15 Geometric Proofs using Vectors

Calculator Assumed

1. [6 marks: 2, 2, 2,]

Given that $\boldsymbol{a}$ and $\boldsymbol{b}$ are non-parallel vectors. Find α and β if:

(a) $2\boldsymbol{a} + (\beta - 2)\,\boldsymbol{b} = (1 - \alpha)\,\boldsymbol{a}$

(b) $\alpha\,(3\boldsymbol{a} - 4\boldsymbol{b}) = 6\boldsymbol{a} + \beta\boldsymbol{b}$

(c) $\alpha\boldsymbol{a} + 5\boldsymbol{b}$ is parallel to $3\boldsymbol{a} + \beta\boldsymbol{b}$

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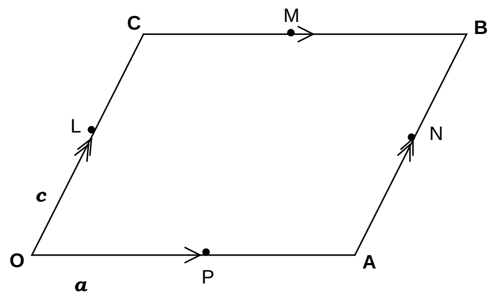
2. [4 marks: 1, 1, 2]



OABC is a parallelogram. **OA** = $\boldsymbol{a}$ and **OC** = $\boldsymbol{c}$. L, M, N and P are the midpoints of OC, CB, BA and AO respectively.

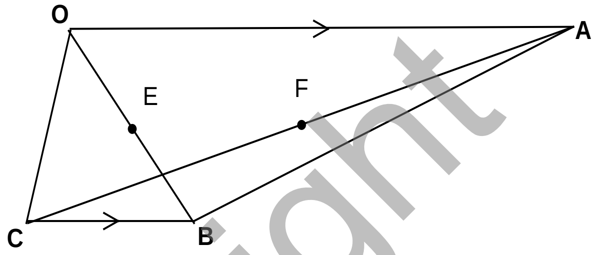
(a) Find **LM** in terms of **a** and **c** .

(b) Find **PN** in terms of **a** and **c**.

(c) Hence, use a vector method to show that LMNP is a parallelogram.

Calculator Assumed

3. [8 marks: 2, 4, 2]



OABC is a trapezium with **OA** = 3**CB**. **OA** = ***a*** and **OC** = ***c*** . E and F are midpoints of OB and CA respectively.

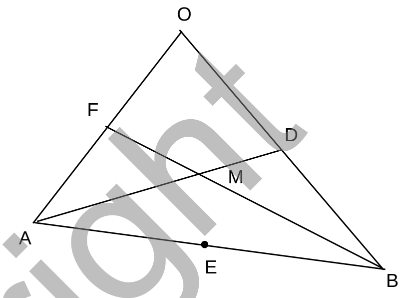
(a) Find **OE** in terms of ***a*** and ***c***.

(b) Find **EF** in terms of ***a*** and ***c***.

(c) Prove that CEFB is a parallelogram.

Calculator Assumed

4. [14 marks: 2, 2, 5, 3, 2]



OAB is a triangle with **OA** = ***a*** and **OB** = ***b***. D, E and F are the midpoints of OB, AB, and OA respectively. **AM** = α**AD** and **MF** = β**BF**.

(a) Find **AD** and **BF** in terms of ***a*** and ***b***.

(b) Find **AM** and **MF** in terms of ***a*** and ***b***.

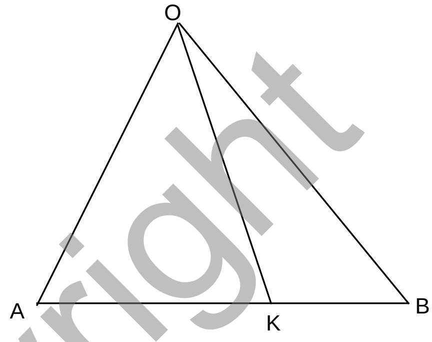
(c) Use your answers in (b) to find α and β.

(d) Show that **OM** = μ**OE** giving the value of μ .

(e) Comment on the significance of the location of M in terms of the lines OE, AD and BF.

Calculator Assumed

5. [8 marks: 2, 4, 2]



In triangle OAB, K divides AB in the ratio λ:μ (that is AK:KB = λ:μ).

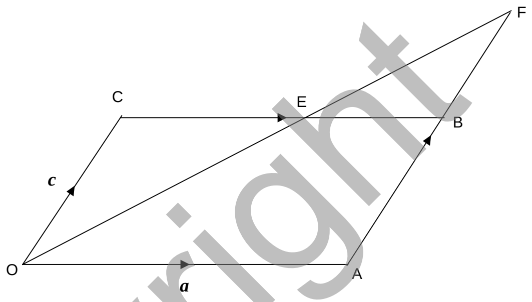
(a) Find **AK** in terms of **OA** and **OB** .

(b) Hence, or otherwise. prove that **OK** = $\left[\frac{1}{\lambda+\mu}\right]$[λ**OB** + μ**OA**] .

(c) Use the result above to find the position vector of a point that divides the line connecting A (1, 2) to B (6, 12) in the ratio 2: 3 .

Calculator Assumed

6. [8 marks]



OABC is a parallelogram. The point E divides CB in the ratio α : β. That is, the point E is such that

EB = $\frac{\beta}{\alpha+\beta}$**CB**. OE extended meets the AB extended at F. Use vector methods to prove that:

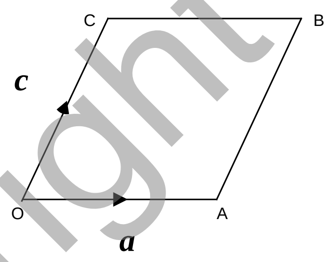
Area of ΔFEB = $\left(\frac{\beta}{\alpha+\beta}\right)^2$ × Area of ΔFOA.

[Hint: Let **EF** = λ**OF** and **BF** = μ**AF**.]

Calculator Assumed

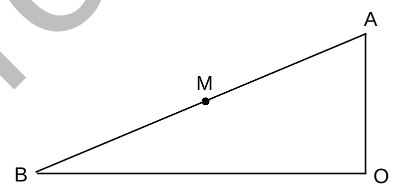
7. [4 marks]

OABC is a rhombus. **OA** = ***a*** and **OC** = ***c*** .
Use a vector method to show that the diagonals of a rhombus are perpendicular to each other.



8. [8 marks: 1, 3, 4]

OAB is a right angled triangle with ∠ AOB = 90^o.
OA = ***a*** and **OB** = ***b***. M is the midpoint of AB.



(a) Explain why ***a*** **.** ***b*** = 0 .

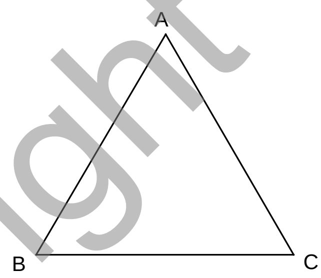
(b) Find $|\mathbf{BM}|^2$ in terms of *a* and *b*, where $|\boldsymbol{a}| = a$ and $|\boldsymbol{b}| = b$.

(c) Hence, prove that M is the centre of a circle passing through A, B and O.

Calculator Assumed

9. [12 marks: 2, 3, 4, 3]

ABC is an isosceles triangle with AB = AC.
Also, **BA** = $\boldsymbol{a}$ and **CB** = $\boldsymbol{b}$.



(a) Show that $|\boldsymbol{a} + \boldsymbol{b}| = |\boldsymbol{a}|$

(b) Show that $|\boldsymbol{b}|^2 = -2\,\boldsymbol{a} \cdot \boldsymbol{b}$.

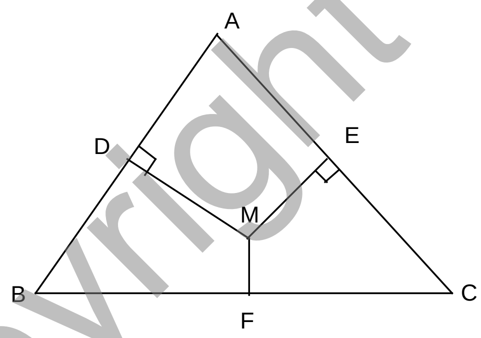
(c) Show that $\cos C = \dfrac{-\boldsymbol{a} \cdot \boldsymbol{b}}{|\boldsymbol{a}||\boldsymbol{b}|}$.

(d) Hence, using a vector method, prove that the base angles of an isosceles triangle are equal.

Calculator Assumed

10. [10 marks: 1, 2, 2, 3, 2]

DM and EM are respectively the perpendicular bisectors of sides AB and AC of triangle ABC. F is midpoint of BC. Also, **AB** = ***b*** , **AC** = ***c*** and **MD** = ***d***.



(a) Find **ME** in terms of ***b***, ***c*** and ***d***.

(b) Use your answer in (a) to show that $[\boldsymbol{d} + \frac{1}{2}(\boldsymbol{c} - \boldsymbol{b})] \cdot \boldsymbol{c} = 0$

(c) Find **MF** in terms of ***b***, ***c*** and ***d***.

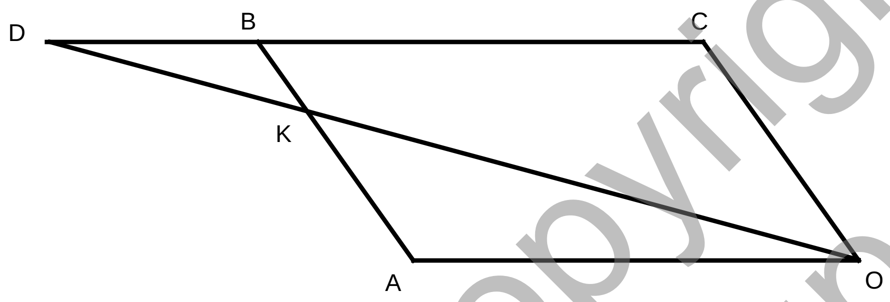
(d) Show that **MF . BC** = 0.

(e) State the significance of the result **MF . BC** = 0 .

Calculator Assumed

11. [5 marks: 2, 3] [TISC]

OABC is a parallelogram with **OA** = ***a*** and **OC** = ***c***.
The point K divides AB in the ratio 2 : 1. OK extended meets the line CB extended at D. **OK** = α**OD** and **CD** = β**CB**.



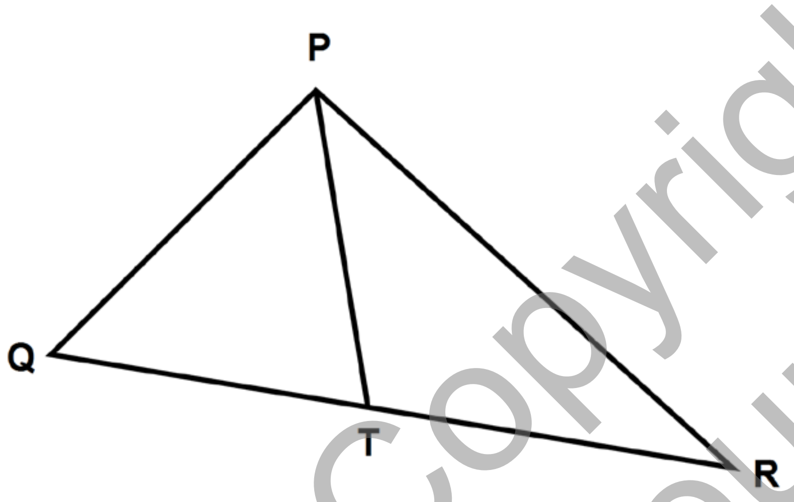
(b) Find **AK** and **OK** in terms of ***a*** and ***c***.

(c) Use vector methods to prove that B divides the line CD in the ratio 2 : 1.

Calculator Assumed

12. [8 marks: 3, 5] [TISC]

In ΔPQR drawn below, the point T is the midpoint of QR. Let **PT** = ***a*** and **TR** = ***b***.



(a) Find **PR** and **PQ** in terms of ***a*** and ***b***.

(b) If T is equidistant to P and R, use a vector method to prove that ∠QPR = 90º.