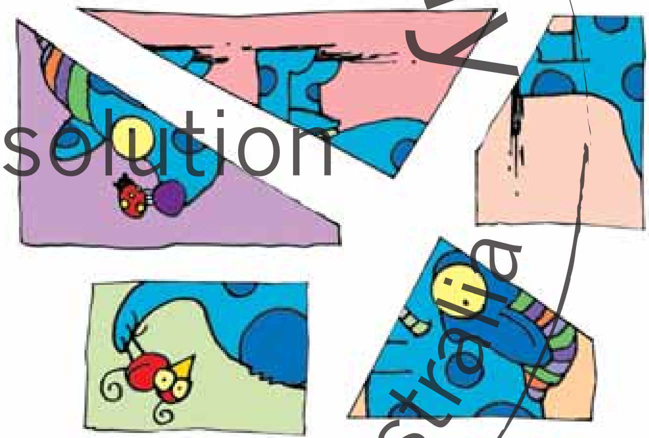
AGES 10+

PROBLEM SOLVING

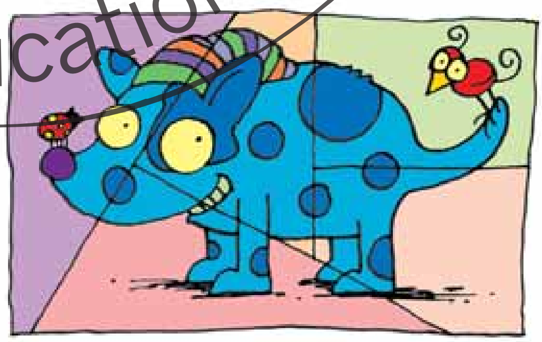
All you need to TEACH...

Problem Solving

Turning problems



into solutions



Peter Maher

MACMILLAN

All you need to TEACH...

PROBLEM SOLVING

Ages 10+

Turning problems into solutions

Peter Maher

MACMILLAN

DEDICATION:

To Tom and Frieda, who taught me that no problem is too difficult to overcome.

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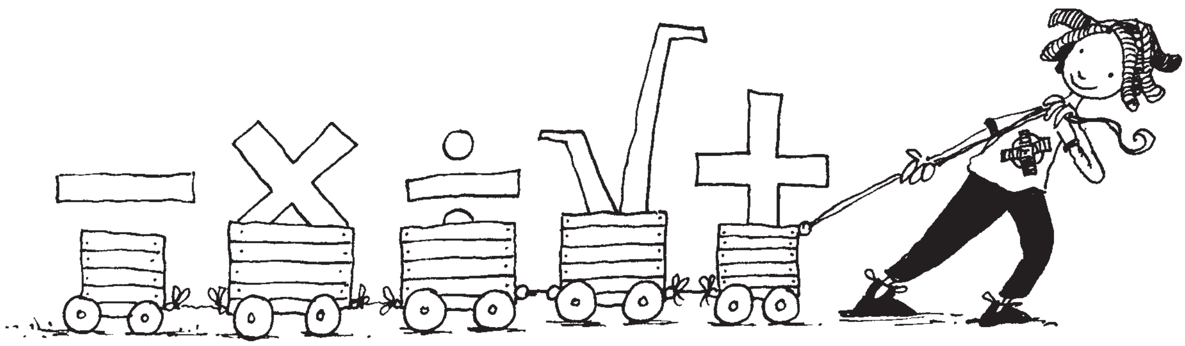


How to Use This Book

All You Need to Teach Problem Solving Ages 10+ is the third in a series of three books designed to help teachers develop the capabilities to strengthen logical and creative thinking skills in the students under their care. This book caters for teachers of students in the sixth and seventh years of schooling and is in four parts.

All The Teaching Tips You Need presents the strategies and techniques that need to be developed and applied by students to solve the range of problems in the books. It also suggests ways to implement a successful problem-solving program in the classroom.

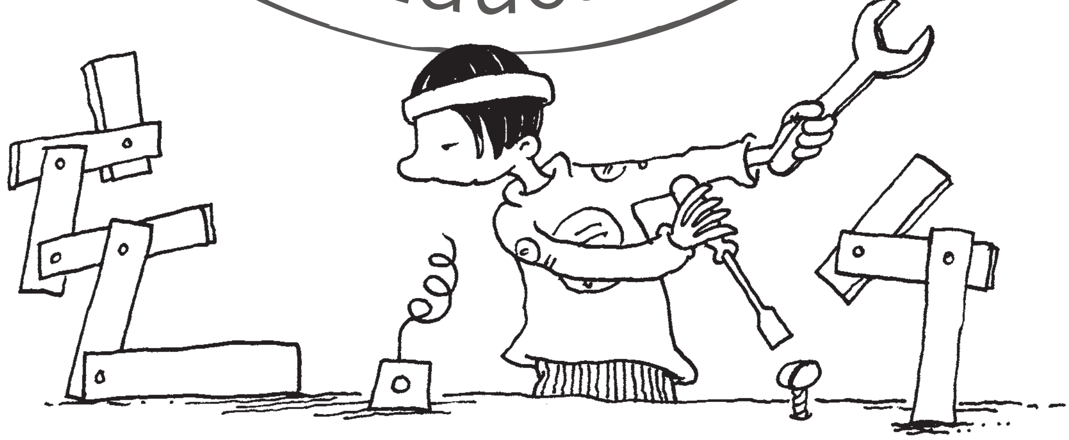
All The Lesson Plans and Worksheets You Need contains 16 lesson plans with accompanying blackline masters. The lesson plans outline the theoretical background of the problems and suggest the best manner to present them to the students. The blackline masters give students the opportunity to draw and describe the strategies and working they used to solve problems.

All The Task Cards You Need contains 16 task cards designed to be photocopied and laminated. Each card presents a variation or extension of the problem found on the blackline master of the same number. The task cards provide an ideal way to assess the development of each student's problem solving capabilities.

All The Answers You Need offers solutions to both the blackline masters and the task cards.

These lesson plans, blackline masters and task cards are designed to be practical, intellectually stimulating and to contain high motivational appeal.

As your own problem solving capabilities grow, your ability to successfully teach problem solving will be similarly enhanced. Problem solving also offers the opportunity to have an enormous amount of fun in the classroom. Mathematics is a discipline that offers a number of opportunities for excitement and stimulation. The *All You Need to Teach Problem Solving* series offers the potential to clearly demonstrate this fact.



Scope and Sequence Chart

	Cracking the Code BLM 1 Task Card 1	Doing Your Block BLM 2 Task Card 2	Taking a Fence BLM 3 Task Card 3	Counting on Atnep BLM 4 Task Card 4	Fraction Attraction BLM 5 Task Card 5	Time's Up BLM 6 Task Card 6	Multo BLM 7 Task Card 7	Eggs Galore BLM 8 Task Card 8	Alpha-Numerics BLM 9 Task Card 9	Breeding like Rabbits BLM 10 Task Card 10	What's My Rule? BLM 11 Task Card 11	Anyone for Tennis BLM 12 Task Card 12	The Matrix BLM 13 Task Card 13	Book-worms BLM 14 Task Card 14	Better than Average BLM 15 Task Card 15	Shirts n' Shorts BLM 16 Task Card 16
Locate key words	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@	@
Look for a pattern	@	@		@		@	@		@	@	@			@		@
Assume a solution			@					@	@		@			@		@
Create a table or chart		@								@		@	@			@
Make a drawing			@			@						@				
Work in reverse								@							@	
Find a similar but simpler problem				@	@			@								@
Make a model				@	@											
Think logically	@						@		@		@	@	@	@	@	@

All the TEACHING TIPS You Need

What is Problem Solving?

Problem solving is the application of previously acquired skills and knowledge to an unfamiliar situation. Numerical equations presented as story or worded problems are often mistaken for problem solving. A typical example of what is not problem solving would be the transference of 5 cents + 5 cents + 10 cents + 10 cents into:

> Four children emptied out their pockets. Kate found five cents. Jake found five cents. Jason found ten cents and Sarah found ten cents. How much money did the four children find altogether?

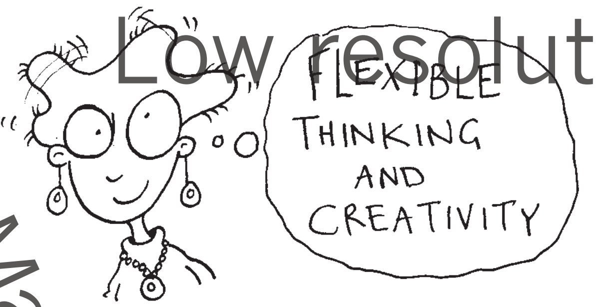
This is not problem solving because the technique required to solve this story problem (addition) is easily identified and requires little creative thought.

Questions that transfer numerical problems into a practical context are an essential part of any effective mathematics program. It is vital that students are constantly shown why they are learning mathematical skills.

A related problem-solving question could be:

> In how many different ways can 30 cents be made in our money system?

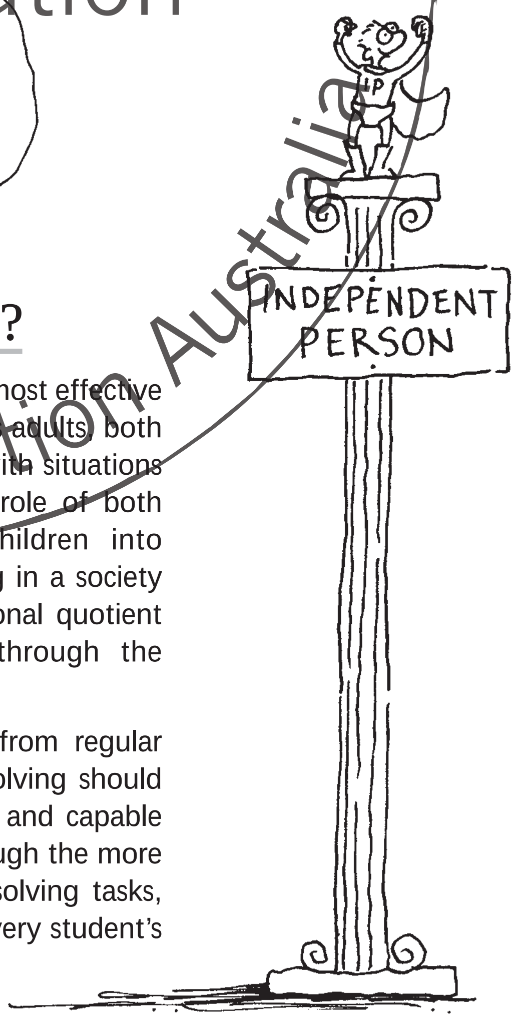
This question requires the student, in a logical manner, to search for a strategy to solve the problem and to apply the previously acquired skill of addition and their knowledge of the coin denominations available in Australian currency. The story problem offers a context. The problem solving exercises the student's ability to work flexibly, creatively and logically.



Why Teach Problem Solving?

It can be argued that problem solving should be the most effective and significant aspect of any mathematics course. As adults, both at work and at home, our everyday lives are filled with situations that demand flexible thinking and creativity. The role of both parents and teachers is to turn dependent children into independent people who are capable of functioning in a society that demands resilience, intelligence, a high emotional quotient and tractability. Such traits are best fostered through the development of an ability to problem solve.

It therefore follows that all students will benefit from regular exposure to problem solving in schools. Problem solving should not lie solely in the domain of the most intelligent and capable students, which is, regrettably, often the case. Although the more intelligent students may achieve best on problem solving tasks, regular problem solving should feature strongly in every student's learning experiences.



Classroom Atmosphere

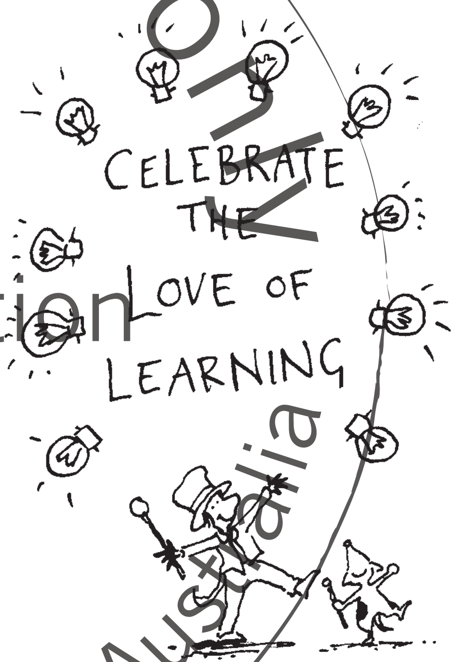
It is essential for students to believe in their own capabilities and to have healthy self-esteem. They need to understand that to have a go, even if their attempt is wrong, is far preferable to not attempting a question at all. Making mistakes plays a vital role in the learning process.

Encourage students to view intellectual challenges as opportunities to demonstrate how much they have learned and how bright they are becoming. Problem solving should come to be seen by students as not just important, but a great source of fun.

This positive atmosphere can best be engendered by teachers who are confident in their own ability to teach problem solving. It is as true for the teacher as for the student: Practice may not necessarily make perfect, but it will lead to improvement. Improvement leads to increased success and greater self-confidence. Success leads to enjoyment!

Raising the Bar

The capabilities of young students are often quite remarkable. They come to school today with far greater confidence and knowledge than any previous generation. In a classroom with a positive atmosphere and a school that celebrates learning, students will not only rise to intellectual challenges, they will thrive on them.



The concept of raising the bar refers to the idea that students should be extended until their full intellectual capabilities are reached, regardless of supposed appropriate year level standards. Every student deserves the opportunity to strive for his or her intellectual best. Problem solving is an excellent adjunct for the teacher to assess such potential. Present the students with challenges and step back to observe the outcomes. I can assure you that, in the appropriate learning environment, most students will exceed expectations.

Timetabling Problem Solving

A problem solving approach to teaching mathematics should be adhered to in each classroom for the reasons outlined on page 7. Whenever lessons involving 'core material' are conducted, every attempt to include open-ended questions should be taken.

If, for example, in a Year 5 classroom the concept of 2 x 2 digit multiplication is being taught, the following question could be introduced to extend the concept:

Given four different digits, say 2, 4, 7 and 1, what is the biggest possible product?

I believe that it can be strongly argued for a lesson a week to be devoted to strengthening problem solving skills. This four day a week core material, one day a week problem solving ratio would complement each component of the mathematics course very well. This is especially the case if the questions posed for the problem solving sessions could be related to the core material topic under review at the time.

STRUCTURING THE LESSONS

Each problem to be solved should be preceded, where appropriate, by a background discussion concerning the context of the problem and the previously acquired skills or knowledge necessary to successfully complete the task. A question, for example, asking 'How many rectangles can be found on a soccer pitch?' could prompt a discussion of the concept of a rectangle and how a rectangle can be formed by using pre-existing rectangles. Point out that a square is merely a special type of rectangle (opposite sides equal and four right angles).

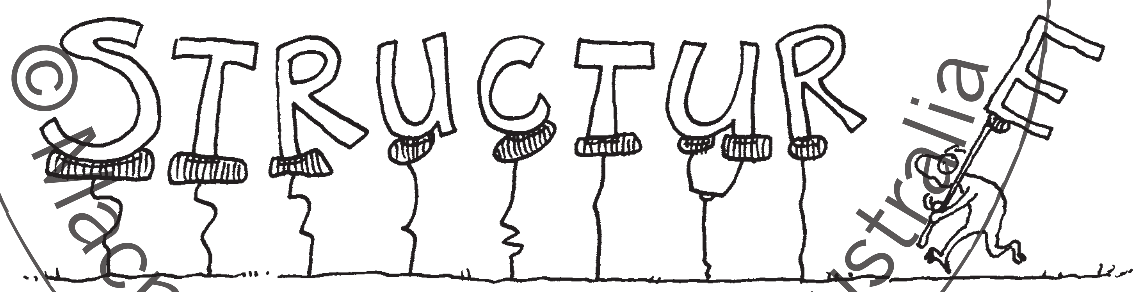
The problem should be read aloud, either by the teacher or a competent student. Ask students to select key words, underline them and write them down. At this point some of the brightest will be eager to get into the problem. Let them do so. For others in the class, a discussion of appropriate strategies that could be employed is valuable. Soon, many other students will be ready to begin.

For those still in need of guidance, it may be necessary to commence an appropriate strategy together. It may take some time, but, by following this ploy, each student will be on the right track.

Not all will finish the problem – some may only make a start. However, this is a step in the right direction and should be praised.

Some students prefer to problem solve individually, some enjoy the cut and thrust that cooperative group learning brings. Either approach is suitable, providing that within the group, each participant has a role to play.

Some problems lend themselves better to group work. Fermi problems such as 'How much money is spent on pets each year in Australia?' are best 'solved' by sharing ideas, estimates and general knowledge.

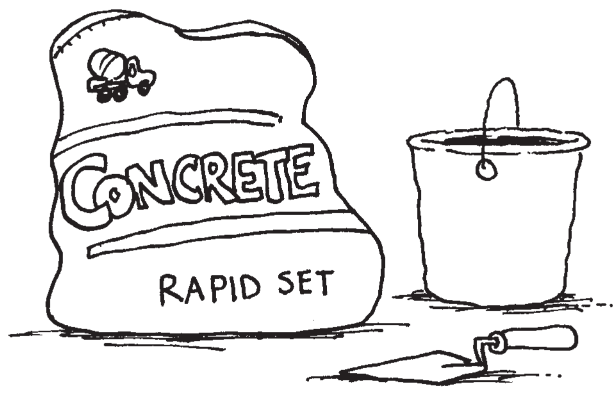


THE STUDENT AS A CONCRETE OPERATOR

Developmental theory has clearly shown that the great majority of primary school aged children, and especially the very young or the mathematically less able, benefit markedly from the manipulation of concrete materials when dealing with mathematical concepts.

For many students, the converse of this argument has dramatic results. The more abstract a concept or question is, the less likely that such a student will understand it and, hence, be able to solve it.

The message for primary teachers is clear: provide concrete materials whenever the students under your care attempt problem solving tasks to achieve the best possible results.



Reflection

Following the completion of each blackline master, gather the class for shared reflection. Encourage students to describe the strategies they have used and to outline the mathematics contained in their technique. Many problems can be solved with more than one strategy, as this reflection will demonstrate. This time will be especially valuable when problems were solved in a hit and miss fashion by students who were unable to recognise a pattern. Celebrating differences is a very healthy classroom activity and should be highlighted at this time.

This will also enable you to offer students praise and encouragement for their efforts. Keep their self confidence and mathematical self-esteem as high as possible. Praise their attempts at all times, even if they may be misdirected. Bear in mind that problem solving is intellectually challenging both for children and adults alike. Remember that any attempt is far preferable to no attempt at all.

Ask students the following types of questions:

- What helped you understand what the question was asking you to do?
- What strategy or strategies did you use in attempting to solve the problem?
- Have we used this/these strategies anywhere before?
- When else might you use this strategy?
- How did you feel when you solved the problem?
- Do you think that your problem solving skills are getting better?
- Do you think Mum or Dad might be able to solve the problem with your help?
- Did you find it useful to work with a partner on the problem?
- Could you make up a problem of your own like this one?
- Was the problem as difficult as it first appeared?

You may also consider encouraging students to record their progress in a journal. As well as providing a useful teacher reference, this can help students to see what they have learned.

Fast Finishers

The students who are likely to enjoy problem solving the most will be those who are more mathematically able. Because of their innate capabilities they will be the first to finish their work. Please do not give these students extra drill and practice examples to do! This is far more likely to dull their enjoyment of the subject and dim their creativity. Make space in your room for fast finishers and offer them a corner full of problem solving tasks, games or puzzles to do to encourage their love of challenges.

WHICH STRATEGY TO USE

There are relatively few types of problem solving questions. As a consequence, the more a student practises these strategies, the more comfortable they will become with problem solving in general. The strategies are similar across the three books in this series for a very sound reason – they are as relevant to a five-year old as they are to an adult. The two key strategies of problem solving are locating key words and looking for a pattern. These are fundamental to almost all problem solving tasks. The other strategies, while still relevant to all learners, are more question specific. The sooner key strategies can be exercised and practised, the better the problem solving skills will become.

For some questions, once students understand what needs to be done, just one strategy will be sufficient. For other questions, more than one strategy will need to be used. Sometimes a range of different strategies may be appropriate. In some instances, two totally different strategies may successfully solve the same problem in two logical and equally creative ways.

LOCATE KEY WORDS

The instructions involved in a problem must be understood before students can begin to attempt the question. Often a student will not attempt a problem because no strategy is immediately apparent. Getting started can often be the toughest part.

The technique of underlining and then writing down the key words in a question (committing something to the page) is an excellent way for students to gather their thoughts and to make a start. This strategy will be emphasised in every problem in this series.

Take the question: ‘What is the smallest number of children in a family to ensure that each child has both a brother and a sister?’

Underlining and then writing down ‘smallest number children – each a brother and sister’, can focus a student’s thinking. This précis of the problem can also make it appear, from a psychological point of view, easier to handle.

Once they understand the problem, encourage students to work in a logical manner while offering suggestions for the students to build upon. Point out that a sensible way of commencing the problem would be to assume a possible solution and then to use pen and paper to test its validity.

Another different, but equally legitimate, approach could be to act out or model the problem by using students to represent brothers and sisters. This approach will also enable students to assess the assumed solution.

Number of Boys	Number of Girls	Boys’ Siblings	Girls’ Siblings
1	1	Sister	Brother
1	2	Sister	Brother/Sister
2	2	Sister/Brother	Brother/Sister

The table enables us to see that the minimum number of children in the family, so that each child has both a brother and a sister, is four.

This problem emphasises that often more than one strategy can be employed to solve a problem and, in fact, that a combination of strategies may be used at the same time.

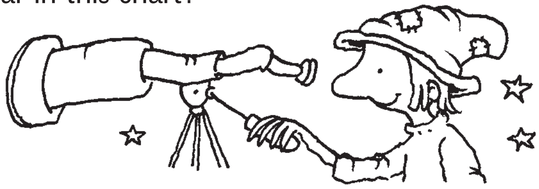
Such success and such a logical approach to the problem obviously requires that students understand the question and can identify the problem’s key components.

Look for a Pattern

This strategy, used effectively, is often the shortcut to success and can greatly simplify problems that may initially appear very difficult. Take the following problem for example:

In which column would the Year 2006 appear in this chart?

A	B	C	D	E
4	2	5	3	1
9	7	10	8	6
14	12	15	13	11



Students should recognise that the five times table can be found in column C. One beyond this table is located in column E. 2006 is one beyond the five times table, therefore it will be located in column E. Compare this approach to counting one at a time!

The discovery of this pattern has saved time and effort. It can be argued that pattern recognition is the most apparent trait in the brightest problem solvers. This fact is often frustrating for both teachers and students. Students who can't see patterns readily are the ones who most need short cuts.

Successful problem solvers will be the first to recognise the patterns in times tables. They see quickly that all numbers in the nine times table have their digits summing to nine or a multiple of nine (18 = 1 + 8 = 9, 234 = 2 + 3 + 4 = 9, 585 = 5 + 8 + 5 = 18 and 1 + 8 = 9, and so on). Students who appreciate this pattern will readily see that 17342 has its digits summing to 17, which is eight beyond a number in the nine times table and so is not a multiple of nine and will give a remainder of eight if divided by nine. These students will also recognise that the two closest numbers in the nine times table to 17342 must be 17343 and 17334, both summing to 18.

Assume a Solution (Guess and Check)

This approach encourages the student to have a go and is particularly useful in problems involving variables. By assuming a solution, patterns often appear, suggesting a suitable pathway to success.

An example of a problem in which assuming a solution is a suitable strategy would be:

I bought a rabbit and a kilogram of rabbit food from 'Perfect Pets' which cost me $38.

My friend Jade bought two rabbits and three kilograms of rabbit food for $79.

How much does a rabbit cost at 'Perfect Pets'?

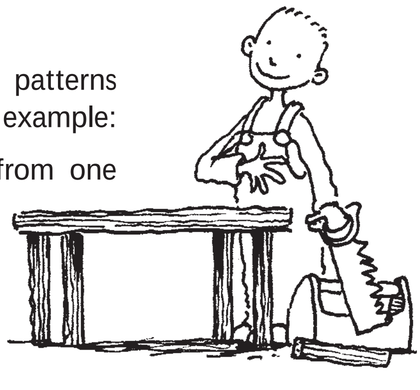
A first attempted solution might be that the rabbit cost $30 and so the food must be $8 per kilogram. This is a logical attempt that uses common sense regarding the relative values of the two products. Common sense should always be used in these types of problems.

However, when these two values are applied to the second purchase, the total is too large. The student must then decide if it is the rabbit or the food which has been overestimated. This will become clear when two new values are assumed and the discrepancy between the projected total and $79 is calculated. Eventually, through trial and error, the correct solution of $35 for the rabbit and $3 for the food should be reached.

Although this approach is not the easiest or fastest way of solving the problem, at least the student who uses this technique is putting something on paper. Even if they don't get the right answer, at least they have made some progress.

Create a Table or Chart

Tables and charts can often be useful in making potential patterns more perceivable to students. Take the following question for example:



A triangle can have no possible diagonal lines drawn from one corner to another.

A rectangle has two possible diagonal lines.

A pentagon has five possible diagonal lines.

How many possible diagonal lines does an octagon have?

This may initially require the drawing of a hexagon and its nine potential diagonals. The student will then perceive that this technique is very time-consuming and becoming quite unwieldy. The creation of a chart will lead far more easily to a pattern being recognised (the difference between the number of diagonals increases by one), and the solution of 20 being found.

Polygon Name	Number of Sides	Number of Possible Diagonals
Triangle	3	0
Rectangle	4	2
Pentagon	5	5
Hexagon	6	9
Heptagon	7	14
Octagon	8	20

For younger students, the table or the chart should be provided. The student's task should be to fill it in rather than to construct it. In later primary years when students have had exposure to the structure and nature of a table, it is appropriate to ask them to form the structure themselves.

Make a Drawing

Drawing is a particularly useful ploy for beginning problem solvers, for students who are visually oriented in their thinking and learning styles, or when applied to certain spatial questions. Take the following problem for example:

My rectangular swimming pool has a surface area of 24 square metres. The pool's side lengths are all in whole metres. What might my swimming pool's perimeter be?

This can be solved effectively by quickly drawing rectangular representations of the pool and inserting appropriate dimensions.

Making a drawing is also an excellent way of getting students started. In a similar way to verbalising and discussing the structure of a question, drawing helps to crystallise what needs to be done.

When asked to solve, for example, problems involving 2D or 3D shapes, it certainly helps to sketch the shape under review. As a result, dimensions, faces, vertices and edges become more apparent and often shed significant light on the problem.

Work in Reverse

This strategy is relevant to a specific type of problem, usually numerical and in many parts.

The question 'Which number, when doubled and tripled, and then reduced by 5, equals 55?' can best be solved by starting with the solution and then working in reverse. This is preferable to using the assume a solution strategy. The missing number that began the process can be obtained by using the converse operations to those given in the problem, in the following manner:

$55 + 5 \div 3 \div 2 = 10$

Whenever a solution to this type of problem is obtained, encourage students to superimpose the solution back into the original problem to verify its accuracy:

$10 \times 2 \times 3 - 5 = 55.$

Find a Similar but Simpler Problem

This is a particularly valuable strategy when applied to what can appear to be very complex problems. Take the following question:

$$\frac{27 \times 35 + 35}{35}$$

This may initially appear to be well out of the range of a primary school student's capabilities. However, students in the upper school should have the mathematical skills to cope with the question from an operational perspective. It is the numbers used that make the question daunting.

Consider a similar, but simpler problem:

$$\frac{2 \times 4 + 4}{4}$$

The concepts contained within this question are the same, only the numerical values have been reduced to make the problem far more accessible.

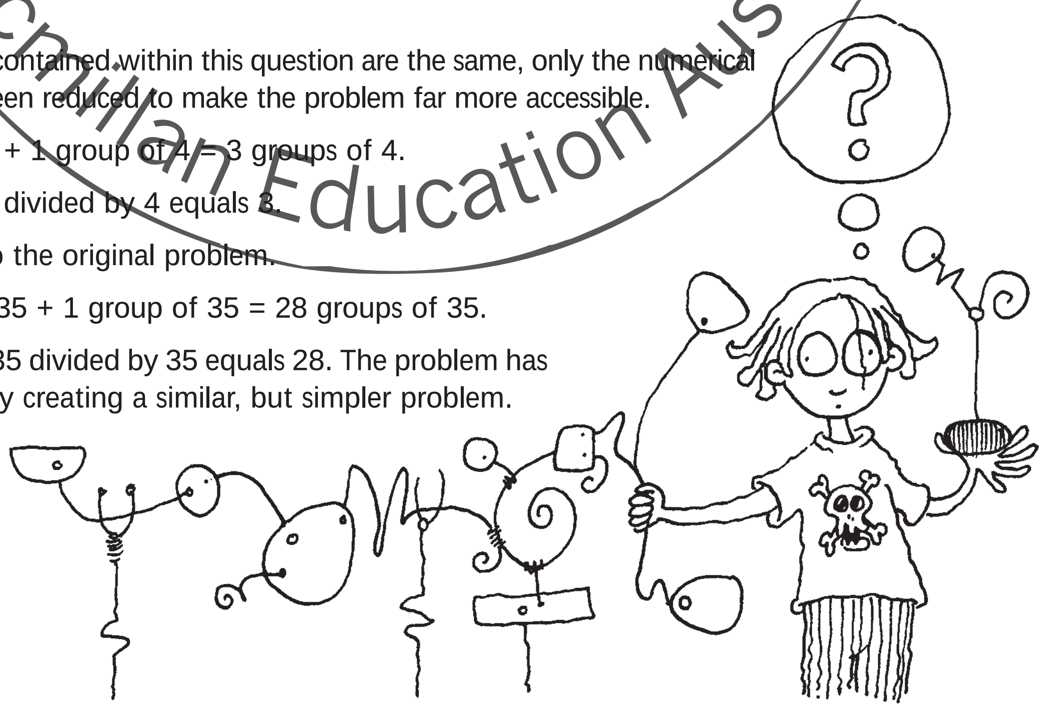
2 groups of 4 + 1 group of 4 = 3 groups of 4.

3 groups of 4 divided by 4 equals 3.

Now return to the original problem.

27 groups of 35 + 1 group of 35 = 28 groups of 35.

28 groups of 35 divided by 35 equals 28. The problem has been solved by creating a similar, but simpler problem.



Make a Model

This strategy is often employed by concrete thinkers whose innate spatial sense may not lend itself to tackling geometric or space-oriented questions in any other way. Take the following question for example:

> A 3 x 3 x 3 cube is constructed and then immersed in a bucket of red paint. How many of the cube's 27 blocks have paint on at least one of their faces?

Many students may solve this best by actually making the cube from connecting blocks and rotating it to count the exposed faces. It is only the centre cube that remains untouched by the paint.

This problem can be extended to a 4 x 4 x 4 cube to reveal that a 2 x 2 x 2 cube remains untouched within, and to a 5 x 5 x 5 cube to discover that a 3 x 3 x 3 cube within will be untouched, and so on.

These extension exercises should be utilised at every opportunity. From the strategy of making a model we have progressed to discovering a pattern. Making a model is often time consuming but, when all else fails, this is better than not obtaining a solution at all. Until basic number facts develop to a near automatic level, concrete material in problem solving, such as connecting blocks, is a wonderful adjunct to learning.

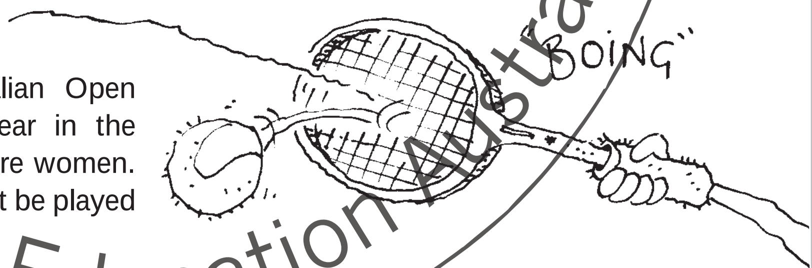
Think Logically

Many problems with numerous potential solutions can be solved by using deductive reasoning to identify and eliminate impossible options. Games like 'Guess Who', '20 Questions' and 'Mastermind' are examples of activities that use this strategy.

Thinking logically requires common sense and creativity in thinking. It is a strategy that tends to be used most frequently by the best problem solvers but can be developed and strengthened in all students.

Consider the following problem:

> 256 players enter the Australian Open Tennis Championships each year in the singles. 128 are men and 128 are women. How many singles matches must be played each year in the tournament?



Many students may start by considering that there must be 64 matches in both the men's and the women's draw in round one and 32 in round two and so on. Others may start by considering the fact that the winner and the runner up must play in seven rounds while the semi-final losers must only play in six rounds each and so on. Other techniques may also be employed, all of which can be time consuming and mathematically difficult to calculate.

The creative, logical thinker will appreciate that, for every match, their must be a winner and a loser. The tournament can only have one winner, and so there must be 127 'losers' and, therefore, 127 matches in both the men's and women's championships. Thus, the Australian Open consists of 254 singles matches.

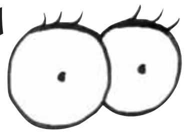
The deductive thinker has cut to the chase. But be sure to praise students who have looked for a pattern, made a table or tried to create a similar but simpler problem. In problem solving several different approaches can work, it's just that some are more direct than others.

THE NINE PROBLEM SOLVING STRATEGIES

1. Locate the key words

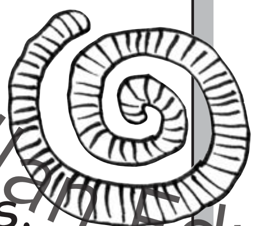
- Underline or write these words down.
- Read these key words a few times.
- Ensure that I know what needs to be done.

2. Look for a pattern



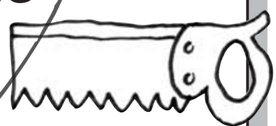
- Does the question contain a number pattern that I can see?
- Can I predict the next answer that works?
- Will this pattern hold for any possible answer?

3. Assume a solution



- Think of a sensible answer that might work.
- Put it into the problem to see if it works.
- If it doesn't, try another.
- Am I getting closer or further from the solution?

4. Make a table or chart



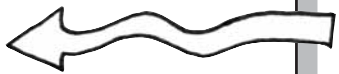
- Will a structure like this help me?
- Does this table help me to see a pattern?

5. Make a drawing

- Can I draw something useful about the problem?
- Will a sketch help me to understand and solve the problem?

6. Work in reverse

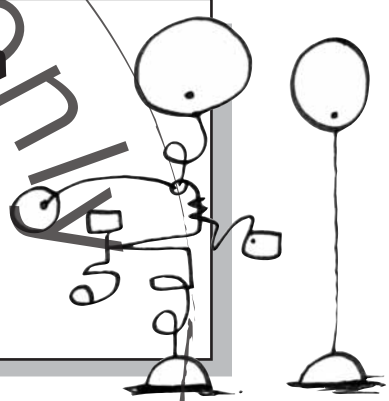
- Can I start at the end of the problem and work towards the start?



- Do I need to change the maths operations to get an answer?

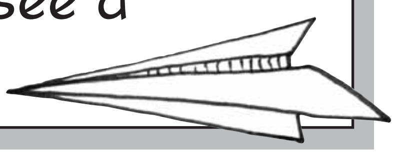
7. Try a similar but simpler problem

- Can I make the question easier to help me see what has to be done?
- Will this help me to see a pattern?



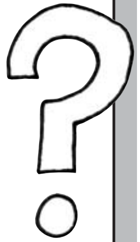
8. Make a model

- Will a model made out of paper or blocks help me to understand the problem?
- Will a model help me to see what needs to be done?
- Will a model make it easier to see a pattern?



9. Think logically

- What is obvious about the answer to the problem?
- What type of answers are obviously wrong?
- How do I know this?



WHEN I PROBLEM SOLVE I MUST REMEMBER THESE THINGS

- Read the question carefully.

- Locate the key words.
- Ensure that I know what needs to be done.

- Find a suitable strategy or strategies to use.

- Check if my answer works by putting it back into the problem.

- Explain the methods I used to solve the problem.

All the Lesson Plans and Worksheets You Need

Cracking the Code

Strategies
- Locate key words
- Look for a pattern
- Think logically

Background

Many of the world's greatest mathematical thinkers have devoted their talents to the creation and breaking of codes. The Enigma project in World War II is a notable example.

Codes are mathematically based and rely on the creation and application of sequences or number patterns. Sequences have rules and each member of a sequence is called a term. Thus, the sequence 2, 4, 6, 8 ... has the rule of + 2 with the fourth term being 8. This lesson asks students to work with number patterns from the contexts of both creation and application.

Resources:
- pencil and paper

Orientation

Advise students that the best way to disguise a message, especially a secret or sensitive one, is to put it into code. Codes can take the form of letters or numbers and they all follow a mathematically based pattern. Discuss the history and nature of codes and give students a simple example, such as a name written backwards. Ask them to memorise some key letters and their position in the alphabet, a ploy that will save a lot of time. J = 10 and T = 20 are two helpful 'markers'. K must, therefore, be 10 + 1 or 11th in the alphabet and V must be T + 2, or 22nd in the alphabet, and so on.

Guided Discovery with BLM 1

Read the first question with the class. The key words are the code itself. Ask students what pattern they recognise in the code. Have them write the alphabet with the number value next to each letter. Some students will recognise that the word starts and ends with T, and may identify the answer as Tibet. Encourage them to keep the answer to themselves. Ask why this code is not suitable for transmitting top secret information.

Move on to the second code. Discuss the pattern of going back two letters. Have students write out a revised alphabet. Ask what they notice in the answer: three letters are repeated and the first name ends and surname begins with the same letter.

The third code is more complex. Write the pattern on the board to help identify the rule of increasing the difference between the terms by one. Have students write down the alphabet and its values. Ask why we don't need to progress beyond the letter R. Remind them that speed and efficiency are vital when code cracking. What do we notice about the answer? Two repeated letters, starts with B and is the name of a town with six letters. The solution is Broome in Western Australia.

Further Exploration

Task Card 1

Read the question with the class: 'Make up a secret code. Use your code to disguise the name of someone in your class and the name of a famous singer or movie star. Ask a friend to try and decode it.'

Help students to identify the key words: Secret code. Someone in your class, famous singer, movie star. Decode it.

This task card can extend the concept according to students' understanding. Students can have fun challenging their classmates while exploring different kinds of codes and patterns. Encourage them to be creative with their codes, and to introduce different elements to make them more complex.

Name Date

Cracking the Code

In a secret code A = 1, B = 2, C = 3 ... Z = 26.
What country is 20, 9, 2, 5, 20?

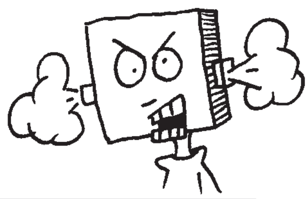
In another code A = Y, B = Z, C = A ... Z = X.
Who is PMYJB BYFJ?

In another code A = 1, B = 2, C = 4, D = 7 ... Z = 326.
In which state is 2, 154, 106, 106, 79, 11?

Strategies Locate key words Look for a pattern Think logically

Lesson Plan 2

Doing Your Block



Strategies

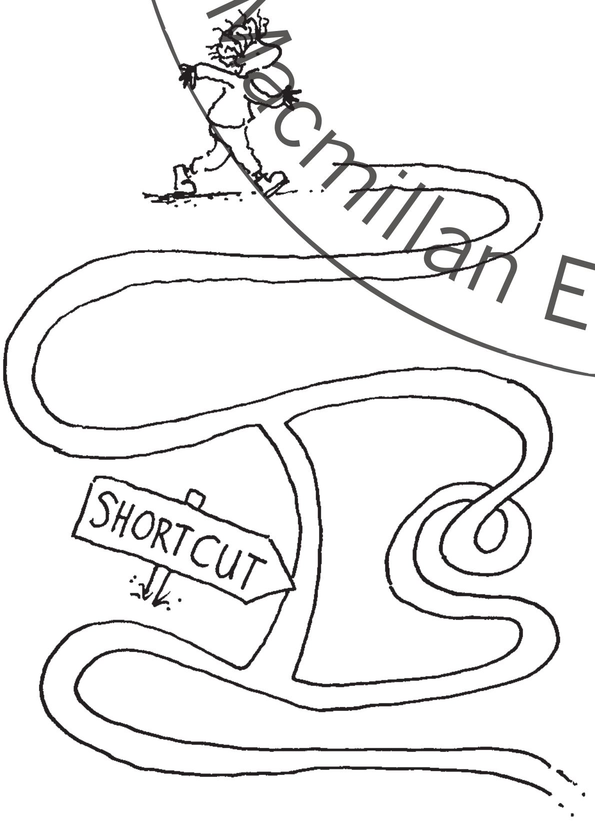
- Locate key words
- Look for a pattern
- Create a table or chart

Background

The 17th century French mathematician Blaise Pascal (1623–1672) was a mathematical prodigy. Pascal created the first mechanical calculator and was one of the first to investigate probability concepts. The famous Pascal's Triangle is often studied at school for its potential mathematical patterns and geometric relationships.

Orientation

Explain to students that shortcuts in mathematics are not only useful, but are, at times, essential for the successful solution of certain problems. Formulae, such as the area of rectangles = L x W, avoid the need to rule up lines and count squares, which can be very time consuming and often inaccurate. Multiplication is a shortcut to repeated addition in the same way that division is a shortcut to repeated subtraction.



Guided Discovery with BLM 2

Read the question and help students to locate the key words: Andrew and Ben, two blocks. Andrew and Chloe, three blocks. Andrew and David, four blocks.

Ensure that the three pathways located between Andrew's and Chloe's houses are only a distance of three blocks.

Ask students to solve the next question by listing all the pathways that start with a move North and then all those that start heading East. Explain that the two sets of pathways are mirror images.

Help students identify the pattern by writing:

2 blocks apart: 2 pathways

3 blocks apart: 3 pathways

4 blocks apart: 6 pathways

5 blocks apart: ? pathways

Test suggestions by drawing maps and listing the pathways located. When the solution of ten is found, ask again for a possible pattern. Ensure all students understand before moving on.

Further Exploration

Task Card 2

Read the question: 'A famous French mathematician named Blaise Pascal created the following famous triangle. Copy this down and see if you can complete the next two lines of the triangle.'

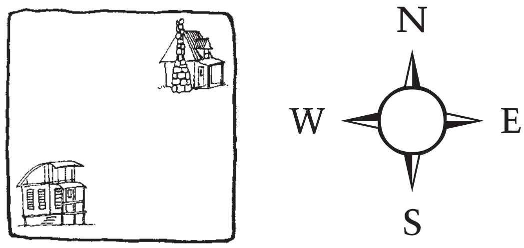
Help students to locate key words: Complete next two lines.

Ask students to underline the number of the row and the highest number in each row. For the fourth row, 6 should be underlined, for the fifth, 10, and so on. Can students see how this number can be worked out from the previous row? Ask them to predict what the next two rows might look like. The highest number is the sum of the two highest numbers in the previous row.

Now ask students to relate this to **BLM 2**. The number of possible shortest pathways is the highest number in each row. The number of the row represents the number of blocks between the two houses.

Doing Your Block

What you need: pencil

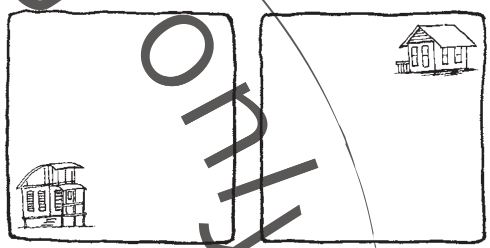


Andrew and Ben live two blocks away from each other.

To get to Ben's house, Andrew must walk North then East or East then North.

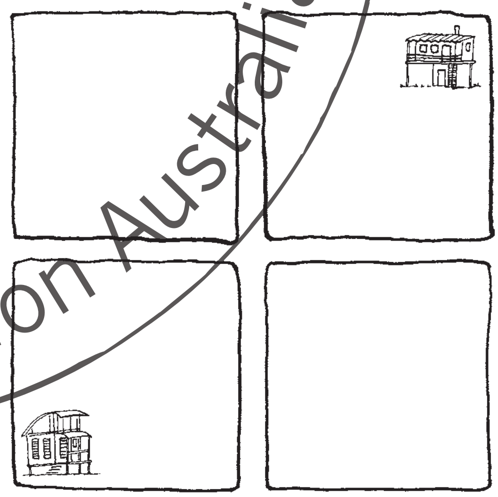
Andrew and Chloe live three blocks away from each other.

List all the three-block pathways from Andrew's house to Chloe's house.



Here is a plan of Andrew and David's houses, which are four blocks apart.

Find all of the four-block pathways between these two houses.



Can you see a pattern forming?

What is your prediction for the number of shortest pathways for two people living five blocks apart?

Strategies Locate key words Look for a pattern Create a table or chart

Taking a Fence

Strategies
- Locate key words
- Assume a solution
- Make a drawing

Background

Once a student is familiar with the concepts of length, perimeter and area and their interrelationships and differences, opportunities arise to strengthen these understandings through application in open-ended situations.

This lesson provides such application to relevant, but unfamiliar contexts.

Resources:

- classroom

Orientation

Demonstrate that the concepts of both perimeter and area can be readily seen in the classroom. The skirting board around the edges of the room is an example of a perimeter. The carpet on the floor is a practical example of area. The amount of carpet used would have determined the cost of the job when being laid. The number of square metres of carpet on the floor (e.g. 6 m x 8 m = 48 sq m) could also equate to a different sized room, say, one which was 12 m x 4 m. Would this imaginary room require the same amount of timber for its skirting board? If not, why not?

Guided Discovery with BLM 3

Read the question and locate the key words: One-metre lengths of fencing. 24 metres – largest area possible.

Ensure that students grasp the difference between perimeter and area. This question is an excellent way to focus thinking on the differences. Ensure that students also grasp the difference between a metre and a square metre – space around and space within.

The drawings that students make may not be to scale. Quick sketches are quite adequate.

Ask students what they notice about the assumed solutions that they are dealing with. What is similar about the sum of the length and width of each of the drawings they have made? (L + W = 12 metres on each occasion – half of the perimeter).

The solution of 36 square metres raises the question 'Is a square a rectangle?' It is, because a rectangle is a quadrilateral with four right angles and opposite sides equal in length.

Ask students to consider whether this result is unusual. Why is it that, given a perimeter, the largest possible area will always be a square, or the closest shape to a square that is possible?

Further Exploration

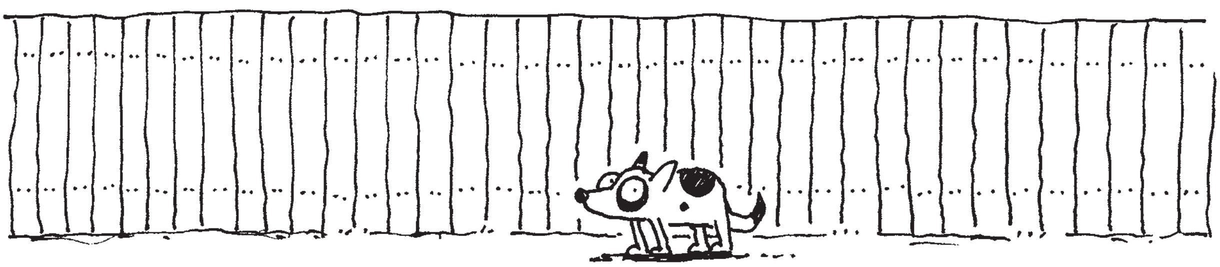
Task Card 3

Read the question with the class: 'Use all 24 iceblock sticks or MAB longs to make as many different rectangles as possible. Each rectangle must use all 24 sticks. Which rectangle had the smallest area? Which rectangle had the largest area?'

Help students to locate the key words: As many different rectangles as possible. All 24 sticks. Smallest and largest area?

Again, students must appreciate that a square is a rectangle and that the length and the width must equal 12 for each rectangle constructed.

Encourage students to move from the smallest possible (11 x 1) to the largest (6 x 6) as systematically as possible. The order of 11 x 1, 10 x 2, 9 x 3, 8 x 4, 7 x 5, 6 x 6 increases the areas progressively. Random rectangle construction may not demonstrate this pattern and may well result in some possible rectangles being omitted.



Name Date BLM 3

Taking a Fence

Madison wants to make a vegetable patch in her backyard, using one-metre lengths of fencing as a border.

She has 24 metres of fencing and wants to enclose the largest area possible.

Draw what the vegetable patch might look like. Include the lengths of the sides in your drawings.

The largest area possible is: ______________________

Strategies Locate key words Assume a solution Make a drawing

Counting on Atnep

Strategies

- Locate key words
- Look for a pattern
- Find a similar but simpler problem
- Make a model

Background

Our Hindu-Arabic system of counting is based on the number ten. Using the digits from 0 to 9 we can create any number, no matter how large or small. The fact that we have ten fingers, sometimes referred to as digits, generated this system of numeration. It was first created in India by the Hindus and was spread around the world by Arab traders, who appreciated its obvious benefits, during their commercial odysseys.

Studying bases other than ten is an excellent manner of appreciating the way that our own numeration system works. 'Counting on Atnep' ('penta' backwards) uses base 5 in its structure.

Resources:

- MAB blocks

Orientation

Explain to the class that we count in tens in our number system but that it is quite possible to count in other bases as well. Computers and calculators use a binary system based on the digits 0 and 1. Rather than ones, tens, hundreds, thousands and so on, the binary system uses ones, twos, fours, eights, sixteens and so on. Show that one to ten in binary is written as:

1, 10, 11, 100, 101, 110, 111, 1000, 1001 and 1010. How does this work?

Guided Discovery with BLM 4

Read the question with the class and locate the key words: 5 Pents = 1 Penta. 5 Pentas = 1 Pentag. 5 Pentags = 1 Pentago. How many Pents = a Pentago?

Get out an MAB 1, 10, 100 and 1000 block and demonstrate that the values of the four blocks can be worked out by multiplying the blocks' dimensions together, finding their volumes:

The unit block: L = 1, W = 1, H = 1, 1 x 1 x 1 = 1

The long or 10 block: L = 10, W = 1, H = 1, 10 x 1 x 1 = 10

The flat or the 100 block: L = 10, W = 10, H = 1, 10 x 10 x 1 = 100

The cube or 1000 block: L = 10, W = 10, H = 10, 10 x 10 x 10 = 1000.

Ask students how many one blocks make a ten block, how many tens make a 100 block and how many 100 blocks make a 1000 block. Therefore, how many tens make a 1000 and how many ones make a 1000?

Now return to the question. Refer back to the MAB exercise. Can we use the patterns identified to discover a pattern in the new problem? Hopefully, the relationship of 5 x 5 x 5 on Atnep will be recognised as a variation of our 10 x 10 x 10.

Further Exploration

Task Card 4

Ensure that students understand the question: 'On the planet Atnep their place value system is structured around base 5 instead of base 10 like ours. Get MAB 1, 10, 100 and 1000 blocks and look at the edges. Now use connecting blocks to make a model of a 1, 5, 25 and 125 block. Look at the edges of these Atnep blocks. What do you notice?'

Help students identify the key words: Base 5. MAB blocks – edges. Make model of 1, 5, 25, 125 block. What do you notice?

Encourage students to observe the edges of the four MAB blocks to see how their volumes relate to the given numerical value of the blocks.

Now have them construct the models of the 1, 5, 25 and 125 blocks. The edge x edge x edge calculations should be done with a calculator if necessary.

Extensions to bases other than ten and five strengthen students' grasp of their own base 10 numeration concepts.

Name Date BLM 4

Counting on Atnep

What you need: MAB blocks

On the planet Atnep:

5 Pents equal 1 Penta,

5 Pentas equal 1 Pentag and

5 Pentags equal 1 Pentago.

Therefore, how many Pents must equal a Pentago?

How did you work this out? Show your working here.

Strategies Locate key words Look for a pattern Find a similar but simpler problem Make a model

Lesson Plan 5

Fraction Attraction

Strategies

- Locate key words
- Find a similar but simpler problem
- Make a model

Background

The study of rational numbers (any number that can be expressed as a fraction, such as one half, 0.43 or 75%) plays a significant part in any mathematics program in the upper school. Fraction concepts need to be developed hierarchically, starting with the concept and meaning of the numerator and denominator and then moving on to equality, simplest form and common denominators.

This lesson presents students with problems that draw upon previously acquired skills such as the concepts of equality, common denominators and comparative size.

Resources:

- fraction wall or fraction strips

Orientation

Demonstrate to students that fractions are infinite, just as whole numbers are. The concept of an infinite set of fractions can best be seen with the use of a fraction wall or fraction strips. Depending on the structure of the kit, ask if the final available set of bricks represents the smallest possible set of fractions. The answer of 'no' will raise the issue of potentially smaller and smaller fractions and also the fact that there are a potentially infinite number of fractions between any two fractions. Between $\frac{1}{4}$ and $\frac{3}{4}$ the students will see numerous bricks below on the wall representing fractions larger than $\frac{1}{4}$ but smaller than $\frac{3}{4}$.

Guided Discovery with BLM 5

Read the question with the class and help students to locate the key words: $\frac{1}{3}$, —, $\frac{1}{2}$. What fraction between?

Explore the problem with concrete materials. Pizza slices, fraction strips or fraction walls will demonstrate the relative values of the two given fractions. You could also fold a piece of paper in half and an identical piece into thirds to demonstrate the discrepancy. This should help students appreciate that a unit fraction (a fraction with one as the numerator) will not work as a possible answer. What if the two fractions were $\frac{1}{2}$ and $\frac{1}{4}$? The answer would be $\frac{3}{8}$ in between, revealing that a common denominator must be found.

A suggestion of six as the common denominator leads to $\frac{2}{6}$ and $\frac{3}{6}$ with no space in between. Multiplying the two fractions by $\frac{2}{2}$, a name for one whole number (any fraction multiplied by one must remain the same in magnitude), will lead to $\frac{4}{12}$ and $\frac{6}{12}$ with $\frac{5}{12}$ fitting in between.

Further Exploration

Task Card 5

Read the question with the class: 'Fold a piece of A4 paper in half, then in half again, then in half again and again. How many equal pieces do you think the sheet has been divided into? Unfold it to see if you were right. What fraction of the original sheet does each piece represent? If you folded the sheet two more times than you did originally, how many equal pieces will be seen when unfolded, and what fraction will each piece represent?'

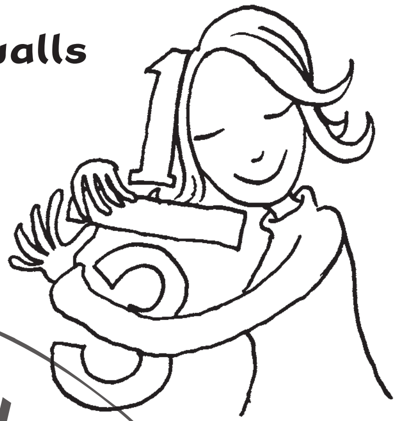
Identify the key words: Fold in half, then again, then again, and again. How many equal pieces? What fraction does each piece represent? Two more times – how many pieces? What fraction?

Students need to be aware of the effect of progressively halving and its converse relationship to doubling. Show students the connection between multiplying by two and dividing by two, and between multiplying and dividing by four.

Once solved, the problem could be extended as far as you like with a calculator.

Name Date BLM 5

Fraction Attraction



What you need: fraction strips or fraction walls

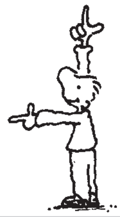
The first term in a sequence is $\frac{1}{3}$.
The third term in the same sequence is $\frac{1}{2}$.
What fraction comes in between?

$\frac{1}{3}, \quad \frac{\quad}{\quad}, \quad \frac{1}{2}$

Use this space to show your working.

Strategies Locate key words Use a similar but simpler problem Make a model

Time's Up



Strategies

- Locate key words
- Look for a pattern
- Make a drawing

Background

The way the hands on a clock face move around the dial and the way that digital devices display time provide great scope for potential mathematical questions, applications and problem solving.

Sweep-hand or analogue clocks afford questions relating to the discrepancy or angles between the hands as they move around the dial. Digital timing devices use light bars to display numbers on their screens:

0 is made up of six light bars, 1 of two light bars, 2 of five light bars, 3 of five light bars, 4 of four light bars, 5 of five light bars, 6 of six light bars, 7 of three light bars, 8 of all available seven light bars, and 9 of six light bars. It is only on a calculator that 7 is formed with four light bars. On all other digital displays only three are used.

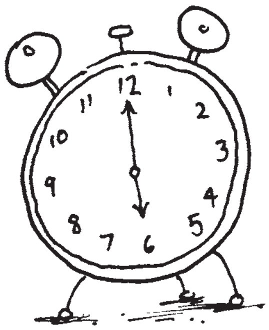
This lesson is centred upon these facts.

Resources:

- sweep-hand clock

Orientation

Use a clock face to demonstrate that, as the hands move around the dial, angles are formed. A right angle can be seen at three o'clock and again at nine o'clock. Explain that this angle is equal to 90 degrees. At two o'clock an acute angle is formed (an angle less than 90 degrees) and at five o'clock an obtuse angle is created (between 90 and 180 degrees). At six o'clock we create a straight angle, which is equal to 180 degrees.



Guided Discovery with BLM 6

Help students locate the key words: 6.00, straight line. What other times?

It is a trap to assume that the next time the hands form a straight line must be 7:05, then 8:10, then 9:15 and so on. Suggest that students begin by considering the hands on the face at about a quarter past 9. This is not a straight line because the hour hand has moved a quarter of the way towards the 10. Therefore, the minute hand must be a quarter of the way between the 3 and the 4 to compensate.

Next ask the students to consider 12:30. It doesn't work, for a similar reason. However, we can see that the hour hand must be halfway between the 12 and the 1. Therefore, the minute hand must be halfway between the 6 and the 7, to compensate.

This should enable students to see a pattern forming or, at least, to make sensible estimates regarding the remaining solutions.

Avoid using a real clock face because students can simply manipulate the hands. Drawing hands on the picture of the clock face on **BLM 6** will enable students to recognise the pattern.

Further Exploration

Task Card 6

Begin by ensuring that the students know how light bars work. Read the question: 'At 12:00, a 12 hour digital clock uses 19 light bars in its display. Use iceblock sticks or MAB longs to show this time. There is a time before 12:00 when a 12 hour digital clock uses 21 light bars in its display. What time is it when this occurs?'

Help students to locate the key words: 12:00, 19 light bars. Before 12.00, 21 light bars. What time?

Ask students which will be the most useful numbers to help solve this problem (0 and 8).

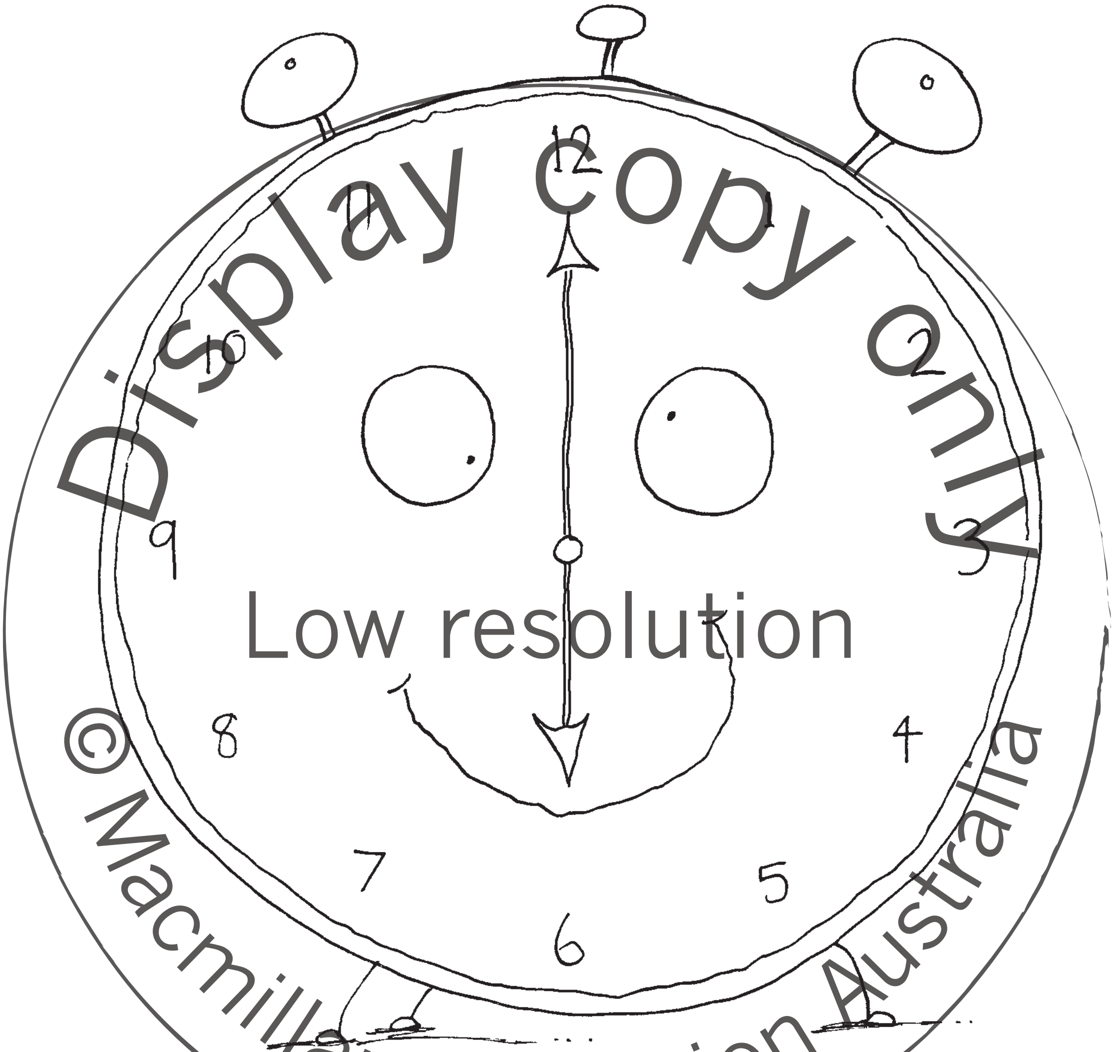
Which digit has to be placed in the far left column? (1)

Which digit is the best to use next? (0), and so on.

If it was a 24 hour digital clock, is 10:08 still the time that will use the greatest number of light bars? (No, 18:08 would use 22 light bars.)

Time's Up

At 6:00, the hands on a clock face form a straight line. At what other times does this occur?



Strategies Locate key words Look for a pattern Make a drawing

Multo

Strategies

- Locate key words
- Look for a pattern
- Think logically

Background

Games can be very valuable when teaching mathematics. Games offer the opportunity to add excitement and a high motivational appeal to your lessons. The best games combine both luck and skill, giving students of all abilities a chance to shine. The game of **Multo** offers both a challenge and the potential for great fun.

Resources:

- a deck of playing cards

Orientation

Teach the class to play **Multo**. Remove the picture cards and jokers from a deck of cards. Each card has face value and the ace is worth one. The game should be played in groups of three to five players. Each player is dealt three cards, face down. Players take turns, round by round, to select a target between 0 – 100. The object of the game is to make an equation with the three cards in your hand that equals the target (two points), or a total closer than your opponent's (one point). The players may use any maths signs they know to help with the equation building.

A sample game with a target of 34 and a hand of 4, 4 and 8 could produce an equation of 44 – 8 to equal 32, or 4 x 8 + √4 to equal 34. If 32 was the closest in the group the player would receive one point. The equation of 34 would receive two points. The winner of the game is the first player in the group to get ten points.

Guided Discovery with BLM 7

Read the question with the class and identify the key words: Target 27. Dealt 2, 4 and 5. Target 73. Dealt 3, 8 and 9.

Ensure that students are familiar with **Multo**. Have them brainstorm maths signs and symbols and encourage them to be creative. Don't be afraid to throw some in yourself to extend their grasp of number theory. Square root, decimal and factorial symbols add greatly to the potential creativity (4! is read as 4 factorial and equals 4 x 3 x 2 x 1 or 24. 3! equals 3 x 2 x 1 or 6, and so on). Encourage students to club digits together. Given a hand of 3, 8 and 9, 38, 83, 39, 93, 89 or 98 could form part of the equation.

Let the challenge begin!

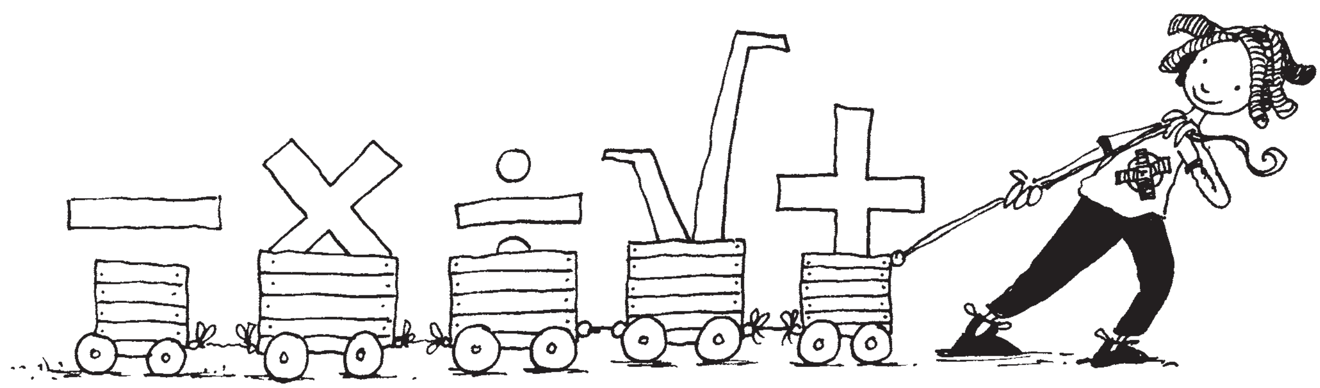
Further Exploration

Task Card 7

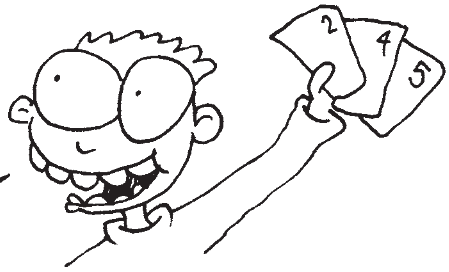
This sets up three games of **Multo** for the students and offers a further challenge. Read the question with the class: 'Play three games of **Multo** with a partner. Deal three cards each and use any maths signs you know to get as close to the following targets as you can: Game 1: 10. Game 2: 80. Game 3: zero.'

Help students to locate key words: Three games **Multo**. Use any maths signs. Targets 10, 80, 0.

Possible extensions could include allowing students to play the game whenever they have spare time. The promise of a **Multo** class challenge can also work very well.



Multo



What you need: calculator

In the game of **Multo**, the target is 27 and you have been dealt 2, 4 and 5.

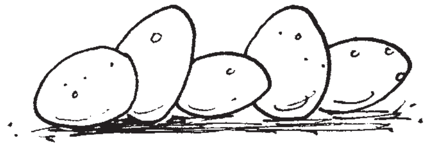
How close can you get to the target?

In another game of **Multo** the target is 73 and you are dealt 3, 8 and 9.

How close can you get to this target?

Strategies Locate key words Look for a pattern Think logically

Eggs Galore



Strategies

- Locate key words
- Assume a solution
- Work in reverse
- Find a similar but simpler problem

Background

The problem solving strategy of working in reverse tends to be more question specific than most of the other strategies outlined in this text. Nonetheless, it is useful and, when applied in the correct context, is very effective.

This strategy can often be complemented by finding a similar but simpler problem. Efficient mathematicians often assume a solution, superimposing the answer gained back into the problem to check its accuracy.

Resources:

- deck of playing cards

Orientation

Select four students at random and give the first student 48 playing cards (remove the jokers and the kings from a standard deck). Do not tell the class how many cards the student has been given. Ask the student to give half of his or her cards to the second student in the line. Ask that student to give half of their cards to the third student and then ask them to give half of those cards to the fourth student. The fourth student then tells the class that he or she has six cards. Ask the rest of the class to work out how many cards started the activity. Working in reverse is the most efficient way of solving the problem.

Guided Discovery with BLM 8

Encourage students to read the question carefully and identify key words: 20 to Joel. 60 to Jemma. Half of what was left. A quarter of what was left. 60 remaining eggs. How many start with?

This may initially appear quite difficult, which may prompt students to guess at an answer. This can often lead to a dead end.

Try a similar but simpler problem, such as: 'I had ten playing cards. I gave half to Ben. I gave two to Jessica. How many did I have left?'

Standard arithmetic will solve this. But what if the question was: 'I had some playing cards. I gave half to Ben. I gave two to Jessica. I had three left for myself. How many did I start with?'

The best approach would be to start at the end and work backwards.

3 + 2 = half of the cards, therefore, I must have started with ten.

Now return to the problem at hand: 60 eggs were left after a quarter were given away, so 60 must represent the complement of $\frac{1}{4}$, which is $\frac{3}{4}$ of an amount. Therefore, 80 must have been the number Claire received (half being given to her before Matthew's 20 and the remaining 60 were distributed). Add Jemma's 60 to Joel's 20 and the original total of 240 is revealed.

Now encourage students to check their solution:

$240 - 20 = 220$

$220 - 60 = 160$

$160 \times \frac{1}{2} = 80$

$80 - \frac{1}{4}$ of $80 = 60$

I was left with 60 eggs.

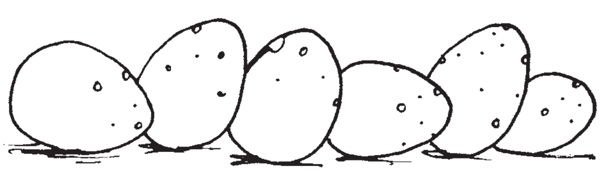
Further Exploration

Task Card 8

Read the question with the class: 'What day was three days after two days before yesterday? When you have answered this, make up your own time tongue twister of this type. Ask a friend to try to solve the riddle.'

Identify the key words: Three days after two days before yesterday. Make up your own tongue twister.

Again, encourage the students to work in reverse. Clarifying what needs to be done is half the battle with this question. The back three, forward three approach can soon be seen to cancel each other out.



Eggs Galore

Last Easter, Andrew was given so many Easter eggs that he decided to give most of them away.

He gave 20 to Joel.

He then gave 60 to Jemma.

Then he gave half of what was left to Claire.

He then gave a quarter of what was left to Matthew.

Andrew kept the 60 remaining eggs for himself.

How many Easter eggs did Andrew start with?

Working:

- Andrew kept 60 so:
- Matthew had: ____________
- Claire had: ____________
- Jemma had: ____________
- Joel had: ____________
- So Andrew must have started with: ____________

Strategies Locate key words · Assume a solution · Work in reverse · Find a similar but simpler problem

Alpha-Numerics!

Strategies
- Locate key words
- Look for a pattern
- Assume a solution
- Think logically

Background

Alpha-numeric problems can often be quite baffling to many students. Solving these types of problems requires arithmetic knowledge and previously understood number theory concepts to be applied to a specific numerical context, set in an alphabetic situation.

3 + 4 = 7, when transferred into the context of 3 + n = 7 is a simple but relevant demonstration of these types of problems.

Half the battle with alpha-numeric questions is to convince the students that they already have the necessary skills and knowledge to solve them.

Orientation

Explain that equations such as 3 + _ = 7 can be transferred into an algebraic form very easily by replacing the blank with a letter of the alphabet such as x or a. Although most of this type of work is done in the secondary school, the great majority of children in Years 6 and 7 can grasp such concepts easily. Ask the students what d might represent in the equation d + d = 12. How did they know this was correct? Watch your students' faces when you tell them they're doing high school maths!

Guided Discovery with BLM 9

First, ensure students identify the key words: AB – BA = 54. Three possible solutions. What might A and B stand for?

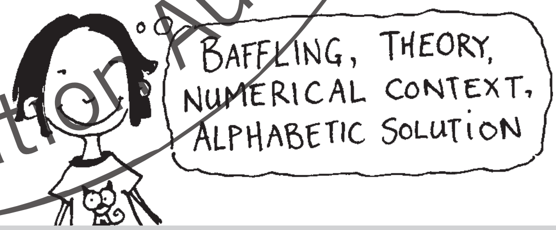
Encourage students to assume a solution. This should lead to valuable discoveries, such as:

- The further apart the values for A and B are, the larger the difference will be.
- No matter what values A and B are given, their difference will always be a multiple of 9.
- A must be larger than B, otherwise the differences between the numbers will be negative.
- These negative numbers are also in the nine times table (–1 x 9 = –9, –7 x 9 = –63, and so on).

When the three possible solutions (A = 7 and B = 1, A = 8 and B = 2 or A = 9 and B = 3) have been found, ask if these could have been achieved more logically. A must be larger than B to produce a positive difference, therefore the A in the tens place must be decomposed in the standard renaming manner. The difference between B and A in the units must be 4. B can't have the value of 0 because BA is a two-digit number, so B minus A will be 11 – 7, 12 – 8, or 13 – 9.

This should lead to the three possible solutions of 93 – 39, 82 – 28 and 71 – 17.

Therefore, the three possible products are 27, 16 or 7 and the three possible sums are 12, 10 and 8.



Further Exploration

Task Card 9

Read the question with the class: 'Make up five of your own letter/number problems like BC x D = 120 or JJ x E = 77 and ask a friend to try and solve them.'

Ensure that students recognise that a given value for a letter must remain constant for the equations created and that if a letter is used more than once in a number it is to represent a repeated digit.

Also remind students that many alpha-numeric problems can have more than one possible solution.

Be sure to praise the achievements of the students when solving these types of problems. Although not as tough as they may first appear, they are not easy and do require abstract thinking.

Alpha-Numerics!

Letters can sometimes stand for numbers in mathematics problems.
For example, 20 could become CK.

$$\begin{array}{rr} & 20 \\ + & 20 \\ + & \underline{20} \\ = & 60 \end{array} \qquad \begin{array}{rr} & CK \\ + & CK \\ + & \underline{CK} \\ = & BK \end{array}$$

In this problem C stands for 2, K stands for 0 and B stands for 6.

In the problem:

AB – BA = 54

there are three possible solutions.

What might A and B stand for?

What is the value of A + B? ______________________

What is the value of A × B? ______________________

Strategies Locate key words · Look for a pattern · Assume a solution · Think logically

Breeding Like Rabbits

Strategies
- Locate key words
- Look for a pattern
- Create a table or chart

Background

Leonardo of Pisa (1180–1250), who is better known today as Fibonacci, was the mathematical colossus of his day. He is credited with changing the numeration system in Europe from the Roman to our Hindu-Arabic system, which he encountered following his extensive travels in North Africa.

Fibonacci observed the arrangement of the petals of flowers, and on the segments of a pineapple and on a pine cone. He observed the patterns located in these natural objects and created the sequence that now bears his name to describe their structures.

The rabbit problem that follows is one that Fibonacci used to exemplify his discovered pattern.

Orientation

Introduce the issue of overcrowding and diminishing resources due to the population explosion in the world. Present the following scenario of a couple producing four children per 25 year generation and progressively dying at 75 years of age, starting at the year 2000:

2000: 2 people

2025: 6 people (2 parents, 4 children)

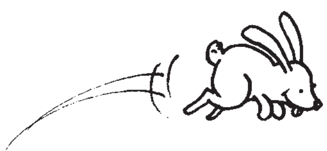
2050: 22 people (2 grandparents, 4 parents, 16 grandchildren)

2075: 84 people (2 great grandparents die, leaving 4 grandparents, 16 parents and 64 children), and so on.

Don't these numbers build up rapidly? What can be done about this population problem?

Guided Discovery with BLM 10

Locate the key words: Breed from two months old. Produce buck and doe. How many pairs after ten months?



Discuss the issue of rabbits in the Australian bush to demonstrate the relevance of mathematics to the real world.

Remind students that every number pattern has a rule that must hold for every term of the sequence. After they have created their tables, ask students to copy down the terms of the sequence and to look for a pattern (each term is the sum of the previous two terms). Have them explain the rule and make sure that it is logical. Once located, the pairs of rabbits for the months to ten can be entered.

Extensions to a year (144) and beyond can be made.

You may like to ask further questions, such as:

Is this pattern increasing steadily? If not, why not?

When will there be 1000 pairs of rabbits? (After 17 months.) Will a calculator help us work this out?

Some students may question the validity of the problem – great! What is it about the problem that is somewhat unrealistic?

- Rabbits won't always produce a male and a female.
- Sometimes rabbits die.

This is an excellent opportunity for a mathematical debate!

Further Exploration

Task Card 10

Read the question with the class: 'A number pattern or sequence must follow a rule. Create your own sequence. Make sure it has a pattern that will hold for any term in the sequence. Give the sequence a name, like the 'Stella' or the 'Angus' sequence, and list the first four terms. Now ask a friend to try and find the rule of your sequence.'

Help students identify key words: Create sequence. Pattern will hold for any term. Give it a name. Ask a friend to find the rule.

Ensure that the sequence students have created has a sufficient number of terms for the hidden rule to be discovered. This can be extended to suit the abilities and the interests of the students.

Name Date

Breeding Like Rabbits

What you need: calculator

Rabbits breed at an amazing rate. A pair of rabbits can breed from when they are two months old. Let's assume that every time these rabbits breed, they produce a buck (male) and a doe (female).

This will make the following pattern, called the Fibonacci sequence:

MONTHS	PAIRS OF RABBITS
1	1
2	1
3	2 (first pair breeds)
4	3 (first pair breeds)
5	5 (both the first pair and their babies breed)
6	8 (first pair, their babies and their 'grandchildren' breed)

Make a table to help find the maths behind this pattern and see if you can find out how many pairs of rabbits there will be after ten months. Draw your table here.

Strategies Locate key words Look for a pattern Create a table or chart

What's My Rule?

Strategies
- Locate key words
- Look for a pattern
- Assume a solution
- Think logically

Background

Used effectively, the calculator can be an excellent adjunct to learning and it lends itself very well to problem solving. The automatic constant function is standard on basic calculators. Simply press a digit button, then one of the four operations buttons (+, –, x or ÷) and then the equals button repeatedly. If, for example, we wanted to add three automatically on our calculators to show the three times table, we would simply press 3 + 3 = = = = = = = = =, to get the numbers 6, 9, 12, 15, 18, 21, 24, 27 and 30 displayed.

The automatic constant function works in a similar way for subtraction, multiplication and division.

Resources:
- calculators

Orientation

What's My Rule? is played in pairs with a calculator. Students take turns to disguise the use of the automatic constant function, such as 3 – =. The opponent then enters numbers into the calculator and presses equals to try to discover the hidden rule. For example:

Player A types 3 – =. At this point –3 is shown on the screen, giving the operation away. Player A should type in 3 = to leave zero on the screen before passing the calculator to the opponent (3 – = will automatically take three away from whatever number is entered).

Player B enters 6 = and sees 3 on the screen, so suggests that the rule is divided by two. This is logical, but incorrect. Player B then enters 10 = and sees 7 on the screen. These two attempts should lead to the answer of the rule being – 3.

This game can be more complex. The rule of 7 divided by = will give 6 digit remainders, 6 times out of 7. The rule of 6.98 x = will require considerable thought to be solved. Cater the degree of difficulty to suit your students.

Guided Discovery with BLM 11

Before introducing **BLM 11**, ensure that students are familiar with **What's My Rule?** and have already played it in pairs. It is still important to read the question carefully with the class and help students to locate the key words: Input and output. Fill in the gaps.

Ensure that the students realise that the reduction of an entry will usually be the result of subtraction or division, while the increase of an entry will usually suggest addition or multiplication. Don't be afraid to point out that this is not a strict rule. What about multiplying by a decimal or dividing by a decimal? The task card games may well produce surprising results.

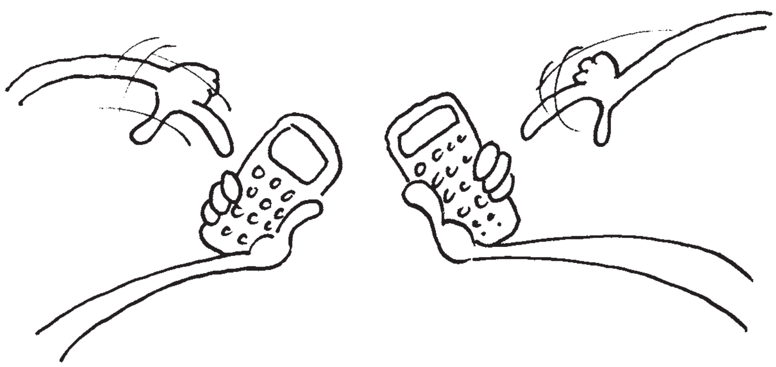
Further Exploration

Task Card 11

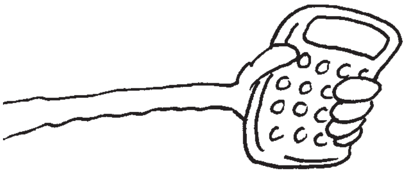
As with **BLM 11**, it is important to ensure that students are familiar with **What's My Rule?** before attempting the Task Card. Read the question with the class: 'Use the automatic constant function on your calculator to play a game of **What's My Rule?** with a partner. See who can find the hidden rule in as few moves as possible.'

Help students to locate the key words: Play **What's My Rule?** Hidden rule. As few moves as possible.

If students become quite adept at the game and need an extension, encourage them to have races to see how quickly they can identify the hidden rule.



Name Date BLM 11



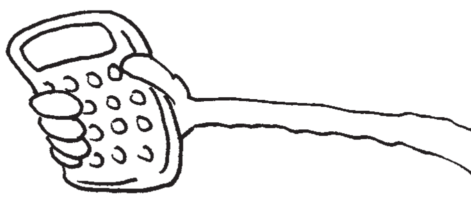
What's My Rule?

What you need: calculator

Here are the input and output numbers in two games of **What's My Rule?** Fill in the gaps.

Input	Output	Possible Rule
3	12	× 4 or + 9
7	28	____
____	44	____
9	____	____

Input	Output	Possible Rule
12	3	____ or ____
8	2	____
10	____	____
____	1.75	____



Strategies Locate key words Look for a pattern Assume a solution Think logically

Anyone for Tennis?

Strategies
- Locate key words
- Create a table or chart
- Make a drawing
- Think logically

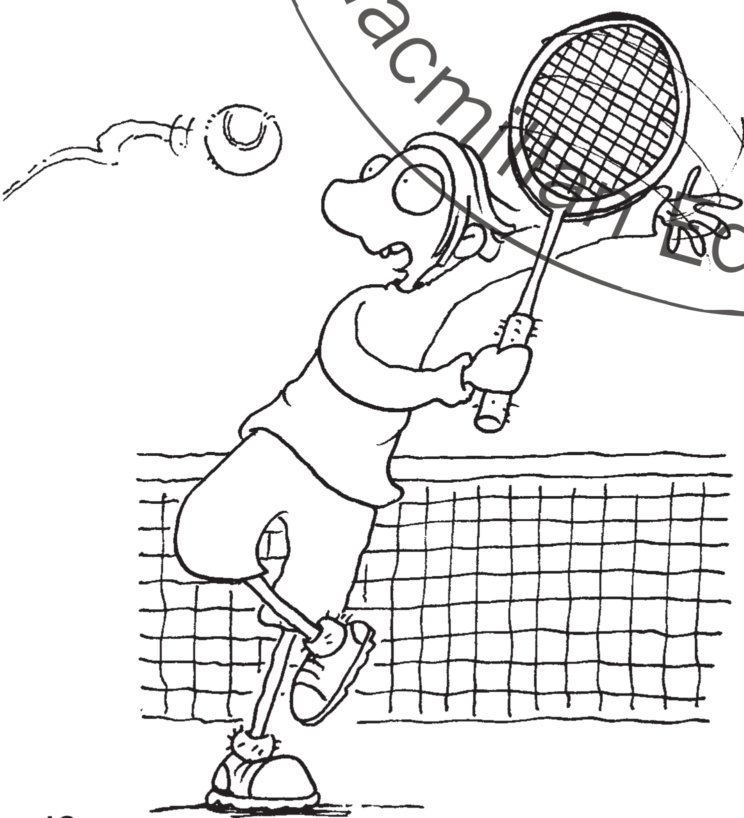
Background

Problems that ask to locate rectangles within rectangles or triangles within triangles require a very systematic approach. Without such an approach, it is highly unlikely that all possible polygons will be found and it will often be the case that the student will not remember which polygons have already been counted. The use of a table, when combined with logic and drawing or colouring, will afford the greatest chance of success.

Resources:
- tennis court, foursquare court or downball court

Orientation

Take students into the playground to see a tennis court, a foursquare court or a downball court. Observe the squares and rectangles found within the court. Discuss the way they overlap and can form part of many rectangles as well as their own individual polygon.



Guided Discovery with BLM 12

Read the problem with the class and help students locate key words: Singles tennis court. How many rectangles? Draw up a table.

Remind students that a square also fits the definition of a rectangle. Explain the structure of a singles tennis court and the meaning of the lines within it.

Call for estimates of the total number of possible rectangles contained within the court and then for a suggested strategy to solve the problem. The clue provided is the key to success. Guide students through the problem from the single rectangles to the six-rectangle polygon using a table structured as follows (see answers section, p. 63 for full table):

RECTANGLES CONTAINED WITHIN RECTANGLES	NUMBER OF POSSIBLE RECTANGLES
1	6

As an extension, introduce the doubles court. This is far more complicated, but is great fun to try and solve.

Further Exploration

Task Card 12

Read the question with the class: ‘Fold a piece of A4 paper in two and then unfold it. Can you see that you have created three rectangles, the two half-sheet rectangles and the full sheet itself? Now fold the same sheet in half and in half again. Unfold the piece of paper. How many rectangles have you created now? Try one more fold in half and try to count the number of possible rectangles that can now be seen.’

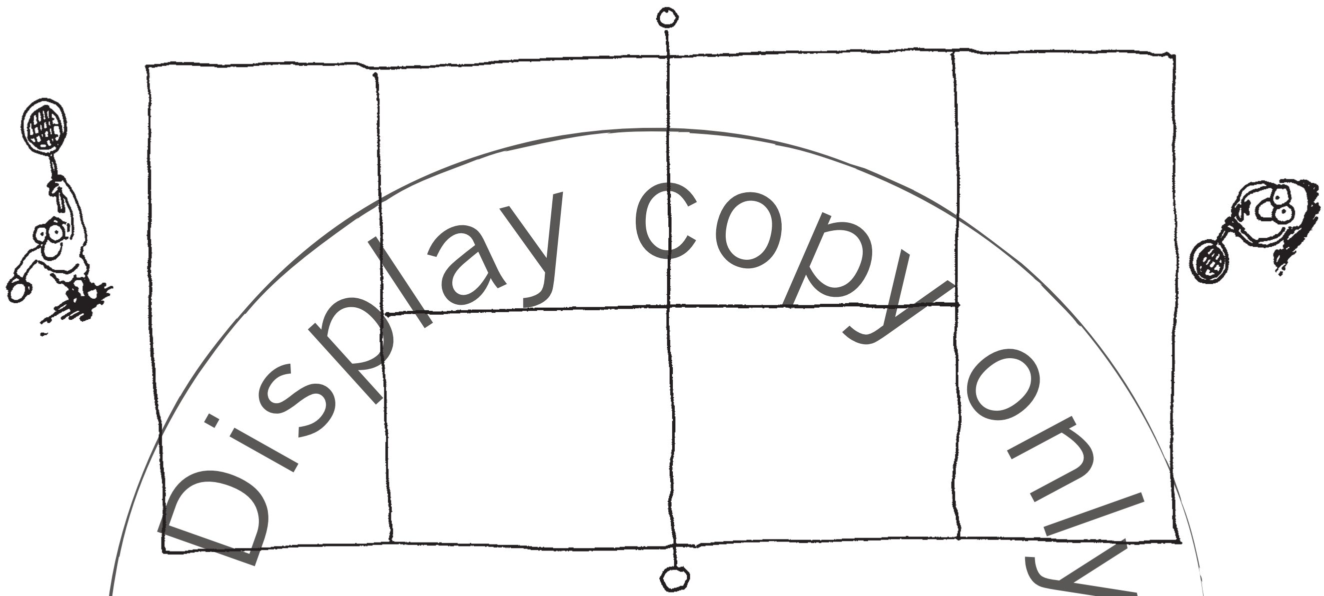
Help students identify the key words: Fold A4 in half and in half again. How many rectangles? One more fold. How many rectangles?

Suggest that students colour the overlapping rectangles to help them to keep their place and avoid counting rectangles twice. The result of nine for the first fold is relatively simple to count. As in **BLM 12**, the use of a table for the next fold is essential. Advise that overlaps are acceptable, especially for the two, three, four and six rectangles within rectangles.

Name Date BLM 12

Anyone for Tennis?

Here is a drawing of a singles tennis court:



How many rectangles can be found in this picture?

Clue: Draw up a table to record the number of single rectangles, rectangles made up of two separate rectangles, rectangles made up of three separate rectangles, and so on).

Strategies Locate key words Create a table or chart Make a drawing Think logically

The Matrix

Strategies

- Locate key words
- Create a table or chart
- Think logically

Background

A matrix is a regular array of symbols. It can be viewed as a grid from which form takes place – literally, something that gives structure. A matrix, when filled, will create enough information for conclusions to be drawn.

The obvious reference to the movie trilogy will almost certainly be made by students and will add both colour and relevance to the lesson's introduction. In the three films, the matrix gave computer generated structure and form to the human world. In the same way, a mathematical matrix can give structure to numbers.

Resources:

- noughts and crosses grid, battleships game or a street directory

Orientation

Use examples such as noughts and crosses, the game of battleships or the layout of a street directory to demonstrate the way that grids form a part of our everyday lives. Identify the use of the X and Y coordinates to locate position and, in the example of the street directory, how the detail contained within the pages forms a highly specific image of a location.

Guided Discovery with BLM 13

Help students locate the key words: Sport every Saturday. Basketball, golf, hockey, swimming, tennis. Anna can't swim. Beau no net. Chelsea no stick or racquet. Net taller than David. No ball in Emma's sport.

Key words are especially effective with these matrix type problems, as they tend to contain several pieces of information that each need to be worked through.

Encourage students to fill in the matrix by putting a cross in the boxes as they eliminate options. For example, Anna can't swim, so put a cross in the box linking Anna and swimming, and so on.

Even after four of the five clues have been revealed, we still have no definite answers. It is the clue about Emma that reveals all. Being in a sport with no ball, she must be a swimmer. We can then see that Chelsea plays basketball, David, therefore, must play hockey, Anna plays tennis and Beau plays golf.

Further Exploration

Task Card 13

Read the question with the class: 'Place counters over the numbers that will not fit the following clues. The number is bigger than three. The number is odd. The number is in the second column. The number has two digits. The units digit is greater than the tens digit. What is the magic number?

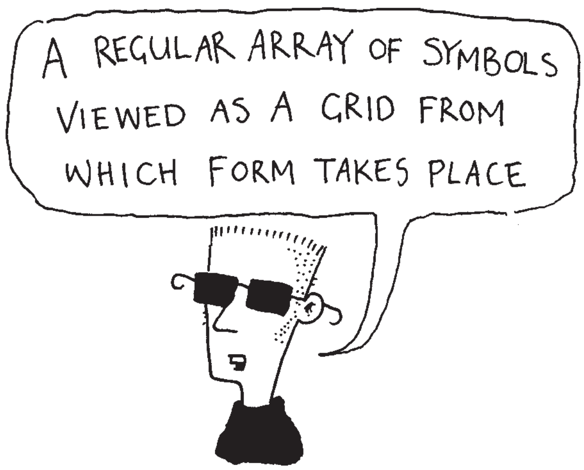
Now, using this grid of numbers, make up clues to reveal your own magic number. See if a friend can find it.'

Have students locate the key words: Magic number. Bigger than three, odd, in second column, two digits. Units greater than tens. Make up own clues.

Encourage students to eliminate numbers on the grid, one at a time, according to the clues provided.

The second part of the problem is the real challenge. The creation of clues will require imagination and clear thinking.

An extension to this task would be for students to draw up their own grid and insert their own numbers as well as their own clues.



The Matrix

Anna, Beau, Chelsea, David and Emma play sport every Saturday.

One plays basketball, one plays golf, one plays hockey, one swims and one plays tennis.

Put the following clues into the matrix provided to find out who plays which sport.

Anna can't swim.

Beau's sport doesn't use a net.

Chelsea doesn't use a stick or a racquet in her sport.

In the sport David plays, the net is taller than he is.

There is no ball involved in Emma's sport.

	Basketball	Golf	Hockey	Swimming	Tennis
Anna					
Beau					
Chelsea					
David					
Emma					

SPORTS:

Anna: ____________________

Beau: ____________________

Chelsea: ____________________

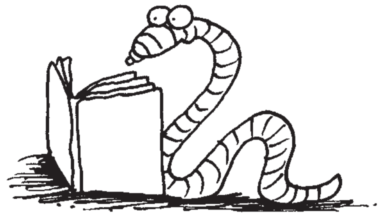
David: ____________________

Emma: ____________________

Strategies Locate key words · Create a table or chart · Think logically

Lesson Plan **14**

Bookworms



Strategies
- Locate key words
- Look for a pattern
- Assume a solution
- Think logically

Background

This lesson is an excellent example of the application of acquired skills to an unfamiliar situation, which is the essence of problem solving. Both the BLM and the Task Card presuppose a basic grasp of times tables, but the use of concrete materials can circumvent any difficulty in this area. It would be useful if the concept of the lowest common multiple, or at least of common multiples, was introduced prior to attempting the activities. In a similar way, the concept of a factor should be introduced prior to attempting the Task Card.

Resources:
- three stopwatches

Orientation

Select three students and present each with a stopwatch. Advise the class that these students represent three different lighthouses. The first student is to say 'flash' every five seconds, representing the time between flashes of his or her lighthouse. The second is to say 'flash' every ten seconds and the third every 15 seconds. Ask the class how long it will take for all three 'flashes' to occur simultaneously. Start the three students at the same time. Once the answer of 30 seconds has been found, ask the class when this will occur again and direct their attention to the times when just two of the lighthouses flashed together, i.e. at ten, 15 and 20 seconds.

Guided Discovery with BLM 14

Make sure everyone has read the question and located the key words: Xavier every three days, Yanni every four, Zoe every five. All borrowed today, how many days before all at library on same day again? If on March 1st, when?

Encourage students to assume a solution. Listing the three, four and five times tables is an excellent way to begin.

Have students circle common multiples, until the answer of 60 days is revealed. Could this have been found more quickly? What do we know after reading the question?

If it is in the five times table it must end in 5 or 0.

If it is in the three times table the digits must sum to 3, 6 or 9.

If it is in the four times table it must be even.

Therefore it must be an even multiple of 5, so it must end with a 0. If it is in both the three and four times tables, it must be in the 12 times table, and it must be the first number ending in 0, so it must be 60.

For the second part, simply add 60 to March 1. Removing the days of March from March 61 quickly reveals April 30.

Further Exploration

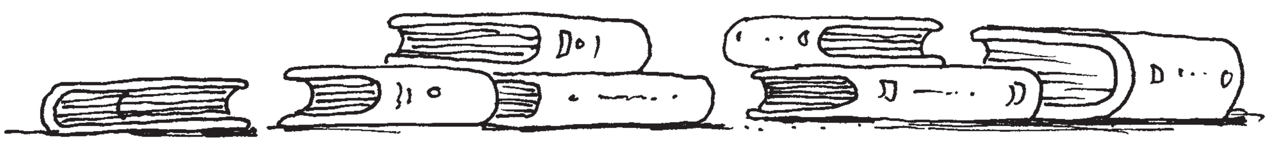
Task Card 14

Read the question with the class: 'In a game of cards, all 52 cards in the deck were dealt out and each player had the same number of cards. How many people could have been playing the game? (Clue: There are six possible answers.)

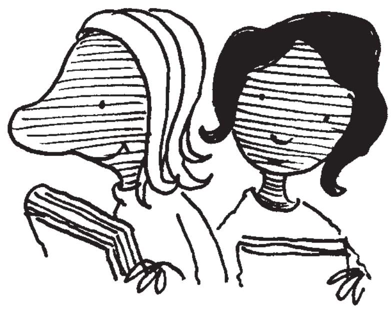
Now remove the four Kings to leave a deck of 48 cards. How many people could be playing in this game? (Clue: There are ten possible answers).'

Locate the key words: 52 cards, each player same number. How many players? 48 cards, each player same number, how many players?

What is similar to **BLM 14**? Does multiplication or counting in groups apply here? These tasks deal with division, looking at factors rather than multiples. Ask students which numbers divide evenly into 52 and 48. A deck of cards will help those less confident with tables or more concrete in their thinking.



Bookworms



Xavier, Yanni and Zoe are three friends who love reading and regularly borrow books from their local library.

Xavier borrows books every three days, Yanni borrows books every four days and Zoe borrows books every five days.

If they all borrowed books today, after how many days will they all be at the library on the same day again? Show your working here.

If they were all at the library on March 1st, on what date would they all be together at the library again?

Strategies: Locate key words · Look for a pattern · Assume a solution · Think logically

Better than Average

Strategies
- Locate key words
- Work in reverse
- Think logically

Background

Finding averages, or the statistical mean, is a standard part of the upper primary school mathematics curriculum. It is usually presented in the form of gathered data and involves the application of division in the format of:

$$\frac{\text{total of entries}}{\text{number of entries}}$$

This lesson tests students' grasp of the concept with the use of a non-standard context.

Orientation

Encourage students to discuss their understanding of the concept of an average and where it is used in the broader community. Sport is bound to come up and you may wish to mention the test batting average of Sir Donald Bradman. This average of 99.94 represents the fact that he was dismissed 70 times and scored 6996 runs in his 20-year career. By using a calculator, students can see how this average was produced. You can also bring up the concept of rounding off in the process.

Guided Discovery with BLM 15

Encourage students to read the question and locate the key words: Batted six times, average 12 runs per innings. First five scores 10, 1, 14, 25 and 3. How many runs sixth innings? How many runs to raise average to 13?

The average is given in this question, but not all the data, so it requires a more creative approach. The number of runs must be the average multiplied by the number of innings played: 6 x 12 = 72. The first five innings produced 53 runs, therefore the sixth innings must have been 72 – 53 = 19.

Part B requires the same approach. To average 13 runs in seven innings, the total number of runs must be 13 x 7 = 91. 72 runs were scored up to this point, therefore 19 are needed to reach this average.

Further Exploration

Task Card 15

Read the question with the class: 'Make up a team of 11 players. Each player's score will be worked out by rolling both dice and multiplying the two numbers together. If, say, you rolled a seven and a five, you would score 35 for your batsman. After ten rolls, add up your total and find the average for each batsman. Do you think you had good luck, average luck or bad luck in the game?

Help students locate the key words: 11 players. Score = roll dice, multiply numbers together. Ten rolls, add up total, find average. Good, average or bad luck?

The perceived probability behind the game is important. There are 100 possible outcomes, from 0 with 0 resulting in a product of 0, to nine and nine resulting in the highest possible product of 81. Ask students what an average result per innings might be. Many will suggest the midpoint of about 40. Test this by writing the 100 possible outcomes and products on the board. 42% of the possible outcomes are single digits.

Most students will feel they have had bad luck. Keep a tally of all students' performances, gather a class average and demonstrate that this game is not as easy as it may seem.

Better than Average

Part A:

So far this season Bill has batted six times for the Clearwater Creek Cricket Team and has an average of 12 runs per innings.

His first five scores were:

10, 1, 14, 25 and 3.

How many runs did Bill score in his sixth innings?

Part B:

How many runs must Bill score in his next innings to raise his average to 13?

Strategies Locate key words Work in reverse Think logically

Shirts n' Shorts

Strategies

- Locate key words
- Look for a pattern
- Assume a solution
- Create a table or chart
- Find a similar but simpler problem
- Think logically

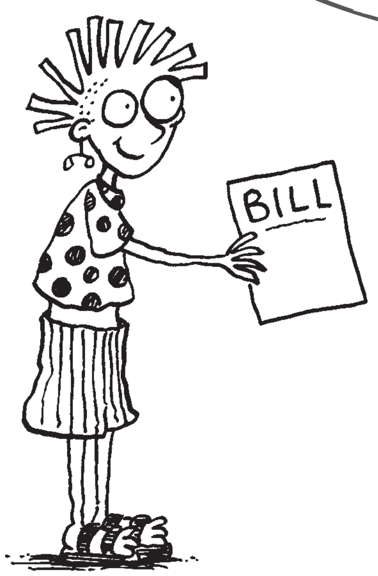
Background

Simultaneous equations are a relatively common but nonetheless challenging form of problem encountered in upper primary school. Simultaneous equations are essentially number sentences that must each have their variables satisfied by the same values of the unknowns in the given question. For example, in **BLM 16** the price paid for the shirts and the shorts in the two purchases must be the same for the two shoppers.

This question is yet another example of a problem that could be solved in a number of different ways utilising a number of different strategies or combinations of strategies.

Orientation

Advise the class that you are selling calculators and decks of playing cards. Select a student and ask them to purchase two calculators and one deck of cards. Present the student with a bill for $24 and then ask the class what the cost of each item, in whole dollars, might have been. To find the definite answer to this question, what other information would be required? How would the purchase of an extra item help?



Guided Discovery with BLM 15

Read the question and locate the key words: Three shirts, four pairs shorts, $96. Two shirts, two pairs shorts, $54. How much are shirts and shorts?

One approach is to use logical thinking as follows:

Caitlin bought two shirts and two pairs of shorts for $54, so one of each must cost a total of $27.

If Tim had bought three times as much, the cost would have been 3 x $27 = $81. One extra pair of shorts brought the total to $96, so shorts must cost $15 and shirts $12.

The best approach uses logic to cancel out one of the two unknowns, thus revealing the other. Double Caitlin's purchase and compare it to Tim's to see that:

4 shirts + 4 shorts = $108

3 shirts + 4 shorts = $96

The difference is one shirt. The difference in the prices is $12.

Further Exploration

Task Card 16

Read the question with the class: 'Shirts n' Shorts are having a "pick and add" sale. Shoppers simply choose two playing cards numbered from one to ten. These two numbers are then added together to represent the cost in dollars of a shirt or a pair of shorts. If, say, you selected a four and a five, you would pay $9 for your item of clothing.

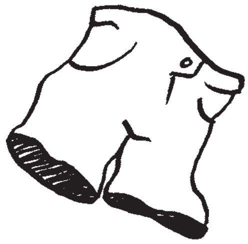
Buy five items of clothing in this manner. How could you work out if you had been lucky or unlucky with your five purchases?'

Locate the key words: Cards numbered one to ten. Pick two. Add to represent cost. Five purchases. Lucky or unlucky?

Ask students what the cheapest price possible is? What will the most expensive item cost? Will finding the average of the five purchases help?

Create a table listing all 90 possible combinations. $11 is the average of the highest and lowest possible prices. Thus, standard luck in the sale would result in five purchases equalling $55.

Shirts n' Shorts



Shirts n' Shorts are having a huge stocktake sale.

Tim bought three shirts and four pairs of shorts for $96.

Caitlin bought two shirts and two pairs of shorts for $54.

How much do shirts and shorts cost in the stocktake sale? Show how you reached your answer.

Strategies Locate key words · Look for a pattern · Assume a solution · Create a table or chart · Find a similar, but simpler problem · Think logically

All the Task Cards You Need

Problem Solving Task Card 1

Secret Identity

What you need:

- Pencil, paper and a calculator

Make up a secret code. Use your code to disguise the name of someone in your class and the name of a famous singer or movie star. Ask a friend to try and decode it.

Problem Solving Task Card 2

Pascal's Problem

What you need:

- Pencil and paper

A famous French mathematician named Blaise Pascal created the following famous triangle:

Top line:	1					
1st line:	1	1				
2nd line:	1	2	1			
3rd line:	1	3	3	1		
4th line:	1	4	6	4	1	
5th line:	1	5	10	10	5	1

Copy this down and see if you can complete the next two lines of the triangle.

Problem Solving Task Card 3

Rectangle Tangle

What you need:

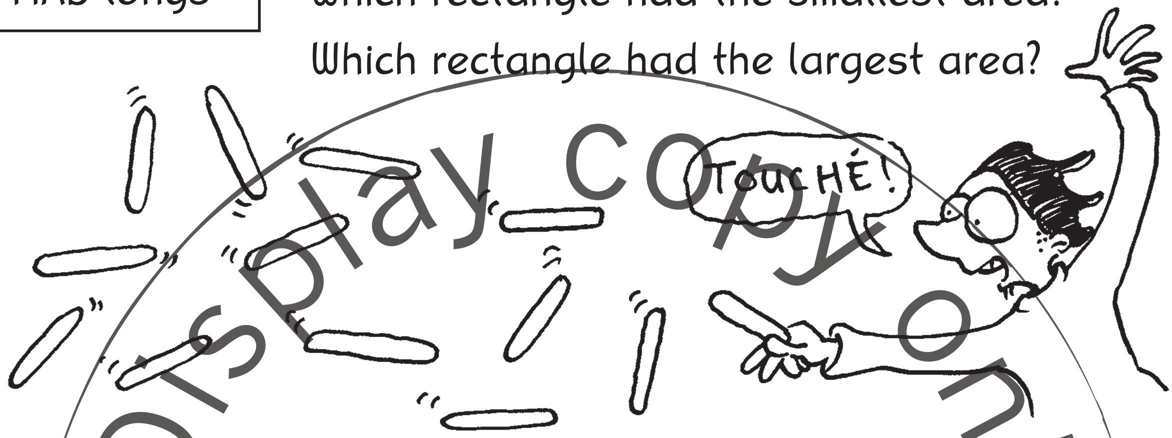
- 24 iceblock sticks or 24 MAB longs

Use all 24 iceblock sticks or MAB longs to make as many different rectangles as possible.

Each rectangle must use all 24 sticks.

Which rectangle had the smallest area?

Which rectangle had the largest area?

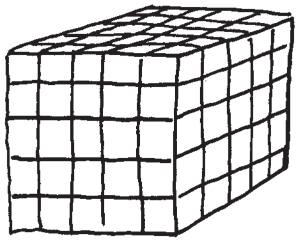


Problem Solving Task Card 4

Blocked Edges

What you need:

- MAB blocks and connecting blocks

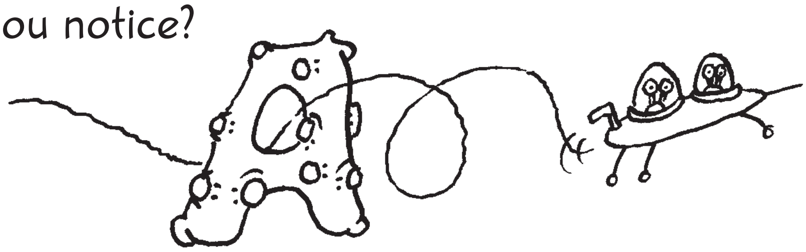


On the planet Atnep their place value system is structured around base 5 instead of base 10 like ours.

Get MAB 1, 10, 100 and 1000 blocks and look at the edges.

Now use connecting blocks to make a model of a 1, 5, 25 and 125 block.

Look at the edges of these Atnep blocks. What do you notice?



PROBLEM SOLVING TASK CARD 5

Folded Fractions

WHAT YOU NEED:

- Piece of A4 paper and a pencil to record your results

Fold a piece of A4 paper in half, then in half again, then in half again and again.

How many equal pieces do you think the sheet has been divided into?

Unfold it to see if you were right.

What fraction of the original sheet does each piece represent?

If you folded the sheet two more times than you did originally, how many equal pieces will be seen when unfolded, and what fraction will each piece represent?

PROBLEM SOLVING TASK CARD 6

Light Times

WHAT YOU NEED:

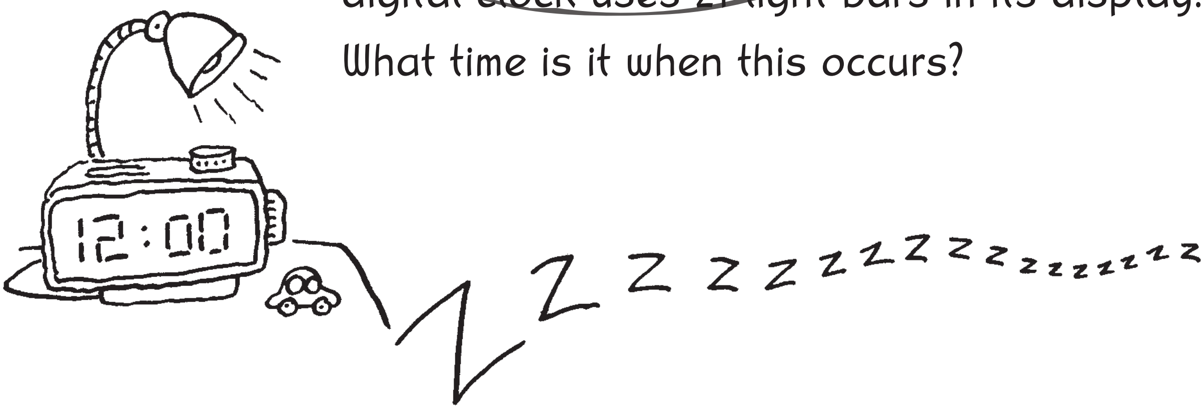
- 21 iceblock sticks or MAB longs

At 12:00, a 12 hour digital clock uses 19 light bars in its display.

Use iceblock sticks or MAB longs to show this time.

There is a time before 12:00 when a 12 hour digital clock uses 21 light bars in its display.

What time is it when this occurs?



Problem Solving Task Card 7

Right on Target

What you need:
- Deck of playing cards

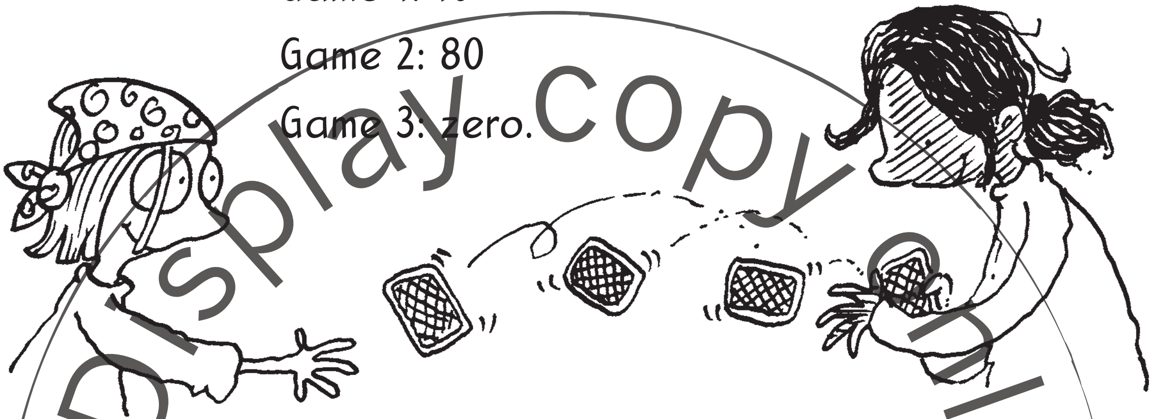
Play three games of **Multo** with a partner.

Deal three cards each and use any maths signs you know to get as close to the following targets as you can:

Game 1: 10

Game 2: 80

Game 3: zero.



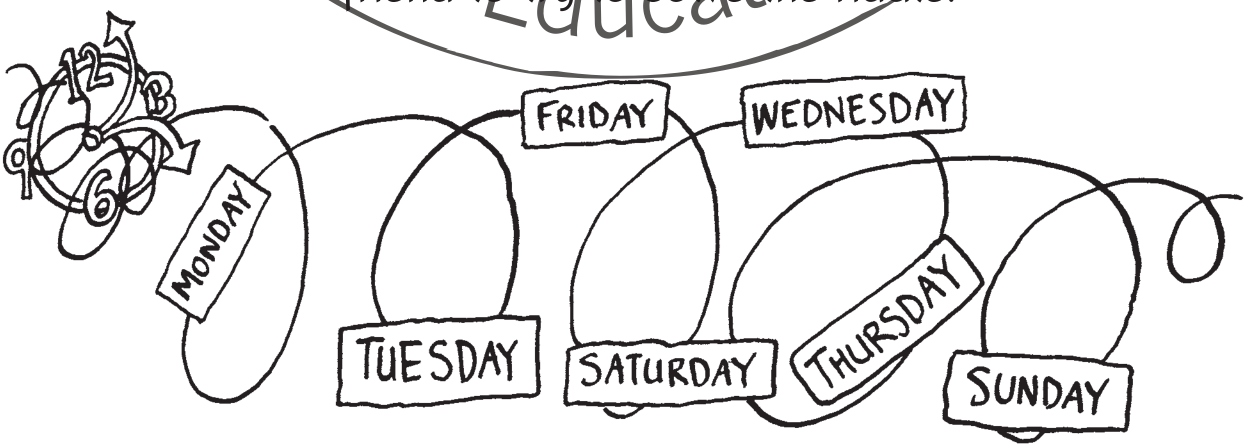
Problem Solving Task Card 8

Twisting Time

What you need:
- Pencil and paper

What day was three days after two days before yesterday?

When you have answered this, make up your own time tongue twister of this type. Ask a friend to try to solve the riddle.



Problem Solving Task Card 9

Alphabet Equations

What you need:

- Pencil and paper

In letter/number problems the letters from A to J could stand for the digits from 0 to 9, in any order. So, BA could equal 25 or 70 or any two-digit number without repeated digits. CC could stand for 11 or 99 or any two-digit number with repeated digits.

If BC × D = 120, then B could equal three and D could equal 0 and D could equal four (30 × 4 = 120).

Make up five of your own letter/number problems like BC × D = 120 or JJ × E = 77 and ask a friend to try and solve them.

Problem Solving Task Card 10

Snappy Sequences

What you need:

- Pen and paper

A number pattern or sequence must follow a rule.

e.g. 1, 3, 7, 15 has the rule of × 2 + 1, or + 2, + 4, + 8, and so on.

Create your own sequence. Make sure it has a pattern that will hold for any term in the sequence. Give the sequence a name, like the 'Stella' or the 'Angus' sequence, and list the first four terms.

Now ask a friend to try and find the rule of your sequence.

PROBLEM SOLVING TASK CARD 11

Calculator Chaos

WHAT YOU NEED:

- Calculator, a pencil and a piece of paper

Use the automatic constant function on your calculator to play a game of **What's My Rule?** with a partner.

See who can find the hidden rule in as few moves as possible.

PROBLEM SOLVING TASK CARD 12

Folding Frenzy

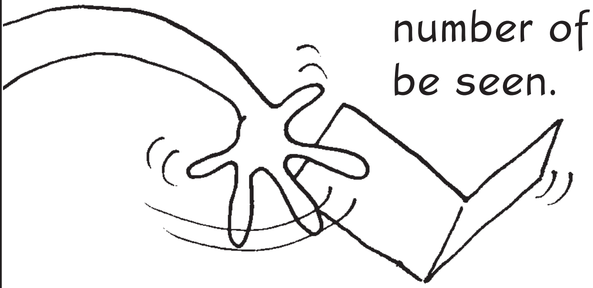
WHAT YOU NEED:

- Piece of A4 paper and coloured pencils

Fold a piece of A4 paper in two and then unfold it. Can you see that you have created three rectangles, the two half-sheet rectangles and the full sheet itself?

Now fold the same sheet in half and in half again. Unfold the piece of paper. How many rectangles have you created now?

Try one more fold in half and try to count the number of possible rectangles that can now be seen.



Problem Solving Task Card 13

Magic Number

1. Place counters over the numbers that will not fit the following clues:

The number is bigger than three.

The number is odd.

The number is in the second column.

The number has two digits.

The units digit is greater than the tens digit.

What is the magic number?

2. Now, using this grid of numbers, make up clues to reveal your own magic number. See if a friend can find it.

What you need:

- 16 counters, pencil and paper

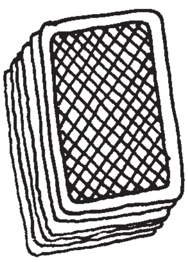
7	5	96	2
32	22	19	5
64	45	84	22
1	61	51	33

Problem Solving Task Card 14

Playing Games

What you need:

- Deck of 52 playing cards, pencil and piece of paper



1. Use the 52 card deck to answer this question:

   In a game of cards, all 52 cards in the deck were dealt out and each player had the same number of cards. How many people could have been playing the game?

   (Clue: There are six possible answers.)

2. Now remove the four Kings to leave a deck of 48 cards.

   In another game all the cards were dealt out and every player in the game had the same number of cards. How many people could be playing in this game?

   (Clue: There are ten possible answers.)

Problem Solving Task Card 15

Lucky Batsman

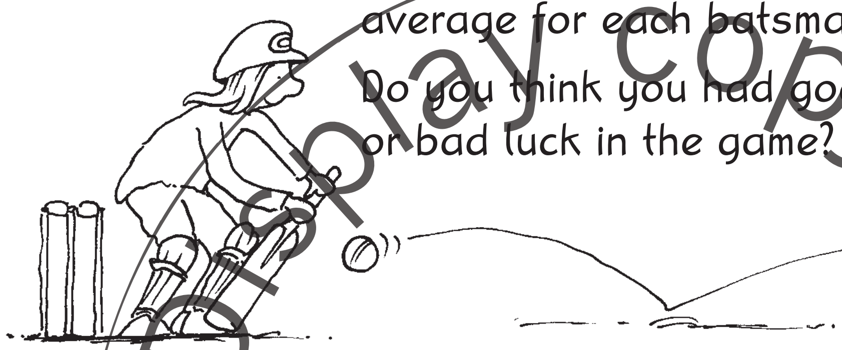
What you need:

- Two 10-sided dice, a pencil and paper

Make up a cricket team of 11 players. Each player's score will be worked out by rolling both dice and multiplying the two numbers together. If, say you rolled a seven and a five, you would score 35 for your batsman.

After ten rolls, add up your total and find the average for each batsman.

Do you think you had good luck, average luck or bad luck in the game?



Problem Solving Task Card 16

Pick and Add

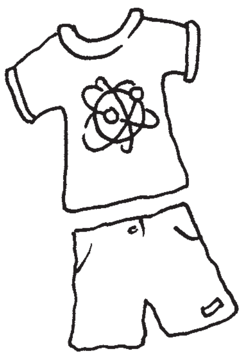
What you need:

- Playing cards from 1-10, pen and paper

Shirts n' Shorts are having a 'pick and add' sale.

Shoppers simply choose two playing cards numbered from one to ten. These two numbers are then added together to represent the cost in dollars of a shirt or a pair of shorts. If, say, you selected a four and a five, you would pay $9 for your item of clothing.

Buy five items of clothing in this manner. How could you work out if you had been lucky or unlucky with your five purchases?



All the ANSWERS You Need

Blackline Masters

BLM 1: Cracking the Code

Tibet

Roald Dahl

Broome

BLM 2: Doing your Block

North East East

East North East

East East North

North North East East

North East North East

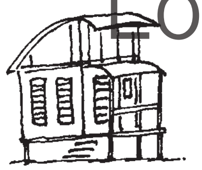
North East East North

East North North East

East North East North

East East North North

Five blocks give ten possible shortest pathways.



BLM 3: Taking a Fence

The largest area possible is 36 square metres.

BLM 4: Counting on Atnep

125 Pents equals 1 Pentago (5 x 5 x 5).

BLM 5: Fraction Attraction

The second term is $\frac{5}{12}$.

BLM 6: Time's Up

6:00, 7:05, 8:11, 9:16, 10:22, 11:27, 12:00, 12:32/33, 1:38, 2:43, 3:49, 4:54.

BLM 7: Multo

25 + √4 = 27

3 x 8 x √9 = 72

8 x 9 + 0.3 = 72.3

83 – 9 = 74

BLM 8: Eggs Galore

Matthew had 20 eggs.

Claire had 80 eggs.

Jemma had 60 eggs.

Joel had 20 eggs.

Andrew started with 240 eggs.

BLM 9: Alpha-Numerics

A = 7, B = 1 or A = 8, B = 2 or A = 9, B = 3

A + B = 8 or 10 or 12

A x B = 7 or 16 or 27

BLM 10: Breeding like Rabbits

Months	Pairs of Rabbits
1	1
2	1
3	2
4	3
5	5
6	8
7	13
8	21
9	34
10	55

The tenth term is 55 pairs of rabbits.

BLM 11: What's My Rule?

Game 1: The rule is multiplied by 4. The missing numbers are 11 and 36.

Game 2: Possible rules are minus 9 or divided by 4. The rule is divided by 4. The missing numbers are 2.5 and 7.

BLM 12: Anyone for Tennis?

Rectangles contained within rectangles	Number of possible rectangles
1	6
2	4
3	2
4	1
5	2
6	1

16 rectangles can be found on a singles tennis court.

Rectangles contained within rectangles	Number of possible rectangles
1	10
2	6
3	2
4	5
5	4
6	1
7	0
8	2
9	0
10	1

31 rectangles can be found on a doubles court.

BLM 13: The Matrix

Anna: Tennis

Beau: Golf

Chelsea: Basketball

David: Hockey

Emma: Swimming

BLM 14: Bookworms

The three friends will be at the library together every 60 days.

April 30.

BLM 15: Better than Average

A. Bill scored 19 runs in his sixth innings.

B. Bill must score 19 runs in his seventh innings.

BLM 16: Shirts n' Shorts

Shirts cost \$12. Shorts cost \$15.

Task Cards

Task Card 1: Secret Identity

Answers will vary.

Task Card 2: Pascal's Problem

1 6 15 20 15 6 1

1 7 21 35 35 21 7 1

This relates to **BLM 2**. The largest number in each row refers to the number of possible shortest pathways for houses the number of the row, in blocks, apart. Thus, for houses seven blocks apart, there are 35 possible shortest pathways, because 35 is the biggest number in row 7.

Task Card 3: Rectangle Tangle

Smallest possible area: 11 square units.

Largest possible area: 36 square units.

Task Card 4: Blocked Edges

L x W x D = 125

Task Card 5: Folded Fractions

16 equal pieces.

$\frac{1}{16}$

64 equal pieces.

$\frac{1}{64}$

Task Card 6: Light Times

At 10:08, 21 light bars are used.

Task Card 7: Right on Target

Answers will vary.

Task Card 8: Twisting Time

Today.

Answers will vary.

Task Card 9: Alphabet Equations

Answers will vary.

Task Card 10: Snappy Sequences

Answers will vary.

Task Card 11: Calculator Chaos

Answers will vary.

Task Card 12: Folding Frenzy

Two folds produce nine rectangles.

Three folds produce 30 rectangles.

Number of rectangles within rectangles	Number of possible rectangles
1	8
2	10
3	4
4	5
5	0
6	2
7	0
8	1

Task Card 13: Magic Number

The magic number is 45.

Answers will vary.

Task Card 14: Playing Games

1, 2, 4, 13, 26, 52 people could be playing.

1, 2, 3, 4, 6, 8, 12, 16, 24, 48 people could be playing.

Task Card 15: Lucky Batsman

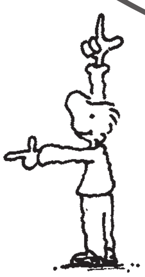
Answers will vary.

Task Card 16: Pick and Add

There are 90 combinations from 1 + 2 to 10 + 9.

The midpoint between 1 and 10 is 5.5.

5.5 + 5.5 = 11. Therefore, $11 is the average price of items in the sale. Five purchases will equal $55 with average luck.



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