers has a 3 in it.

What might my 2 numbers be? Give as many answers as you can.

LEARNING FOCUS

Even though the learning task asks students to use the hundreds chart to solve additive problems, it's really about using and interpreting place value.

To indicate the intended learning, you might write: 'Use patterns in the hundreds chart to answer addition questions.'

KEY MATHEMATICAL LANGUAGE

Introduce or revise the necessary language, such as *more than*, *difference*, *addition* and *sum*.

INTRODUCTION

Give students a hundreds chart (a large classroom version will suffice). Ask students to use the chart to answer questions such as:

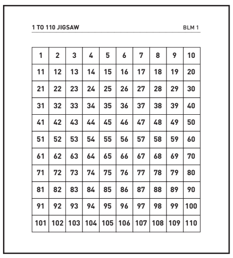
- What is: 13 + 5? 13 + 10? 13 + 8?
- What is the difference between: 18 and 22? 18 and 33?
- How much do I have to add to 24 to get to: 32? 44? 51?

This introduction could be treated as a lesson in itself.

PEDAGOGICAL CONSIDERATIONS

It's assumed that students will use a hundreds chart for these tasks. It can be one you have in the classroom or one you print for them, such as **BLM 1: 1 to 110 jigsaw** (p. 170). If students have a printed chart, they can use counters to mark the number pairs and make patterns more obvious.

The learning task has a number of pieces of information or steps that the students need to use. If this seems complex, pose the task with fewer steps initially. It's also possible to allocate each student in a group a number and they 'look after' one of the pieces of information.


1 TO 110 JIGSAW BLM 1

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100
101	102	103	104	105	106	107	108	109	110

ENABLING PROMPT

Tell students: *I am thinking of 2 numbers on the hundreds chart. One number is 5 more than the other. What might my 2 numbers be?*

EXTENDING PROMPT

Have students convince you that they have all the possible answers, and describe any patterns they can see.

SUPPLEMENTARY TASKS

TWO NUMBERS ON THE HUNDREDS CHART

Tell students that you are thinking of 2 numbers on the hundreds chart. The difference between the 2 numbers is 13. One of the numbers has a 5 in it. Ask: *What might my 2 numbers be?*

Have students give as many answers as they can.

HOW MANY PENCILS?

Tell students that boxes of pencils hold 10 pencils. You have 4 full boxes and some extra pencils. Your friend has 16 more pencils than you. Ask: *How many boxes and how many extra pencils might my friend have?*

HOW MANY EGGS?

Tell students that some egg cartons hold 10 eggs. Amy has some full cartons and some loose eggs. Becky has 6 full cartons and some loose eggs. Becky has 2 more full cartons than Amy does. Amy has 15 fewer eggs that Becky. Ask: *How many eggs might Amy and Becky have?*

FINDING ADDITION PAIRS

Tell students that you are thinking of 2 numbers on the hundreds chart. The numbers are 2 rows apart. The sum of the numbers is 52. Ask: *What might be my numbers?*

Have students give as many answers as they can.

POSSIBLE STUDENT SOLUTIONS TO THE LEARNING TASK

There are four sequences of possibilities:

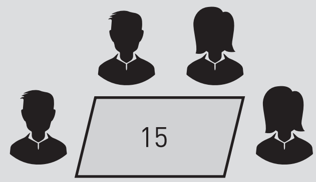
- 3 and 18; 13 and 28; 23 and 38 etc.
- 8 and 23; 18 and 33; 28 and 43 etc.
- 15 and 30; 16 and 31; 17 and 32 etc.
- 30 and 45; 31 and 46; 32 and 47 etc.

There may be debate about the meaning of 'one of my numbers'.

SHARING COUNTERS

LEARNING TASK

Four students have 15 counters between them. All students have a different number of counters. How many counters might each student have?



LEARNING FOCUS

This lesson intends to move students beyond adding two numbers to adding four numbers together. To do this, students add numbers and find the difference between their progressive total and the target. The requirement to have a different number of counters allows students to be systematic about searching for various solutions.

To indicate the intended learning, you might write: 'Find ways to use the hundreds chart to work out the sum of four numbers.'

KEY MATHEMATICAL LANGUAGE

Introduce or revise the necessary language, such as *between*, *different*, *add*, *subtract* and *total*.

INTRODUCTION

Ask students to sort themselves into four groups (without talking or pointing), with each group having a different number of students.

PEDAGOGICAL CONSIDERATIONS

Allow students to read the question for themselves if they can. Some students might miss the requirement that the number of counters is different. Do not stress this at the start but encourage them to re-read carefully if they have missed it.

While the question does not state this, explain that all students have at least one counter. In the supplementary question about fishing, they can catch zero fish.

Interestingly, there are only six possible solutions, assuming everyone has at least one counter. Do not tell the students this but encourage them to find more if they have not yet found them all. Afterwards, emphasise the importance of being systematic.

Some students will be comfortable using symbols, some will draw pictures and some might need counters. Note that counters can make it harder for students to be systematic.

ENABLING PROMPT

Tell students that two students have 15 counters between them. They have different numbers of counters. Ask: *How many counters might each student have?*

EXTENDING PROMPT

Have students convince you that they have all the possible answers.

SUPPLEMENTARY TASKS

21 COUNTERS

Tell students that 5 students have 21 counters between them. All have different numbers of counters. Ask: *How many counters might each student have?*

ODDS AND EVENS

Tell students that 4 students have 15 counters between them. All have different numbers of counters. Ask:

Is it possible that all students have an odd number of counters?
Is it possible that all students have an even number of counters?
What's the maximum number of students with an even number of counters?
What's the maximum number of students with an odd number of counters?

FISHING

Tell students that 4 people go fishing. They catch 15 fish between them. They each catch different numbers of fish. Ask: *How many fish might each person have caught?*

POSSIBLE STUDENT SOLUTIONS TO THE LEARNING TASK

There are six possible solutions to the learning task:

1, 2, 3, 9
1, 2, 4, 8
1, 2, 5, 7
1, 3, 4, 7
1, 3, 5, 6
2, 3, 4, 6

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100
101	102	103	104	105	106	107	108	109	110