Australian Curriculum
Number and Algebra
Number and place value

chapter 1

Integers

What you will learn

Public key encryption

Most of the world's electronic commercial transactions are encrypted so that important information does not get into the wrong hands. The encryptions use an algorithm that uses prime numbers, division and remainders, equations and the 2300-year-old Euclidean division algorithm to complete the task. If it wasn't for Euclid (about 300 BCE) and the prime numbers, today's electronic transactions would not be secure.

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Pre-test

1 Put the following terms under the headings of addition (+), subtraction (–), multiplication (×) or division (÷).

a sum **b** of **c** and
d less than **e** total **f** into
g more than **h** increase **i** quotient

2 Complete these additions.

a $12 + 7$ **b** $50 + 19$ **c** $42 + 31$
d $146 + 213$ **e** $15 + 19 + 23$ **f** $\begin{array}{r} 123 \\ +\ 39 \\ \hline \end{array}$

3 Complete these subtractions.

a $12 - 8$ **b** $50 - 28$ **c** $47 - 29$
d $12 - 6 - 6$ **e** $784 - 163$ **f** $\begin{array}{r} 336 \\ -289 \\ \hline \end{array}$

4 Complete these multiplications.

a 9×4 **b** 5×8 **c** 12×11
d 15×5 **e** $\begin{array}{r} 121 \\ 9 \\ \hline \end{array}$ **f** $\begin{array}{r} 338 \\ 14 \\ \hline \end{array}$

5 Complete these divisions.

a $28 \div 4$ **b** $99 \div 3$ **c** $18 \div 6$
d $72 \div 12$ **e** $3\overline{)453}$ **f** $7\overline{)364}$

6 a List the first 5 multiples of 6.
b List the first 4 multiples of 9.
c What is the lowest common multiple (LCM) of 6 and 9?

7 a List all the factors of 12.
b List all the factors of 15.
c What is the highest common factor (HCF) of 12 and 15?

8 Prime numbers have exactly two factors. Copy these numbers into your workbook and circle the prime numbers. The first prime is circled for you.

1 (2) 3 4 5 6 7 8 9 10 11 12 13 14 15

9 Answer the following as true or false.

a $2 + 3 \times 4 = 2 + 12$ **b** $10 - 8 \div 2 = 10 - 4$ **c** $(5 - 2) \times 7 = 3 \times 7$
d $9 \times 3 + 5 = 9 \times 8$ **e** $9 \times (3 + 5) = 9 \times 8$ **f** $12 \div 3 \times 4 = 1$

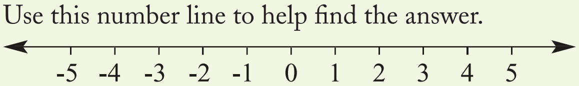
10 Copy and complete this table.

a	$2 \times 2 =$ □	$\sqrt{4} = 2$	**e**	$9 \times 9 =$ □	$\sqrt{□} = 9$
b	$3 \times 3 =$ □	$\sqrt{9} =$ □	**f**	$10 \times 10 =$ □	$\sqrt{□} = 10$
c	$4 \times 4 =$ □	$\sqrt{16} =$ □	**g**	□ × □ $= 49$	$\sqrt{49} =$ □
d	$6 \times 6 =$ □	$\sqrt{36} =$ □	**h**	□ × □ $= 144$	$\sqrt{144} =$ □

11 What are the next two numbers in each of these patterns?

a 3, 2, 1, ___, ___ **b** 2, 0, -2, ___, ___ **c** -9, -10, -11, ___, ___

12 Use this number line to help find the answer.



a $2 - 5$ **b** $0 - 3$ **c** $-4 + 6$ **d** $-2 + 7$

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1.1 Whole number addition and subtraction

The number system that we use today is called the Hindu–Arabic or decimal system. It uses the digits 0, 1, 2, 3, 4, 5, 6, 7, 8 and 9. The value of each digit depends on its place in the number, so, for example, the 4 in 3407 has a place value of 400. Whole numbers include 0 (zero) and the counting (natural) numbers 1, 2, 3, 4, … We can add or subtract whole numbers.

Let's start: Sum and difference

Use a guess-and-check method to try to find a pair of numbers described by these sentences.

- The sum of two numbers is 41 and their difference is 11.
- The sum of two numbers is 41 and their difference is 1.

Describe the meaning of the words 'sum' and 'difference'.

Discuss how you found the pair of numbers in each case.

Key ideas

- You can add in any order.
  e.g. $7 + 5 = 5 + 7$
  $9 + 3 + 1 = 9 + 1 + 3$
  – This is called the **commutative law** for addition.
- You cannot subtract in any order.
  e.g. $7 - 5 \neq 5 - 7$
- If the numbers are large, write addition and subtraction as algorithms as shown.

$$\begin{array}{r} 431 \\ +\ 165 \\ \hline 596 \end{array} \qquad \begin{array}{r} 394 \\ -\ 153 \\ \hline 241 \end{array}$$

Commutative law When adding and multiplying, the order in which two numbers are combined does not matter

Exercise 1A

Understanding

1 Match each of the questions in the left-hand column to the working out in the right-hand column.

a the total of 156, 94 and 6 — **I** $\begin{array}{r} 2491 \\ +\ 945 \\ \hline \end{array}$

b take 856 away from 2491 — **II** $2491 - 856$

c 945 more than 2491 — **III** $156 + 94 + 6$

d 945 less 863 — **IV** $\begin{array}{r} 945 \\ -\ 863 \\ \hline \end{array}$

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2 Write each of the following as an addition (+) or as a subtraction (–).

a 26 plus 17
b 43 take away 9
c 134 minus 23
d 451 add 50
e the sum of 19 and 29
f the sum of 111 and 236
g the difference between 59 and 43
h the difference between 339 and 298
i 36 more than 8
j 142 more than 421
k 32 less than 49
l 120 less than 251

3 Copy and complete.

a

+	2	5	7	10	12
5					
0					
18					
58					

b

+	3	9		
15				30
		10		
6			24	
2				

4 Are these additions and subtractions true or false?

a $15 + 6 = 6 + 15$
b $29 - 6 = 6 - 29$
c $95 + 0 = 95$
d $81 - 81 = 0$
e $15 + 6 + 4 = 15 + 10$
f $41 - 6 + 4 = 41 - 10$

Fluency

Example 1 Using mental arithmetic

Evaluate this difference and these sums mentally.

a $347 - 39$
b $125 + 127$
c $28 + 13$

Solution	Explanation	
a $347 - 39 = 308$	$347 - 39 = 347 - 40 + 1$ $= 307 + 1$ $= 308$	This method is called compensating. e.g. $134 + 29 = 134 + 30 - 1$
b $125 + 127 = 252$	$125 + 127 = 2 \times 125 + 2$ $= 250 + 2$ $= 252$	This method is called doubling. e.g. $127 = 125 + 2$
c $28 + 13 = 41$	$28 + 13 = 28 + 12 + 1$ $= 40 + 1$ $= 41$	This method is called counting on. e.g. $28 + 13 = 28 + 12 + 1$

5 Complete these additions.

a $21 + 5$
b $3 + 14$
c $17 + 13$
d $298 + 2$
e $35 + 11$
f $16 + 19$
g $21 + 5$
h $6 + 18$

Do these without a calculator or algorithm.

6 Complete these subtractions.

a $5 - 2$
b $16 - 4$
c $16 - 14$
d $21 - 21$
e $16 - 3$
f $45 - 13$
g $52 - 12$
h $52 - 14$

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7 Evaluate these sums and differences mentally.

a 94 – 62 **b** 146 + 241 **c** 1494 – 351 **d** 36 + 19
e 138 + 25 **f** 251 – 35 **g** 99 – 20 **h** 441 – 50
i 350 + 351 **j** 115 + 114 **k** 80 – 41 **l** 320 – 159

Example 2 Using an algorithm

Use an algorithm to find this sum and difference.

a $\begin{array}{r} 938 \\ +\ 217 \\ \hline \end{array}$ **b** $\begin{array}{r} 141 \\ -\ 86 \\ \hline \end{array}$

Solution

a $\begin{array}{r} 9\,{}^{1}3\,8 \\ +\ 2\,1\,7 \\ \hline 1\,1\,5\,5 \end{array}$

b $\begin{array}{r} 1\,{}^{3}\not{4}\,{}^{1}1 \\ -\ \ 8\,6 \\ \hline 5\,5 \end{array}$

Explanation

a: 8 + 7 = 15 (carry the 1 to the tens column)
1 + 3 + 1 = 5
9 + 2 = 11

b: Borrow from the tens column then subtract 6 from 11. Now borrow from the hundreds column and then subtract 8 from 13.

8 Use an algorithm to find these sums and differences.

a $\begin{array}{r} 128 \\ +\ 46 \\ \hline \end{array}$ **b** $\begin{array}{r} 94 \\ +\ 337 \\ \hline \end{array}$ **c** $\begin{array}{r} 9014 \\ +\ 927 \\ +\ 421 \\ \hline \end{array}$ **d** $\begin{array}{r} 814 \\ +\ 1439 \\ +\ 326 \\ \hline \end{array}$

e $\begin{array}{r} 94 \\ -\ 36 \\ \hline \end{array}$ **f** $\begin{array}{r} 421 \\ -\ 204 \\ \hline \end{array}$ **g** $\begin{array}{r} 1726 \\ -\ 1699 \\ \hline \end{array}$ **h** $\begin{array}{r} 14072 \\ -\ 328 \\ \hline \end{array}$

i $\begin{array}{r} 428 \\ +\ 314 \\ +\ 107 \\ +\ 29 \\ \hline \end{array}$ **j** $\begin{array}{r} 1004 \\ +\ 2407 \\ +\ 9116 \\ +\ 10494 \\ \hline \end{array}$ **k** $\begin{array}{r} 3017 \\ -\ 2942 \\ \hline \end{array}$ **l** $\begin{array}{r} 10024 \\ -\ 936 \\ \hline \end{array}$

Carry the 1 for totals larger than 9 and borrow 'ten' for subtraction.

Problem-solving and Reasoning

9 A racing bike's odometer shows 21 432 km at the start of a race and 22 110 km at the end of the race. How far was the race?

Casey Stoner racing at the Malaysian Grand Prix

10 Kristian has \$246 more than Sally. David has \$56 less than Sally. If Sally has \$492, how much do Kristian and David have?

11 Callum walks 15 km on Monday and 3 km more each day. How many kilometres does Callum walk on Thursday?

12 The sum of two numbers is 39 and their difference is 5. What is the larger number?

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Magic triangles and tricky additions and subtractions

13 a Write the digit missing from these sums and differences.

i $$\begin{array}{r} 2\ 3\ 7 \\ +\ \ \ 4\ \square \\ \hline 2\ 7\ 9 \end{array}$$

ii $$\begin{array}{r} 4\ 9 \\ +\ 3\ 8 \\ \hline 8\ \square \end{array}$$

iii $$\begin{array}{r} 4\ 9\ 3 \\ +\ 2\ 1\ 4 \\ \hline 7\ \square\ 7 \end{array}$$

iv $$\begin{array}{r} 1\ \square\ 4 \\ +\ 3\ 9\ 2 \\ \hline 5\ 5\ 6 \end{array}$$

v $$\begin{array}{r} 3\ 8 \\ -\ 1\ 9 \\ \hline 1\ \square \end{array}$$

vi $$\begin{array}{r} 1\ 2\ 8 \\ -\ \ \ 8\ \square \\ \hline 3\ 9 \end{array}$$

vii $$\begin{array}{r} 3\ \square\ 4 \\ -\ 1\ 6\ 2 \\ \hline 1\ 4\ 2 \end{array}$$

viii $$\begin{array}{r} 2\ 5\ 1 \\ -\ 1\ \square\ 4 \\ \hline 8\ 7 \end{array}$$

b Find the missing digits in these sums and differences.

i $$\begin{array}{r} 2\ 3\ \square \\ +\ \square\ 9\ 4 \\ \hline 6\ \square\ 1 \end{array}$$

ii $$\begin{array}{r} \square\ 3\ \square \\ +\ \ \ \square\ 2 \\ \hline 2\ 1\ 9 \end{array}$$

iii $$\begin{array}{r} \square\ 3\ 7 \\ +\ 4\ 9\ \square \\ \hline 7\ \square\ 2 \end{array}$$

iv $$\begin{array}{r} \square\ 3 \\ -\ 2\ 9 \\ \hline 6\ \square \end{array}$$

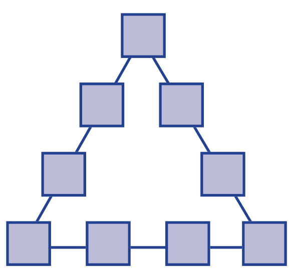
v $$\begin{array}{r} 3\ \square\ 2 \\ -\ \square\ 3\ \square \\ \hline 1\ 0\ 4 \end{array}$$

vi $$\begin{array}{r} 2\ \square\ \square\ 5 \\ -\ \ \ 6\ 8\ \square \\ \hline \square\ 3\ 1\ 8 \end{array}$$

c The sides of a magic triangle all sum to the same total.

i Show how it is possible to arrange all the digits from 1 to 9 so that each side adds to 17.

ii Show how it is possible to arrange the same digits to a different total. How many different totals can you find?



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1.2 Whole number multiplication and division

Multiplying and dividing numbers without a calculator is useful in many situations such as finding the cost of 9 tickets at $109 each or the number of trucks needed to carry 280 tonnes of coal.

A typical large mining truck has a capacity of 140 tonnes.

Let's start: Multiplication or division?

In solving many problems it is important to know whether multiplication or division should be used. Decide if the following situations require the use of multiplication or division. Discuss them in a group or with a partner.

- The number of cookies 4 people get if a packet of 32 cookies is shared equally between them.
- The cost of paving 30 square metres of courtyard at a cost of $41 per square metre.
- The number of sheets of paper in a shipment of 4000 boxes of 5 reams each (1 ream is 500 sheets).
- The number of hours I can afford a plumber at $75 per hour if I have a fixed budget of $1650.

Make up your own situation that requires the use of multiplication and another for division.

Key ideas

- Another word for multiplication is **product**.
- You need to know your multiplication tables.
- Multiplication can be done:
  - mentally
    e.g. $6 \times 5 = 30$
  - set out
    e.g.
$$\begin{array}{r l}
217 & \\
26 & \\
\hline
1302 & \leftarrow 217 \times 6 \\
4340 & \leftarrow 217 \times 20 \\
\hline
5642 & \leftarrow 1302 + 4340
\end{array}$$
- You can multiply numbers in any order.
  e.g. $6 \times 5 = 30$ and $5 \times 6 = 30$
  - This is the commutative law for multiplication.
- Using division results in finding a **quotient** and a **remainder**.
  e.g. $38 \div 11 = 3$ and 5 remainder
  (38 = dividend, 11 = divisor, 3 = quotient)
  $$7\overline{)5 1 ^2 2 ^1 4} \quad 7\,3\,2$$
- The **distributive law** is helpful when multiplying.
  e.g. $\mathbf{5} \times (97 + 3) = \mathbf{5} \times 97 + \mathbf{5} \times 3$

Product The result of multiplication

Quotient The result of division

Remainder The amount left over after division, when one number cannot be divided exactly into another

Distributive law Adding numbers *then* multiplying the total gives the same answer as multiplying each number first *then* adding the products

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Exercise 1B

Understanding

1 Match each of the questions to the working out on the right.

a	the product of 9 and 6	**I**	15×12
b	36 divided by 12	**II**	$15 \div 5$
c	15 lots of 12	**III**	9×6
d	the quotient when 15 is divided by 5	**IV**	$15 \div 12$
e	divide 12 into 15	**V**	$36 \div 12$

2 Copy and complete these multiplication grids.

a

×	1	2	3	4	5	6	7
1							
2							
3							
4							
5							
6							
7							

b

×	2	5	7	9
	6			
		20		
			63	
				90

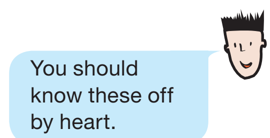
3 Use your knowledge of the multiplication table to answer the following.

a 5×8 **b** 11×9 **c** 6×7 **d** 9×8

e 11×6 **f** 12×11 **g** 8×4 **h** 7×9

i $100 \div 10$ **j** $88 \div 8$ **k** $121 \div 11$ **l** $144 \div 12$

m $56 \div 7$ **n** $33 \div 3$ **o** $65 \div 5$ **p** $78 \div 6$



4 Are these simple equations true or false?

a $4 \times 13 = 13 \times 4$

b $2 \times 7 \times 9 = 7 \times 9 \times 2$

c $6 \div 3 = 3 \div 6$

d $60 \div 20 = 30 \div 10$

e $14 \div 2 \div 7 = 7 \div 2 \div 14$

f $51 \times 7 = (50 \times 7) + (1 \times 7)$

g $79 \times 13 = (80 \times 13) - (1 \times 13)$

h $93 \div 3 = (90 \div 3) + (3 \div 3)$

Fluency

Example 3 Using mental strategies for multiplication

Use a mental strategy to evaluate the following.

a 5×160 **b** 7×89 **c** $5 \times 43 \times 2$

Solution	Explanation
a $5 \times 160 = 800$	To multiply by 5 you can multiply by 10 then halve the result. $160 \times 10 = 1600$, $1600 \div 2 = 800$
b $7 \times 89 = 623$	$89 = 90 - 1 \therefore 7 \times 89 = 7 \times 90 - 7 \times 1 = 630 - 7 = 623$ (this is the distributive law)
c $5 \times 43 \times 2 = 430$	$5 \times 43 \times 2 = 5 \times 2 \times 43$ (look for easy pairs) $= 10 \times 43$ $= 430$

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5 Use a mental strategy to evaluate the following.

a 15×3	**b** 18×4	**c** $6 \times 5 \times 2$	**d** 7×20
e 16×4	**f** 99×7	**g** 79×3	**h** 42×5
i $5 \times 13 \times 2$	**j** $2 \times 26 \times 5$	**k** 4×35	**l** 17×4
m 17×1000	**n** 136×100	**o** 59×7	**p** 119×6
q 9×51	**r** 6×61	**s** 4×252	**t** 998×6

Example 4 Using mental strategies for division

Use a mental strategy to evaluate the following.

a $464 \div 4$ **b** $480 \div 5 \div 2$

Solution	Explanation
a $464 \div 4 = 116$	To divide by 4 you can divide by 2 twice. $464 \div 4 = 464 \div 2 \div 2$ ($\div 2$ is the same as halving the number) $= 232 \div 2$ $= 116$
b $480 \div 5 \div 2 = 48$	Dividing by 5 and then by 2 is the same as dividing by 10. $480 \div 10 = 48$

6 Use a mental strategy to evaluate the following.

a $64 \div 2$	**b** $64 \div 4$	**c** $640 \div 4$
d $492 \div 4$	**e** $185 \div 5$	**f** $1980 \div 5 \div 2$
g $128 \div 8$	**h** $252 \div 4$	**i** $123 \div 3$
j $508 \div 4$	**k** $96 \div 6$	**l** $1016 \div 8$

Example 5 Using multiplication and division setting out

Use an algorithm to evaluate the following.

a 412
25

b $938 \div 13$

Solution	Explanation
a 412 25 2060 8240 10300	$412 \times 5 = 2060$ and $412 \times 20 = 8240$ Add these two products to get the final answer.
b 7 2 Rem 2 $13\overline{)93^{2}8}$	$93 \div 13 = 7$ and 2 remainder $28 \div 13 = 2$ and 2 remainder
So $938 \div 13 = 72$ and 2 remainder. $= 72\frac{2}{13}$	We write remainders as fractions $72\frac{2}{13}$.

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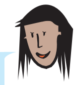
7 Use setting out to evaluate the following.

a $\begin{array}{r}67\\ \underline{9}\end{array}$ b $\begin{array}{r}129\\ \underline{4}\end{array}$ c $\begin{array}{r}294\\ \underline{13}\end{array}$ d $\begin{array}{r}1004\\ \underline{90}\end{array}$

e $\begin{array}{r}690\\ \underline{14}\end{array}$ f $\begin{array}{r}96\\ \underline{12}\end{array}$ g $\begin{array}{r}58\\ \underline{24}\end{array}$ h $\begin{array}{r}163\\ \underline{52}\end{array}$



Use the setting out described in Example 5.

8 Use the short division setting out to evaluate the following.

a $3\overline{)85}$ b $7\overline{)214}$ c $10\overline{)4167}$ d $15\overline{)207}$

e $6\overline{)15084}$ f $3\overline{)1236}$ g $12\overline{)2520}$ h $12\overline{)8892}$

Problem-solving and Reasoning

9 A university student earns $550 for 20 hours work. What is the student's pay rate per hour?

10 Packets of biscuits are purchased by a supermarket in boxes of 12. The supermarket orders 220 boxes and sells 89 boxes in one day. How many boxes are left? How many packets of biscuits remain in the supermarket?

11 Riley buys a fridge, which he can pay for by the following options.

A 9 payments of $183

B $1559 up front

Which option is cheaper and by how much?

12 The shovel of a giant excavator can move 6 tonnes of rock in each load. How many loads are needed to shift 750 tonnes of rock?

13 Tom saves $362 a week. How much will he save in 52 weeks?

Maximum tickets

14 A child ticket to a theatre is $7 and an adult ticket is $12.

a Find the cost of 2 adults and 3 children tickets.

b Find the cost of 1 adult and 5 children tickets.

c Gen spends exactly $90 to buy child tickets and adult tickets. Find the maximum number of tickets that Gen could purchase.

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1.3 The order of operations

As we saw last year, when working with more than one operation, such as multiplication and addition, a particular order needs to be followed.

Let us look at the simple sum of $5 + 4 \times 5 = 25$.

If we did the addition first, then $5 + 4 \times 5 = 9 \times 5 = 45$, but we know that this is not true. We need to be consistent with our order of operations to ensure we all get the same answer for each problem.

Let's start: How many?

How many ways can you get $36 - 20 = 16$?

See if you can come up with at least five different statements using the four operations ($+ - \times \div$) and brackets that give the same subtraction above. One example is $9 \times 4 - (24 - 4)$.

Key ideas

Order of operations

- Deal with the **grouping symbols** or brackets first.
- Do any multiplication ($\times$) and division ($\div$) next, working across the question from left to right.
- Do any addition ($+$) and subtraction ($-$) next, again working from left to right.

NOTE: Within any brackets the order of operations still needs to be followed.

Grouping symbols Parentheses ( ), brackets [ ] and braces { } are used to collect terms and operations together

Exercise 1C

Understanding

1 Copy each question into your books. By following the order of operations, underline the operation that needs to be done first.

a $2 + 3 \times 9$

b $10 - 2 \div 2$

c $1 \times 3 + 5$

d $6 \times (9 - 6)$

e $(12 + 6) \div 2$

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2 Match each of the questions on the left to the correct working on the right.

a	$10 + 7 \times 3$	**I**	$10 + 21$
b	$15 - 9 \div 3$	**II**	$5 - 4$
c	$(9 - 4) \times 6$	**III**	$15 - 3$
d	$(9 - 4) - (10 - 6)$	**IV**	$2 + 10$
e	$18 \div 9 + 5 \times 2$	**V**	5×6

Example 6 Two operations

Find the answers to each of the following.

a $10 + 5 \times 3$ **b** $18 \div 6 \times 2$ **c** $15 - (7 - 3)$

Solution	Explanation
a $10 + 5 \times 3 = 10 + 15$ $= 25$	Multiplication (×) is done BEFORE addition (+). $5 \times 3 = 15$
b $18 \div 6 \times 2 = 3 \times 2$ $= 6$	Division (÷) and multiplication (×) are done as they appear from left to right. $18 \div 6$ is done first then $\times 2$ last.
c $15 - (7 - 3) = 15 - 4$ $= 11$	Brackets need to be done first $(7 - 3) = 4$. Then do the subtraction $15 - 4$.

3 Find the answers to each of the following.

a	$12 + 5 \times 2$	**b**	$24 - 6 \times 3$
c	$10 \times 2 + 6$	**d**	$15 \div 3 - 2$
e	$(9 - 2) \times 4$	**f**	$18 - (12 - 8)$
g	$28 \div (2 \times 7)$	**h**	$56 - 5 \times 10$
i	$120 + 200 \div 5$	**j**	$88 \times 2 \div 8$
k	$12 \div (18 \div 6)$	**l**	$16 - 18 \div 9$
m	$55 \div 11 \times 5$	**n**	$55 - 25 \div 5$
o	$240 \div 10 \times 2$	**p**	$58 + 100 \div 20$
q	$100 - 25 \div 5$	**r**	$(24 - 9) \times 3$

First: brackets
Next: × or ÷
Last: + or −

4 Find the answer to these problems by first writing the sentence using numbers and symbols.

a Double the sum of 3 and 7
b Double the quotient of 24 and 8
c The product of 5 and 7 plus 4
d 8 more than the product of 12 and 5
e 10 less than the quotient of 66 and 3
f Triple the difference between 18 and 12

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Example 7 Several steps

Find the answers to each of the following.

a $4 \times 5 - 3 \times 2$ **b** $(7 + 2) \times 5 - 6$ **c** $10 + (2 \times (6 - 4))$

Solution	Explanation
a $4 \times 5 - 3 \times 2$ $= 20 - 6$ $= 14$	Both sets of multiplication (×) need to be done first. Then do the subtraction (–).
b $(7 + 2) \times 5 - 6$ $= 9 \times 5 - 6$ $= 45 - 6$ $= 39$	Do the brackets first $(7 + 2)$. Next do the multiplication 9×5. Then the subtraction $45 - 6$.
c $10 + (2 \times (6 - 4))$ $= 10 + (2 \times 2)$ $= 10 + 4$ $= 14$	Start with the inner most brackets $(6 - 4)$. Finish working with the brackets – we follow the order of operations within the brackets (2×2). Then the addition $10 + 4$.

Fluency

5 Find the answers to the following.

a $2 \times 4 - 4 \div 2$ **b** $13 + 4 \times 5 - 3$ **c** $(14 - 12) \times 4 + 11$
d $(12 - 5) \times (6 + 3)$ **e** $5 \times 6 + 12 \times 3$ **f** $25 - 20 \div 5 + 2$
g $25 - 20 \div 5 + 2 \times 5$ **h** $(10 + 10) \div (25 - 5)$ **i** $(10 \times 10 + 5) \div 5$
j $(20 - 8) \times 12 - 4$

Show steps of working as in the examples.

6 Simplify.

a $5 \times 4 + 8 \times 4$ **b** $24 \div 4 \times 6 - 8$ **c** $(15 - 5) \times 8 + 200$
d $6 \times 4 - 2 \times 6 + 12$ **e** $96 \div (12 \times 8)$ **f** $5 + (12 \times (23 - 6))$
g $1 + 4 + 3 \times (8 - 5)$ **h** $(12 - 5) \times (22 - 12)$ **i** $12 + (18 - (12 - 5))$
j $15 \times (24 \div 6 \times 2)$

7 Evaluate.

a $56 - 4 \times 6$ **b** $96 \div 4 + 3 \times 6$
c $150 - (7 \times (10 - 3 \times 2))$ **d** $(12 \times (13 - 8) \times (24 - 18))$

Problem-solving and Reasoning

8 True or false?

a $5 + 9 = 5 + 3 \times 3$ **b** $10 + 2 \times 7 = 12 + 7$ **c** $18 - 6 + 5 = 12 + 5$
d $3 \times 5 \times 6 = 15 \times 6$ **e** $120 \div 6 \times 2 = 20 \times 2$ **f** $(5 + 3) \times 9 = 8 \times 9$
g $15 \div 5 \times 3 = 1$

9 Insert brackets into each of the following statements to make it true.

a $12 - 8 \times 2 = 8$ **b** $4 \times 5 + 6 = 44$
c $16 \div 2 \times 8 = 1$ **d** $6 \times 2 + 6 \times 1 = 48$
e $15 \times 4 - 2 = 30$

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10 Insert operation symbols (+, –, ×, ÷) between the numbers to make each of the following statements true.

a 5 ___ 4 ___ 9 = 0

b 5 ____ 4 ____ 9 = 11

c 5 ______ 4 _____ 9 = 41

11 Write each of the following situations into mathematical symbols and numbers, and then calculate.

a Murray receives four dollars from his mum and seven dollars from his dad as pocket money each week for 12 weeks. How much money does he have at the end of the twelve weeks?

b A raffle prize consists of \$5000 cash and 6 shopping vouchers each worth \$500. What is the total value of the raffle prize?

c Sally has fifty dollars. She buys four pens at two dollars each and eight exercise books at three dollars each. How much change does Sally get?

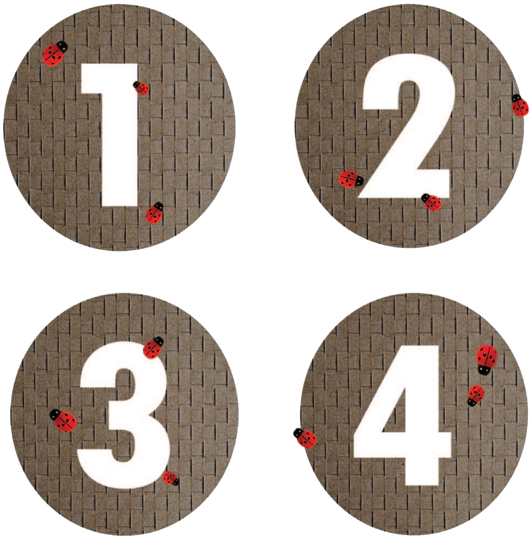
12 Decide if the brackets in each of the following are really needed.

a 10 + (9 × 8) b 12 + (3 + 4) c 12 – (3 + 4)

d 25 × (3 – 1) e (100 – 4 × 3)

Make ten from four

13 Can you make the first 10 counting numbers (1, 2, 3, 4, 5, 6, 7, 8, 9 and 10) using only the four digits 1, 2, 3 and 4 (once each), brackets and any of the four operations?



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1.4 Squares, cubes and other powers

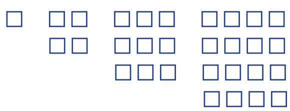
In mathematics there is often a way to abbreviate your work.

Using repeated addition, $4 + 4 + 4 + 4 + 4$ becomes multiplication 5×4.

Using repeated multiplication, $3 \times 3 \times 3 \times 3$ becomes index notation 3^4.

We read 3^4 as 3 to the power of 4.

Let's start: Square numbers



Can you explain why we call the numbers 1, 4, 9 and 16 square numbers?

Draw the next two square numbers in your book.

Use centicubes to build the first three cube numbers. Write down the next cube number.

Key ideas

- **Index** notation

  index or power

  3^4 $\qquad$ $3^4 = 3 \times 3 \times 3 \times 3$

  base

  The **base** of 3 shows the factor that is repeating in multiplication and the **power** or **index** is the number of times it repeats.
- The **square** of a number is written a^2 and it means $a \times a$.

  e.g. 5^2 means 5×5 (we say 5 squared, the square of 5, or 5 to the power of 2)
- The opposite of squaring is the **square root** of a number. The symbol $\sqrt{\ }$ means square root.

  e.g. $\sqrt{9} = 3$ as $3^2 = 9$

  – The square root of a number is always positive.
- The **cube** of a number a is $a^3 = a \times a \times a$.

  e.g. $5^3 = 5 \times 5 \times 5$ (we say 5 cubed, or, 5 to the power of 3)
- The opposite of cubing is taking the **cube root** of a number. The symbol for cube root is $\sqrt[3]{\ }$.

  e.g. $\sqrt[3]{8} = 2$ as $2^3 = 2 \times 2 \times 2 = 8$

Base The number or pronumeral that is being raised to a power

Index The number of times the base number is repeated under multiplication

Square To multiply a number by itself

Square root The opposite operation of squaring

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Exercise 1D

Understanding

1 Write each of the following in abbreviated form.

a 2×2 b 4×4 c 5×5
d $5 \times 5 \times 5$ e $6 \times 6 \times 6 \times 6$ f $7 \times 7 \times 7$

2 Match each expression in words to an expression in symbols, given on the right.

a The square of 10 — I $\sqrt{16}$
b The cube of 1 — II $\sqrt[3]{1}$
c The square of 12 — III $\sqrt{1}$
d The square root of 1 — IV 10^2
e The cube root of 1 — V 1^3
f The square root of 16 — VI 12^2

The cube of 2 is $2^3 = 2 \times 2 \times 2 = 8$

3 Copy and complete.

$1^2 = 1 \times 1 = 1$
$2^2 = 2 \times 2 = 4$
$3^2 =$
$4^2 =$
$5^2 =$
$6^2 =$
$7^2 =$
$8^2 =$
$9^2 =$
$10^2 =$

4 Copy and complete.

$1^3 = 1 \times 1 \times 1 = 1$
$2^3 = 2 \times 2 \times 2 = 8$
$3^3 =$
$4^3 =$
$5^3 =$
$6^3 =$

Example 8 Using index notation

Write each product in index notation.

a $8 \times 8 \times 8$ b $7 \times 7 \times 7 \times 7 \times 7 \times 7$

Solution	Explanation
a $8 \times 8 \times 8 = 8^3$	The number 8 is repeating in multiplication 3 times. We write 8 to the power of 3.
b $7 \times 7 \times 7 \times 7 \times 7 \times 7 = 7^6$	The 7 is repeating in multiplication 6 times. We write 7 to the power of 6.

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5 Write each of the following products in index notation.

a $7 \times 7 \times 7$ **b** $10 \times 10 \times 10 \times 10$ **c** 8×8

d $4 \times 4 \times 4$ **e** $2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$ **f** $6 \times 6 \times 6 \times 6 \times 6 \times 6 \times 6$

g 12×12 **h** $5 \times 5 \times 5 \times 5 \times 5 \times 5$ **i** 6

Example 9 Expanded notation and evaluating index notation

a Write 5^4 in expanded form. **b** Find the value of 5^4.

Solution	Explanation
a $5^4 = 5 \times 5 \times 5 \times 5$	The power of 4 tells us that the number 5 repeats in multiplication 4 times. $5^4 = 5 \times 5 \times 5 \times 5$
b $5^4 = 625$	$5^4 = 5 \times 5 \times 5 \times 5$ (multiply the 5 by itself four times) $= 25 \times 5 \times 5$ $= 125 \times 5$ $= 625$

Fluency

6 Write each index notation in expanded form.

a 8^5 **b** 3^4 **c** 9^2

d 4^4 **e** 2^8 **f** 11^2

$5 \times 5 \times 5$ is the expanded form of 5^3.

7 Find the value of each index notation.

a 2^3 **b** 2^4 **c** 3^3

d 10^4 **e** 5^3 **f** 1^4

Example 10 Finding squares, cubes, square roots and cube roots

Evaluate the following.

a 6^2 **b** $\sqrt{81}$ **c** 3^3 **d** $\sqrt[3]{64}$

Solution	Explanation
a $6^2 = 6 \times 6$ $= 36$	Find the product of 6 with itself.
b $\sqrt{81} = 9$	$9^2 = 9 \times 9 = 81$ so $\sqrt{81} = 9$
c $3^3 = 3 \times 3 \times 3$ $= 27$	In general $x^3 = x \times x \times x$.
d $\sqrt[3]{64} = 4$	$4^3 = 4 \times 4 \times 4 = 64$ so $\sqrt[3]{64} = 4$

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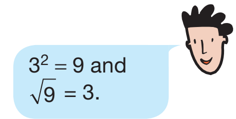
8 Evaluate these squares and square roots.

a 4^2	**b** 10^2	**c** 13^2	**d** 15^2
e 100^2	**f** 20^2	**g** $\sqrt{25}$	**h** $\sqrt{49}$
i $\sqrt{121}$	**j** $\sqrt{900}$	**k** $\sqrt{1600}$	**l** $\sqrt{256}$



9 Evaluate these cubes and cube roots.

a 2^3	**b** 4^3	**c** 7^3	**d** 5^3
e 6^3	**f** 10^3	**g** $\sqrt[3]{27}$	**h** $\sqrt[3]{8}$
i $\sqrt[3]{125}$	**j** $\sqrt[3]{512}$	**k** $\sqrt[3]{729}$	**l** $\sqrt[3]{1000000}$

Problem-solving and Reasoning

10 Decide which of the following is larger.
- **a** 2^3 or 3^2
- **b** 2^4 or 3^2
- **c** 2^5 or 5^2

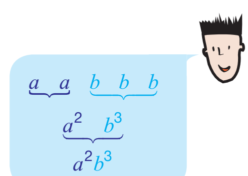
11 Copy and complete.
- **a** If $13^2 = 169$, then $\sqrt{169} = \square$
- **b** If $15^2 = 225$, then $\sqrt{225} = \square$
- **c** If $\sqrt{625} = 25$, then $25^2 = \square$
- **d** If $9^3 = 729$, then $\sqrt[3]{729} = \square$
- **e** If $\sqrt[3]{1331} = 11$, then $11^3 = \square$

12 Given $5 \times 5 \times 5 \times 4 \times 4$ is written as $5^3 \times 4^2$ (the different bases of 5 and 4 are kept separate), write each of the following in index form.

a $6 \times 6 \times 7 \times 7 \times 7 \times 7$	**b** $5 \times 5 \times 5 \times 5 \times 2 \times 2$
c $3 \times 3 \times 8 \times 8$	**d** $11 \times 9 \times 9 \times 9 \times 9$
e $12 \times 12 \times 4 \times 4 \times 4$	**f** $2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3$

Algebraic bases

13 Write each of the following in index form. Remember, different bases cannot be collected.
- **a** $m \times m \times m$
- **b** $a \times a \times a \times a \times a$
- **c** $n \times n \times n \times n \times n \times n \times n$
- **d** $p \times p \times p \times p \times p \times p \times p \times p \times p \times p$
- **e** $p \times p \times p \times q \times q$
- **f** $a \times a \times a \times a \times b \times b$
- **g** $a \times a \times b \times b \times b \times b$
- **h** $x \times x \times x \times x \times y$



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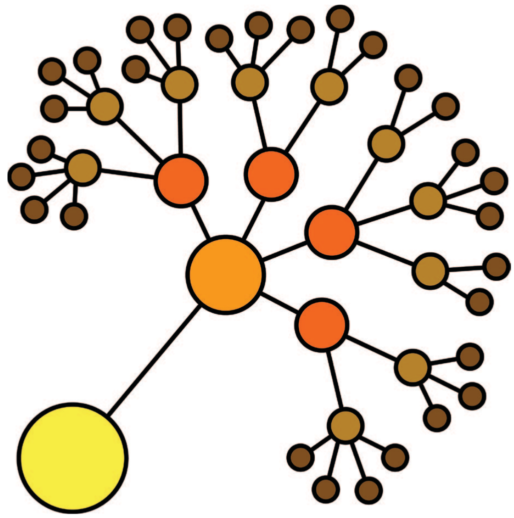
1.5 The index laws

In this section we will look at the rules needed when working with numbers written in index notation.

We call these rules the index laws.

Let's start: Investigating the first two rules

Write out 3^7 in expanded notation.
Now write out 3^4 in expanded notation.
What do you get when 3^7 is multiplied by 3^4? How many times does the base of 3 repeat in this product?
What do you get when 3^7 is divided by 3^4? How many times does the base of 3 repeat in this quotient?



Index notation has wide application, particularly in modelling growth and decay, in science, economics and computer applications.

Key ideas

Index law 1: $a^m \times a^n = a^{m+n}$

- Use when multiplying numbers written in **index notation**. If the base is the same, you keep the base and add the powers together.
  - e.g. $2^3 \times 2^2 = (2 \times 2 \times 2 \times 2 \times 2)$
    $= 2^5$ (here the base of 2 repeats 5 times $(3 + 2)$)

Index law 2: $a^m \div a^n = a^{m-n}$

- Use when dividing numbers written in index notation. If the base is the same, you keep the base and subtract the powers together.
  - e.g. $2^6 \div 2^2 = (2 \times 2 \times 2 \times 2 \times 2 \times 2) \div (2 \times 2)$
    $= \dfrac{2\ \ 2\ \ 2\ \ 2\ \ 2\ \ 2}{2\ \ 2}$
    $= 2^4$ (here the base of 2 repeats 4 times $(6 - 2)$)

Index law 3: $(a^m)^n = a^{m \times n}$

- Use when a number written in index notation is raised to another power. The base remains the same and the two powers (indices) are multiplied together.
  - e.g. $(2^3)^4 = 2^3 \times 2^3 \times 2^3 \times 2^3$
    $= 2^{3+3+3+3}$
    $= 2^{12}$ (here the base of 2 repeats in total 12 times (3×4))

The zero power: $a^0 = 1$

- Any non-zero number raised to the power of zero gives an answer of one.
  - e.g. $2^0 = 1$
    e.g. $2^3 \div 2^3 = 2^{3-3} = 2^0$ (but $2^3 \div 2^3 = 1$ so this must mean that $2^0 = 1$)

Index notation
Method of writing numbers that are multiplied by themselves

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Exercise 1E

Understanding

1 Which of the following is the same as $4^3 \times 4^4$?

A $4 \times 4 \times 4 \times 4 \times 4 \times 4 \times 4$ **B** $16 \times 16 \times 16 \times 16 \times 16 \times 16 \times 16$
C 16^7 **D** 16^{12}

2 Which of the following is equal to $3^6 \div 3^2$?

A $3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3$ **B** $3 \times 3 \times 3$
C $3 \times 3 \times 3 \times 3$ **D** 1^4

3 Write the following in your workbook using index notation.

a 6 raised to the power of 2
b 7 raised to the power of 0
c $(5 \times 5 \times 5 \times 5 \times 5 \times 5) \times (5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5)$
d $(6 \times 6 \times 6 \times 6 \times 6 \times 6 \times 6 \times 6 \times 6) \div (6 \times 6 \times 6)$

4 Which of the following is the same as $(2^2)^3$?

A 2^5 **B** 4^5
C $(2 \times 2) \times (2 \times 2) \times (2 \times 2) = 2^6$ **D** 2^{23}

Fluency

Example 11 The first two index laws

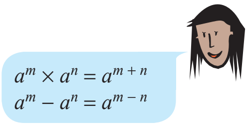
Simplify each of these, leaving your answer in index form.

a $6^5 \times 6^4$ **b** $5^7 \div 5^4$

Solution	Explanation
a $6^5 \times 6^4 = 6^9$	Use index law 1: $a^m \times a^n = a^{m+n}$ (keep the base and add the powers) $6^5 \times 6^4$ (the base of 6 repeats 5 times in the first term and 4 times in the next term) The base of 6 repeats 9 times in the product.
b $5^7 \div 5^4 = 5^3$	Use index law 2: $a^m \div a^n = a^{m-n}$ $5^7 \div 5^4 = 5^{7-4}$ $= 5^3$

5 Copy and complete the following.

a $7^4 \times 7^2 = 7^{\square}$ **b** $8^2 \times 8^1 = 8^{\square}$ **c** $9^6 \times 9^3 = 9^{\square}$
d $5^4 \times 5^3 = 5^{\square}$ **e** $2^{10} \times 2^3 = 2^{\square}$ **f** $2^{\square} \times 2^9 = 2^{15}$
g $5^8 \div 5^2 = 5^{\square}$ **h** $6^4 \div 6^1 = 6^{\square}$ **i** $2^{12} \div 2^8 = 2^{\square}$
j $1^{16} \div 1^{13} = 1^{\square}$ **k** $8^{\square} \div 8^4 = 8^2$ **l** $10^7 \div 10^{\square} = 10^2$



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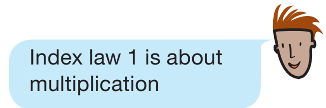
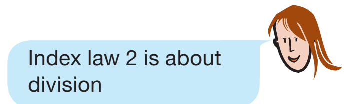
6 Simplify each of the following using the index law for multiplication.

a $3^4 \times 3^2$ **b** $2^2 \times 2^3$ **c** $10^3 \times 10^1$
d $9^6 \times 9^4$ **e** $4^4 \times 4$ **f** $2^3 \times 2^9$
g $8^7 \times 8^3$ **h** $12^9 \times 12$ **i** $16^5 \times 16^3$



7 Simplify each of the following using the index law for division.

a $3^4 \div 3^2$ **b** $2^7 \div 2^5$ **c** $9^6 \div 9^2$
d $4^5 \div 4^2$ **e** $17^{26} \div 17^{20}$ **f** $11^9 \div 11^3$



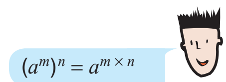
Example 12 Raising powers

Simplify $(4^3)^3$.

Solution	Explanation
$(4^3)^3 = 4^9$	Use index law 3: $(a^m)^n = a^{m \times n}$ $(4^3)^3 = 4^{3 \times 3}$ The base of 4 stays the same and the powers are multiplied together.

8 Copy and complete.

a $(2^3)^4 = 2^{\square}$ **b** $(3^2)^5 = 3^{\square}$ **c** $(5^2)^2 = 5^{\square}$
d $(2^4)^3 = 2^{\square}$ **e** $(7^3)^2 = 7^{\square}$ **f** $(8^4)^5 = 8^{\square}$



9 Simplify the following.

a $(7^2)^2$ **b** $(2^5)^4$ **c** $(3^7)^2$ **d** $(8^4)^2$
e $(3^4)^2$ **f** $(10^6)^5$ **g** $(9^2)^7$ **h** $(5^5)^3$

Example 13 The power of zero

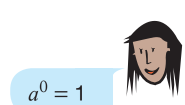
Simplify.

a 9^0 **b** $(3 \times 2)^0$ **c** 4×5^0

Solution	Explanation
a $9^0 = 1$	A number (except zero) raised to the power of zero equals one.
b $(3 \times 2)^0 = 1$	As the overall power on the brackets is zero – the expression equals one.
c $4 \times 5^0 = 4 \times 1$ $= 4$	$5^0 = 1$ so the product of 4 and 5^0 is the same as 4×1.

10 Simplify the following.

a 5^0 **b** 6^0 **c** 19^0 **d** 15^0
e $(27 \times 25)^0$ **f** $5^0 + 7$ **g** $8 - 3^0$ **h** 10×2^0
i $5^0 \times 6^0$ **j** $5^0 + 6^0$ **k** $6^0 + 5$ **l** $12^0 \times 3$



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Problem-solving and Reasoning

11 Complete the following.

- **a** Given $4 = 2^2$, write the product $2^7 \times 4$ as $2^{\square}$.
- **b** Write $5^4 \times 25$ as $5^{\square}$.
- **c** Write down the numerical value of $6^{14} \div 6^{12}$.
- **d** What do you notice about $(3^4)^2$ and $(3^2)^4$?
- **e** Write down the numerical value of $4^2 \times 3^2$. Is it the same as 7^2 or 12^2?

12 Simplify the following.

- **a** $2^7 \times 2^4 \div 2^3$
- **b** $(2^3)^3 \times 2^4$
- **c** $10^7 \div 10^2 \div 10^2$
- **d** $7^9 \times 7^3 \times 7^2$
- **e** $6^4 \times 6^5 \div 6^8$
- **f** $3^7 \times 3 \times 3$

Combine the index laws where required.

Algebraic bases

13 Use the four index laws to complete these index law questions involving pronumeral bases.

- **a** $a^7 \times a^4$
- **b** $m^4 \times m^3$
- **c** $a^5 \times a^4$
- **d** $x^5 \times x^8$
- **e** $n^7 \times n^4$
- **f** $m^6 \times m^7 \times m$
- **g** $n^9 \div n^3$
- **h** $a^{10} \div a^7$
- **i** $m^6 \div m^4$
- **j** $a^7 \times a^2 \times a^3$
- **k** $w^{12} \div w^3$
- **l** $p^8 \times p^2 \div p^6$

Remember, the base stays the same.
$m^{20} \times m^4$
$= m^{20+4}$
$= m^{24}$

14 Simplify these using the given hint.

- **a** $5m^4 \times m^3$
- **b** $6m^2 \times 4m^6$
- **c** $8m^6 \times 2m^4$
- **d** $3a^2 \times 4a^7$
- **e** $7x^3 \times 3x^4$
- **f** $5x^9 \times 4x^3$

$5x^7 \times 3x^2$
$= 5 \times 3 \times x^7 \times x^2$
$= 15 \times x^{7+2}$
$= 15x^9$

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1.6 Further number properties

Knowing the properties of numbers helps us with our problem-solving work.

A prime number, for example, only has two factors, and can help us with our division.

What number properties do you remember from last year?

Into how many equal groups could these people be divided?

Let's start: How many in 60 seconds?

In 60 seconds, write down as many numbers as you can that fit each description.

- Multiples of 7
- Factors of 144
- Prime numbers

Compare your lists with the results of the class. What is the biggest prime number that the class came up with?

Key ideas

- A **multiple** of a number is obtained by multiplying the number by the **counting numbers** 1, 2, 3, …
  e.g. Multiples of 9 include 9, 18, 27, 36, 45, … (think of your multiplication tables)
- The **lowest common multiple** (LCM) is the smallest multiple of two or more numbers that is common.
  e.g. Multiples 3 are 3, 6, 9, 12, (15), 18, …
  e.g. Multiples of 5 are 5, 10, (15), 20, 25, …
  The LCM of 3 and 5 is therefore 15.
- A **factor** of a number has a remainder of zero when divided into the given number.
  e.g. 11 is a factor of 77 since 77 ÷ 11 = 7 with 0 remainder.

Multiple The multiple of a number is the product of that number and any other whole number

Counting numbers The set of whole numbers starting at 1

Factor A whole number that will divide into another number exactly

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- The **highest common factor** (HCF) is the largest factor of two or more numbers that is common.
  - Factors of 24 are 1, 2, 3, 4, 6, 8, 12, 24.
  - Factors of 36 are 1, 2, 3, 4, 6, 9, 12, 18, 36.

  The HCF of 24 and 36 is therefore 12.
- **Prime numbers** have only two factors, the number itself and 1.
  - 2, 13 and 61 are examples of prime numbers.
  - 1 is not considered to be a prime number. (It has only **one** factor)
- **Composite numbers** have more than two factors.
  - 6, 20 and 57 are examples of composite numbers.

Prime number An integer greater than 1 that only has two factors, itself and 1

Composite number A number that has at least three factors

Exercise 1F

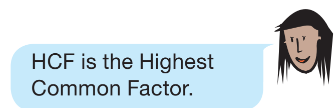
Understanding

1 Write down the factors of each number.

a 4 **b** 6 **c** 12 **d** 15 **e** 20

2 Write down the next term in each of these multiplication table results.

a 2, 4, 6, 8, __ **b** 3, 6, 9, 12, __ **c** 5, 10, 15, 20, 25, __
d 7, 14, 21, __ **e** 6, 12, 18, __ **f** 11, 22, 33, 44, __



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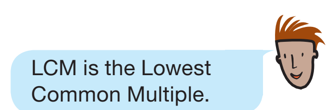
The factors of 16 are 1, 2, 4, 8, 16.
The factors of 24 are 1, 2, 3, 4, 6, 8, 12, 24.
The factors of 18 are 1, 2, 3, 6, 9, 18.
The factors of 30 are 1, 2, 3, 5, 6, 10, 15, 30.
The factors of 8 are 1, 2, 4, 8.

Using the information given in the table, write down the highest common factor (HCF) of each pair of numbers.

a 16 and 24 **b** 24 and 30 **c** 18 and 30
d 16 and 8 **e** 24 and 18 **f** 8 and 24
g 16 and 18 **h** 18 and 8

4 Use the first six multiples of the numbers given to find the LCM of each pair of numbers.

Number	Multiples
2	2, 4, 6, 8, 10, 12
4	4, 8, 12, 16, 20, 24
3	3, 6, 9, 12, 15, 24
5	5, 10, 15, 20, 25, 30
6	6, 12, 18, 24, 30, 36



a 2 and 4 **b** 4 and 3 **c** 3 and 6
d 4 and 6 **e** 4 and 5 **f** 5 and 6

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Fluency

Example 14 Primes and composites

Decide whether each of the following is a prime number or a composite number.

a 29 **b** 117

Solution	Explanation
a 29 is a prime number	29 has only 2 factors 1 and 29. It is a **prime** number.
b 117 is a composite number	117 has factors 1, 3, 9, 13, 39, 117

5 Decide whether each of the following numbers is prime or composite.

a 7	**b** 12	**c** 27	**d** 69
e 105	**f** 28	**g** 15	**h** 11
i 31	**j** 37	**k** 49	**l** 99

Primes have exactly two factors, composites have more than two factors.

6 Copy the following list of the first 30 counting numbers and circle the prime numbers.
1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30.

Example 15 Finding the LCM

Find the LCM of 6 and 8.

Solution	Explanation
Multiples of 6 are: 6, 12, 18, **24**, 30, …	First, list some multiples of 6 and 8. Continue the lists until there is at least one in common.
Multiples of 8 are: 8, 16, **24**, 32, 40, …	Choose the smallest number that is common to both lists.
The LCM is 24.	

7 Find the LCM of these pairs of numbers.

a 2, 3	**b** 5, 9	**c** 8, 12	**d** 4, 8
e 25, 50	**f** 4, 18	**g** 8, 60	**h** 12, 20
i 5, 7	**j** 10, 15	**k** 4, 12	**l** 12, 18

Example 16 Finding the HCF

Find the HCF of 36 and 48.

Solution	Explanation
Factors of 36 are: 1, 2, 3, 4, 6, 9, **12**, 18, 36	First, list factors of 36 and 48.
Factors of 48 are: 1, 2, 3, 4, 6, 8, **12**, 16, 24, 48 The HCF is 12.	Choose the largest number that is common to both lists.

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8 Find the HCF of these pairs of numbers.

a 6, 8	**b** 18, 9	**c** 16, 24	**d** 24, 30
e 7, 13	**f** 19, 31	**g** 72, 36	**h** 108, 64
i 6, 4	**j** 6, 12	**k** 8, 24	**l** 15, 25

Problem-solving and Reasoning

9 Find:

a the LCM of 8, 12 and 6
b the LCM of 7, 3 and 5
c the HCF of 20, 15 and 10
d the HCF of 32, 60 and 48

10 A teacher has 64 students to divide into equal groups of greater than 2 with no remainder. In how many ways can this be done?

11 Three sets of traffic lights (A, B and C) all turn red at 9.00 am exactly. Light set A turns red every 2 minutes, light set B turns red every 3 minutes and light set C turns red every 5 minutes. How long does it take for all three lights to turn red again at the same time?

12 Below are the numbers 1 to 100. Copy the grid and highlight all the prime numbers. How many numbers less than 100 are prime numbers?

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

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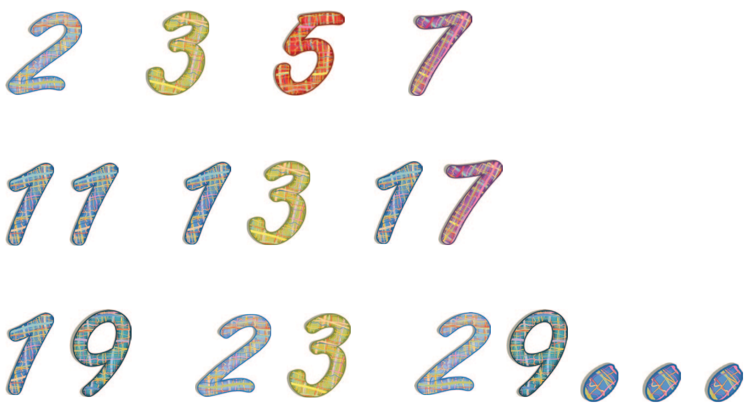
Goldbach's conjecture and twin primes

13 *Goldbach's conjecture* is a famous mathematical statement that says that every even number greater than four can be written as the sum of two prime numbers.

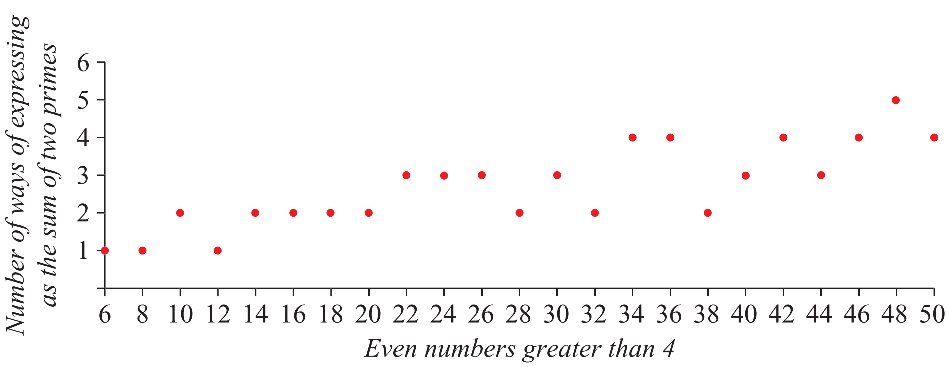
The even numbers 4, 6 and 8 have been written as the sum of two primes.

Show how the even numbers 10 to 30 can be written as the sum of two primes. Some can be done in more than one way.

4 = 2 + 2
6 = 3 + 3
8 = 3 + 5
10 =
12 =
14 =
16 =
18 =
20 =
22 =
24 =
26 =
28 =
30 =



The first ten prime numbers.



A graph illustrating Goldbach's conjecture up to and including 50, is obtained by plotting the number of ways of expressing even numbers greater than 4 as the sum of two primes.

14 Twin primes are pairs of prime numbers that differ by 2. It has been suggested that there are infinitely many twin primes. Use the table of primes you created in question **12** of this exercise and list the pairs of twin primes less than 100.

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1.7 Divisibility and prime factorisation

The basic rule of arithmetic says that every whole number greater than 1 can be written as a product of prime numbers, e.g. $6 = 3 \times 2$ and $20 = 2 \times 2 \times 5$. Writing numbers as a product of prime numbers can help to simplify expressions and determine other properties of numbers or pairs of numbers.

Prime numbers can be thought of as the building blocks or foundations of all other whole numbers.

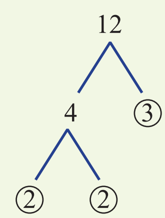
Let's start: Remembering divisibility tests

To test if a number is divisible by 2, we simply need to see if the number is even or odd. All even numbers are divisible by 2. Try to remember the divisibility tests for each of the following. As a class, can you describe tests for any of the following?

- Divisible by 3
- Divisible by 4
- Divisible by 5
- Divisible by 6
- Divisible by 8
- Divisible by 9
- Divisible by 10

Key ideas

- **Prime factorisation** uses a **factor tree**, or similar, to write a number as a product of its prime factors.
  e.g. $12 = 2 \times 2 \times 3$ or $2^2 \times 3$ (using indices)



- The **Highest Common Factor** (HCF) can be found using prime factors.
  The **HCF** = All common primes raised to the smallest power
  e.g. $12 = 2^2 \times 3$ $\quad 20 = 2^2 \times 5$ $\quad \therefore$ HCF $= 2^2$ or 4.
- The **Lowest Common Multiple** (LCM) can be found using prime factors.
  The LCM = All different primes raised to the highest power
  e.g. $12 = 2^2 \times 3$ $\quad 20 = 2^2 \times 5$ $\quad \therefore$ LCM $= 2^2 \times 3 \times 5$
- **Divisibility tests**
  A number is:
  - divisible by **2** if it is even (ends with the digit 0, 2, 4, 6 or 8), e.g. 24
  - divisible by **3** if the sum of all the digits is divisible by 3
    e.g. 162 where $1 + 6 + 2 = 9$, which is divisible by 3
  - divisible by **4** if the number formed by the last two digits is divisible by 4
    e.g. 148 where 48 is divisible by 4
  - divisible by **5** if the last digit is a 0 or 5
    e.g. 145 or 2090

Factor tree
An illustrated breakdown of a number into its prime factors

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– divisible by **6** if it is divisible by both 2 and 3
  e.g. 456 where 6 is even and $4 + 5 + 6 = 15$, which is divisible by 3
– divisible by **8** if the number formed from the last 3 digits is divisibly by 8
  e.g. 2112 where 112 is divisible by 8
– divisible by **9** if the sum of all the digits are divisible by 9
  e.g. 3843 where $3 + 8 + 3 + 3 = 18$ which is divisible by 9
– divisible by **10** if the last digit is a 0
  e.g. 4230

There is no simple test for 7.

Exercise 1G

Understanding

1 Write down all the factors of these numbers.
a 15 **b** 24 **c** 40 **d** 84

2 Write down the first 10 prime numbers. Note that 1 is not a prime number.

3 Write using powers.
a $3 \times 3 \times 3 \times 3$ **b** 5×5 **c** $7 \times 7 \times 7 \times 7$
d $2 \times 2 \times 3 \times 3 \times 3$ **e** $2 \times 2 \times 5 \times 5$ **f** $2 \times 2 \times 3 \times 3 \times 5$

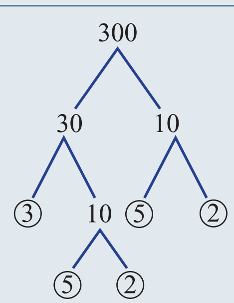
4 Evaluate.
a $2^2 \times 3$ **b** $2 \times 3^2 \times 5$ **c** $2^3 \times 5 \times 7$ **d** $3^3 \times 7$

Fluency

Example 17 Finding prime factor form

Use a factor tree to write 300 as a product of prime factors.

Solution



300 = 2 2 3 5 5
$= 2^2$ 3 5^2

Explanation

First, divide 300 into the product of **any** two factors. Choose the easiest pair $300 = 30 \times 10$.

Continue dividing numbers into two factors until the factors are prime.

Circle the prime factors.

Write the factors in ascending order.

Use index notation (powers) to abbreviate your answer.

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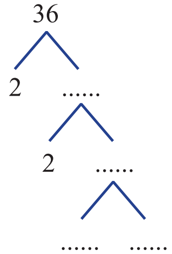
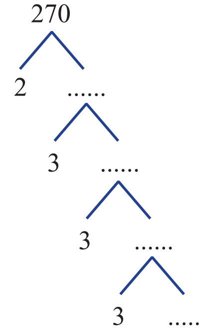
5 Copy and complete these factor trees to help write the prime factor form of the given numbers.

a



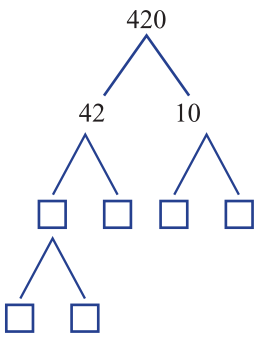
$\therefore 36 = 2^2 \times$ ......

b



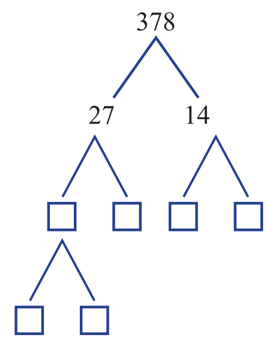
$\therefore 270 = 2 \times$ ...... $\times$ ......

c



420 = ____

d



378 = _______

6 Use a factor tree to find the prime factor form of these numbers.

a 20 **b** 28 **c** 40 **d** 90
e 280 **f** 196 **g** 360 **h** 660

Example 18 Testing for divisibility

Use divisibility tests to decide if the number 627 is divisible by 2, 3, 4, 5, 6, 8 or 9.

Solution	Explanation
Not divisible by **2** since 7 is odd.	The last digit needs to be even.
Divisible by **3** since 6 + 2 + 7 = 15 and this is divisible by 3.	The sum of all the digits needs to be divisible by 3.
Not divisible by **4** as 27 is not divisible by 4.	The number formed from the last two digits needs to be divisible by 4.
Not divisible by **5** as the last digit is not a 0 or 5.	The last digit needs to be a 0 or 5.
Not divisible by **6** as it is not divisible by 2.	The number needs to be divisible by both 2 and 3.
Not divisible by **8** as the last 3 digits together are not divisible by 8.	The number formed from the last three digits needs to be divisible by 8.
Not divisible by **9** as 6 + 2 + 7 = 15 is not divisible by 9.	The sum of all the digits needs to be divisible by 9.

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7 Use divisibility tests to decide if these numbers are divisible by 2, 3, 4, 5, 6, 8 or 9.

a 51 **b** 126 **c** 248 **d** 387
e 315 **f** 517 **g** 894 **h** 3107

Do the seven tests on each number.

Example 19 Finding the LCM and HCF

Find the LCM and HCF of 105 and 90, using prime factorisation.

Solution	Explanation
$105 = 3 \times 5 \times 7$ $90 = 2 \times 3^2 \times 5$	First, express each number in prime factor form. Note that 3 and 5 are common primes.
$\text{LCM} = 2 \times 3^2 \times 5 \times 7$ $= 630$	For the LCM include all the different primes, raising the common primes to their highest power.
$\text{HCF} = 3 \times 5$ $= 15$	For the HCF include only the common primes raised to the lowest power. 105 and 90 **both** have one 3 and one 5.

8 Copy and complete this table of LCM and HCF.

	Number 1	Number 2	LCM	HCF
a	**48** $= 2^4 \times 3$	**30** $= 2 \times 3 \times 5$		
b	**250** $= 2 \times 5^3$	**900** $= 2^2 \times 3^2 \times 5^2$		
c	**54** $= 2 \times 3^3$	**96** $= 2^5 \times 3$		
d	**245** $= 5 \times 7^2$	**350** $= 2 \times 5^2 \times 7$		
e	**198** $= 2 \times 3^2 \times 11$	**693** $= 3^2 \times 7 \times 11$		

9 Find the highest common prime factors of these pairs of numbers.

a 10, 45 **b** 42, 72 **c** 24, 80 **d** 539, 525

10 Find the LCM and the HCF of these pairs of numbers, using prime factorisation.

a 10, 12 **b** 14, 28 **c** 15, 24
d 12, 15 **e** 20, 28 **f** 13, 30
g 42, 9 **h** 270, 420

Problem-solving and Reasoning

11 What is the smallest number that can be divided, without giving a remainder, by all of the following four numbers?

a 2, 3, 4 and 6
b 2, 6, 8 and 9
c 2, 5, 15 and 6

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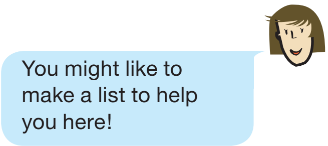
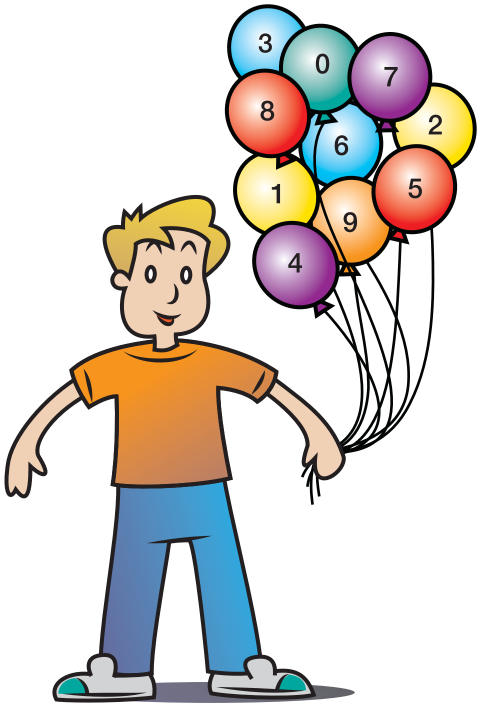
12 Nana Magoo's two grandchildren love to visit her. Lachlan visits her every 8 days while Bryce visits every 18 days. They both visited her last Monday. How many days will it be before they both visit her on the same day again?



Find the missing digit

13 Use the divisibility rules given to you at the start of this section to find the missing digit for each of the following.

- **a** 2 ☐ 6 if the number is divisible by 3 (can you have more than one answer?)
- **b** 1 ☐ 35 if the number is divisible by 9.
- **c** 4 ☐ 3 if the number is divisible by 3.
- **d** 4 ☐ 3 if the number is divisible by 3 and 9.
- **e** 276 ☐ if the number is divisible by 2.
- **f** 276 ☐ if the number is divisible by 2 and 5.



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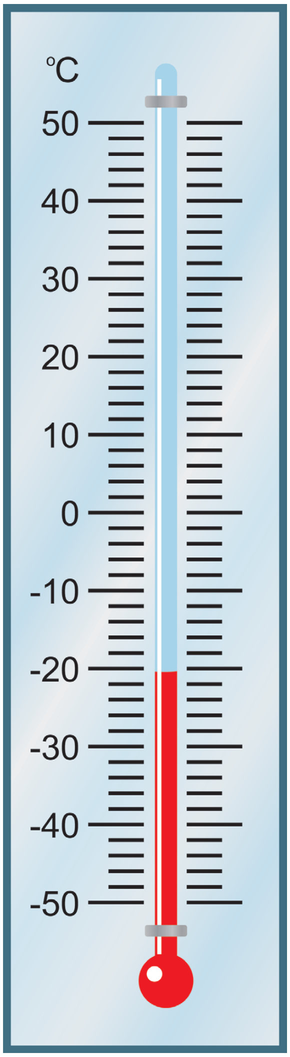
1.8 Negative numbers

The Indian mathematician Brahmagupta set out rules for negative numbers in the 7th century.

Today, negative numbers are used in science, engineering and business. They help us describe opposites such as left and right, up and down, profit and loss, and temperatures above and below freezing.



Let's start: A negative world

Describe how to use negative numbers in these situations.

- 6°C below zero
- A loss of \$4200
- 150 m below sea level
- A turn of 90° anticlockwise
- The solution to the equation $x + 5 = 3$

Can you describe another situation in which you might make use of negative numbers?

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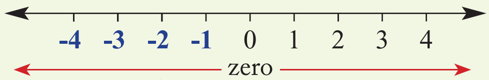
Key ideas

- **Negative numbers** are numbers **less than zero**.
- The **integers** are …, -4, -3, -2, -1, 0, 1, 2, 3, 4 …
  These include positive integers (natural numbers), zero and negative integers. These are illustrated clearly on a number line.

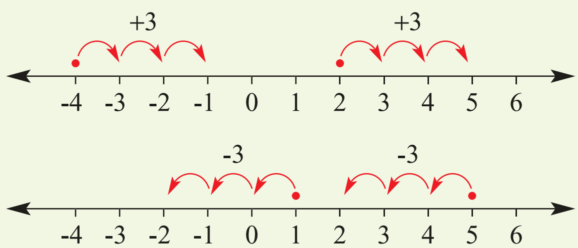


negative numbers — Values decreases as you move to the left along the number line

positive numbers — Values increase as you move to the right along the number line

Integers The set of positive and negative whole numbers, including zero

- Adding or subtracting a positive integer can result in a positive or negative number.
  – Adding a positive integer
    e.g. 2 + 3 = 5
    -4 + 3 = -1
  – Subtracting a positive integer
    e.g. 1 – 3 = -2
    5 – 3 = 2

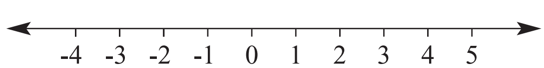


Exercise 1H

Understanding

1 Write down the number suggested by:
  a 2 above zero
  b 5 above zero
  c 3 below zero
  d 10 below zero
  e 1 below zero

2 Copy the number line below and mark (with a dot) the integers -3, -1, 1, 3 and 5.



3 Write the symbol < (less than) or > (greater than) to make these statements true.
  a 5 ___ -1 **b** -3 ___ 4 **c** -10 ___ 3 **d** -1 ___ -2
  e -20 ___ -24 **f** -62 ___ -51 **g** 2 ___ -99 **h** -61 ___ 62

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4 What is the final temperature?

a 10°C is reduced by 12°C

b 32°C is reduced by 33°C

c -11°C is increased by 2°C

d -4°C is increased by 7°C

Fluency

Example 20 Adding a positive integer

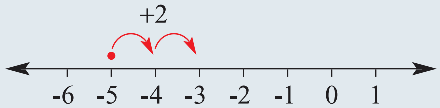
Evaluate the following.

a -5 + 2

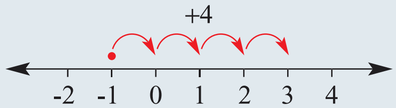
b -1 + 4

Solution | **Explanation**

a -5 + 2 = -3



b -1 + 4 = 3



5 Evaluate the following.

a -1 + 2
b -3 + 7
c -10 + 11
d -4 + 12
e -20 + 35
f -6 + 4
g -7 + 2
h -15 + 8
i -26 + 19
j -38 + 24
k -10 + 15
l -2 + 9
m -7 + 3
n -7 + 7
o -6 + 9
p -6 + 1

Start with the left number and move right on the number line.

Example 21 Subtracting a positive integer

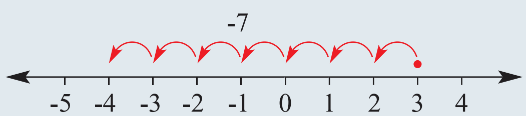
Evaluate the following.

a 3 – 7

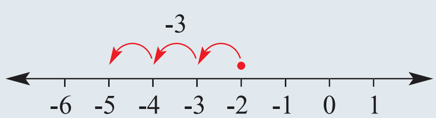
b -2 – 3

Solution | **Explanation**

a 3 – 7 = -4



b -2 – 3 = -5



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6 Evaluate the following.

a 4 – 5	**b** 10 – 15	**c** 0 – 26	**d** 14 – 31
e 6 – 8	**f** 10 – 9	**g** -4 – 7	**h** -11 – 20
i -14 – 15	**j** -10 – 100	**k** -11 – 6	**l** 0 – 12
m -15 – 5	**n** 3 – 12	**o** 8 – 4	**p** -8 – 4

Start with the left number and move left on the number line.

7 Evaluate the following.

a -9 + 6	**b** -9 – 6	**c** -12 + 12	**d** -12 – 12
e -7 – 7	**f** -7 + 0	**g** 15 – 14	**h** 15 – 16
i -9 – 10	**j** -9 + 10	**k** 9 – 15	**l** -20 + 10
m 100 – 101	**n** -50 – 50	**o** -5 + 25	**p** -9 + 40

8 Work from left to right to evaluate the following.

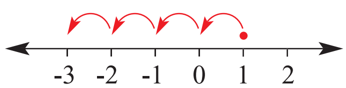
a -3 + 4 – 8 + 6 **b** 0 – 10 + 19 – 1

c 26 – 38 + 14 – 9 **d** 9 – 18 + 61 – 53

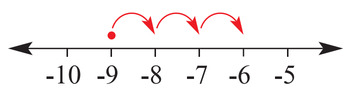
Problem-solving and Reasoning

9 Write the sum (e.g. -3 + 4 = 1) or difference (e.g. 1 – 5 = -4) to match these number lines.

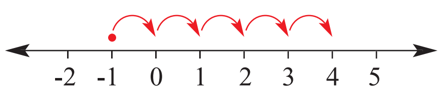
a



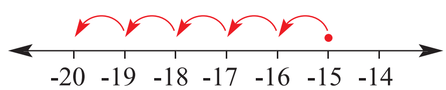
b



c



d



10 Write the missing number.

a -1 + ___ = 5 **b** ___ + 30 = 26

c ___ + 11 = -3 **d** -32 + ___ = -21

e 5 – ___ = -10 **f** ___ – 17 = -12

g ___ – 4 = -7 **h** -26 – ___ = -38

11 In a high-rise building there are 8 floors above ground level and 6 floors below ground level. A lift starts at the 2nd floor and moves 4 floors up, then 7 floors down before moving down a further 3 floors.
At what floor does the lift finish?

12 On Monday Milly borrows $35 from a friend. On Tuesday she pays her friend $40. On Friday she borrows $42 and pays back $30 that night. How much does Milly owe her friend then?

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Budgets and zero

13 **a** Complete Suzanne's account for the week shown.
A credit is an addition (+) and a debit is a subtraction (–).

Spending and earning	Credit (+)	Debit (–)	Balance
opening balance			$500
pays 1 weeks rent of $375		375	
earns $80 baby sitting			
receives $100 from her parents for her birthday			
buys a pair of jeans for $90			
buys a top for $45			
pays her monthly mobile phone bill $49			
gives $25 to charity			

b How much would Suzanne need to deposit (credit) into her account so that she can pay the rent for the next week?

14 Find what positive integer needs to be added or subtracted to each so that the end result is always zero.

a -6 ____ = 0
b -8 ____ = 0
c 16 ____ = 0
d 10 – 7 ____ = 0
e -9 + 7 ____ = 0
f -9 – 7 – 2 ____ = 0

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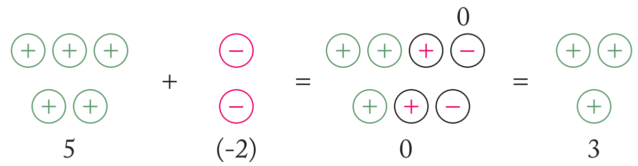
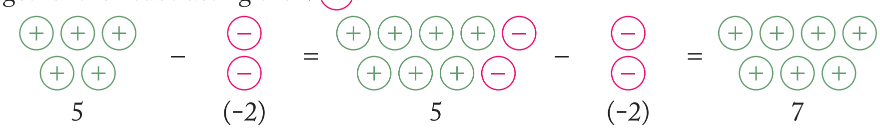
1.9 Addition and subtraction of negative integers

If (+) represents +1 and (−) represents -1 then (+) (−) added together has a value of zero. Using these symbols 5 + (−2) = 3 could be illustrated as the addition of 2 (−), leaving a balance of 3.



So 5 + (−2) is the same as 5 – 2.
Also 5 – (−2) = 7 could be illustrated first as 7 (+) and 2 (−) together then subtracting the 2 (−).



So 5 – (−2) is the same a 5 + 2.
When adding or subtracting negative integers we follow the rules set out by the above two illustrations, as well as the patterns below.

Let's start: Looking at patterns for adding and subtracting negative numbers

Copy and complete.

A

6 + 4	10	
6 + 3	9	
6 + 2	8	
6 + 1		
6 + 0		
6 + (-1)		→ same as 6 – 1 = 5
6 + (-2)		→ same as 6 ☐ 2 =
6 + (-3)		→ same as 6 ☐ 3 =
6 + (-4)		→ same as 6 ☐ 4 =

B

6 – 4	2	
6 – 3	3	
6 – 2	4	
6 – 1		
6 – 0		
6 – (-1)		→ same as 6 + 1 =
6 – (-2)		→ same as ☐
6 – (-3)		→ same as ☐
6 – (-4)		→ same as ☐

Key ideas

- Adding a negative number is the same as subtracting its opposite.
  e.g. 2 + (-3) = 2 – 3 = -1 two opposite signs give a subtraction/minus
  -4 + (-7) = -4 – 7 = -11
- Subtracting a negative number is the same as adding its opposite.
  e.g. 2 – (-5) = 2 + 5 = 7 two like signs give an addition/plus
  -6 – (-4) = -6 + 4 = -2

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Exercise 1I

Understanding

1 -3 and 3 are opposites. Write down the opposites of these numbers.
a -6 b 10 c 38 d -46
e -32 f 88 g 673 h -349

2 Write the words 'add' or 'subtract' to suit each sentence.
a To add a negative number ____________ its opposite.
b To subtract a negative number ____________ its opposite.

3 Are the following statements true or false?
a 5 + (-2) = 5 + 2 b 3 + (-4) = 3 – 4 c -6 + (-4) = -6 – 4
d -1 + (-3) = 1 – 3 e 8 – (-3) = 8 + 3 f 2 – (-3) = 2 – 3
g -3 – (-1) = 3 + 1 h -7 – (-5) = -7 + 5 i -6 – (-3) = 6 + 3

4 Rewrite each of the following with only a (+) or (–) between the two numbers.
a 7 + (-3) b 10 + (-5) c 8 – (-1)
d 6 – (-8) e 15 + (-20) f -3 – (-4)
g -9 – (-9) h 0 + (-5) i 18 + (-18)

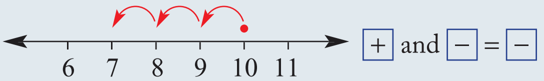
6 + (-9) = 6 – 9
3 – (-7) = 3 + 7

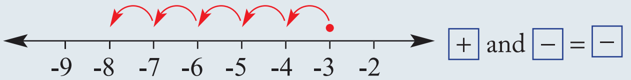
Fluency

Example 22 Adding negative numbers

Evaluate the following.
a 10 + (-3) b -3 + (-5)

Solution	Explanation
a 10 + (-3) = 10 – 3 = 7	Adding -3 is the same as subtracting 3.
b -3 + (-5) = -3 – 5 = -8	Adding -5 is the same as subtracting 5.





5 Evaluate the following.
a 6 + (-2) b 4 + (-1) c 7 + (-12) d 20 + (-5)
e 2 + (-4) f 26 + (-40) g -3 + (-6) h -16 + (-5)
i -18 + (-20) j -36 + (-50) k -83 + (-22) l -120 + (-10)
m 7 + (-8) n -9 + (-12) o 6 + (-12) p -6 + (-12)
q -8 + (-8) r 5 + (-5) s -70 + (-15) t -100 + (-6)

To add a negative, subtract its opposite.

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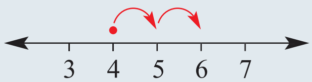
Example 23 Subtracting negative numbers

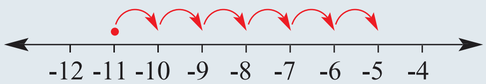
Evaluate the following.

a 4 – (-2) **b** -11 – (-6)

Solution	Explanation
a 4 – (-2) = 4 + 2 = 6	Subtracting -2 is the same as adding 2.
b -11 – (-6) = -11 + 6 = -5	Subtracting -6 is the same as adding 6.



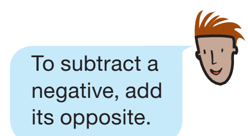
[–] and [–] = [+]



[–] and [–] = [+]

6 Evaluate the following.

a 2 – (-3) **b** 4 – (-4) **c** 15 – (-6) **d** 24 – (-14)
e 59 – (-13) **f** 147 – (-320) **g** -5 – (-3) **h** -8 – (-10)
i -13 – (-16) **j** -10 – (-42) **k** -88 – (-31) **l** -125 – (-5)
m 60 – (-5) **n** -60 – (-5) **o** -12 – (-12) **p** -10 – (-18)
q 41 – (-41) **r** 48 – (-52) **s** -46 – (-8) **t** -170 – (-12)



7 Simplify.

a 46 – 50 **b** 46 + (-50) **c** 9 – 12
d 9 + (-12) **e** -8 + 6 **f** -8 – (-6)
g 81 – 15 **h** 81 + (-15) **i** 7 + (-7)

Problem-solving and Reasoning

8 Write down the missing number.

a 4 + ___ = 1 **b** 6 + ___ = 0
c -2 + ___ = -1 **d** ___ + (-8) = 2
e ___ + (-5) = -3 **f** ___ + (-3) = -17
g 12 – ___ = 14 **h** 8 – ___ = 12
i -1 – ___ = 29 **j** ___ – (-7) = 2
k ___ – (-2) = -4 **l** ___ – (-436) = 501

9 An ice cube is removed from a freezer at -25°C and placed into a glass of juice at 7°C.
What is the difference in the two temperatures?

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10 Kelvin owes the bank \$450 000. What must he deposit into his account to only owe \$270 000?

11 What must be added or subtracted to each of the following to obtain an answer of zero?

a $-6 + \square = 0$ **b** $7 - \square = 0$ **c** $-18 - \square = 0$

12 If $a = -5$ and $b = -3$. Find the value of:

a $a + (-3)$ **b** $a - (-2)$ **c** $b - (-4)$

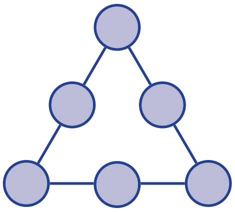
d $a + b$ **e** $a - b$ **f** $b - a$

Replace the pronumeral in the statement with the number it represents.
e.g. $a = -2$
then $a + (-5)$
$= -2 + (-5)$
$= -2 - 5$
$= -7$

Puzzles with negatives

13 Place the integers from -3 to 2 in this magic triangle so that each side adds to the given number.

a -3 **b** 0



14 A magic square has each row, column and main diagonal adding to the same magic sum. Complete these magic squares.

a

		1
0	-2	-4

b

-12		
	-15	
	-11	-18

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1.10 Multiplication and division of integers

As a repeated addition, the product $3 \times (-2)$ can be written as $-2 + (-2) + (-2) = -6$. So $3 \times (-2) = -6$ and, since $a \times b = b \times a$ for all numbers a and b, then -2×3 is also equal to -6.

For division we can write the product $3 \times 2 = 6$ as a quotient $6 \div 2 = 3$.

So, if $3 \times (-2) = -6$ then $-6 \div (-2) = 3$.

Also if $-2 \times 3 = -6$ then $-6 \div 3 = -2$.

The quotient of two negative numbers results in a positive number. The product or quotient of two numbers of opposite sign is a negative number.

$6 \div (-2) = -3$ can also be rearranged to $-3 \times (-2) = 6$. The product of two negative numbers is a positive number.

Let's start: Repeated additions

Rewrite each of these as a multiplication and then find the value of each.

- $7 + 7 + 7 + 7$
- $(-7) + (-7) + (-7) + (-7)$
- $3 + 3 + 3 + 3 + 3 + 3$
- $(-3) + (-3) + (-3) + (-3) + (-3) + (-3)$
- $(-10) + (-10) + (-10)$

Quotients. Complete these statements.

- If $10 \div (-2) = (-5)$, then $(-2) \times (-5) = \square$
- If $18 \div (-6) = (-3)$, then $(-3) \times (-6) = \square$

What do these observations tell us about multiplying and dividing positive and negative numbers?

Key ideas

- The product **or** quotient of two integers of the **same sign is a positive integer**.
  - Positive × Positive = Positive
  - Positive ÷ Positive = Positive
  - Negative × Negative = Positive
  - Negative ÷ Negative = Positive
- The product or quotient of two integers of **opposite signs is a negative integer**.
  - Positive × Negative = Negative
  - Positive ÷ Negative = Negative
  - Negative × Positive = Negative
  - Negative ÷ Positive = Negative

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Exercise 1J

Understanding

1 Write the missing numbers in these tables. You should create a pattern in the third column.

a

□	△	□ × △
3	5	15
2	5	
1	5	
0	5	
-1	5	
-2	5	
-3	5	

b

□	△	□ × △
3	-5	-15
2	-5	-10
1	-5	
0	-5	
-1	-5	
-2	-5	
-3	-5	

2 Write the missing numbers in these sentences. Use the tables in question **1** to help.

a $3 \times 5 =$ ___ so $15 \div 5 =$ ___
b $-3 \times 5 =$ ___ so $-15 \div 5 =$ ___
c $3 \times (-5) =$ ___ so $15 \div (-5) =$ ___
d $-3 \times (-5) =$ ___ so $15 \div (-5) =$ ___

3 Without finding the answer to these products decide if the answer would be positive or negative.

a 109×4
b -76×5
c $15 \times (-9)$
d $-6 \times (-13)$
e 89×104
f -74×8
g $-94 \times (-5)$
h $80 \times (-7)$
i -37×-3

4 Without finding the answer to these quotients decide if the answer would be positive or negative.

a $16 \div 2$
b $24 \div (-3)$
c $78 \div (-2)$
d $-56 \div 2$
e $-81 \div 9$
f $-99 \div (-11)$

Fluency

Example 24 Finding products

Evaluate the following.

a $3 \times (-7)$
b $-4 \times (-12)$

Solution	Explanation
a $3 \times (-7) = -21$	The product of two numbers of opposite sign is negative. $\boxed{+} \times \boxed{-} = \boxed{-}$
b $-4 \times (-12) = 48$	-4 and -12 are both negative and so the product will be positive. $\boxed{-} \times \boxed{-} = \boxed{+}$

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1J

5 Evaluate the following.

a $4 \times (-5)$	**b** $6 \times (-9)$	**c** -4×10	**d** -11×9
e $-2 \times (-3)$	**f** -6×7	**g** -9×8	**h** $-11 \times (-9)$
i $20 \times (-2)$	**j** -16×4	**k** $-5 \times (-7)$	**l** $8 \times (-4)$
m $-10 \times (-6)$	**n** $44 \times (-1)$	**o** $-9 \times (-1)$	**p** -5×12

Example 25 Finding quotients

Evaluate the following.

a $-63 \div 7$ **b** $-121 \div (-11)$

Solution	Explanation
a $-63 \div 7 = -9$	The two numbers are of opposite sign so the answer will be negative. $\boxed{-} \div \boxed{+} = \boxed{-}$
b $-121 \div (-11) = 11$	-121 and -11 are both negative so the quotient will be positive. $\boxed{-} \div \boxed{-} = \boxed{+}$

6 Evaluate the following.

a $-10 \div 2$	**b** $-38 \div 19$	**c** $-60 \div 15$	**d** $-120 \div 4$
e $32 \div (-16)$	**f** $-6 \div 2$	**g** $6 \div (-2)$	**h** $-6 \div (-2)$
i $-12 \div 6$	**j** $-24 \div (-3)$	**k** $-45 \div 5$	**l** $-45 \div (-9)$
m $-66 \div (-6)$	**n** $-5 \div (-5)$	**o** $-8 \div 1$	**p** $-8 \div (-1)$

Example 26 Order of operations

a $-7 + 6 \times (-5)$ **b** $-4 \times 6 \div (-2)$

Solution	Explanation
a $-7 + 6 \times (-5)$ $= -7 + (-30)$ $= -7 - 30$ $= -37$	The order of operation multiplication first $\boxed{+} \times \boxed{-} = \boxed{-}$ $6 \times (-5) = -30$ Lastly addition of a negative = subtraction $-7 + (-30) = -7 - 30$
b $-4 \times 6 \div (-2)$ $= -24 \div (-2)$ $= 12$	Multiplication and division work from left to right -4×6 *First* $\boxed{-} \times \boxed{+} = \boxed{-}$ $-24 \div -2$ *Last* $\boxed{-} \div \boxed{-} = \boxed{+}$

7 Follow the order of operation to find the following.

a $10 + (-6) \times 5$	**b** $15 - 3 \times (-2)$
c $18 \times (-2) \div 3$	**d** $-9 \times 2 + (-5)$
e $45 - 50 \div (-10)$	**f** $9 - 6 \times 3$
g $-10 \div (-2) \times (-3)$	**h** $9 \times 3 - 6 \times (-2)$
i $18 \div (-3) + 3 \times (-4)$	**j** $-9 \times (-2) + (-10)$

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8 If $(-2)^2 = -2 \times -2 = 4$, find the value of the following.

a $(-5)^2$ **b** $(-6)^2$ **c** $(-7)^2$
d $(-8)^2$ **e** $(-9)^2$ **f** $(-10)^2$

Problem-solving and Reasoning

9 Write the missing number.

a ___ $\times 3 = -9$ **b** ___ $\times (-7) = 35$ **c** ___ $\times (-4) = -28$
d $-3 \times$ ___ $= -18$ **e** $-19 \times$ ___ $= 57$ **f** ___ $\div (-9) = 8$
g ___ $\div 6 = -42$ **h** $85 \div$ ___ $= -17$ **i** $-150 \div$ ___ $= 5$

10 Will $(-2)^3$ give a positive or negative answer?

11 Insert a × sign and/or ÷ sign to make these equations true.

a -2 ___ 3 ___ (-6) = 1 **b** 10 ___ (-5) ___ (-2) = 25
c 6 ___ (-6) ___ 20 = -20 **d** -14 ___ (-7) ___ (-2) = -1

12 The product of two numbers is -24 and their sum is -5. What are the two numbers?

Further substitution with integers using brackets

13 Evaluate these expressions using $a = -2$ and $b = 1$.

a $a + b$ **b** $a - b$ **c** $2a - b$ **d** $b - a$
e $a - 4b$ **f** $3b - 2a$ **g** $b \times (2 + a)$ **h** $(2b + a) - (b - 2a)$

14 Evaluate these expressions using $a = -3$ and $b = 5$.

a ab **b** ba **c** $a + b$ **d** $a - b$
e $b - a$ **f** $3a + 2b$ **g** $(a + b) \times (-2)$ **h** $(a + b) - (a - b)$

15 Evaluate these expressions using $a = -3$ and $b = 5$.

a $a + b^2$ **b** $a^2 - b$ **c** $b^2 - a$ **d** $b^3 + a$
e $a^3 - b$ **f** $a^2 - b^2$ **g** $b^3 - a^3$ **h** $(b - a^2)^2$

16 Evaluate these expressions using $a = -4$ and $b = -3$.

a $3a + b$ **b** $b - 2a$ **c** $4b - 7a$
d $-2a - 2b$ **e** $4 + a - 3b$ **f** $ab - 4a$
g $-2 \times (a - 2b) + 3$ **h** $ab - ba$ **i** $3a + 4b + ab$
j $a^2 - b$ **k** $a^2 - b^2$ **l** $b^3 - a^3$

17 Insert brackets in these statements to make them true.

a $-2 + 1 \times 3 = -3$ **b** $-10 \div 3 - (-2) = -2$
c $-8 \div (-1) + 5 = -2$ **d** $-1 - 4 \times 2 + (-3) = 5$
e $-4 + (-2) \div 10 + (-7) = -2$ **f** $20 + 2 - 8 \times (-3) = 38$
g $1 - (-7) \times 3 \times 2 = 44$ **h** $4 + (-5) \div 5 \times (-2) = -6$

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Puzzles and games

1 Hey, do you know what a wisecracker is?

A	-6 – 4	**R**	-8 – (-2)	**I**	-4 + 7 – 10	**Y**	-17 – 6
E	8 – 10	**M**	-6 – 7 – 4	**S**	20 – 7	**K**	16 – (-6)
O	6 – (-4)	**C**	46 + (-6) – 8	**V**	12 + (-3) – 6	**T**	-13 – 7 – 6 + 8

Complete the sums above to unlock the puzzle code.

–10				

3	–2	–6	–23

13	–17	–10	–6	–18

32	10	10	22	–7	–2

2 What explosive event was in the year 1000 AD?

Answer the following directed number multiplications and divisions to work out the puzzle code. Write your answer on another sheet of paper.

K	-3 × 4	**N**	8 ÷ -4
A	-1 × 6	**S**	$\frac{-36}{6}$
C	100 ÷ -5	**U**	-9 × -7
L	-8 × -6	**G**	$\frac{10}{-2}$
W	40 ÷ 8 × -2	**D**	-2 × 2
H	4 × -4	**O**	-12 + 5
R	$(-10)^2$	**P**	$(-4)^2$
E	0 × -5	**V**	-5 × -4
M	-16 ÷ -8	**F**	$(-3)^2$
I	24 ÷ 8	**T**	-3 × -2 × -4

–20	–16	3	–2	–6

–4	0	20	0	48	–7	16	6

–5	63	–2	16	–7	–10	–4	0	100

–24	–16	3	6

48	0	–6	–4	6

–24	–7

–24	–16	0

2	–6	–2	63	9	–6	–20	–24	63	100	0

–7	9

9	3	100	0	–10	–7	100	–12	6

3 Using the symbols +, –, ×, ÷, make as many sums as you can that have -5 as their answer.

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Chapter summary

Integers {..., -3, -2, -1, 0, 1, 2, 3, ...}

Whole numbers 0, 1, 2, 3, ...

Addition and subtraction

$$\begin{array}{r} 2^{1}47 \\ +108 \\ \hline 355 \end{array} \qquad \begin{array}{r} {}^{8}\not{9}^{13}\not{4}2^{1} \\ -368 \\ \hline 574 \end{array}$$

Multiplication and division

$$\begin{array}{rl} 167 & \\ \times 15 & \\ \hline 835 & 5 \times 167 \\ 1670 & 10 \times 167 \\ \hline 2505 & 835 + 1670 \end{array}$$

$$7\overline{)41^{6}6}\ \text{gives}\ 59$$

$\therefore 416 \div 7 = 59$ and 3 remainder

Mental strategies

- $156 + 79 = 156 + 80 - 1 = 235$
- $45 + 47 = 45 + 45 + 2 = 92$
- $3 \times 22 = 3 \times 20 + 3 \times 2 = 66$
- $4 \times 88 = 2 \times 176 = 352$
- $164 \div 4 = 82 \div 2 = 41$
- $297 \div 3 = (300 \div 3) - (3 \div 3) = 99$

Negative number operations

$4 - (-3) = 4 + 3$
$4 + (-3) = 4 - 3$

Multiplication and division

$+ \times + = +$
$- \times - = +$
$+ \times - = -$
$- \times + = -$
$+ \div + = +$
$- \div - = +$
$+ \div - = -$
$- \div + = -$

Substitution

$a = -2,\ b = 5,\ c = -4$

$2c - ab = 2 \times (-4) - (-2) \times 5$
$= -8 - (-10)$
$= 2$

use brackets with negatives

Order of operations

- Brackets, $\times$ and $\div$ then $+$ and $-$

$10 \times (-3) + 7$
$= -30 + 7$
$= -23$

Properties

Primes (2 factors)

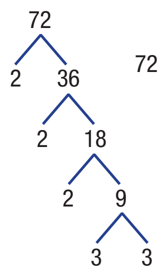
2, 3, 5, 7, 11, 13 ...

Composite

more than 2 Factors

4, 6, 8, 12

Prime factorisation



$72 = 2^3 \times 3^2$

Lowest Common Multiple LCM

Multiples of 3: 3, 6, 9, **12** ...
Multiples of 4: 4, 8, **12** ...
$\therefore$ LCM = **12**

Highest Common Factor HCF

Factors of 8: 1, 2, **4**, 8
Factors of 28: 1, 2, **4**, 7, 14, 28
$\therefore$ HCF = **4**

Divisibility

- 2 Even number.
- 3 Sum of digits divisible by 3.
- 4 Number from last 2 digits divisible by 4.
- 5 Last digit 0 or 5.
- 6 Divisible by 2 and 3.
- 8 Number from last 3 digits divisible by 8.
- 9 Sum of digits divisible by 9.
- 10 Last digit 0.

Powers and indices

$9 \times 9 \times 9 \times 9 = 9^4$

Squares and cubes

$4^2 = 16,\ \sqrt{16} = 4$
$3^3 = 27,\ \sqrt[3]{27} = 3$

Index laws

$4^2 \times 4^3 = 4^5$ $\quad 4^9 \div 4^3 = 4^6$
$(2^4)^3 = 2^{12}$ $\quad 5^0 = 1$

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Multiple-choice questions

1 $400 \div 5 \times 2$ is the same as:
A $400 \div 10$ **B** 80×2 **C** 16 **D** $400 \div 2 \times 5$

2 The sum and difference of 97 and 49 are:
A 146 and 58 **B** 246 and 48 **C** 136 and 58 **D** 146 and 48

3 561 is divisible by:
A 5 **B** 2 **C** 3 **D** 9

4 89×5 is the same as:
A 90×4 **B** $90 \times 5 - 1 \times 5$
C $89 \times 10 \times 2$ **D** 178×10

5 $2 \times 2 \times 2 \times 2 \times 5 \times 5$ is:
A $2^4 \times 5^2$ **B** $2 \times 4 + 5 \times 2$ **C** $2^4 + 5^2$ **D** 10^7

6 The LCM of $2^2 \times 3 \times 5$ and 2×7 is:
A 2
B $2^2 \times 3 \times 5 \times 7$
C $2 \times 3 \times 5 \times 7$
D $2^3 \times 3 \times 5 \times 7$

7 $6^9 \div 6^2$ equals:
A 6^{11} **B** 1^7 **C** 12^{11} **D** 6^7

8 $-6 + (-4)$ is the same as:
A $-6 - 4$ **B** $-6 + 4$
C $-4 + 6$ **D** $6 + 4$

9 If $18^2 = 324$, then $\sqrt{324}$ equals:
A 162 **B** 102976
C 18 **D** 9

10 $16^3 \times 16^2$ equals:
A 32^5 **B** 16^6 **C** 16^5 **D** 256^5

Short-answer questions

1 Use a mental strategy to evaluate the following.
a $324 + 173$ **b** $592 - 180$ **c** $89 + 40$ **d** $135 - 68$
e $55 + 57$ **f** $280 - 141$ **g** $1001 + 998$ **h** $10\,000 - 4325$

2 Use a mental strategy to find these sums and differences.
a $\begin{array}{r} 392 \\ +\ 147 \\ \hline \end{array}$ **b** $\begin{array}{r} 1031 \\ +\ 999 \\ \hline \end{array}$ **c** $\begin{array}{r} 147 \\ -\ 86 \\ \hline \end{array}$ **d** $\begin{array}{r} 3970 \\ -\ 896 \\ \hline \end{array}$

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3 Use a mental strategy for these products and quotients.

a $2 \times 17 \times 5$ **b** 3×99 **c** 8×42 **d** 141×3
e $164 \div 4$ **f** $357 \div 3$ **g** $618 \div 6$ **h** $1005 \div 5$

4 Find these products and quotients using setting out.

a $\begin{array}{r} 139 \\ \underline{12} \end{array}$ **b** $\begin{array}{r} 507 \\ \underline{42} \end{array}$ **c** $3\overline{)843}$ **d** $7\overline{)854}$

5 Find the remainder when 673 is divided by these numbers.

a 5 **b** 3 **c** 7 **d** 9

6 Write using powers.

a $6 \times 6 \times 6$ **b** $8 \times 8 \times 8 \times 8$ **c** $2 \times 2 \times 5 \times 5 \times 5 \times 5$

7 Evaluate.

a $\sqrt{81}$ **b** $\sqrt{121}$ **c** 7^2 **d** 20^2
e $\sqrt[3]{27}$ **f** $\sqrt[3]{64}$ **g** 5^3 **h** 10^3

8 Simplify these powers.

a $4^9 \times 4^2$ **b** $3^4 \div 3^2$ **c** 5^0 **d** $(3^4)^5$

9 **a** Find all the factors of 60.
b Find all the multiples of 7 between 110 and 150.
c Find all the prime numbers between 30 and 60.
d Find the LCM of 8 and 6.
e Find the HCF of 24 and 30.

10 Write these numbers in prime factor form. You may wish to use a factor tree.

a 36 **b** 84 **c** 198

11 Use divisibility tests to decide if these numbers are divisible by 2, 3, 4, 5, 6, 8 or 9.

a 84 **b** 155 **c** 124 **d** 621

12 Write the numbers 20 and 38 in prime factor form and then use this to help find the following.

a LCM of 20 and 38
b HCF of 20 and 38

13 Evaluate.

a $-6 + 9$ **b** $-24 + 19$ **c** $5 - 13$ **d** $-7 - 24$
e $-62 - 14$ **f** $-194 - 136$ **g** $-111 + 110$ **h** $-328 + 426$

14 Evaluate.

a $5 + (-3)$ **b** $-2 + (-6)$ **c** $-29 + (-35)$ **d** $162 + (-201)$
e $10 - (-6)$ **f** $-20 - (-32)$ **g** $-39 - (-19)$ **h** $37 - (-55)$

15 Evaluate.

a -5×2 **b** $-11 \times (-8)$ **c** $9 \times (-7)$ **d** $-100 \times (-2)$
e $-10 \div (-5)$ **f** $48 \div (-16)$ **g** $-32 \div 8$ **h** $-81 \div (-27)$

16 Evaluate using the order of operations.

a $2 + 3 \times (-2)$ **b** $-3 \div (11 + (-8))$
c $-2 \times 3 + 10 \div (-5)$ **d** $-20 \div 10 - 4 \times (-7)$

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17 Let $a = -2$, $b = 3$ and $c = -5$ and evaluate these expressions.

a $ab + c$ **b** $a^2 - b$ **c** $ac - b$ **d** $a + b + c$

18 Copy and complete.

a $1^2 =$ _____ **b** $(-1)^2 =$ _____
c $2^2 =$ _____ **d** $(-2)^2 =$ _____
e $3^2 =$ _____ **f** $(-3)^2 =$ _____

Extended-response questions

1 A monthly bank account show deposits as positive numbers and purchases and withdrawals (P + W) as negative numbers.

Details	P + W	Deposits	Balance
Opening balance	–	–	\$250
Water bill	-\$138	–	*a*
Cash withdrawal	-\$320	–	*b*
Deposit	–	*c*	\$115
Supermarket	*d*	–	-\$160
Deposit	–	\$400	*e*

a Find the values of *a*, *b*, *c*, *d* and *e*.
b If the water bill amount was \$150, what would be the new value for letter *e*?
c What would the final deposit need to be if the value for *e* was \$0? Assume the original water bill amount is \$138 as in the table above.

2 Two teams compete at a club games night. Team A has 30 players while team B has 42 players.
a How many players are there in total?
b Write both 30 and 42 in prime factor form.
c Find the LCM and HCF of the number of players representing the two teams.
d Teams are asked to divide into groups with equal numbers of players. What is the largest group size possible if team A and team B must have groups of the same size?

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