# Exploring the Concept of Multiplication

## CONTENT DESCRIPTION

**NA031** Recognise and represent multiplication as repeated addition, groups and arrays

## MATHEMATICAL BACKGROUND

In this unit, students work with the set (equal groups) and area (array) models for multiplication. They use connecting cubes to model the situations. The cubes foster the opportunity to explore the linear model of multiplication that is also important at this stage. The area model is used to establish many of the key properties and thinking strategies for multiplication in later stages. The array is the basis for the turnaround concept so students understand that the factors in a multiplication number sentence can be written in any order. Students discover many multiplication facts, including turnarounds, and write many number sentences using the '×' symbol. However, there is no emphasis on memorising these facts or learning 'tables'.

## LESSON OVERVIEW

**12.1** Representing Multiplication – Equal Groups

**12.2** Representing Multiplication – Array

**12.3** Working with the '×' Symbol

**12.4** Interpreting Multiplication Sentences

**12.5** Introducing Turnaround Facts

## LANGUAGE

Students will use and develop the following language:
group, equal group, multiply, multiplication, multiplication symbol (×), array, row, turnaround fact

## MATERIALS

***Lesson 12.1***

- *GM ACE* student journal, page 52
- *GM ACE* mentals workbook, page 23
- 4 paper plates
- Connecting cubes

***Lesson 12.2***

- *GM ACE* student journal, page 53
- Sticky notes
- Flash cards showing these facts:

*9 – 3 = ___* *9 – 4 = ___*
*9 – 5 = ___* *9 – 6 = ___*
*9 – 7 = ___* *8 – 6 = ___*
*8 – 5 = ___* *8 – 4 = ___*
*8 – 3 = ___* *8 – 2 = ___*

***Lesson 12.3***

- *GM ACE* student journal, page 54 and, Tear Out 3, page 143
- 2 cubes R with these numbers on the faces:
  - Cube 1: 3, 4, 5, 6, 7, 8
  - Cube 2: 2, 3, 4, 2, 3, 4
- Flash cards showing these facts:

*11 – 6 = ___* *12 – 5 = ___*
*12 – 7 = ___* *13 – 6 = ___*
*13 – 7 = ___* *14 – 6 = ___*
*14 – 8 = ___* *15 – 7 = ___*
*15 – 8 = ___* *16 – 7 = ___*

***Lesson 12.4***

- *GM ACE* student journal, page 55
- 2 cubes R from the previous lesson
- Connecting cubes
- Flash cards from 12.2 and 12.3

***Lesson 12.5***

- *GM ACE* student journal, page 56
- *GM ACE* mentals workbook, page 24
- Several overhead transparencies
- Overhead projector
- Non-permanent marker

***Optional Digital Resources and Program Blackline Masters***
The lessons in this program are further supported by optional online resources.
Go to **www.origoeducation.com/go-maths-ace-support** for further information about the program blackline masters and these resources.

***Further Support***
Lessons in this unit are also supported by the ORIGO Big Book, *The Big Bug Band*. This full-colour storybook develops the mathematical language for multiplication (arrays).

# Assessment

## CONTENT INDICATORS

On completion of this unit, the students should be able to

**NA031**

- **A** represent and solve multiplication problems involving arrays
- **B** represent and solve multiplication problems involving equal groups
- **C** solve multiplication facts and draw array models to match
- **D** solve multiplication facts and write the related turnaround facts
- **E** use the multiplication symbol to describe and solve problems

## TECHNIQUES

The following tools can be used to assess the content indicators.

### 1. Written Test A B C D

Allow time for the students to complete the written test for Unit 12. Consider administering the test one or two weeks after completion of the unit.

### 2. Diagnostic Probe A C E

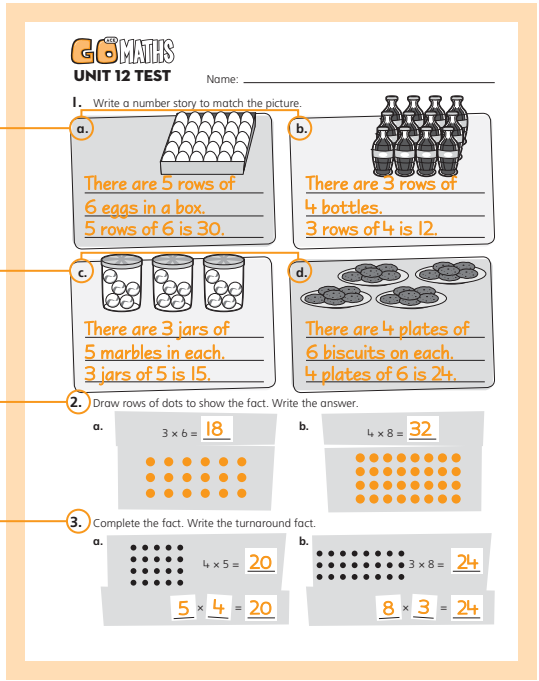
Give the student 12 counters and ask them to arrange them into equal rows.

If successful, ask the student to write a number sentence to match the array.

If successful, ask for a different arrangement and number sentence.

Write ***4 × 5 =*** ___ on a piece of paper. Ask the student to use counters (supply more than 20) to make an array to solve the problem.

***Unit 12 Written Test***




## RECORDING

### Content Strands

Record each student's achievement of the content indicators in the box(es) for this unit alongside the relevant content description(s) on a copy of the Progress Record (page xii).

| | Year 2 |
|---|---|
| NA031 | 12 |

### Proficiency Strands

Record significant observations in the Progress Record (page xiv).

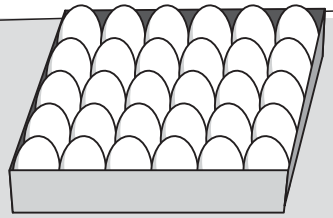
GO MATHS ACE

# UNIT 12 TEST

Name: _____

**1.** Write a number story to match the picture.

**a.**



_____

_____

_____

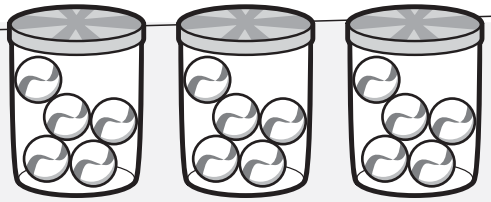
**b.**

_____

_____

_____

**c.**



_____

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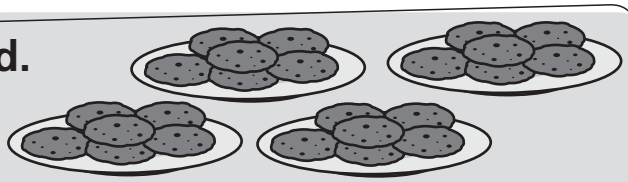
**d.**



_____

_____

_____

**2.** Draw rows of dots to show the fact. Write the answer.

**a.** $3 \times 6 =$ _____

**b.** $4 \times 8 =$ _____

**3.** Complete the fact. Write the turnaround fact.

**a.**

$4 \times 5 =$ _____

_____ × _____ = _____

**b.**

$3 \times 8 =$ _____

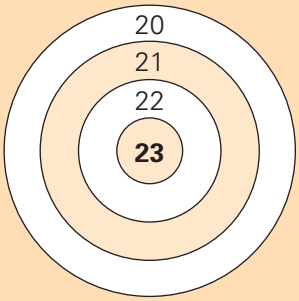
_____ × _____ = _____

# Representing Multiplication – Equal Groups

In this lesson, students use concrete materials to represent equal groups in a familiar real-world situation. Then they draw pictures of equal groups.

## DAILY NUMBER SENSE

On the board, draw the target on the right. Ask: *If two darts hit the target, what could their total be? How do you know?* Invite a volunteer to choose a pair of numbers, give the total and explain her or his thinking. Repeat the discussion with another volunteer. Make sure the students realise that both darts can hit the same number. Encourage them to describe how they could use doubles to help them work out most of the totals.




## ACTIVITY

1. Explain to the students that they need to plan a party for four people. They need to work out the number of items that must be made or bought for the four guests so that each guest receives the same number of each item.
2. Ask four volunteers to each hold a plate. Invite other students to suggest different items and how many of each item the guests might receive, such as five biscuits for each guest. Write three of the suggestions on the board in this format: '4 plates of 5 biscuits', '4 plates of 3 muffins' etc. Then ask: *What if one more person is invited to the party? How will we change what is written on the board?* Encourage the students to give expressions for five people (e.g. 5 plates of 5 biscuits) and write these on the board.
3. Now discuss the expressions for four people on the board. Ask: *How can we work out the total number that we need to buy for each type of item?* Provide the students with connecting cubes and discuss one example as a whole class. For example, for 4 plates of 5 biscuits the students can make four lots of five joined cubes and then add the number of cubes. Encourage them to explain how the numbers are combined to form the total. On the board, complete the expression and write ***4 plates of 5 is 20 biscuits.***
4. Ask the students to work with a partner to copy and complete the other expressions in the same way.
5. Have the students work independently to complete page 52 of the *GO Maths ACE* student journal.

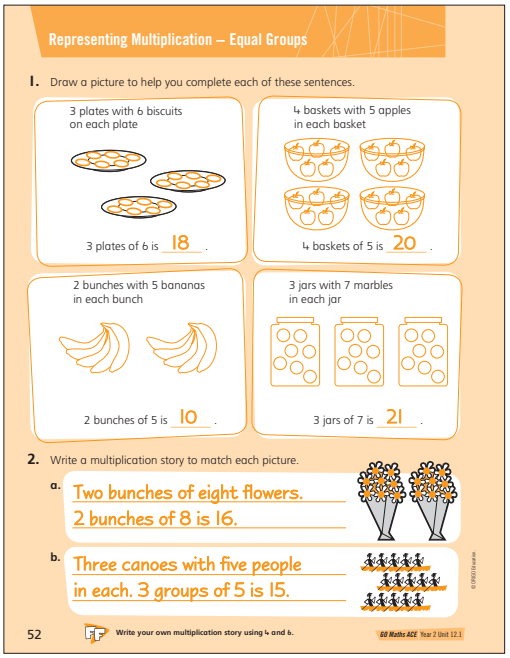
## REFLECTION

Discuss the number sentences the students completed from the board. Ask: *What does the number you worked out for each number sentence say about the items?* Bring out the fact that the answer to a multiplication number sentence comes from combining all of the items in all of the groups.

## MATERIALS

- *GM ACE* student journal, page 52
- *GM ACE* mentals workbook, page 23
- 4 paper plates
- Connecting cubes

*Student Journal*


Representing Multiplication – Equal Groups

1. Draw a picture to help you complete each of these sentences.

3 plates with 6 biscuits on each plate — 3 plates of 6 is 18.

4 baskets with 5 apples in each basket — 4 baskets of 5 is 20.

2 bunches with 5 bananas in each bunch — 2 bunches of 5 is 10.

3 jars with 7 marbles in each jar — 3 jars of 7 is 21.

2. Write a multiplication story to match each picture.

a. Two bunches of eight flowers. 2 bunches of 8 is 16.

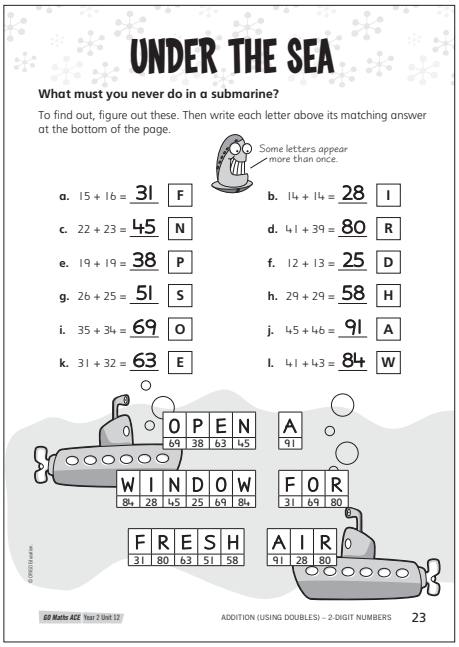
b. Three canoes with five people in each. 3 groups of 5 is 15.

52 Write your own multiplication story using 4 and 6. GO Maths ACE Year 2 Unit 12.1

## DAILY COMPUTATION PRACTICE

Use page 23 of the *GM ACE* mentals workbook.

*Mentals Workbook*


UNDER THE SEA

What must you never do in a submarine?

To find out, figure out these. Then write each letter above its matching answer at the bottom of the page.

Some letters appear more than once.

a. 15 + 16 = 31 F
b. 14 + 14 = 28 I
c. 22 + 23 = 45 N
d. 41 + 39 = 80 R
e. 19 + 19 = 38 P
f. 12 + 13 = 25 D
g. 26 + 25 = 51 S
h. 29 + 29 = 58 H
i. 35 + 34 = 69 O
j. 45 + 46 = 91 A
k. 31 + 32 = 63 E
l. 41 + 43 = 84 W

OPEN A
WINDOW FOR
FRESH AIR

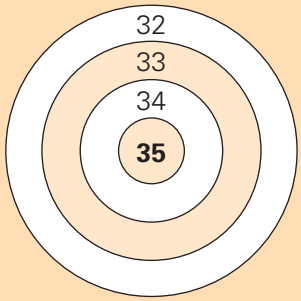
GO Maths ACE Year 2 Unit 12 ADDITION (USING DOUBLES) – 2-DIGIT NUMBERS 23

# Representing Multiplication – Array

In this lesson, students use rows of equal length to represent multiplication situations. (*Note:* There is no universal agreement about the direction of rows in an array. While some individuals and cultures describe rows vertically, the convention in Australia is to indicate rows horizontally. In the last lesson of this unit, students will use the 'rows of' language to describe an array in each direction.)

## DAILY NUMBER SENSE

On the board, draw the target on the right. Ask: *If two darts hit the target, what could their total be? How do you know?* Invite a volunteer to choose a pair of numbers, give the total and explain their thinking. Repeat the discussion with another volunteer. Make sure the students realise that both darts can hit the same number. Encourage the students to describe how they could use doubles to help them work out most of the totals.




## ACTIVITY

1. On the board, draw a picture of a bookcase with four shelves. Say: *Imagine that each sticky note is a book. How many books will we put on the first shelf?* Invite a volunteer to stick any number of notes on the first shelf. Then say: *Whatever number of books we put on the first shelf, we must put that number on every shelf. How many books do you think there will be altogether? What number sentence will we write about the picture?* Have other volunteers stick the 'books' on the other shelves. Then encourage the students to suggest a number sentence such as '4 rows of 6 books is 24 books'.
2. Repeat the discussion as time allows for three, four or five shelves.
3. Ask: *What are some other things that you see in equal rows? Where would you see these rows?* Invite the students to suggest objects they see at stores or at home that are arranged in equal rows. Write some of the examples on the board, such as '2 rows of 6 eggs is 12 eggs' or '3 rows of 4 drinks is 12 drinks'.
4. Have the students work independently to complete page 53 of the *GO Maths ACE* student journal.

## REFLECTION

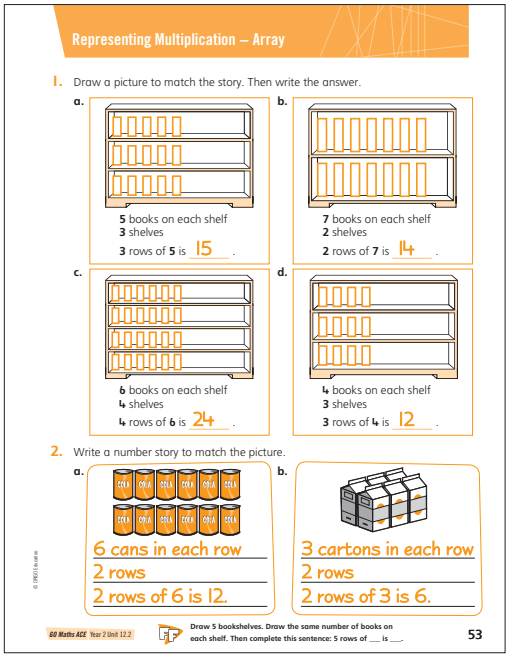
Discuss the students' answers to page 53 of the *GO Maths ACE* student journal. Ask the students to describe the strategies they used to work out the total number.

## MATERIALS

- *GM ACE* student journal, page 53
- Sticky notes
- Flash cards showing these facts:

| | |
|---|---|
| 9 – 3 = ___ | 9 – 4 = ___ |
| 9 – 5 = ___ | 9 – 6 = ___ |
| 9 – 7 = ___ | 8 – 6 = ___ |
| 8 – 5 = ___ | 8 – 4 = ___ |
| 8 – 3 = ___ | 8 – 2 = ___ |

*Student Journal*


Representing Multiplication – Array

1. Draw a picture to match the story. Then write the answer.

a. 5 books on each shelf, 3 shelves. 3 rows of 5 is 15.

b. 7 books on each shelf, 2 shelves. 2 rows of 7 is 14.

c. 6 books on each shelf, 4 shelves. 4 rows of 6 is 24.

d. 4 books on each shelf, 3 shelves. 3 rows of 4 is 12.

2. Write a number story to match the picture.

a. 6 cans in each row. 2 rows. 2 rows of 6 is 12.

b. 3 cartons in each row. 2 rows. 2 rows of 3 is 6.

GO Maths ACE Year 2 Unit 12.2 — Draw 5 bookshelves. Draw the same number of books on each shelf. Then complete this sentence: 5 rows of ___ is ___. 53

## DAILY COMPUTATION PRACTICE

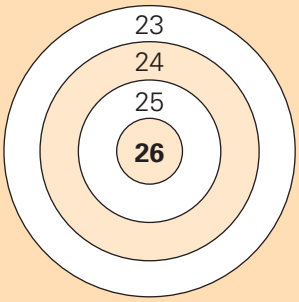
Have the students write the addition fact that helps to work out each answer and circle the number in the addition fact that is the answer. Retain the cards for use throughout this unit.

# Working with the '×' Symbol

In this lesson, the 'rows of' language is extended to the formal symbolic language for multiplication.

## DAILY NUMBER SENSE

On the board, draw the target on the right. Ask: *If two darts hit the target, what could their total be? How do you know?* Invite a volunteer to choose a pair of numbers, give the total and explain their thinking. Repeat the discussion with another volunteer. Make sure the students realise that both darts can hit the same number. Encourage them to describe how they could use doubles to help work out most of the totals.




## ACTIVITY

1. Invite a volunteer to roll the two cubes and use the numbers to draw a picture on the board of equal rows. For example, if the student rolled a '3' and a '4', he or she could draw a picture of three rows of four books. Encourage the students to use real objects from situations they might recall from the previous lesson. Work as a class to write 'rows of' word sentences, such as '3 rows of 4 books is 12 books', to describe each of the pictures.
2. After writing the sentence for each picture, ask: *What is a shorter way to write each of the sentences? What symbol do you already know that you can use for 'is' or 'equals'?* (The '=' symbol.) *What is a symbol we can use to describe equal groups or equal rows?* Invite confident individuals to draw the '×' symbol or, if necessary, draw it yourself on the board.
3. Discuss each of the pictures and write a number sentence using the '×' symbol just under the word sentence.
4. Organise the students into groups of four. Give each group of students a pair of cubes and ask them to number the faces as shown in the materials list. Have them take turns to roll the cubes and use the grid paper to shade pictures of equal rows to match the numbers rolled. Encourage them to write matching number sentences inside the rectangles they shade.
5. Have the students work independently to complete page 54 of the *GO Maths ACE* student journal.

## REFLECTION

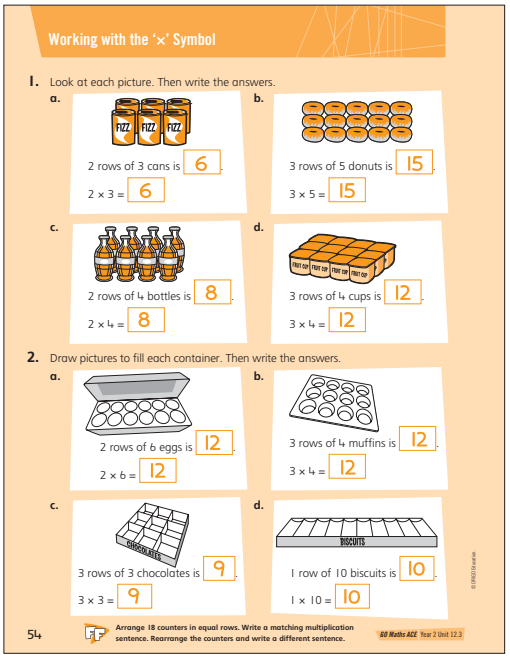
Discuss the second picture on page 54 of the *GO Maths ACE* student journal. Say: *Imagine each row of donuts was placed in a bag. How many bags of donuts would there be?* (Three.) *How many donuts would be in each bag?* (Five.) *What word sentence will we write?* (3 bags of 5 donuts is 15 donuts.) *What number sentence will we write?* (3 × 5 = 15.) Make up similar stories and write sentences for other pictures on the page so the students begin to understand that the '×' symbol can be used with a variety of language.

## MATERIALS

- *GM ACE* student journal, page 54 and Tear Out 3, page 143
- 2 cubes R with these numbers on the faces:
  - Cube 1: 3, 4, 5, 6, 7, 8
  - Cube 2: 2, 3, 4, 2, 3, 4
- Flash cards showing these facts:

| | |
|---|---|
| 11 – 6 = ___ | 12 – 5 = ___ |
| 12 – 7 = ___ | 13 – 6 = ___ |
| 13 – 7 = ___ | 14 – 6 = ___ |
| 14 – 8 = ___ | 15 – 7 = ___ |
| 15 – 8 = ___ | 16 – 7 = ___ |

*Student Journal*


Working with the '×' Symbol

1. Look at each picture. Then write the answers.

a. 2 rows of 3 cans is 6. 2 × 3 = 6

b. 3 rows of 5 donuts is 15. 3 × 5 = 15

c. 2 rows of 4 bottles is 8. 2 × 4 = 8

d. 3 rows of 4 cups is 12. 3 × 4 = 12

2. Draw pictures to fill each container. Then write the answers.

a. 2 rows of 6 eggs is 12. 2 × 6 = 12

b. 3 rows of 4 muffins is 12. 3 × 4 = 12

c. 3 rows of 3 chocolates is 9. 3 × 3 = 9

d. 1 row of 10 biscuits is 10. 1 × 10 = 10

54 Arrange 18 counters in equal rows. Write a matching multiplication sentence. Rearrange the counters and write a different sentence. GO Maths ACE Year 2 Unit 12.3

## DAILY COMPUTATION PRACTICE

Repeat the Daily Computation Practice activity from the previous lesson with the new flash cards. Retain the cards for use in the next lesson.





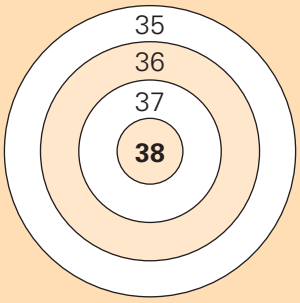
UNDERSTANDING | FLUENCY | PROBLEM SOLVING | REASONING

# Interpreting Multiplication Sentences

In this lesson, students draw array pictures to match a multiplication number sentence and then complete the sentence. They also investigate working backwards to find an array that matches a given total.

## DAILY NUMBER SENSE

On the board, draw the target on the right. Say: *If two darts hit the target, what could their total be? How do you know?* Invite a volunteer to choose a pair of numbers, give the total and explain their thinking. Repeat the discussion with another volunteer. Make sure the students realise that both darts can hit the same number. Encourage them to describe how they could use doubles to help work out most of the totals.




## ACTIVITY

1. Invite a volunteer to roll the cubes and use both to write one multiplication number sentence. For example, if a '6' and a '3' are rolled, the sentence will be 3 × 6 = ___ or 6 × 3 = ___. Ask: *What is a multiplication problem we can make up that matches this sentence? What picture can you draw to help solve the problem?* Have the students work independently to make notes and draw a picture. Encourage them to use equal groups or examples of arrays.
2. Invite individuals to describe the problems they have created. Select examples so that a range of situations and models are discussed. For each example, ask volunteers to model the situation using connecting cubes and work out the answer. Then ask them to describe the steps they used.
3. Say: *I put 18 stars in equal rows. What does my picture look like?* Encourage the students to work with a partner to find different ways to arrange the stars. While they are working, write this blank number sentence, ___ × ___ = **18**, on the board four times. Discuss the different arrangements for the stars (2 × 9, 9 × 2, 6 × 3 and 3 × 6) and complete the blank sentences.
4. Have the students work independently to complete page 55 of the *GO Maths ACE* student journal.

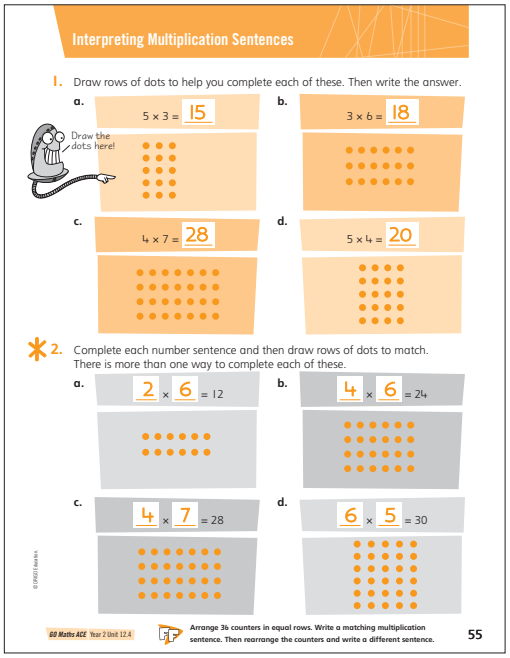
## REFLECTION

Discuss the students' answers to page 55 of the *GO Maths ACE* student journal. Ask the students to write a different number sentence and draw a matching picture for Questions 2b and 2d. Reinforce that the answer (24 or 30) is the total of all of the objects in each array that is drawn.

## MATERIALS

- *GM ACE* student journal, page 55
- 2 cubes R from the previous lesson
- Connecting cubes
- Flash cards from 12.2 and 12.3

*Student Journal*


Interpreting Multiplication Sentences

1. Draw rows of dots to help you complete each of these. Then write the answer.

a. 5 × 3 = 15 b. 3 × 6 = 18

Draw the dots here!

c. 4 × 7 = 28 d. 5 × 4 = 20

2. Complete each number sentence and then draw rows of dots to match. There is more than one way to complete each of these.

a. 2 × 6 = 12 b. 4 × 6 = 24

c. 4 × 7 = 28 d. 6 × 5 = 30

GO Maths ACE Year 2 Unit 12.4

Arrange 36 counters in equal rows. Write a matching multiplication sentence. Then rearrange the counters and write a different sentence.

55

## DAILY COMPUTATION PRACTICE

Repeat the Daily Computation Practice activity from the previous lesson using both sets of flash cards.

## CONSOLIDATION

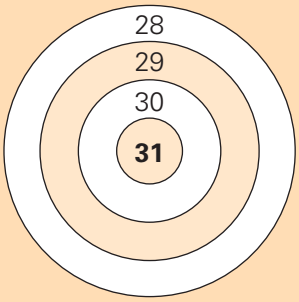
Ask the students to write 7 × 5 = ___ at the top of a sheet of paper and then draw a picture of a real situation to match. Work with confident individuals to draw pictures of 'equal jumps' along a number track or number line. Invite students to write a story about their pictures. Arrange all of the pictures in a display about multiplication.

# Introducing Turnaround Facts

In this lesson, the array is used to show that two multiplication number sentences can be written for most arrays.

## DAILY NUMBER SENSE

On the board, draw the target on the right. Ask: *If two darts hit the target, what could their total be? How do you know?* Invite a volunteer to choose a pair of numbers, give the total and explain her or his thinking. Repeat the discussion with another volunteer. Make sure the students realise that both darts can hit the same number. Encourage them to describe how they could use doubles or double add/subtract 1 or 2 to help work out most of the totals.




## ACTIVITY

1. Write ***5 × 7 =*** ___ on the board and then work with the class to draw and display the matching array on the overhead projector. Ask: *If I turn this picture, what will it look like? Is there another number sentence we can write? What is it?* (7 × 5 = ___.) Invite the students to describe their thinking before the array is turned.
2. Turn the array to check the students' predictions and write ***7 × 5 =*** ___ on the board. Then ask: *Has the total number of dots changed? What is the same about the two sentences? What is different?* Bring out the fact that no dots have been removed or added so the answer is the same but the array does look different when it is turned.
3. Repeat Steps 1 and 2 using ***9 × 2 =*** ___ and ***5 × 8 =*** ___.
4. Have the students work independently to complete page 56 of the *GO Maths ACE* student journal.

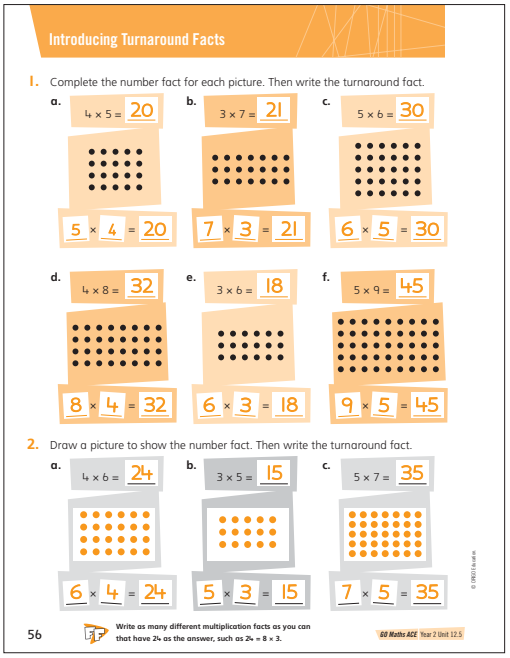
## REFLECTION

Discuss the students' answers to page 56 of the *GO Maths ACE* student journal. Discuss each example on the page using questions such as: *Read both number sentences. What is the same about the sentences? What is different? Did you need to count the dots to work out the total? Was one number sentence easier to work out than the other? Why?* Confident students might explain that a number sentence such as '4 × 5 = ___' is easier to work out because they can quickly count by fives. Reinforce that the answer for one number sentence is the same as the other so once they know one answer, they know the other.

## MATERIALS

- *GM ACE* student journal, page 56
- *GM ACE* mentals workbook, page 24
- Several overhead transparencies
- Overhead projector
- Non-permanent marker

*Student Journal*


Introducing Turnaround Facts

1. Complete the number fact for each picture. Then write the turnaround fact.

a. 4 × 5 = 20 — 5 × 4 = 20
b. 3 × 7 = 21 — 7 × 3 = 21
c. 5 × 6 = 30 — 6 × 5 = 30
d. 4 × 8 = 32 — 8 × 4 = 32
e. 3 × 6 = 18 — 6 × 3 = 18
f. 5 × 9 = 45 — 9 × 5 = 45

2. Draw a picture to show the number fact. Then write the turnaround fact.

a. 4 × 6 = 24 — 6 × 4 = 24
b. 3 × 5 = 15 — 5 × 3 = 15
c. 5 × 7 = 35 — 7 × 5 = 35

56 Write as many different multiplication facts as you can that have 24 as the answer, such as 24 = 8 × 3.

GO Maths ACE Year 2 Unit 12.5

## DAILY COMPUTATION PRACTICE

Use page 24 of the *GM ACE* mentals workbook.

*Mentals Workbook*

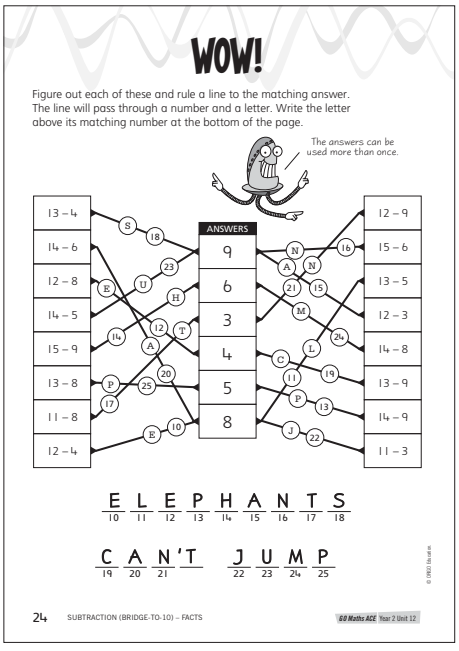

WOW!

Figure out each of these and rule a line to the matching answer. The line will pass through a number and a letter. Write the letter above its matching number at the bottom of the page.

The answers can be used more than once.

ANSWERS: 9, 6, 3, 4, 5, 8

13 – 4, 14 – 6, 12 – 8, 14 – 5, 15 – 9, 13 – 8, 11 – 8, 12 – 4

12 – 9, 15 – 6, 13 – 5, 12 – 3, 14 – 8, 13 – 9, 14 – 9, 11 – 3

E L E P H A N T S
10 11 12 13 14 15 16 17 18

C A N'T J U M P
19 20 21 22 23 24 25

24 SUBTRACTION (BRIDGE-TO-10) – FACTS

GO Maths ACE Year 2 Unit 12