Contents

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enVisionMATHS has an extensive expert author and consultancy team, including nationally and internationally recognised researchers and leading classroom teachers from across Australia.

enVisionMATHS Author Team Year 4

Jane Beams

Jane Beams taught for 20 years in Victorian Catholic, Government and Independent schools across all year levels. Following a further three years as publisher of Australian Primary Maths books and digital components for Pearson Australia, she is currently the Program Manager of Pearson Professional Learning. Jane has presented at national Mathematics conferences and she spends time visiting Maths teachers across Australia, listening to their needs and informing them of the latest developments in the Australian Curriculum. Jane has a Masters in Education with an interest in differentiation, visual learning strategies and inclusion.

Rochelle Manners

Rochelle Manners has worked as a secondary mathematics teacher and trained as a guidance counsellor. She is a mum with three pre-school children who is currently studying editing and publishing while also working in the publishing industry and writing mathematics and science texts for Pearson. She has worked on a number of secondary texts for science, and has been involved with the **enVisionMATHS** project for years 4 to 6.

Carmen Morgan

Carmen Morgan has taught in various education systems for the last 26 years. Her work involves teaching the whole spectrum of students from those who are having learning difficulties to those who need extension and enrichment. She has presented at Maths and IWB conferences for primary publishers, and conducts ongoing professional development and mentoring for teachers.

Carmen has a passion for teaching mathematics and believes the combined elements of the **enVisionMATHS** program, with its provision for differentiation and mathematical skill and thinking development, gives teachers the necessary tools to engage all students in learning mathematics.

Nicole Morrison

Nicole Morrison is a practising classroom teacher. She completed her undergraduate degree and Diploma of Education at Macquarie University and her Graduate Certificate in Training and Development at Charles Sturt University. Nicole has taught at the primary school level in both the public and private systems, and has supervised undergraduate students on professional experience programs. She also spent seven years working in educational publishing as an educational consultant and content developer. Nicole is currently a year 4 classroom teacher and team leader at Knox Grammar Preparatory School in NSW.

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Matt Skoss

Matt Skoss is an experienced classroom teacher based in Alice Springs. He has taught Years 7 to 11 since 1987, as well as enjoying several curriculum roles with a mathematics and ICT focus for NT Department of Education and Training. He also offers professional learning services on a consultancy basis, with a particular interest in supporting remote and country schools. He has a strong belief in making mathematics visual and accessible to all students, using rich mathematical problems to extend students. He likes to make use of learning technologies and Web 2.0 tools to amplify student learning, leading to the development of a community of learners.

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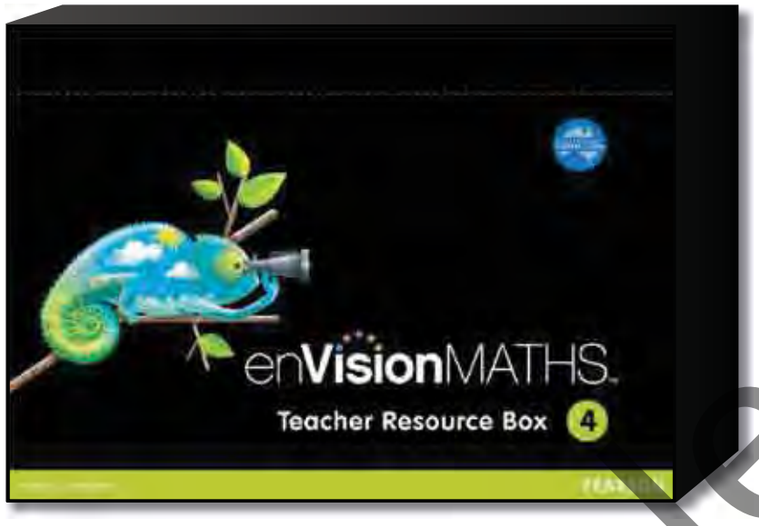
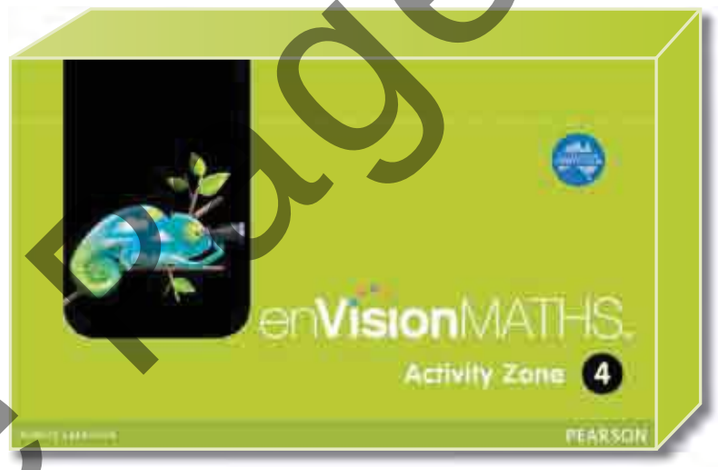
What is enVisionMATHS?

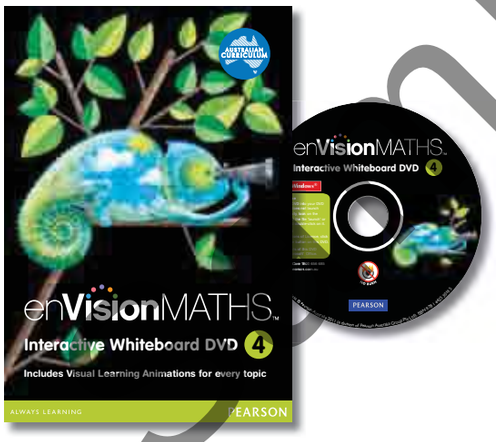
enVisionMATHS is a multi-component, topic-driven print and online teaching and learning program aligned to the Australian Curriculum that gives primary students a deeper conceptual understanding of and higher proficiency in maths. The program incorporates powerful visual learning strategies that make meaningful connections between known and new maths ideas. Differentiated teaching and learning strategies allow teachers to tailor their teaching methods to improve students' learning. The program features extensive teacher support materials, including maths background knowledge, differentiation strategies, instructional questions, suggested lesson sequences and a range of assessment tasks.

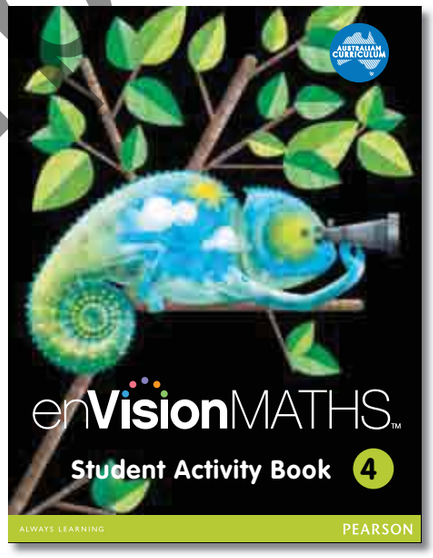
The **enVisionMATHS** program at year 4 includes these elements:

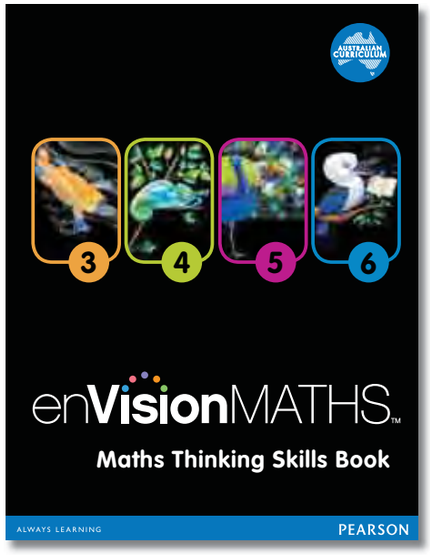
- Teacher Resource Box
- Interactive Whiteboard DVD
- Activity Zone
- Student Activity Book
- Maths Thinking Skills Book.











Topics

enVisionMATHS is designed to be modular with no prescribed sequence, and this is largely achieved through its topic-driven structure. It can be used for whole-school curricular purposes and NAPLAN assessment preparation as required, and is aligned directly to the new Australian Curriculum. The topics, however, are presented in numerical order to provide a suggested appropriate sequence if this is sought.

This topic-based program also allows for easy links to all the **enVisionMATHS** resources provided in the program, as all these resources are connected by topic.

The topics not only provide a range of lessons that suit the Australian Curriculum but include lessons beyond the Curriculum for further exploration and enrichment.

The number of lessons varies in each topic, depending on the year level and the topic's complexity.

Topics

enVisionMATHS

Content Strand Colours

Number and Algebra

Measurement and Geometry

Statistics and Probability

Proficiency Strands

The Proficiency Strands of Fluency, Understanding, Problem Solving and Reasoning are embedded within the lessons throughout this Student Activity Book.

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Provided for each topic:

- Teacher Resource Booklet
- Student Lessons
- Games, Investigations, Mental Computation and Digital Activity cards
- Assessment

Contents page, Student Activity Book 4

Digital Environment

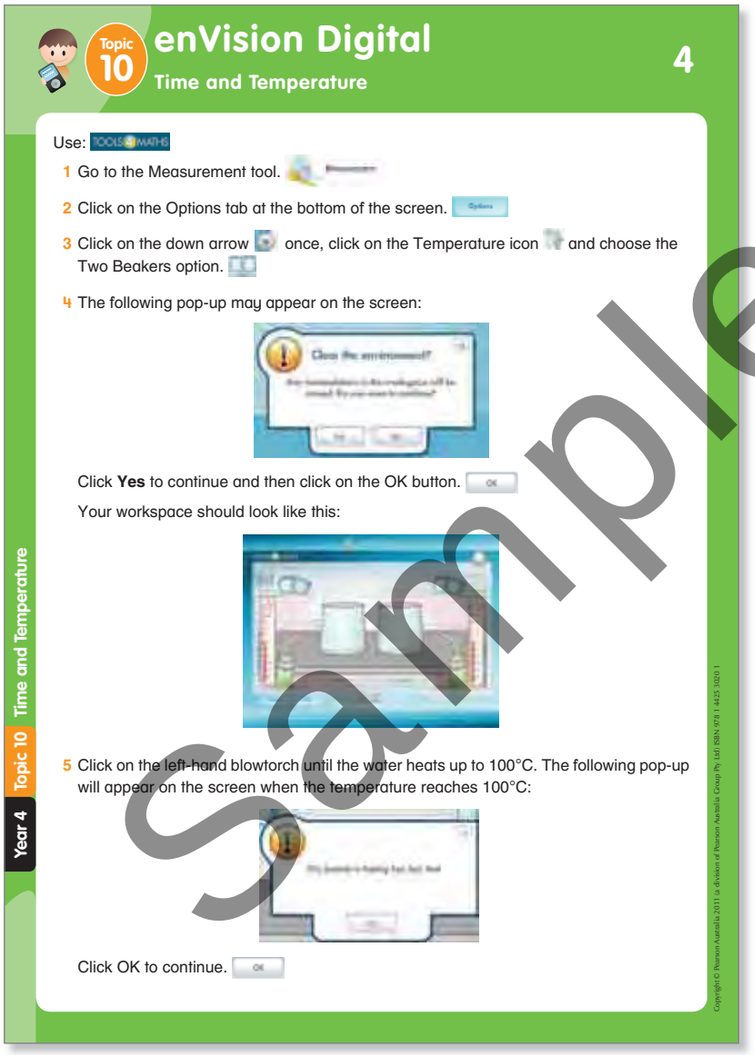
enVisionMATHS includes rich digital elements for both teachers and students:

- Digital Activity Cards (in the Activity Zone)
- Teacher Resource DVD: editable teacher planning, Assessment PDFs, Differentiated Worksheets
- Interactive Whiteboard DVD: Visual Learning Animations, Visual Learning Bridges, Tools4Maths

enVision Digital Cards

enVision Digital Activity Cards are included in the Activity Zone and provide topic-based digital tasks and activities for students to undertake on their own.

These are designed to be used with Tools4Maths.



Topic 10 enVision Digital 4

Time and Temperature

Use: TOOLS4MATHS

1 Go to the Measurement tool.

2 Click on the Options tab at the bottom of the screen.

3 Click on the down arrow once, click on the Temperature icon and choose the Two Beakers option.

4 The following pop-up may appear on the screen:

Click **Yes** to continue and then click on the OK button.

Your workspace should look like this:

5 Click on the left-hand blowtorch until the water heats up to 100°C. The following pop-up will appear on the screen when the temperature reaches 100°C:

Click OK to continue.

Year 4 Topic 10 Time and Temperature

Teacher Resource DVD

The year 4 Teacher Resource DVD (included with this Overview and Implementation Guide) contains editable teacher planning material, Pre- and Post-assessment questions and Differentiated Worksheets for all year 3, 4 and 5 lessons within the **enVisionMATHS** program.

Interactive Whiteboard DVD

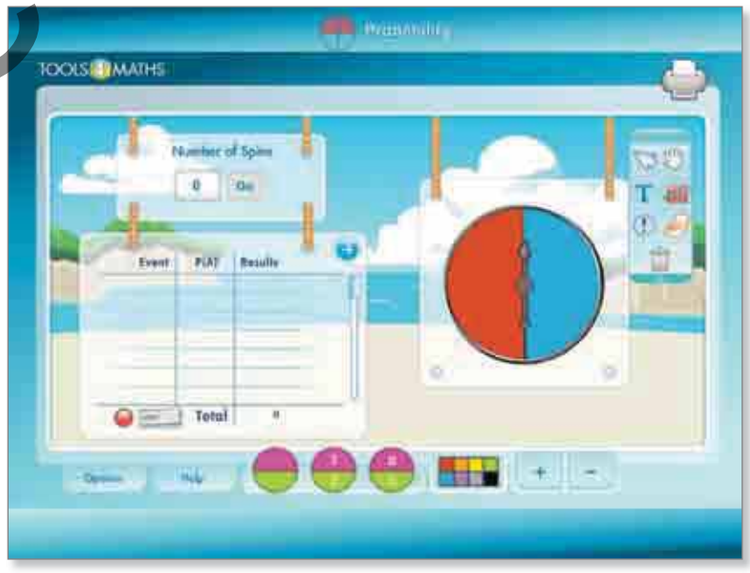
The year 4 IWB DVD contains all Visual Learning Bridges (VLBs) and Animations (VLAs) for year 4, together with Tools4Maths.

Tools4Maths

Tools4Maths is a suite of digital, lively, animated maths learning tools covering place value, counters, money, fractions, shapes and 3D objects, measurement, time, geometry, graphs and probability.

These tools are designed for use either on the class whiteboard or by students (individually or in groups) on their own computers.

Tools4Maths can be used for extension activities to provide further enrichment, or to provide further explanation and practice to aid conceptual understanding.



Conceptual Understanding

enVisionMATHS is designed to foster in students a deep understanding of the mathematical concepts being introduced. This also aligns with the Australian Curriculum's Understanding proficiency strand:

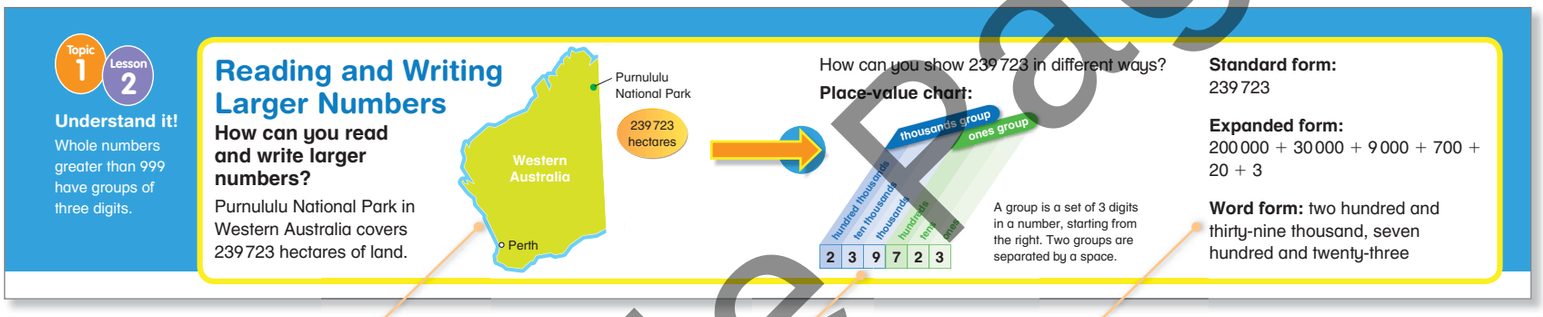
Students build a robust knowledge of adaptable and transferable mathematical concepts. They make connections between related concepts and progressively apply the familiar to develop new ideas. They develop an understanding of the relationship between the 'why' and the 'how' of mathematics. Students build understanding when they connect related ideas, when they represent concepts in different ways, when they identify commonalities and differences between aspects of content, when they describe their thinking mathematically and when they interpret mathematical information.

ACARA, 2010

Extensive research for the **enVisionMATHS** program has shown that daily, problem-based, interactive maths learning, accompanied by visual learning strategies, deepens students' conceptual understanding by making meaningful connections and delivering strong, sequential visual/verbal connections through the Visual Learning Bridge (VLB) in every lesson.

These VLBs provide different ways of looking at mathematical concepts to ensure all learning styles are catered for.

The VLBs are supported and reinforced by accompanying visual learning animations.



Real-life pictorial images ➜ **Mathematical models** ➜ **Mathematical language**

Visual Learning

Research has shown that visual learning strategies deepen conceptual understanding by making meaningful connections for students and delivering strong, sequential visual/verbal connections.

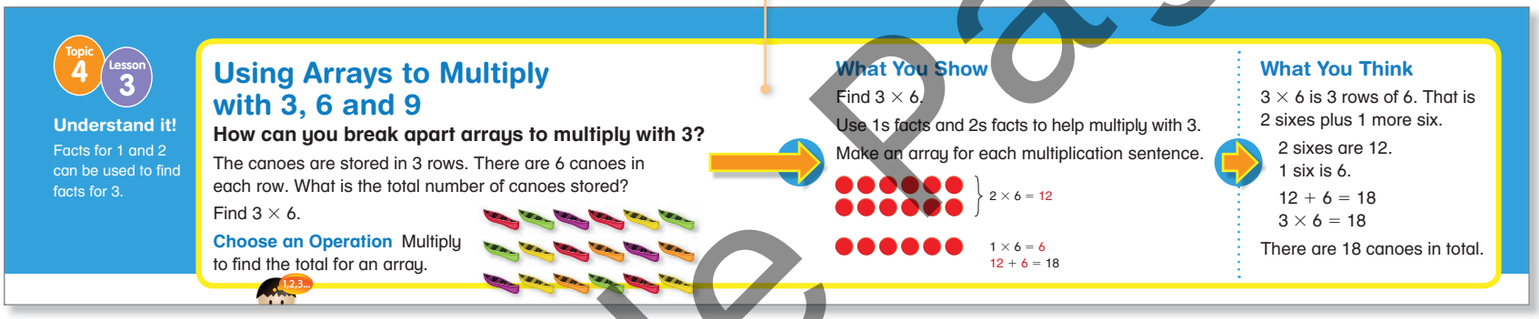
enVisionMATHS focuses strongly on many visual learning strategies. Visual material is used extensively throughout the program: number lines, fractions strips, arrays, Visual Learning Bridges (VLBs), topic-openers, animations (VLAs), bar diagrams and more.

Visual Learning Bridges

The VLBs link the mathematical concepts to real-life situations, thereby making maths less abstract and more meaningful to students.

The VLBs demonstrate the main aspects of a mathematical concept and act as a reference guide for students. They also allow parents to see and understand their child's current maths learning.

Visual Learning Bridge (VLB): presents information in different ways to engage all types of learners.



Visual Learning Animations

The Visual Learning Animations (VLAs) link the mathematical concepts to real-life situations, making maths more meaningful to students. There is one VLA for each lesson in year 4.

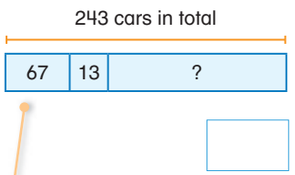
Bar Diagrams

Bar diagrams help students solve problems with visual models.

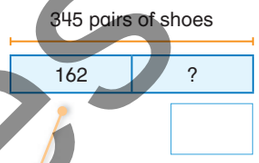
These pictorial bar diagrams are used consistently throughout the program in appropriate formats for each year level. The diagrams are designed to represent abstract maths concepts in a visual image, and to link pictorial models and mathematics symbols. This helps students visualise abstract concepts and aids in conceptual understanding.

Research shows that bar diagrams can be a key to success in problem solving. Bar diagrams help students to understand relationships between quantities in the problem and this helps students to choose a correct operation to solve the problem (Diezmann and English, 2001).

6 A car park had a total of 243 cars enter in one day. By 6:00 am, there were 67 cars in the car park. In the next hour, 13 more cars arrived. How many more cars would enter the car park by the end of the day?



7 Jill's shoe shop sold 162 pairs of shoes. Her goal was to sell 345 pairs. How many pairs of shoes did she **not** sell?



Bar Diagrams

Tools4Maths

Tools4Maths provides another means of adding visual material to the classroom and provides visual explanations that are sometimes not possible to achieve with concrete materials. For example: changing 3D objects into their nets and building them again, breaking down money into its possible denominations and breaking place-value blocks into units.

Differentiation

The amount of differentiation in the classroom depends on the needs of the class and the planning by the teacher.

enVisionMATHS has differentiation possibilities embedded throughout the program. For each topic in year 4, the Teacher Resource Booklets provide extra activity ideas for ESL and additional needs students. Differentiated Worksheets (Replay, Practice, Challenge) are also provided for use with each lesson.

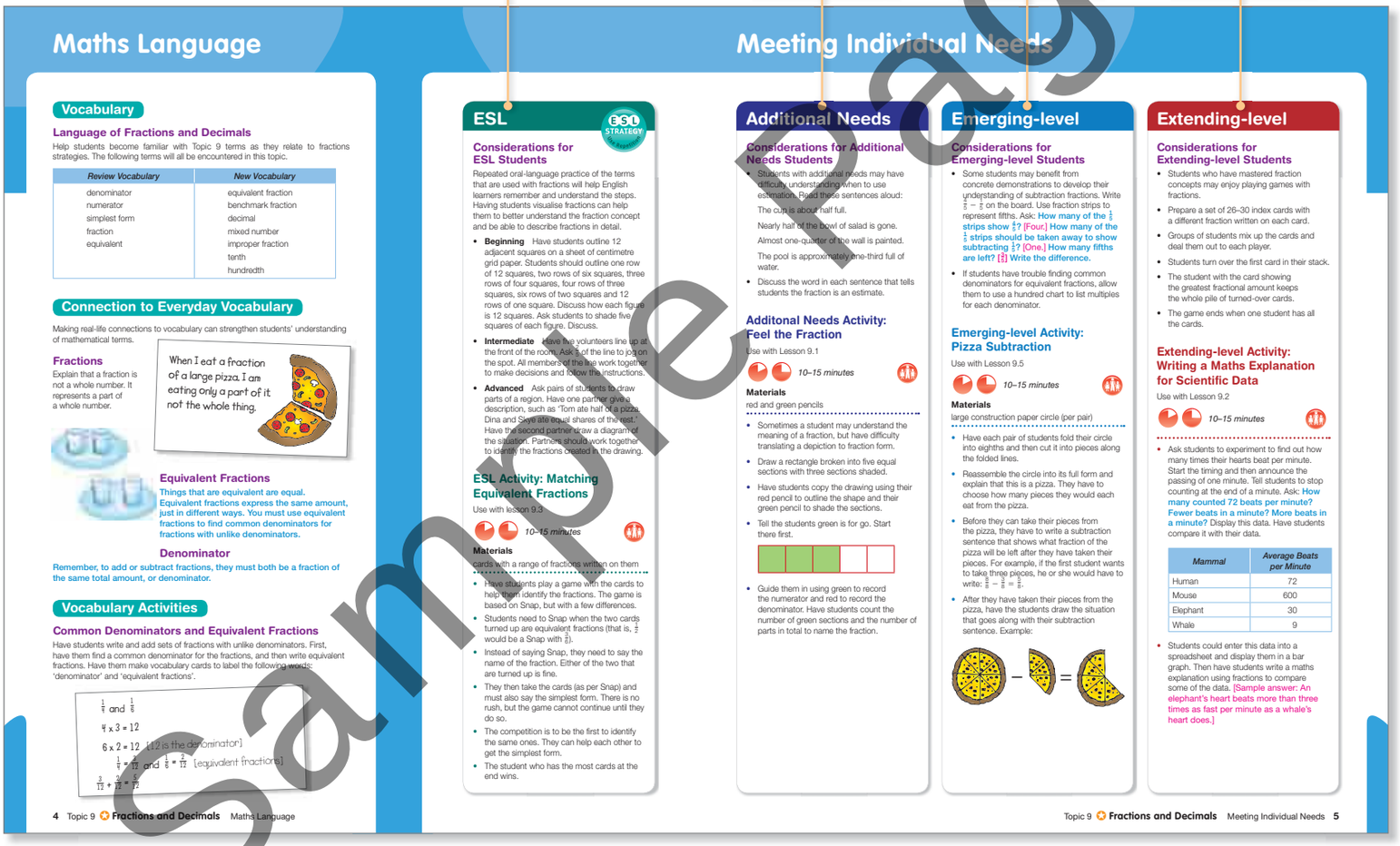
Teacher Resource Booklet

ESL ideas

Additional Needs ideas

Emerging-level activity idea

Extending-level activity idea



Maths Language

Vocabulary

Language of Fractions and Decimals

Help students become familiar with Topic 9 terms as they relate to fractions strategies. The following terms will all be encountered in this topic.

Review Vocabulary	New Vocabulary
denominator	equivalent fraction
numerator	benchmark fraction
simplest form	decimal
fraction	mixed number
equivalent	improper fraction
	tenth
	hundredth

Connection to Everyday Vocabulary

Making real-life connections to vocabulary can strengthen students' understanding of mathematical terms.

Fractions

Explain that a fraction is not a whole number. It represents a part of a whole number.

When I eat a fraction of a large pizza, I am eating only a part of it not the whole thing.

Equivalent Fractions

Things that are equivalent are equal. Equivalent fractions express the same amount, just in different ways. You must use equivalent fractions to find common denominators for fractions with unlike denominators.

Denominator

Remember, to add or subtract fractions, they must both be a fraction of the same total amount, or denominator.

Vocabulary Activities

Common Denominators and Equivalent Fractions

Have students write and add sets of fractions with unlike denominators. First, have them find a common denominator for the fractions, and then write equivalent fractions. Have them make vocabulary cards to label the following words: 'denominator' and 'equivalent fractions'.

$\frac{1}{4}$ and $\frac{1}{6}$

$4 \times 3 = 12$

$6 \times 2 = 12$ [12 is the denominator]

$\frac{1}{4} = \frac{3}{12}$ and $\frac{1}{6} = \frac{2}{12}$ [equivalent fractions]

$\frac{3}{12} + \frac{2}{12} = \frac{5}{12}$

4 Topic 9 ✪ **Fractions and Decimals** Maths Language

Meeting Individual Needs

ESL

ESL STRATEGY

Considerations for ESL Students

Repeated oral-language practice of the terms that are used with fractions will help English learners remember and understand the steps. Having students visualise fractions can help them to better understand the fraction concept and be able to describe fractions in detail.

- **Beginning** Have students outline 12 adjacent squares on a sheet of centimetre grid paper. Students should outline one row of 12 squares, two rows of six squares, three rows of four squares, four rows of three squares, six rows of two squares and 12 rows of one square. Discuss how each figure is 12 squares. Ask students to shade five squares of each figure. Discuss.
- **Intermediate** Have five volunteers line up at the front of the room. Ask 3 of the line to jog on the spot. All members of the line work together to make decisions and follow the instructions.
- **Advanced** Ask pairs of students to draw parts of a region. Have one partner give a description, such as "Tom ate half of a pizza. Dina and Sam ate equal shares of the rest." Have the second partner draw a diagram of the situation. Partners should work together to identify the fractions created in the drawing.

ESL Activity: Matching Equivalent Fractions

Use with lesson 9.3

10–15 minutes

Materials

cards with a range of fractions written on them

- Have students play a game with the cards to help them identify the fractions. The game is based on Snap, but with a few differences.
- Students need to Snap when the two cards turned up are equivalent fractions (that is, $\frac{1}{2}$ would be a Snap with $\frac{3}{6}$).
- Instead of saying Snap, they need to say the name of the fraction. Either of the two that are turned up is fine.
- They then take the cards (as per Snap) and must also say the simplest form. There is no rush, but the game cannot continue until they do so.
- The competition is to be the first to identify the same ones. They can help each other to get the simplest form.
- The student who has the most cards at the end wins.

Additional Needs

Considerations for Additional Needs Students

- Students with additional needs may have difficulty understanding when to use estimation. Read these sentences aloud:

  The cup is about half full.

  Nearly half of the bowl of salad is gone.

  Almost one-quarter of the wall is painted.

  The pool is approximately one-third full of water.
- Discuss the word in each sentence that tells students the fraction is an estimate.

Additonal Needs Activity: Feel the Fraction

Use with Lesson 9.1

10–15 minutes

Materials

red and green pencils

- Sometimes a student may understand the meaning of a fraction, but have difficulty translating a depiction to fraction form.
- Draw a rectangle broken into five equal sections with three sections shaded.
- Have students copy the drawing using their red pencil to outline the shape and their green pencil to shade the sections.
- Tell the students green is for go. Start there first.

- Guide them in using green to record the numerator and red to record the denominator. Have students count the number of green sections and the number of parts in total to name the fraction.

Emerging-level

Considerations for Emerging-level Students

- Some students may benefit from concrete demonstrations to develop their understanding of subtraction fractions. Write $\frac{3}{5} - \frac{2}{5}$ on the board. Use fraction strips to represent fifths. Ask: **How many of the $\frac{1}{5}$ strips show $\frac{3}{5}$? [Four.] How many of the $\frac{1}{5}$ strips should be taken away to show subtracting $\frac{2}{5}$? [One.] How many fifths are left? [$\frac{1}{5}$] Write the difference.**
- If students have trouble finding common denominators for equivalent fractions, allow them to use a hundred chart to list multiples for each denominator.

Emerging-level Activity: Pizza Subtraction

Use with Lesson 9.5

10–15 minutes

Materials

large construction paper circle (per pair)

- Have each pair of students fold their circle into eighths and then cut it into pieces along the folded lines.
- Reassemble the circle into its full form and explain that this is a pizza. They have to choose how many pieces they would each eat from the pizza.
- Before they can take their pieces from the pizza, they have to write a subtraction sentence that shows what fraction of the pizza will be left after they have taken their pieces. For example, if the first student wants to take three pieces, he or she would have to write: $\frac{8}{8} - \frac{3}{8} = \frac{5}{8}$.
- After they have taken their pieces from the pizza, have the students draw the situation that goes along with their subtraction sentence. Example:

Extending-level

Considerations for Extending-level Students

- Students who have mastered fraction concepts may enjoy playing games with fractions.
- Prepare a set of 26–30 index cards with a different fraction written on each card.
- Groups of students mix up the cards and deal them out to each player.
- Students turn over the first card in their stack.
- The student with the card showing the greatest fractional amount keeps the whole pile of turned-over cards.
- The game ends when one student has all the cards.

Extending-level Activity: Writing a Maths Explanation for Scientific Data

Use with Lesson 9.2

10–15 minutes

- Ask students to experiment to find out how many times their hearts beat per minute. Start the timing and then announce the passing of one minute. Tell students to stop counting at the end of a minute. Ask: **How many counted 72 beats per minute? Fewer beats in a minute? More beats in a minute?** Display this data. Have students compare it with their data.

Mammal	Average Beats per Minute
Human	72
Mouse	600
Elephant	30
Whale	9

- Students could enter this data into a spreadsheet and display them in a bar graph. Then have students write a maths explanation using fractions to compare some of the data. [Sample answer: An elephant's heart beats more than three times as fast per minute as a whale's heart does.]

Topic 9 ✪ **Fractions and Decimals** Meeting Individual Needs 5

Research says to give all students access to the same content but level the instruction based on how much support different students need (Cotton, 2001).

The Student Activity Book lesson pages also provide differentiation possibilities through the use of the:

- Visual Learning Bridge
- Another Example feature
- Guided Practice questions
- Independent Practice questions
- Problem-solving questions.

Student Activity Book Lesson Pages

Visual Learning Bridge (VLB): presents information in different ways to engage all types of learners.

Independent Practice: confident students can progress at their own speed.

Topic 9 Lesson 1

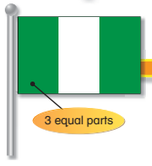
Understand it!
Fractions name parts of a whole or parts of a set of objects.

Using Fractions to Name Parts of Wholes or Sets

How can you name and show parts of a whole and parts of a set?

A **fraction** *is a symbol, such as* $\frac{2}{3}$ *or* $\frac{1}{5}$, *used to name a part of a whole, a part of a set, a location on a number line or a division of whole numbers.*

What fraction of the Nigerian flag is green?



Parts of a Whole

The **numerator** *tells how many equal parts are described.* The **denominator** *tells how many equal parts there are in total.*

$\frac{2}{3}$ ← **Numerator**
← **Denominator**

In the Nigerian flag, $\frac{2}{3}$ of the flag is green.

Parts of a Set

These flags show the first 4 letters in the International Code of Signals:

What fraction of these flags are rectangles?

$\frac{2}{4}$ ← Number that are rectangles
← Total number in set

In this set of 4 flags, $\frac{2}{4}$ are rectangles.

Another Example

How can you draw parts of a whole and parts of a set?

Draw parts of a whole

Draw a flag that is $\frac{3}{5}$ green.

In both flags there are 5 equal parts and 3 of the parts are green.

Both flags are $\frac{3}{5}$ green.

Draw parts of a set

Draw a set of shapes in which $\frac{4}{10}$ of the shapes are small triangles.

There are 4 small triangles out of 10 shapes. So, $\frac{4}{10}$ or four tenths of the shapes are small triangles.

Guided Practice

Write a fraction to describe the part of the whole or set that is green.

1 

2 

Draw a model for each fraction.

3 $\frac{4}{5}$ of a grid

4 $\frac{2}{9}$ of a set

Independent Practice

5 What fraction of the signal flags at the top of the page contain blue?

6 What fraction of the flags contain yellow?

7 Why do these fractions both have the same denominator?

Write a fraction to describe the part of each whole or set that is green.

8 

9 

Problem Solving

10a It rained for $\frac{4}{7}$ of the week, but not on the weekend. On which days might it have rained?

b What fraction of the week didn't it rain and what might the weather have been like on those days?

Curriculum link: NA058

Guided Practice: students apply the concept with teacher support.

Another Example: presents the concept in a different example to consolidate understanding.

Problem Solving: open-ended questions. Capable students can be challenged and less-capable students will also be able to come up with an answer.

Differentiation

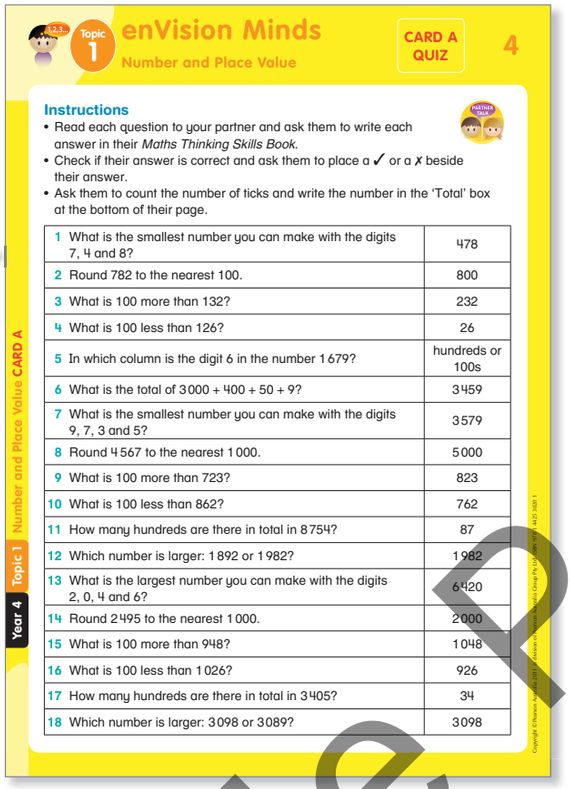
Activity Zone Cards

enVision Minds Cards: quiz questions are in groups of six, with each group being progressively more difficult.

enVision Digital Cards: follow-up tasks are provided to allow the more motivated and capable students to extend the task further. These cards are designed to be used with Tools4Maths.

enVision Investigations Cards: provide real-life investigations related to the topic and are progressively more challenging.

enVision Games Cards: cards are two-sided, with the first side (★) being suitable for all students and the second side (★★) being an extension.



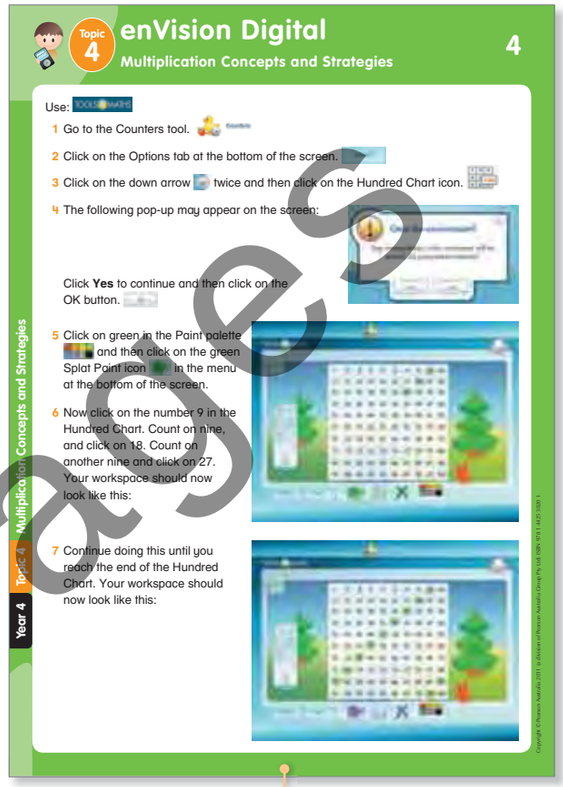
Topic 1 enVision Minds — Number and Place Value — CARD A QUIZ — 4

Instructions

- Read each question to your partner and ask them to write each answer in their *Maths Thinking Skills Book*.
- Check if their answer is correct and ask them to place a ✓ or a ✗ beside their answer.
- Ask them to count the number of ticks and write the number in the 'Total' box at the bottom of their page.

1	What is the smallest number you can make with the digits 7, 4 and 8?	478
2	Round 782 to the nearest 100.	800
3	What is 100 more than 132?	232
4	What is 100 less than 126?	26
5	In which column is the digit 6 in the number 1 679?	hundreds or 100s
6	What is the total of 3 000 + 400 + 50 + 9?	3 459
7	What is the smallest number you can make with the digits 9, 7, 3 and 5?	3 579
8	Round 4 567 to the nearest 1 000.	5 000
9	What is 100 more than 723?	823
10	What is 100 less than 862?	762
11	How many hundreds are there in total in 8 754?	87
12	Which number is larger: 1 892 or 1 982?	1 982
13	What is the largest number you can make with the digits 2, 0, 4 and 6?	6 420
14	Round 2 495 to the nearest 1 000.	2 000
15	What is 100 more than 948?	1 048
16	What is 100 less than 1 026?	926
17	How many hundreds are there in total in 3 405?	34
18	Which number is larger: 3 098 or 3 089?	3 098

Year 4 Topic 1 Number and Place Value CARD A



Topic 4 enVision Digital — Multiplication Concepts and Strategies — 4

Use: Tools4Maths

1 Go to the Counters tool.

2 Click on the Options tab at the bottom of the screen.

3 Click on the down arrow twice and then click on the Hundred Chart icon.

4 The following pop-up may appear on the screen:

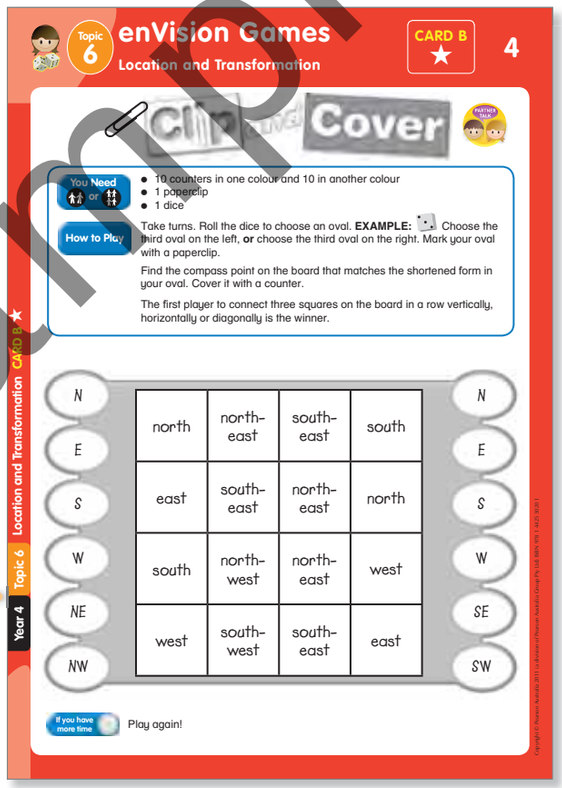
Click **Yes** to continue and then click on the OK button.

5 Click on green in the Paint palette and then click on the green Splat Paint icon in the menu at the bottom of the screen.

6 Now click on the number 9 in the Hundred Chart. Count on nine, and click on 18. Count on another nine and click on 27. Your workspace should now look like this:

7 Continue doing this until you reach the end of the Hundred Chart. Your workspace should now look like this:

Year 4 Topic 4 Multiplication Concepts and Strategies



Topic 6 enVision Games — Location and Transformation — CARD B ★ — 4

Clip and Cover

You Need

- 10 counters in one colour and 10 in another colour
- 1 paperclip
- 1 dice

How to Play

Take turns. Roll the dice to choose an oval. **EXAMPLE:** Choose the third oval on the left, **or** choose the third oval on the right. Mark your oval with a paperclip.

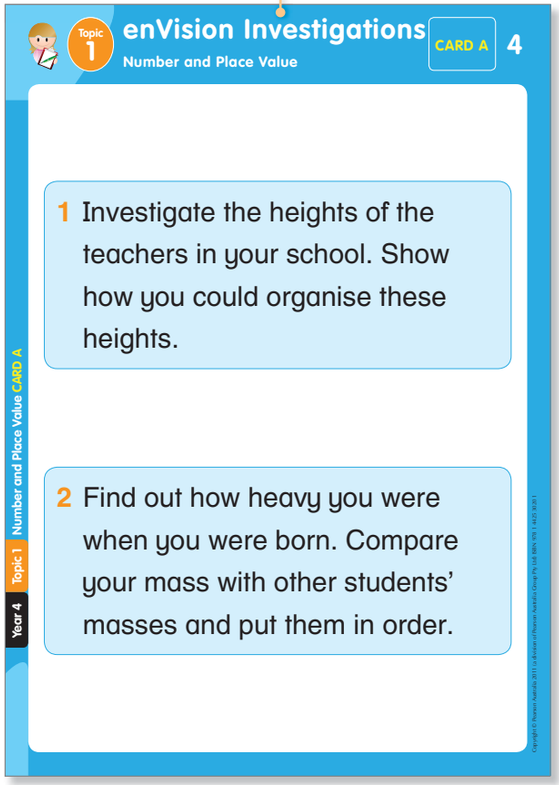
Find the compass point on the board that matches the shortened form in your oval. Cover it with a counter.

The first player to connect three squares on the board in a row vertically, horizontally or diagonally is the winner.

N	north	north-east	south-east	south
E	east	south-east	north-east	north
S	south	north-west	north-east	west
W	west	south-west	south-east	east
NE				
NW				
SE				
SW				

If you have more time — Play again!

Year 4 Topic 6 Location and Transformation CARD B ★



Topic 1 enVision Investigations — Number and Place Value — CARD A — 4

1 Investigate the heights of the teachers in your school. Show how you could organise these heights.

2 Find out how heavy you were when you were born. Compare your mass with other students' masses and put them in order.

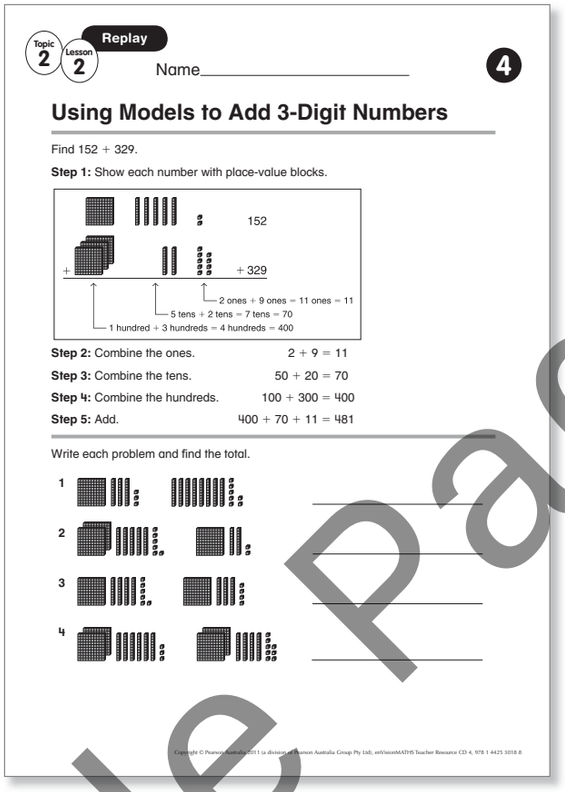
Year 4 Topic 1 Number and Place Value CARD A

Differentiated Worksheets

Worksheets are provided in PDF format on the Teacher Resource DVD (at the back of this book). A Replay, Practice and Challenge worksheet is provided for each lesson within each topic. These are designed for below-level, at-level and above-level students, with the teacher directing students to the appropriate worksheet. The Replay and Practice worksheets are directly related to the topic. Some of the Challenge worksheets aim to extend the students within the topic while others aim to enrich student learning beyond the topic.

Teachers can use these worksheets in a number of ways, as desired: for homework, extra practice, revision classes etc.

Worksheets for years 3, 4 and 5 are provided on the year 4 Teacher Resource DVD.



Topic 2 Lesson 2 **Replay** 4

Name_____

Using Models to Add 3-Digit Numbers

Find 152 + 329.

Step 1: Show each number with place-value blocks.

152

+ 329

2 ones + 9 ones = 11 ones = 11

5 tens + 2 tens = 7 tens = 70

1 hundred + 3 hundreds = 4 hundreds = 400

Step 2: Combine the ones. 2 + 9 = 11

Step 3: Combine the tens. 50 + 20 = 70

Step 4: Combine the hundreds. 100 + 300 = 400

Step 5: Add. 400 + 70 + 11 = 481

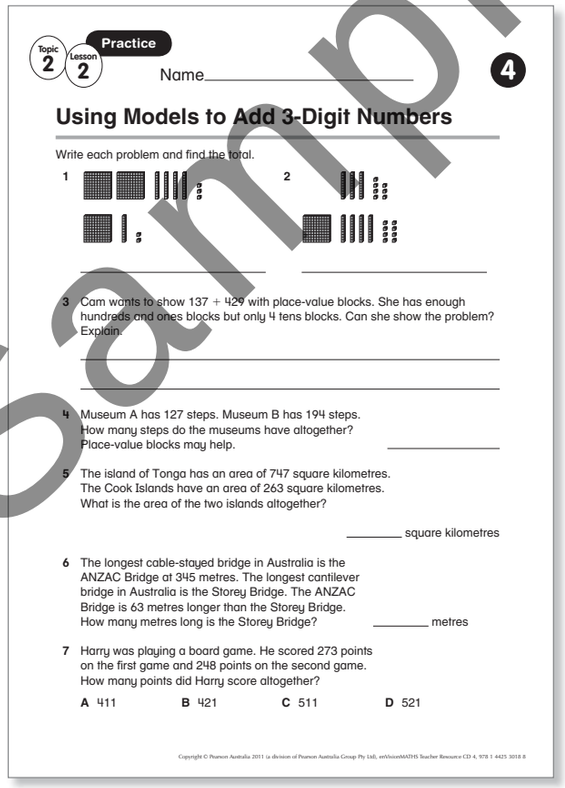
Write each problem and find the total.

1

2

3

4



Topic 2 Lesson 2 **Practice** 4

Name_____

Using Models to Add 3-Digit Numbers

Write each problem and find the total.

1

2

3 Cam wants to show 137 + 429 with place-value blocks. She has enough hundreds and ones blocks but only 4 tens blocks. Can she show the problem? Explain.

4 Museum A has 127 steps. Museum B has 194 steps. How many steps do the museums have altogether? Place-value blocks may help. _____

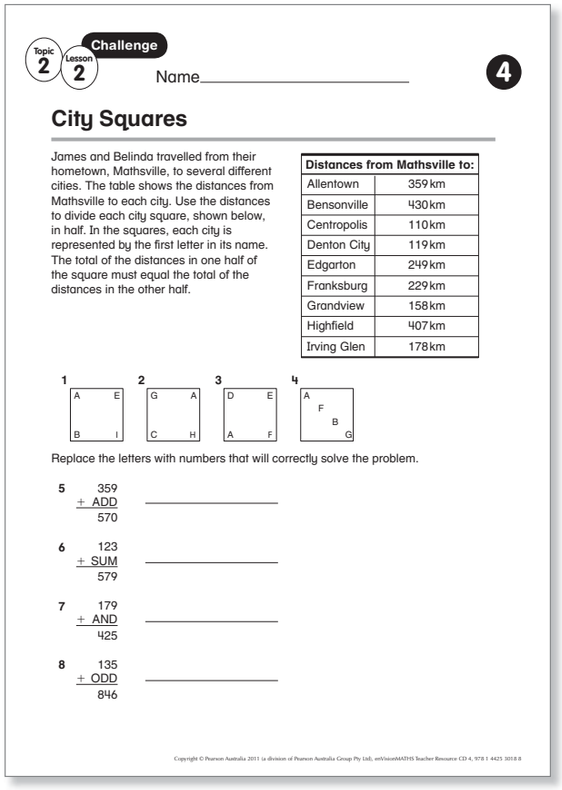
5 The island of Tonga has an area of 747 square kilometres. The Cook Islands have an area of 263 square kilometres. What is the area of the two islands altogether?

_____ square kilometres

6 The longest cable-stayed bridge in Australia is the ANZAC Bridge at 345 metres. The longest cantilever bridge in Australia is the Storey Bridge. The ANZAC Bridge is 63 metres longer than the Storey Bridge. How many metres long is the Storey Bridge? _____ metres

7 Harry was playing a board game. He scored 273 points on the first game and 248 points on the second game. How many points did Harry score altogether?

A 411 **B** 421 **C** 511 **D** 521



Topic 2 Lesson 2 **Challenge** 4

Name_____

City Squares

James and Belinda travelled from their hometown, Mathsville, to several different cities. The table shows the distances from Mathsville to each city. Use the distances to divide each city square, shown below, in half. In the squares, each city is represented by the first letter in its name. The total of the distances in one half of the square must equal the total of the distances in the other half.

Distances from Mathsville to:	
Allentown	359 km
Bensonville	430 km
Centropolis	110 km
Denton City	119 km
Edgarton	249 km
Franksburg	229 km
Grandview	158 km
Highfield	407 km
Irving Glen	178 km

1 A E B I

2 G A C H

3 D E A F

4 A F B G

Replace the letters with numbers that will correctly solve the problem.

5 359 + ADD = 570 _____

6 123 + SUM = 579 _____

7 179 + AND = 425 _____

8 135 + ODD = 846 _____

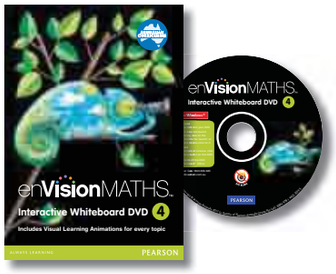
enVisionMATHS Components

enVisionMATHS components cater for the seven year-levels of the Australian Curriculum: Mathematics F–6.

These components can be adapted to all primary maths classrooms around Australia and can be taught in any order.

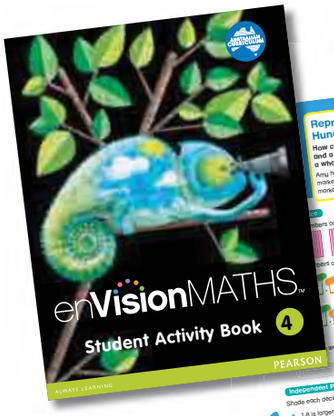
2012 Release

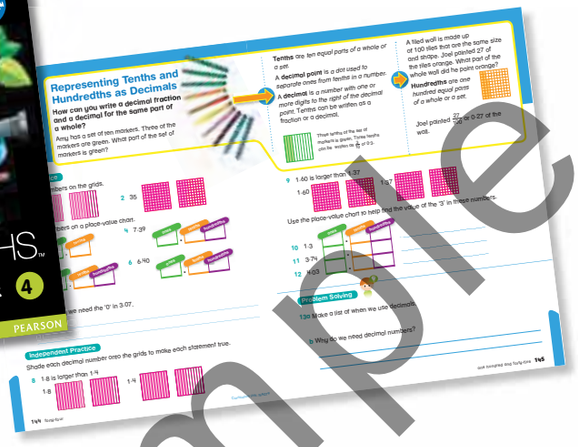
- Teacher Resource Box
- Interactive Whiteboard DVD (including Tools4Maths)
- Student Activity Book
- Activity Zone (investigations cards, mental computation cards, games cards, digital activity cards)
- Maths Thinking Skills Book



Interactive Whiteboard DVD

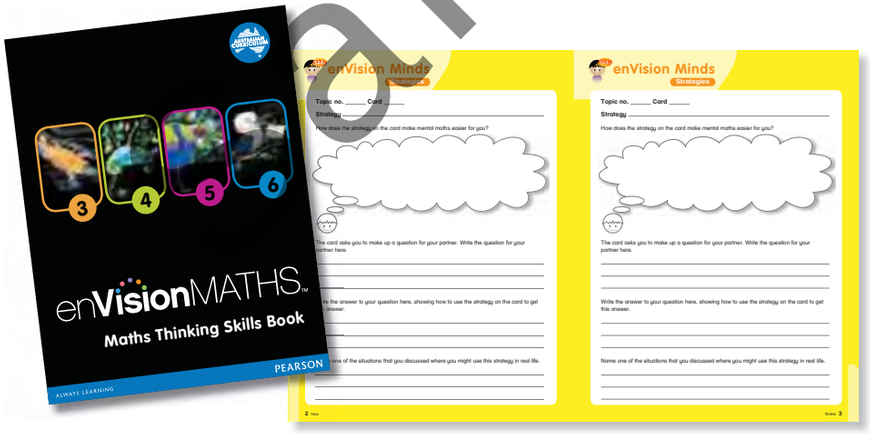
Teacher Resource Box





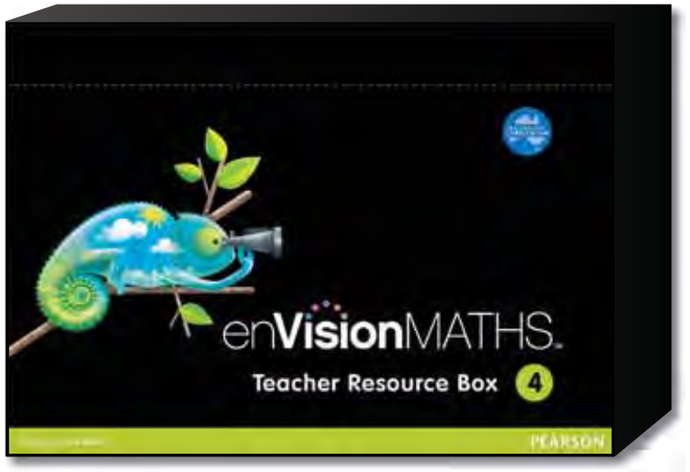
Student Activity Book

Activity Zone



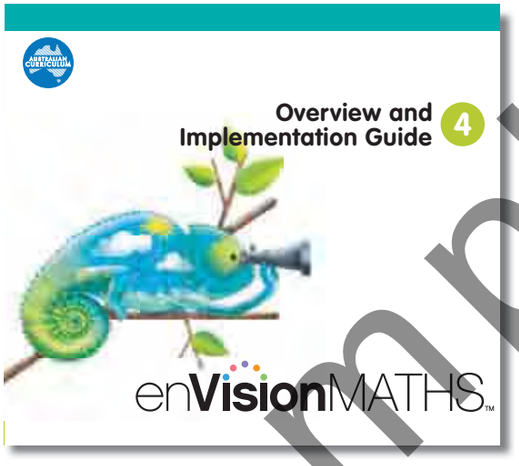
Maths Thinking Skills Book

Teacher Resource Box



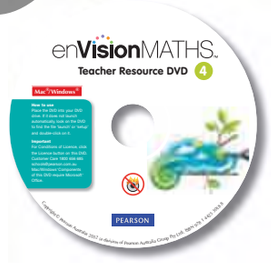
Overview and Implementation Guide

The Overview and Implementation Guide provides the outline of the series for each year level, together with program implementation guidance.



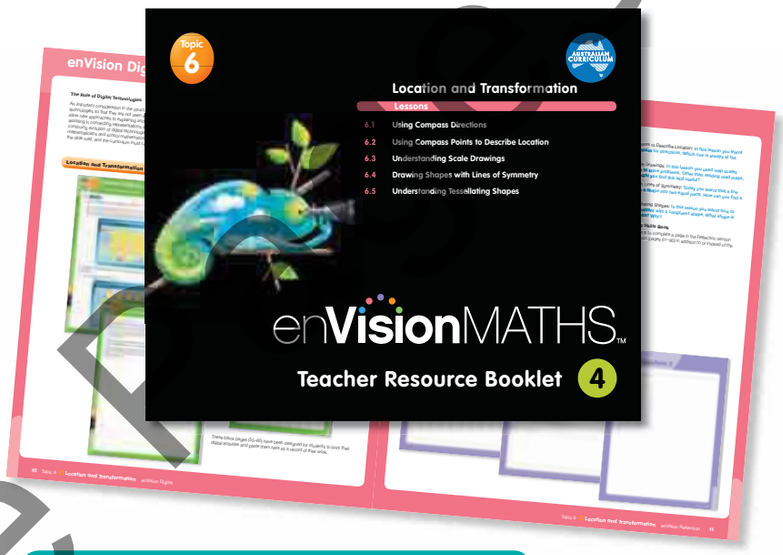
Teacher Resource DVD

The Teacher Resource DVD is in the back of the Overview and Implementation Guide. The year 4 DVD contains all Differentiated Worksheets, Visual Learning Bridges, Pre- and Post-assessments, and editable teacher planning documents for years 3, 4 and 5.



Teacher Resource Booklets

The Teacher Resource Booklets are presented in the Teacher Resource Box for protection and portability. For year 4 the box includes 12 colour-coded topic booklets (and the Overview and Implementation Guide). Each booklet includes the planning and assessment information and tools necessary to implement each topic of the program. Each booklet also includes reduced-sized copies of all components for the topic so that teachers need only take a lightweight booklet with them for their planning when working away from school.



Interactive Whiteboard DVD

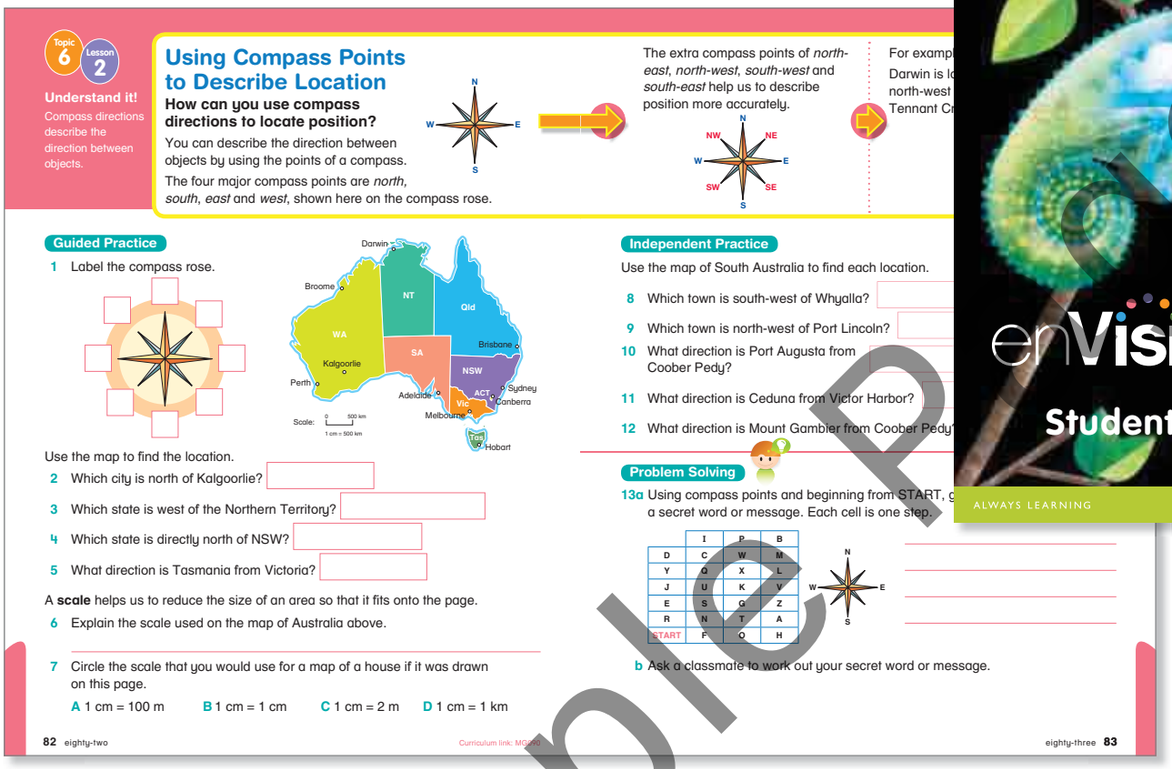
The Interactive Whiteboard DVD includes captivating animations (VLAs) which assist teachers to explicitly teach the underlying mathematical concepts in each topic. The Visual Learning Bridges (VLBs) are also on this DVD. These link each of the important maths concepts to result in a deep conceptual understanding for students. For year 4, there are 89 VLAs and VLBs.

The DVD also includes Tools4Maths—a set of digital maths tools for student or class use on PCs or the whiteboard.

enVisionMATHS Components

Student Activity Book

The Student Activity Book at each year level allows for furthering conceptual understanding, building fluency, reasoning, mental computation and open-ended problem solving. The Student Activity Books are full colour and make extensive use of engaging visual-learning strategies. The Student Activity Book for year 4 covers 12 topics and 89 lessons.



Topic 6 Lesson 2

Understand it!
Compass directions describe the direction between objects.

Using Compass Points to Describe Location

How can you use compass directions to locate position?

You can describe the direction between objects by using the points of a compass.

The four major compass points are *north*, *south*, *east* and *west*, shown here on the compass rose.

The extra compass points of *north-east*, *north-west*, *south-west* and *south-east* help us to describe position more accurately.

Guided Practice

1 Label the compass rose.

Use the map to find the location.

2 Which city is north of Kalgoorlie?

3 Which state is west of the Northern Territory?

4 Which state is directly north of NSW?

5 What direction is Tasmania from Victoria?

A **scale** helps us to reduce the size of an area so that it fits onto the page.

6 Explain the scale used on the map of Australia above.

7 Circle the scale that you would use for a map of a house if it was drawn on this page.

A 1 cm = 100 m B 1 cm = 1 cm C 1 cm = 2 m D 1 cm = 1 km

Independent Practice

Use the map of South Australia to find each location.

8 Which town is south-west of Whyalla?

9 Which town is north-west of Port Lincoln?

10 What direction is Port Augusta from Coober Pedy?

11 What direction is Ceduna from Victor Harbor?

12 What direction is Mount Gambier from Coober Pedy?

Problem Solving

13a Using compass points and beginning from START, g... a secret word or message. Each cell is one step.

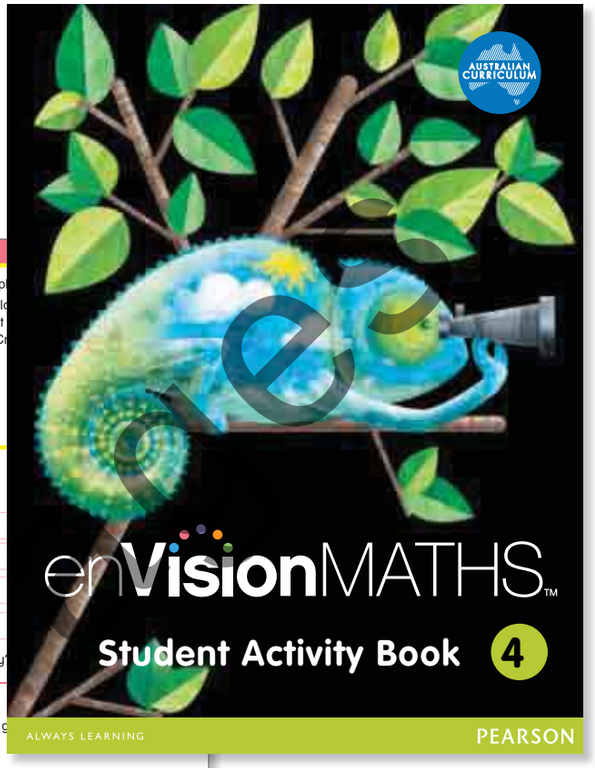
	I	P	B
D	C	W	M
Y	O	X	L
J	U	K	V
E	S	G	Z
R	N	T	A
START	F	Q	H

b Ask a classmate to work out your secret word or message.

82 eighty-two

Curriculum link: MG6-8

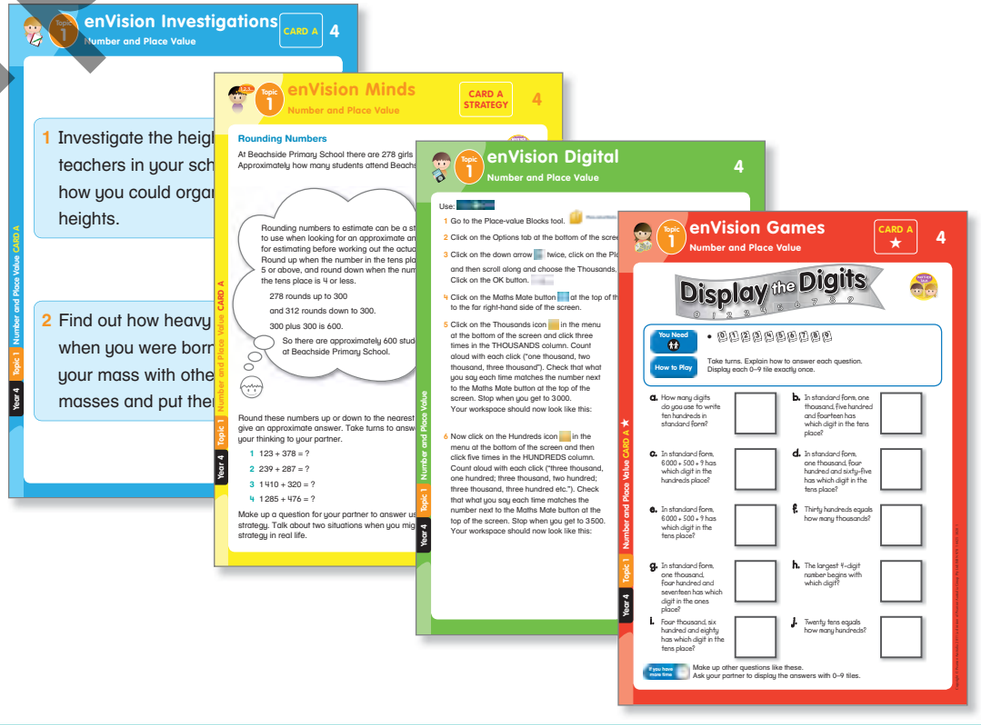
eighty-three 83



Activity Zone

The Activity Zone comprises a box of sturdy, laminated cards that are designed to promote interaction between students and to make their maths learning meaningful and enjoyable. The Activity Zone for year 4 includes 24 Minds cards, 36 Investigations cards, 48 Games cards and 12 Digital cards. Each card is colour-coded and topic-based. Opportunities for differentiation have been embedded by levelling the games. Students record their responses to these cards in the Maths Thinking Skills Book.

The year 4 Activity Zone Box contains one copy of each Games card, two copies of each Minds and Investigations card and four copies of each Digital card for year 4. It also includes sturdy topic dividers that clearly show the topic name and number and how many cards belong with that topic. In some cases, boxes may arrive with the dividers and cards wrapped separately; if so, the cards will need to be collated into their appropriate topics.



Maths Thinking Skills Book

The Maths Thinking Skills Book is aligned directly to the card activities in the Activity Zone. It provides opportunities for students to record their maths thinking in areas such as games strategies, investigations and reflection. This book provides teachers, students and parents with a valuable portfolio of a student's mathematical thinking and their goals for that year's work.

There is one book suitable for use in years 3, 4, 5 or 6 and another book suitable for use in years F, 1 or 2.

How is enVisionMATHS Different?

enVisionMATHS combines modular, print-based teaching resources with breakthrough digital teaching and learning tools. This fully integrated combination of print and digital teaching tools makes **enVisionMATHS** different to other Australian primary mathematics programs.

The use of Visual Learning Bridges and instructional animations is unique to the program and provides for deep conceptual understanding of maths in accordance with the aims of the Australian Curriculum.

- **enVisionMATHS** is a complete program containing all that is needed to provide a variety of learning experiences and differentiation across F to 6. This breadth and the inclusion of elements such as the IWB DVD, Activity Zone and Maths Thinking Skills Book differentiates it from the usual Activity Book and Teacher's Guide of other maths programs. This multi-component approach makes it easier for teachers to find relevant content, and broadens students' experiences by giving them a greater range of materials to work with.
- Everything is organised around topics and everything connects: Teacher support → VLA → Student Book → Activity Zone → Maths Thinking Skills Book → Differentiated Worksheets → Assessment. (This is a different approach to series that take items which were conceived and developed separately then place them together as programs.)
- The Activity Zone and Maths Thinking Skills Book pairing is unique.
- **enVisionMATHS** gives students a voice by providing many opportunities for students to discuss, work in groups and reflect.

enVisionMATHS delivers a truly flexible and comprehensive program through its multi-component but straightforward structure and in-built differentiation and assessment.

enVisionMATHS™

enVisionMATHS and the Australian Curriculum

enVisionMATHS is aligned with the Australian Curriculum maths content and proficiency strands.

Throughout all the **enVisionMATHS** components, the following colours have been allocated to the three Australian Curriculum maths content strands:

Number and Algebra

Measurement and Geometry

Statistics and Probability

These colours show at a glance which strand is being focused on and provide clear links to the curriculum.

The four Australian Curriculum proficiency strands (Understanding, Fluency, Problem solving and Reasoning) are embedded throughout the **enVisionMATHS** program and are specifically addressed as follows and as displayed below.

- Understanding (build and apply knowledge, make connections): use of Visual Learning Animations and Bridges to explain maths concepts; investigations cards
- Fluency (develop skills in applying appropriate procedures, recall and apply facts and concepts; becoming a mathematician): Guided and Independent practice in Student Activity Books, Mental computation and Minds cards, Tools4Maths
- Problem-solving (make choices, interpret, model, communicate): open-ended problem-solving for every lesson in the Student Activity Books; Investigations and Games cards
- Reasoning (reason mathematically by analysing, proving, evaluating, explaining, inferring, justifying and generalising): Reasoning activities are explicit on Student Activity Book pages and in the Maths Thinking Skills Book.

All aspects of these Australian Curriculum content and proficiency strands are included in the Diagnostic Pre-and Post-assessments designed for each topic.

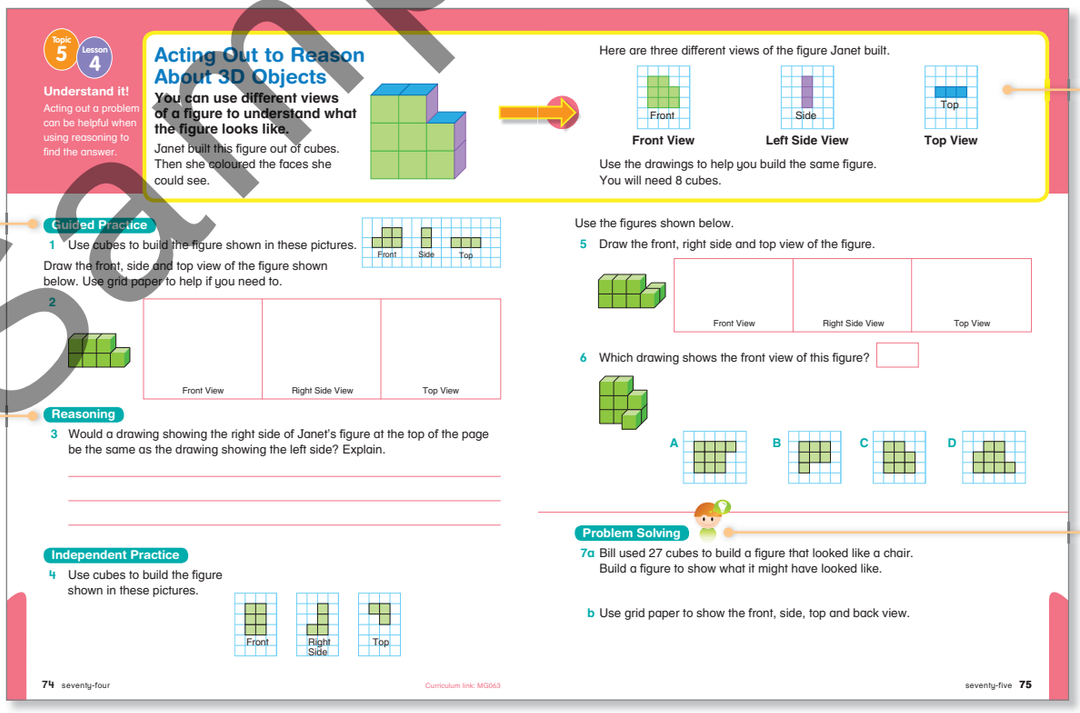
enVisionMATHS also encourages and aids the implementation of the Australian Curriculum's overarching aim of inquiry and active participation and puts in place the means to develop general capabilities.

The seven general capabilities are 'The skills, behaviours and attributes that students need to succeed in life and work in the twenty-first century'. Teachers will be asked to incorporate the general capabilities into their planning. This is how **enVisionMATHS** will help:

- Literacy: focus on vocabulary (page 4 of any Teacher Booklet), Topic Openers, use of Maths Thinking Skills Books, viewing and responding to Visual Learning Animations, reading and understanding problems and investigations
- Numeracy: evident in **enVisionMATHS** content and in working through problems, investigations, use of Maths Thinking Skills Book.
- Competence in information and communication technology: exploration and use of Tools4Maths
- Critical and creative thinking: evident in problem-solving and investigations
- Ethical behaviour: evident in selecting and interpreting data
- Personal and social competence: evident in real-life understandings such as timetables, financial maths; working in groups
- Intercultural understanding: evident in Topic Openers in years 3 to 6, which give real-life maths contexts from around the world.

Students are encouraged to take responsibility for their own learning in maths and to work collaboratively in teams.

Activities relevant to the three cross-curriculum priorities in the Australian Curriculum (Aboriginal and Torres Strait Islander histories and cultures; Asia and Australia's engagement with Asia; and Sustainability) are embedded throughout **enVisionMATHS**. The Australian Curriculum: Mathematics identifies which content descriptions best lend themselves to these priorities.



Fluency: guided and independent practice sections are included on each SAB lesson page.

Reasoning: specific questions are embedded in lessons.

Understanding: Visual Learning Bridges enhance conceptual understanding.

Problem solving: sections appear in each lesson and involve open-ended problem solving.

Research and Development

Extensive research has informed the development of **enVisionMATHS**.

Australian Research Phase

Period	Action
early 2009	Analysis of existing enVision components to determine suitability in Australian environment, conducted by Pearson Australia Primary Maths publishing team
early 2009	Workshop led by Pearson Australia Primary Maths publishing team together with Catherine Pearn (University of Melbourne) and Heidi-Lee Reitsma (Queensland) to formulate list of topics for each year level in line with current Australian teaching practices across the Australian states and early information on shape of maths in the Australian Curriculum
mid-2009	Extensive interviews with Australian primary teachers to verify findings
throughout 2009	Substantial changes formulated for Australia including (but not limited to) introduction of new components Maths Thinking Skills Books and Activity Zones (including investigations and mental computation strategy cards); realignment of topics and year levels; change to more open-ended problem-solving approach and appointment of writer to provide a problem-solving strand across the year levels; change to scrapbook-size workbooks for F–2
November 2009	Workshop with Paul Swan, Linda Marshall, Catherine Attard, Linda Flanagan, Deborah Vietri, Jenny Saunders, Terri Bullock, Carmen Morgan, Heidi-Lee Reitsma, John Sandy, Greg Thomas and Nicole Morrison to review instructional design and commission writers
October 2009–September 2010	Continual consultation and revision over Australian writing phase
March 2010	Phase 1 revision based on Australian Curriculum draft
June 2010	Focus group with primary teachers to problem-solve on draft
November 2010	Assessment pathway revised
December 2010–July 2011	Phase 2 revision based on Australian Curriculum
2011	Reports from pilot schools in Australia will inform future refinements

Original Research Phase

Four distinct phases of research were integrated into the development of the original enVision program in the US.

Phase 1 ✪ Ongoing research

Ongoing research with existing Pearson programs (such as Scott Foresman Mathematics, Scott Foresman/Addison Wesley Math and Sliver Burdett Ginn Mathematics programs) provides a strong basis for success. Scores on standardised tests as well as longitudinal studies prove these programs help raise maths scores.

Phase 2 ✪ Scientific research base

An experienced authorship team incorporated findings from the large body of available scientific research to develop the instructional and assessment tools.

Phase 3 ✪ Formative research

As the program was designed, valuable input was contributed by classroom field studies, as well as leading mathematicians, administrators, teachers and reviewers. Pre- and post-test scores proved the research findings were on track.

Phase 4 ✪ Summative research

Ongoing scientific research, including longitudinal studies in the classroom, further validates the program's effectiveness and supports the commitment to producing the highest quality mathematics materials.

A Research Study

PRES Associates, an independent research firm, conducted a longitudinal randomised control trial study to assess the effectiveness of **enVisionMATH** in helping students attain critical maths skills. The study commenced in 2007–8 with 2nd and 4th grade students and followed these students into 3rd and 5th grades in 2008–9. The final sample comprised 708 students who participated in both years of the study. Fifty-six teachers participated in the first year of the study, and forty-four participated in the second year. The participating schools represented suburban and rural communities from eight US states.

Successful Results enVisionMATH

Students of all ability levels using **enVisionMATH** demonstrated significantly greater improvement than students using other maths programs in the areas of: Maths Computation, Problem Solving and Maths Communication.

Furthermore, the data suggests that increased student and teacher exposure to **enVisionMATH** produces stronger effects with the rate of growth becoming more pronounced in the second year of usage.

Suggested Teaching Sequence

enVisionMATHS provides a flexible sequence with topics that are organised and colour coded by content strand, and which are small enough to be rearranged into a personalised curriculum that matches the preferred sequence of a class/school.

The key to the implementation of **enVisionMATHS** in the classroom is the sequencing of the program elements. A suggested teaching sequence is provided below.

Topic

- Plan lesson (using Teacher Resource Booklet, pages 2–11).
- Introduce topic using topic-opener (Teacher Resource Booklet, page 2).
- Administer Diagnostic Pre-assessment to students (by concept).

Lessons

- Introduce each lesson by setting the purpose.
- Make connections to students' previous learning.
- Watch the Visual Learning Animation and show students the Visual Learning Bridge on the Interactive Whiteboard DVD or at the top of the relevant lesson page in the Student Activity Book.
- Whole-class teaching focus: consolidate the concept and include problem solving.
- Students complete activity in Student Activity Book.
- Small group work: work with students who need further instruction (error intervention, extension) while rest of class work in groups on differentiated learning centre activities from the Activity Zone that are appropriate to their level, recording their work in their Maths Thinking Skills Books.
- Whole-class reflection.
- Students record reflections in the Maths Thinking Skills Book.
- Students do extra practice at home or school using Differentiated Worksheets.

Assessment

- Ongoing and throughout: Use assessment pages from the **enVisionMATHS** Teacher Resource Booklet and DVD, observations and recorded work in Maths Thinking Skills Book.
- Administer Diagnostic Post-assessment to students (by concept).

Note: Teachers are free to adapt this sequence to best suit their classes.

How Long Does a Topic or Lesson Take to Do?

Year 4 has 89 lessons across 12 topics. The topics range from 4 to 10 lessons. A lesson would take about 2 hours (two 1-hour blocks) so about 2 to 3 lessons could be covered in a week. A topic would therefore range from nearly 2 weeks for the shortest topic to about 4 weeks for the longest, with those with an average 6 to 8 lessons taking 2 to 3 weeks each including time needed for assessment.

In summary:

A lesson = 2 hours
A topic = average 3 weeks
12 Topics × 3 weeks = 36 weeks

See pages 26 to 37 for an approximate time allocation per topic.

The year 4 Teacher Resource DVD (at the back of this book) includes editable planning documents related to years 3, 4 and 5 to assist with planning **enVisionMATHS** work throughout the year.

These planning documents are simple, flexible Microsoft® Excel® spreadsheets. They match Australian Curriculum strands and substrands to **enVisionMATHS** lessons, and provide the relevant Student Book and Teacher Booklet page references for each lesson. Other columns in the spreadsheet are left blank for teachers to use as they wish.

In developing **enVisionMATHS**, it was recognised that teachers need to fit in with their whole-school planning, so there is no prescribed sequence. The suggested teaching sequence can be used as a guide but the program has been designed to be flexible and can be implemented as a whole-class program or used with different ability groups.

Note that the Student Activity Books do not include extensive practice examples. The program focuses on understanding concepts and enrichment while providing the flexibility for teachers to introduce their own practice examples through the use of the Differentiated Worksheets, if they wish.

The year 4 contents sequence on pages 26–37 provides an at-a-glance view of the program's components and suggested resources for each topic.

Year 4 Planning Document

Australian Curriculum Reference	**enVisionMATHS** Student Activity Book year 4	SAB 4* (pg no.)	TRB 4* (booklet no: pg no.)
NUMBER AND ALGEBRA			
Number and place value			
NA071 Investigate and use the properties of odd and even numbers	**1.1** Reading and Writing Thousands	4	1:12
	1.3 Comparing and Ordering Whole Numbers	8	1:16
	3.2 Subtracting on a Hundred Chart	28	3:14
	4.4 Multiplying in Any Order	54	4:18
NA072 Recognise, represent and order numbers to at least tens of thousands	**1.1** Reading and Writing Thousands	4	1:12
	1.2 Reading and Writing Larger Numbers	6	1:14
	1.3 Comparing and Ordering Whole Numbers	8	1:16
	1.4 Rounding Whole Numbers	10	1:18
NA073 Apply place value to partition, rearrange and regroup numbers to at least tens of thousands to assist calculations and solve problems	**2.1** Using Mental Maths to Add	14	2:12
	2.2 Using Models to Add 3-Digit Numbers	16	2:14
	2.3 Adding Whole Numbers	18	2:16
	2.4 Adding Three or More Numbers	20	2:18
	3.1 Using Models to Subtract 2-Digit Numbers	26	3:12
	3.2 Subtracting on a Hundred Chart	28	3:14
	3.3 Subtracting 2-Digit Numbers	30	3:16
	3.4 Using Modesl to Subtract 3-Digit Numbers	32	3:18
	3.5 Subtracting 3-Digit Numbers	34	3:20
	3.7 Subtracting with Zero	38	3:24
	3.8 Subtracting with More Than One Zero	40	3:26
	3.9 Solving Subtraction Problems in Two Steps (Revision NA055)	42	3:28
	7.3 Using Materials to Divide 2-Digit Numbers	96	7:16
	7.7 Using Mental Maths to Divide	104	7:24
	7.8 Dividing Multiples of 10, 100 and 1 000	106	7:26
NA074 Investigate number sequences involving multiples of 3, 4, 6, 7, 8 and 9	**4.1** Using Arrays to Multiply	48	4:12
	4.2 Using Arrays to Multiply with 4 and 8	50	4:14
	4.3 Using Arrays to Multiply with 3, 6 and 9	52	4:16
	8.1 Looking for a Pattern	114	8:12
	8.2 Looking for a Pattern to Solve a Problem	116	8:14
NA075 Recall multiplication facts up to 10 × 10 and related division facts	**4.2** Using Arrays to Multiply with 4 and 8	50	4:14
	4.3 Using Arrays to Multiply with 3, 6 and 9	52	4:16
	4.5 Solving Multiplication Problems in Two Steps	56	4:20
	4.8 Multiplying 2-Digit by 1-Digit Numbers	62	4:26
	4.9 Multiplying by Multiples of 10 and 100	64	4:27
	7.4 Relating Multiplication and Division Facts	98	7:18
	7.5 Using Inverse Operations	100	7:20
	7.6 Using Multiplication Facts to Find Division Facts	102	7:22
	7.7 Using Mental Maths to Divide	104	7:24

* SAB 4 = **enVisionMATHS** Student Activity Book Year 4, TRB 4 = **enVisionMATHS** Year 4 Teacher Resource Booklets

(cont.)

Number and place value (cont.)			
NA076 Develop efficient mental and written strategies and use appropriate digital technologies for multiplication and for division where there is no remainder	**4.1** Using Arrays to Multiply	48	4:12
	4.2 Using Arrays to Multiply with 4 and 8	50	4:14
	4.3 Using Arrays to Multiply with 3, 6 and 9	52	4:16
	4.4 Multiplying in Any Order	54	4:18
	4.5 Solving Multiplication Problems in Two Steps	56	4:20
	4.6 Using Mental Maths to Multiply	58	4:22
	4.7 Recording Multiplication in Different Ways	60	4:24
	4.8 Multiplying 2-Digit by 1-Digit Numbers	62	4:26
	4.9 Multiplying by Multiples of 10 and 100	64	4:27
	7.1 Using Diagrams to Divide	92	7:12
	7.2 Using a Table to Show Division	94	7:14
	7.3 Using Materials to Divide 2-Digit Numbers	96	7:16
	7.4 Relating Multiplication and Division Facts	98	7:18
	7.5 Using Inverse Operations	100	7:20
	7.6 Using Multiplication Facts to Find Division Facts	102	7:22
	7.7 Using Mental Maths to Divide	104	7:24
	7.8 Dividing Multiples of 10, 100 and 1 000	106	7:26
	7.9 Dividing with 2-Digit Numbers	108	7:28
	7.10 Dividing with Remainders (Extension NA101, NA121)	110	7:30
	8.5 Working Backwards to Solve Problems	122	8:20
	8.7 Finding Equality	126	8:24
Fractions and decimals			
NA077 Investigate equivalent fractions used in contexts	**9.1** Using Fractions to Name Parts of Wholes or Sets Revision NA058)	132	9:12
	9.2 Writing a Maths Explanation (Revision NA058)	134	9:14
	9.3 Finding Equivalent Fractions	136	9:16
	9.4 Finding More Equivalent Fractions	138	9:18
	9.5 Defining Fractions (Revision NA058)	140	9:20
NA078 Count by quarters, halves and thirds, including with mixed numerals. Locate and represent these fractions on a number line	**9.4** Finding More Equivalent Fractions	138	9:18
	9.6 Using Improper Fractions and Mixed Numbers	142	9:20
NA079 Recognise that the place-value system can be extended to tenths and hundredths. Make connections between fractions and decimal notation.	**9.7** Representing Tenths and Hundredths as Decimals	144	9:22
	9.8 Writing Fractions and Decimals	146	9:24
Money and financial mathematics			
NA080 Solve problems involving purchases and the calculation of change to the nearest five cents with and without digital technologies	**3.10** Calculating Change from Money	44	3:30
Patterns and algebra			
NA081 Explore and describe number patterns resulting from performing multiplication	**4.2** Using Arrays to Multiply with 4 and 8	50	4:14
	4.4 Multiplying in Any Order	54	4:18
	7.7 Using Mental Maths to Divide	104	7:24
	8.1 Looking for a Pattern	114	8:12
	8.3 Describing Geometric Patterns Using a Table	118	8:16
NA082 Solve word problems by using number sentences involving multiplication or division where there is no remainder	**4.3** Using Arrays to Multiply with 3, 6 and 9	52	4:16
	4.4 Multiplying in Any Order	54	4:18
	4.5 Solving Multiplication Problems in Two Steps	56	4:20
	4.8 Multiplying 2-Digit by 1-Digit Numbers	62	4:26
	4.9 Multiplying by Multiples of 10 and 100	64	4:27
	7.1 Using Diagrams to Divide	92	7:12
	7.2 Using a Table to Show Division	94	7:14
	7.3 Using Materials to Divide 2-Digit Numbers	96	7:16
	7.6 Using Multiplication Facts to Find Division Facts	102	7:22
	7.8 Dividing Multiples of 10, 100 and 1 000	106	7:26
	7.9 Dividing with 2-Digit Numbers	108	7:28
	8.2 Looking for a Pattern to Solve a problem	116	8:14
	8.4 Using Expressions to Find Missing Numbers	120	8:18
	8.5 Working Backwards to Solve Problems	122	8:20
	8.6 Using Multiplication and Division Expressions	124	8:22
NA083 Use equivalent number sentences involving addition and subtraction to find unknown quantities	**2.5** Using Diagrams to Connect Addition and Subtraction (Revision NA055)	22	2:20
	3.5 Subtracting 3-Digit Numbers	34	3:20
	3.6 Drawing a Diagram to Solve Subtraction Problems	36	3:22
	8.7 Finding Equality	126	8:24
	8.8 Identifying Equal or Unequal Number Sentences	128	8:26
	10.6 Understanding Temperature	160	10:22

MEASUREMENT AND GEOMETRY			
Using units of measurement			
MG084 Use scaled instruments to measure and compare lengths, masses, capacities and temperatures	**10.6** Understanding Temperature	160	10:22
	11.1 Measuring with Formal Units of Length	164	11:12
	11.2 Finding Perimeter (Extension MG109)	166	11:14
	11.3 Identifying Different Shapes with the Same Perimeter (Extension MG109)	168	11:16
	11.6 Estimating and Measuring with Millilitres and Litres	174	11:22
	11.7 Estimating and Measuring with Grams and Kilograms	176	11:24
	11.8 Introducing Volume	178	11:26
	11.9 Enlarging and Reducing Measurements	180	11:28
	11.10 Reading Scales	182	11:30
MG290 Compare objects using familiar metric units of area and volume	**11.4** Understanding Area (Extension MG109)	170	11:18
	11.5 Estimating and Measuring Area (Extension MG109)	172	11:20
	11.8 Introducing Volume	178	11:26
MG085 Convert between units of time	**10.2** Relating Units of Time	152	10:14
	10.3 Comparing Units of Time	154	10:16
MG086 Use am and pm notation and solve simple time problems	**10.1** Telling Time to the Minute (Revision MG062)	150	10:12
	10.3 Comparing Units of Time	154	10:16
	10.4 Finding Elapsed Time	156	10:18
	10.5 Working Backwards to Solve Time Problems	158	10:20
Shape			
MG087 Compare the areas of regular and irregular shapes by informal means	**5.2** Relating 2D Shapes and 3D Objects (Revision MG063)	70	5:14
	5.3 Describing and Classifying Objects (Revision MG063)	72	5:16
	5.4 Acting Out to Reason About 3D Objects (Revision MG063)	74	5:18
	11.4 Understanding Area (Extension MG109)	170	11:18
	11.5 Estimating and Measuring Area (Extension MG109)	172	11:20
MG088 Compare and describe two-dimensional shapes that result from combining and splitting common shapes, with and without the use of digital technologies	**5.5** Identifying 2D Shapes in Other Shapes	76	5:20
Location and transformation			
MG090 Use simple scales, legends and directions to interpret information contained in basic maps	**6.1** Using Compass Directions	80	6:12
	6.2 Using Compass Points to Describe Location	82	6:14
	6.3 Understanding Scale Drawings	84	6:16
MG091 Create symmetrical patterns, pictures and shapes with and without digital technologies	**6.4** Drawing Shapes with Lines of Symmetry (Revision MG066)	86	6:18
	6.5 Understanding Tessellating Shapes	88	6:20
Geometric reasoning			
MG089 Compare angles and classify them as equal to, greater than or less than a right angle	**5.1** Describing Angles	68	5:12
STATISTICS AND PROBABILITY			
Chance			
SP092 Describe possible everyday events and order their chances of occurring	**12.7** Predicting and Comparing Outcomes	198	12:24
SP093 Identify everyday events where one cannot happen if the other happens	**12.8** Identifying Related Events	200	12:26
SP094 Identify events where the chance of one will not be affected by the occurrence of the other	**12.9** Identifying Independent Events	202	12:28
Data representation and interpretation			
SP095 Select and trial methods for data collection including survey questions and recording sheets	**12.1** Using Surveys	186	12:12
SP096 Construct suitable data displays, with and without the use of digital technologies, from given or collected data. Include tables, column graphs and picture graphs where one picture can represent many data values	**12.1** Using Surveys	186	12:12
	12.2 Organising Data	188	12:14
	12.3 Reading Pie Graphs	190	12:16
	12.4 Making a Graph	192	12:18
	12.5 Understanding Venn Diagrams	194	12:20
SP097 Evaluate the effectiveness of different displays in illustrating data features including variability	**12.4** Making a Graph	192	12:18
	12.5 Understanding Venn Diagrams	194	12:20
	12.6 Using Reasoning to Draw Conclusions	196	12:22

Topic 1

Year 4 Contents Sequence

Number and Place Value

Suggested time: 2 weeks

Topic

Diagnostic Pre-assessment: Topic 1

Lessons

Year 4 ✪ Topic 1 ✪ Number and Place Value

Lesson		SAB* page	TRB* page
1.1	Reading and Writing Thousands	4	12
1.2	Reading and Writing Larger Numbers	6	14
1.3	Comparing and Ordering Whole Numbers	8	16
1.4	Rounding Whole Numbers	10	18

Activity Zone 4
Minds Cards 1A, 1B
Digital Card 1
Games Cards 1A–D (★ and ★★)
Investigations Cards 1A, 1B, 1C

Interactive Whiteboard DVD 4
Visual Learning Bridges 1.1–1.4
Visual Learning Animations 1.1–1.4
Tools4Maths: Counters, Place-value Blocks

Teacher Resource DVD 4
Replay, Practice, Challenge Worksheets Year 4 Topic 1 lessons 1–4

AC Links
NA071, NA072

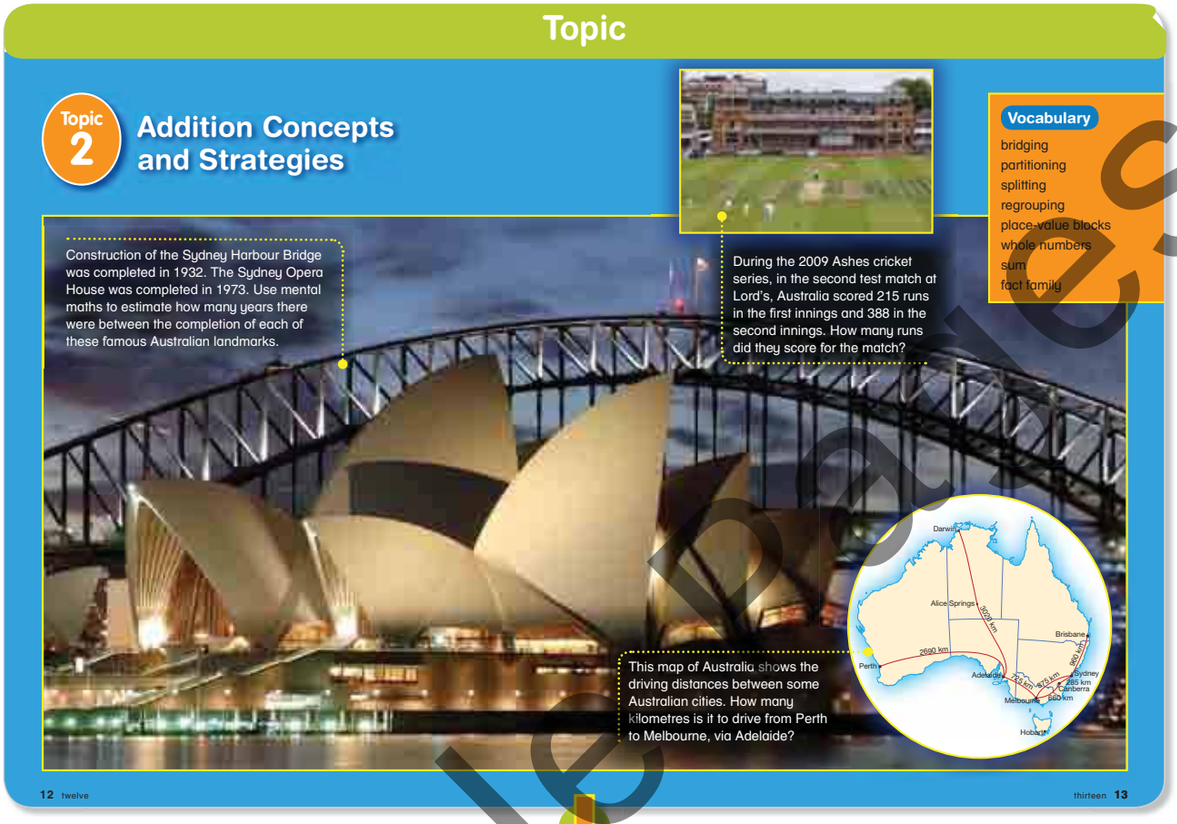
* SAB = Student Activity Book 4; TRB = Year 4 Teacher Resource Booklet 1

Ongoing assessment ● Diagnostic Post-assessment: Topic 1

Addition Concepts and Strategies

Suggested time: 2 weeks

Topic



Diagnostic Pre-assessment: Topic 2

Lessons

Year 4 ✪ Topic 2 ✪ Addition Concepts and Strategies

Lesson		SAB* page	TRB* page
2.1	Using Mental Maths to Add	14	12
2.2	Using Models to Add 3-Digit Numbers	16	14
2.3	Adding Whole Numbers	18	16
2.4	Adding Three or More Numbers	20	18
2.5	Using Diagrams to Connect Addition and Subtraction	22	20

Activity Zone 4
Minds Cards 2A, 2B
Digital Card 2
Games Cards 2A–D (★ and ★★)
Investigations Cards 2A, 2B, 2C

Interactive Whiteboard DVD 4
Visual Learning Bridges 2.1–2.5
Visual Learning Animations 2.1–2.5
Tools4Maths: Notepad, Counters, Place-value Blocks

Teacher Resource DVD 4
Replay, Practice, Challenge Worksheets Year 4
Topic 2 lessons 1–5

AC Links
NA055, NA073, NA083

* SAB = Student Activity Book 4; TRB = Year 4 Teacher Resource Booklet 2

Ongoing assessment ● Diagnostic Post-assessment: Topic 2

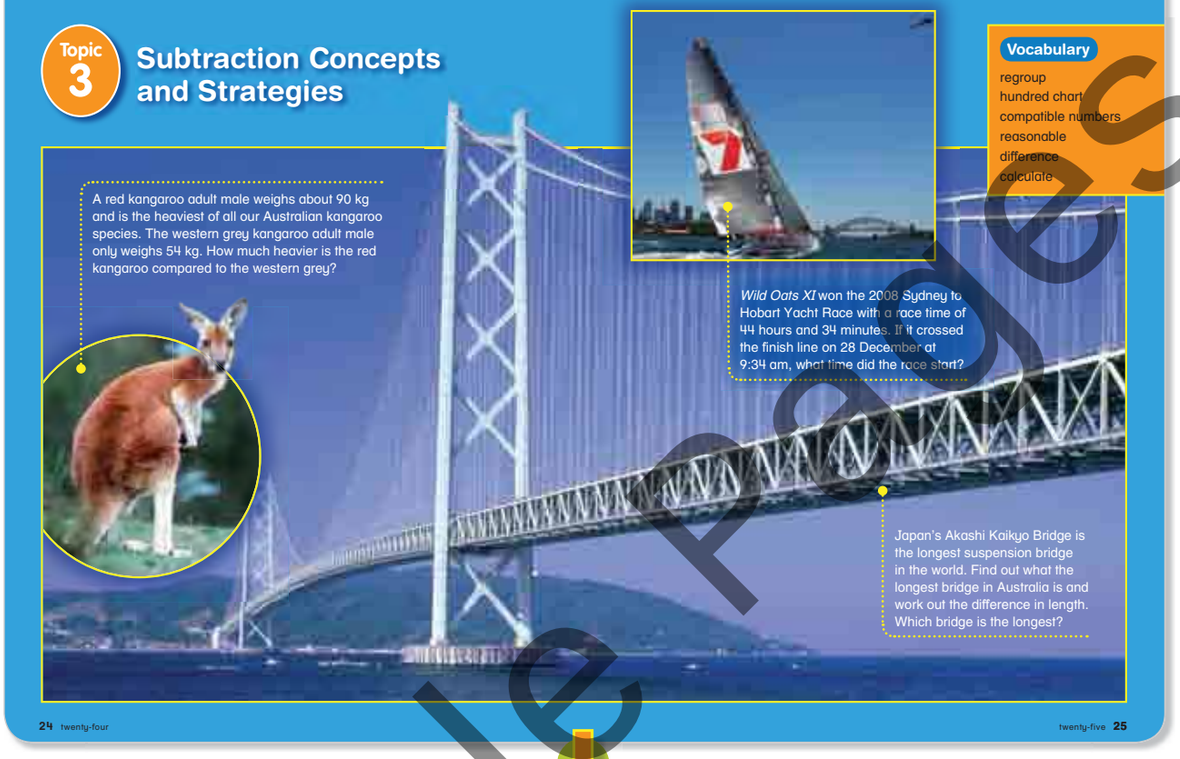
Topic 3

Year 4 Contents Sequence

Subtraction Concepts and Strategies

Suggested time: 4 weeks

Topic



Diagnostic Pre-assessment: Topic 3

Lessons

Year 4 ✪ Topic 3 ✪ Subtraction Concepts and Strategies

Lesson		SAB* page	TRB* page
3.1	Using Models to Subtract 2-Digit Numbers	26	12
3.2	Subtracting on a Hundred Chart	28	14
3.3	Subtracting 2-Digit Numbers	30	16
3.4	Using Models to Subtract 3-Digit Numbers	32	18
3.5	Subtracting 3-Digit Numbers	34	20
3.6	Drawing a Diagram to Solve Subtraction Problems	36	22
3.7	Subtracting With Zero	38	24
3.8	Subtracting With More Than One Zero	40	26
3.9	Solving Subtraction Problems in Two Steps	42	28
3.10	Calculating Change from Money	44	30

Activity Zone 4
Minds Cards 3A, 3B
Digital Card 3
Games Cards 3A–D (★ and ★★)
Investigations Cards 3A, 3B, 3C

Interactive Whiteboard DVD 4
Visual Learning Bridges 3.1–3.10
Visual Learning Animations 3.1–3.10
Tools4Maths: Counters, Place-value Blocks, Money

Teacher Resource DVD 4
Replay, Practice, Challenge Worksheets Year 4 Topic 3 lessons 1–10

AC Links
NA055, NA071, NA073, NA080, NA083

* SAB = Student Activity Book 4; TRB = Year 4 Teacher Resource Booklet 3

Ongoing assessment ● Diagnostic Post-assessment: Topic 3

Multiplication Concepts and Strategies

Suggested time: 4 weeks

Topic

Diagnostic Pre-assessment: Topic 4

Lessons

Year 4 ✪ Topic 4 ✪ Multiplication Concepts and Strategies

Lesson		SAB* page	TRB* page
4.1	Using Arrays to Multiply	48	12
4.2	Using Arrays to Multiply with 4 and 8	50	14
4.3	Using Arrays to Multiply with 3, 6 and 9	52	16
4.4	Multiplying in Any Order	54	18
4.5	Solving Multiplication Problems in Two Steps	56	20
4.6	Using Mental Maths to Multiply	58	22
4.7	Recording Multiplication in Different Ways	60	24
4.8	Multiplying 2-Digit by 1-Digit Numbers	62	26
4.9	Multiplying by Multiples of 10 and 100	64	28

Activity Zone 4
Minds Cards 4A, 4B
Digital Card 4
Games Cards 4A–D (★ and ★★)
Investigations Cards 4A, 4B, 4C

Interactive Whiteboard DVD 4
Visual Learning Bridges 4.1–4.9
Visual Learning Animations 4.1–4.9
Tools4Maths: Counters, Place-value Blocks

Teacher Resource DVD 4
Replay, Practice, Challenge Worksheets Year 4 Topic 4 lessons 1–9

AC Links
NA071, NA074, NA075, NA076, NA081, NA082

* SAB = Student Activity Book 4; TRB = Year 4 Teacher Resource Booklet 4

Ongoing assessment ● Diagnostic Post-assessment: Topic 4

Topic 5

Year 4 Contents Sequence

Angles and Shapes

Suggested time: 2 weeks

Topic

Diagnostic Pre-assessment: Topic 5

Lessons

Year 4 ✪ Topic 5 ✪ Angles and Shapes

Lesson		SAB* page	TRB* page
5.1	Describing Angles	68	12
5.2	Relating 2D Shapes and 3D Objects	70	14
5.3	Describing and Classifying Objects	72	16
5.4	Acting Out to Reason About 3D Objects	74	18
5.5	Identifying 2D Shapes in Other Shapes	76	20

Activity Zone 4
Minds Cards 5A, 5B
Digital Card 5
Games Cards 5A–D (★ and ★★)
Investigations Cards 5A, 5B, 5C

Interactive Whiteboard DVD 4
Visual Learning Bridges 5.1–5.5
Visual Learning Animations 5.1–5.5
Tools4Maths: Shapes and 3D Objects, Geometry

Teacher Resource DVD 4
Replay, Practice, Challenge Worksheets Year 4 Topic 5 lessons 1–5

AC Links
MG063, MG088, MG089

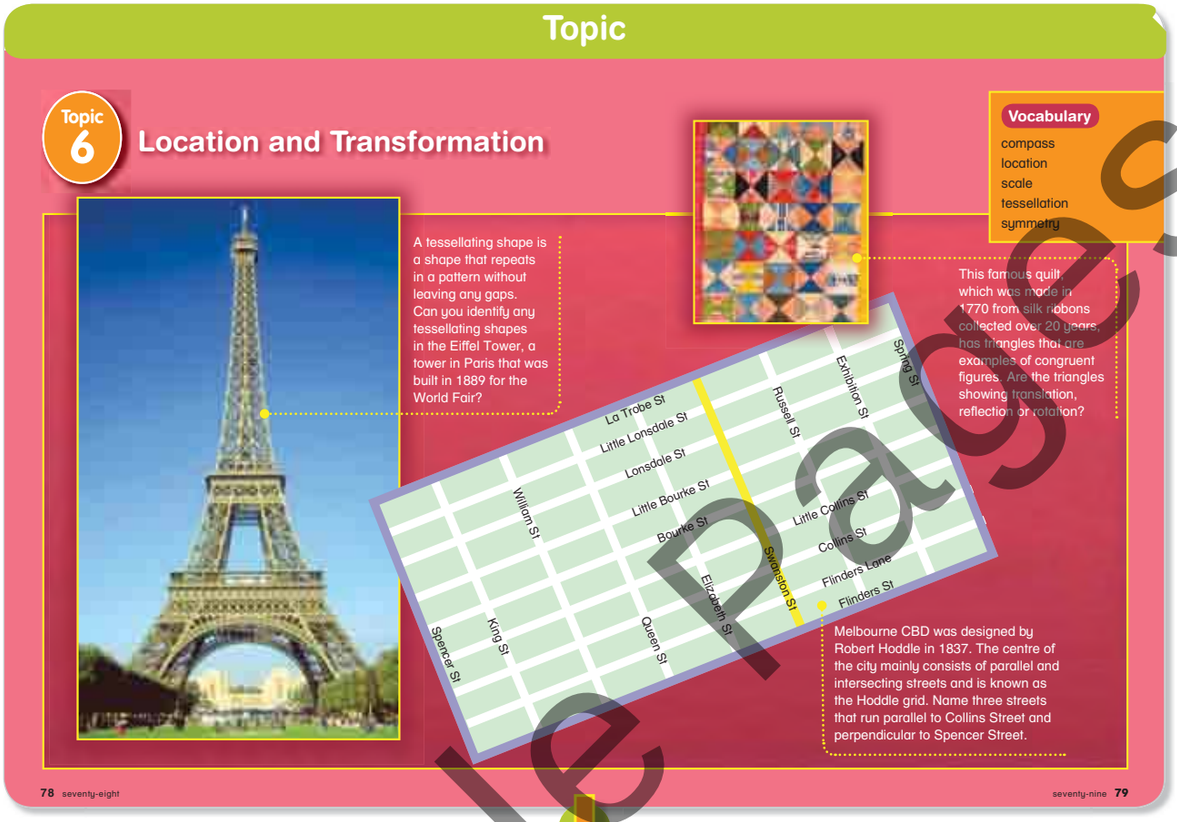
* SAB = Student Activity Book 4; TRB = Year 4 Teacher Resource Booklet 5

Ongoing assessment ● Diagnostic Post-assessment: Topic 5

Location and Transformation

Suggested time: 2 weeks

Topic



Diagnostic Pre-assessment: Topic 6

Lessons

Year 4 ✪ Topic 6 ✪ Location and Transformation

Lesson		SAB* page	TRB* page
6.1	Using Compass Directions	80	12
6.2	Using Compass Points to Describe Location	82	14
6.3	Understanding Scale Drawings	84	16
6.4	Drawing Shapes with Lines of Symmetry	86	18
6.5	Understanding Tessellating Shapes	88	20

Activity Zone 4
Minds Cards 6A, 6B
Digital Card 6
Games Cards 6A–D (★ and ★★)
Investigations Cards 6A, 6B, 6C

Interactive Whiteboard DVD 4
Visual Learning Bridges 6.1–6.5
Visual Learning Animations 6.1–6.5
Tools4Maths: Shapes and 3D Objects

Teacher Resource DVD 4
Replay, Practice, Challenge Worksheets Year 4 Topic 6 lessons 1–5

AC Links
MG066, MG090, MG091

* SAB = Student Activity Book 4; TRB = Year 4 Teacher Resource Booklet 6

Ongoing assessment ● Diagnostic Post-assessment: Topic 6

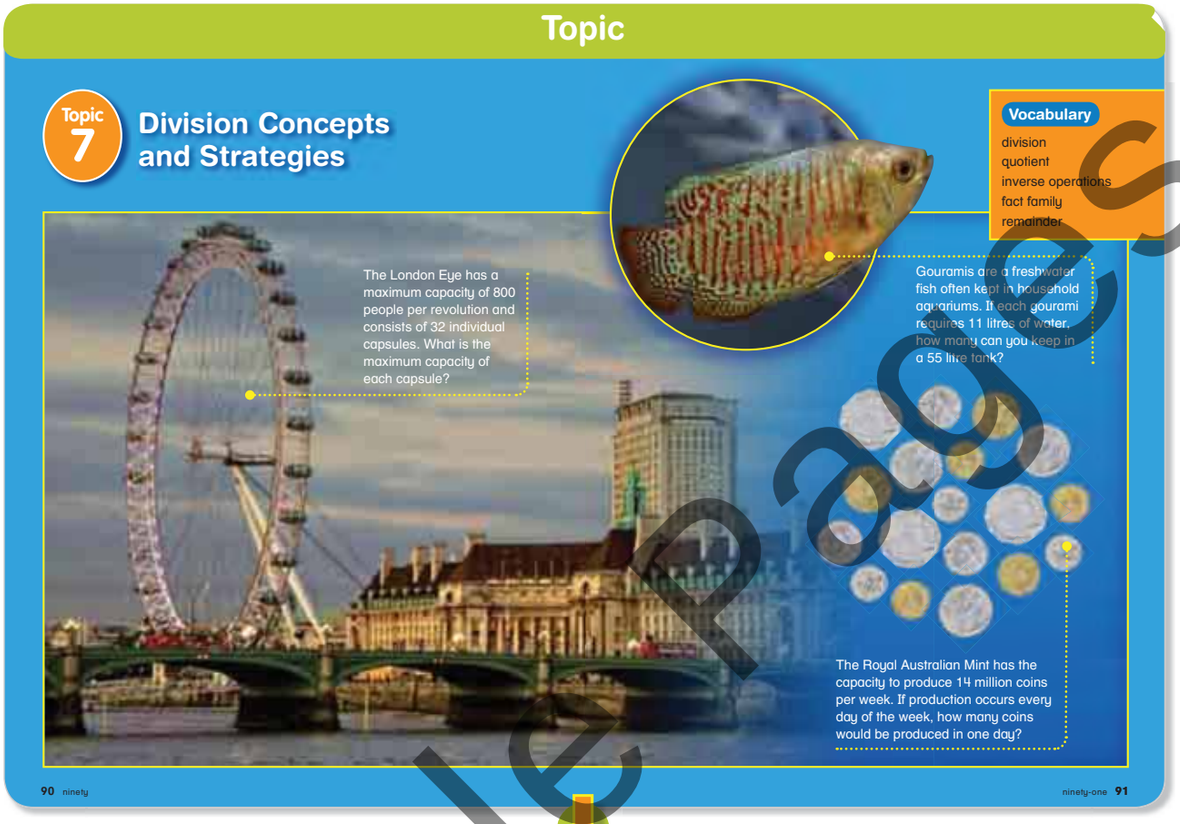
Topic 7

Year 4 Contents Sequence

Division Concepts and Strategies

Suggested time: 4 weeks

Topic



Diagnostic Pre-assessment: Topic 7

Lessons

Year 4 ✪ Topic 7 ✪ Division Concepts and Strategies

Lesson		SAB* page	TRB* page
7.1	Using Diagrams to Divide	92	12
7.2	Using a Table to Show Division	94	14
7.3	Using Materials to Divide 2-Digit Numbers	96	16
7.4	Relating Multiplication and Division Facts	98	18
7.5	Using Inverse Operations	100	20
7.6	Using Multiplication Facts to Find Division Facts	102	22
7.7	Using Mental Maths to Divide	104	24
7.8	Dividing Multiples of 10, 100 and 1 000	106	26
7.9	Dividing with 2-Digit Numbers	108	28
7.10	Dividing with Remainders	110	30

Activity Zone 4
Minds Cards 7A, 7B
Digital Card 7
Games Cards 7A–D (★ and ★★)
Investigations Cards 7A, 7B, 7C

Interactive Whiteboard DVD 4
Visual Learning Bridges 7.1–7.10
Visual Learning Animations 7.1 –7.10
Tools4Maths: Counters, Place-value Blocks

Teacher Resource DVD 4
Replay, Practice, Challenge Worksheets Year 4 Topic 7 lessons 1–10

AC Links
NA073, NA075, NA076, NA081, NA082, NA101, NA121

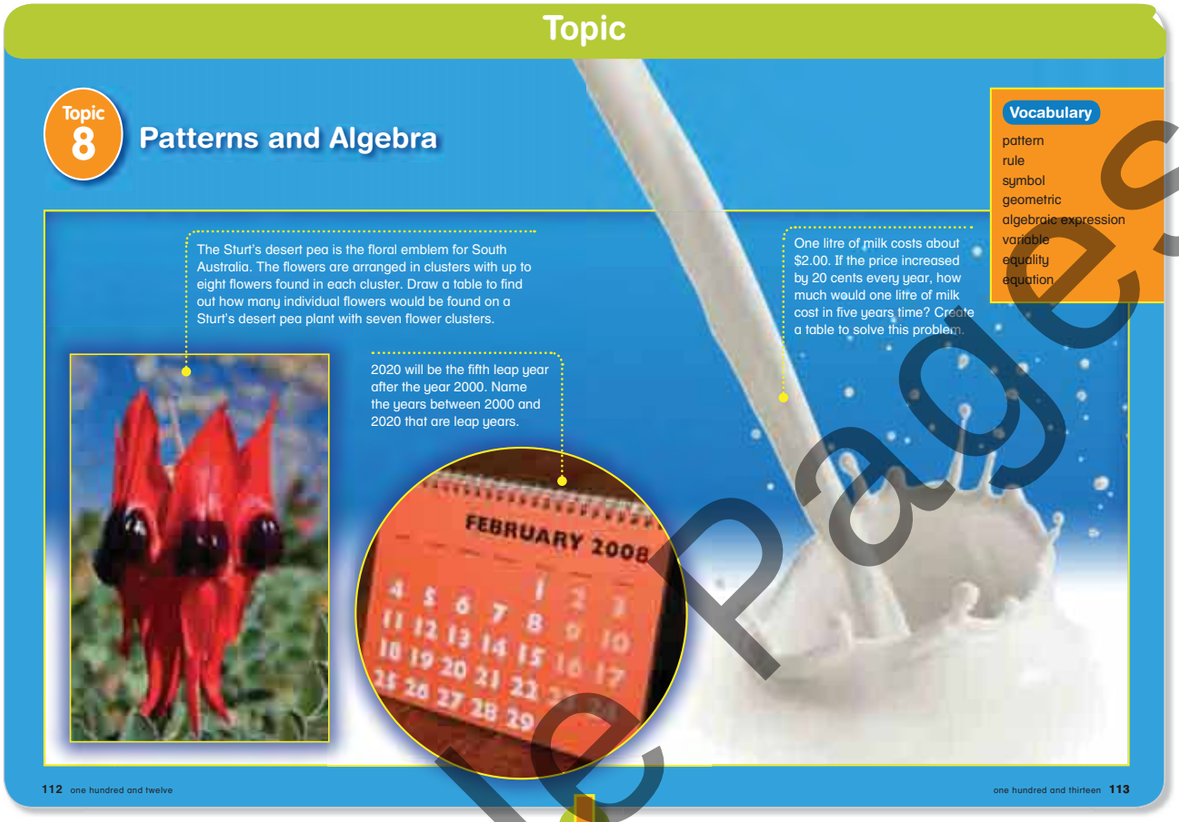
* SAB = Student Activity Book 4; TRB = Year 4 Teacher Resource Booklet 7

Ongoing assessment ● Diagnostic Post-assessment: Topic 7

Patterns and Algebra

Suggested time: 3.5 weeks

Topic



Diagnostic Pre-assessment: Topic 8

Lessons

Year 4 ✪ Topic 8 ✪ Patterns and Algebra

Lesson		SAB* page	TRB* page
8.1	Looking for a Pattern	114	12
8.2	Looking for a Pattern to Solve a Problem	116	14
8.3	Describing Geometric Patterns Using a Table	118	16
8.4	Using Expressions to Find Missing Numbers	120	18
8.5	Working Backwards to Solve Problems	122	20
8.6	Using Multiplication and Division Expressions	124	22
8.7	Finding Equality	126	24
8.8	Identifying Equal or Unequal Number Sentences	128	26

Activity Zone 4
Minds Cards 8A, 8B
Digital Card 8
Games Cards 8A–D (★ and ★★)
Investigations Cards 8A, 8B, 8C

Interactive Whiteboard DVD 4
Visual Learning Bridges 8.1–8.8
Visual Learning Animations 8.1 –8.8
Tools4Maths: Counters

Teacher Resource DVD 4
Replay, Practice, Challenge Worksheets Year 4 Topic 8 lessons 1–8

AC Links
NA074, NA076, NA081, NA082, NA083

* SAB = Student Activity Book 4; TRB = Year 4 Teacher Resource Booklet 8

Ongoing assessment ● Diagnostic Post-assessment: Topic 8

Topic 9

Year 4 Contents Sequence

Fractions and Decimals

Suggested time: 3.5 weeks

Topic

Diagnostic Pre-assessment: Topic 9

Lessons

Year 4 ✪ Topic 9 ✪ Fractions and Decimals

Lesson		SAB* page	TRB* page
9.1	Using Fractions to Name Parts of Wholes or Sets	132	12
9.2	Writing a Maths Explanation	134	14
9.3	Finding Equivalent Fractions	136	16
9.4	Finding More Equivalent Fractions	138	18
9.5	Defining Fractions	140	20
9.6	Using Improper Fractions and Mixed Numbers	142	22
9.7	Representing Tenths and Hundredths as Decimals	144	24
9.8	Writing Fractions and Decimals	146	26

Activity Zone 4
Minds Cards 9A, 9B
Digital Card 9
Games Cards 9A–D (★ and ★★)
Investigations Cards 9A, 9B, 9C

Interactive Whiteboard DVD 4
Visual Learning Bridges 9.1–9.8
Visual Learning Animations 9.1–9.8
Tools4Maths: Fractions

Teacher Resource DVD 4
Replay, Practice, Challenge Worksheets Year 4 Topic 9 lessons 1–8

AC Links
NA058, NA077, NA078, NA079

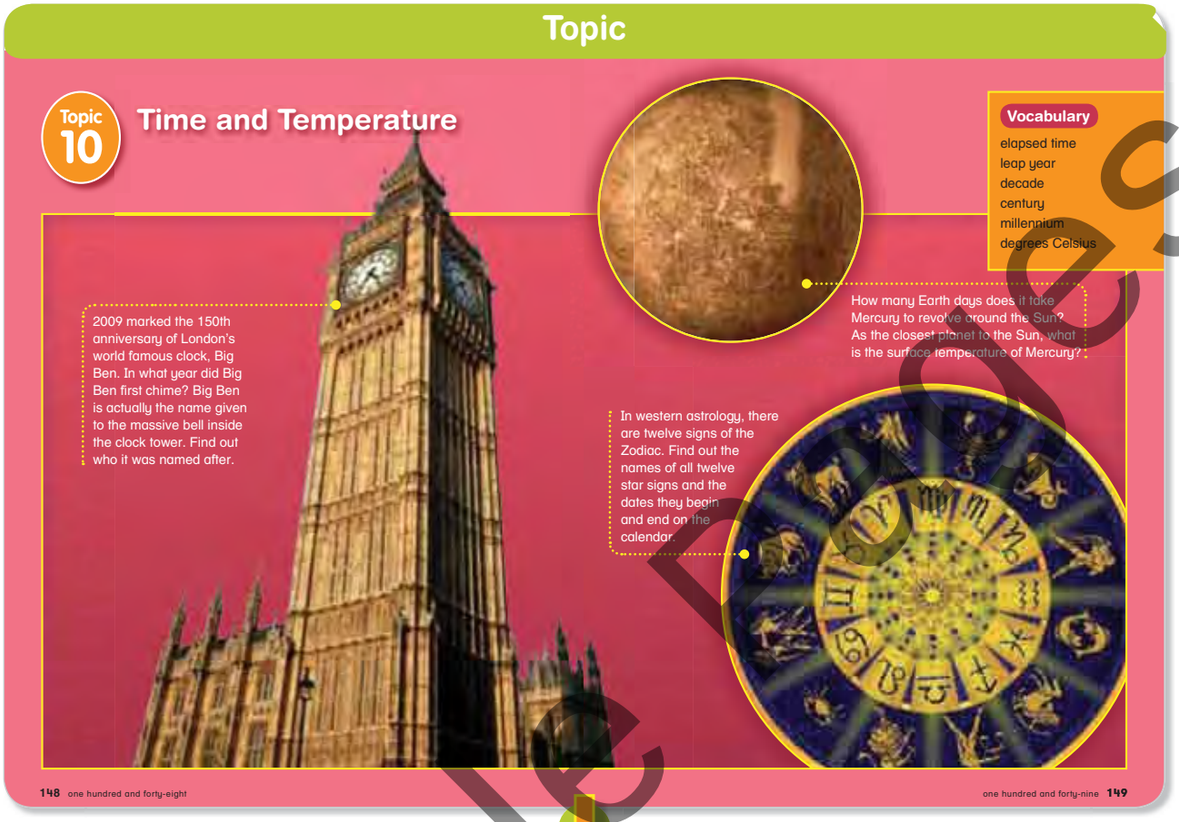
* SAB = Student Activity Book 4; TRB = Year 4 Teacher Resource Booklet 9

Ongoing assessment ● Diagnostic Post-assessment: Topic 9

Time and Temperature

Suggested time: 2.5 weeks

Topic



Diagnostic Pre-assessment: Topic 10

Lessons

Year 4 ✪ Topic 10 ✪ Time and Temperature

Lesson		SAB* page	TRB* page
10.1	Telling Time to the Minute	150	12
10.2	Relating Units of Time	152	14
10.3	Comparing Units of Time	154	16
10.4	Finding Elapsed Time	156	18
10.5	Working Backwards to Solve Time Problems	158	20
10.6	Understanding Temperature	160	22

Activity Zone 4
Minds Cards 10A, 10B
Digital Card 10
Games Cards 10A–D (★ and ★★)
Investigations Cards 10A, 10B, 10C

Interactive Whiteboard DVD 4
Visual Learning Bridges 10.1–10.6
Visual Learning Animations 10.1–10.6
Tools4Maths: Time, Measurement

Teacher Resource DVD 4
Replay, Practice, Challenge Worksheets Year 4 Topic 10 lessons 1–6

AC Links
MG062, MG084, MG085, MG086

* SAB = Student Activity Book 4; TRB = Year 4 Teacher Resource Booklet 10

Ongoing assessment ● Diagnostic Post-assessment: Topic 10

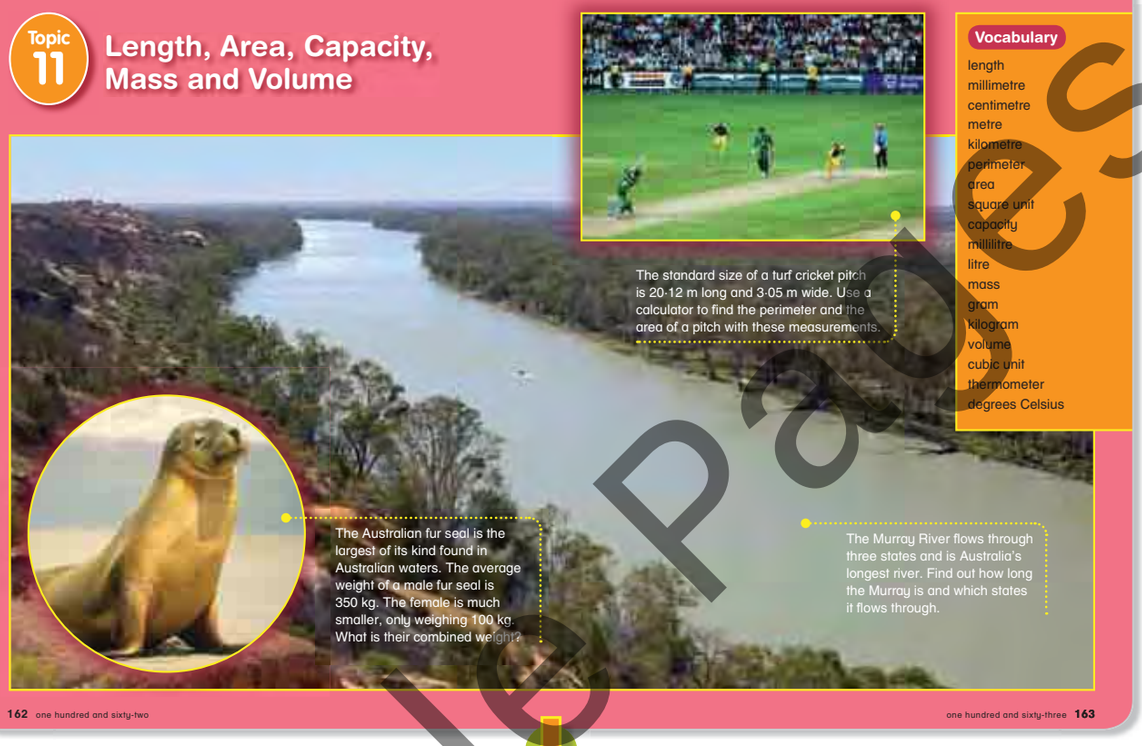
Topic 11

Year 4 Contents Sequence

Length, Area, Capacity, Mass and Volume

Suggested time: 4 weeks

Topic



Diagnostic Pre-assessment: Topic 11

Lessons

Year 4 ✪ Topic 11 ✪ Length, Area, Capacity, Mass and Volume

Lesson		SAB* page	TRB* page
11.1	Measuring with Formal Units of Length	164	12
11.2	Finding Perimeter	166	14
11.3	Identifying Different Shapes with the Same Perimeter	168	16
11.4	Understanding Area	170	18
11.5	Estimating and Measuring Area	172	20
11.6	Estimating and Measuring with Millilitres and Litres	174	21
11.7	Estimating and Measuring with Grams and Kilograms	176	22
11.8	Introducing Volume	178	24
11.9	Enlarging and Reducing Measurements	180	26
11.10	Reading Scales	182	28

Activity Zone 4
Minds Cards 11A, 11B
Digital Card 11
Games Cards 11A–D (★ and ★★)
Investigations Cards 11A, 11B, 11C

Interactive Whiteboard DVD 4
Visual Learning Bridges 11.1–11.10
Visual Learning Animations 11.1–11.10
Tools4Maths: Measurement

Teacher Resource DVD 4
Replay, Practice, Challenge Worksheets Year 4 Topic 11 lessons 1–10

AC Links
MG084, MG087, MG290, MG109

* SAB = Student Activity Book 4; TRB = Year 4 Teacher Resource Booklet 11

Ongoing assessment ● Diagnostic Post-assessment: Topic 11

Data, Graphs and Probability

Suggested time: 4 weeks

Topic

Diagnostic Pre-assessment: Topic 12

Lessons

Year 4 ✪ Topic 12 ✪ Data, Graphs and Probability

Lesson		SAB* page	TRB* page
12.1	Using Surveys	186	12
12.2	Organising Data	188	14
12.3	Reading Pie Graphs	190	16
12.4	Making a Graph	192	18
12.5	Understanding Venn Diagrams	194	20
12.6	Using Reasoning to Draw Conclusions	196	22
12.7	Predicting and Comparing Outcomes	198	24
12.8	Identifying Related Events	200	26
12.9	Identifying Independent Events	202	28

Activity Zone 4
Minds Cards 12A, 12B
Digital Card 12
Games Cards 12A–D (★ and ★★)
Investigations Cards 12A, 12B, 12C

Interactive Whiteboard DVD 4
Visual Learning Bridges 12.1–12.9
Visual Learning Animations 12.1–12.9
Tools4Maths: Graphs, Probability

Teacher Resource DVD 4
Replay, Practice, Challenge Worksheets Year 4 Topic 12 lessons 1–9

AC Links
SP092, SP093, SP094, SP095, SP096, SP097

* SAB = Student Activity Book 4; TRB = Year 4 Teacher Resource Booklet 12

Ongoing assessment ● Diagnostic Post-assessment: Topic 12

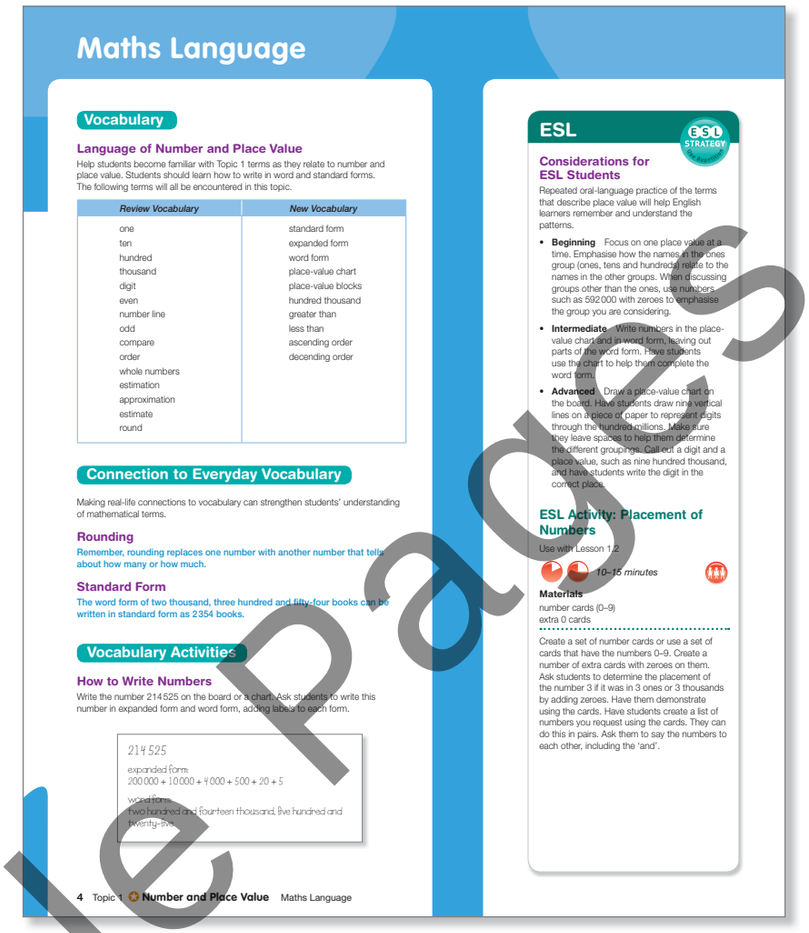
Literacy and Language

enVisionMATHS places a strong emphasis on mathematical language and terminology. The program allows for literacy to be embedded within the mathematical learning and also provides advice on introducing and explaining terminology and vocabulary to ESL students.

Level-appropriate lists of vocabulary are provided for each topic on the Student Activity Book topic-opener pages and on page 4 of the Teacher Resource Booklets. The vocabulary for year 4 has been compiled alphabetically below for easy reference.

Students can record all new maths language in the My Maths Language pages in the Maths Thinking Skills Book.

'Research says mathematics is like a language and some techniques used to learn language can be used to learn the language of maths' (Paris & Cunningham, 1996).



Maths Language

Vocabulary

Language of Number and Place Value

Help students become familiar with Topic 1 terms as they relate to number and place value. Students should learn how to write in word and standard forms. The following terms will all be encountered in this topic.

Review Vocabulary	New Vocabulary
one	standard form
ten	expanded form
hundred	word form
thousand	place-value chart
digit	place-value blocks
even	hundred thousand
number line	greater than
odd	less than
compare	ascending order
order	decending order
whole numbers	
estimation	
approximation	
estimate	
round	

Connection to Everyday Vocabulary

Making real-life connections to vocabulary can strengthen students' understanding of mathematical terms.

Rounding

Remember, rounding replaces one number with another number that tells about how many or how much.

Standard Form

The word form of two thousand, three hundred and fifty-four books can be written in standard form as 2354 books.

Vocabulary Activities

How to Write Numbers

Write the number 214 525 on the board or a chart. Ask students to write this number in expanded form and word form, adding labels to each form.

214 525

expanded form
200 000 + 10 000 + 4 000 + 500 + 20 + 5

word form
two hundred and fourteen thousand, five hundred and twenty-five

4 Topic 1 Number and Place Value Maths Language

ESL

ESL STRATEGY

Considerations for ESL Students

Repeated oral-language practice of the terms that describe place value will help English learners remember and understand the patterns.

- **Beginning** Focus on one place value at a time. Emphasise how the names in the ones group (ones, tens and hundreds) relate to the names in the other groups. When discussing groups other than the ones, use numbers such as 592 000 with zeroes to emphasise the group you are considering.
- **Intermediate** Write numbers in the place-value chart and in word form, leaving out parts of the word form. Have students use the chart to help them complete the word form.
- **Advanced** Draw a place-value chart on the board. Have students draw nine vertical lines on a piece of paper to represent digits through the hundred millions. Make sure they leave spaces to help them determine the different groups. Call out a digit and a place value, such as nine hundred thousand, and have students write the digit in the correct place.

ESL Activity: Placement of Numbers

Use with Lesson 1.2

10–15 minutes

Materials

number cards (0–9)
extra 0 cards

Create a set of number cards or use a set of cards that have the numbers 0–9. Create a number of extra cards with zeroes on them. Ask students to determine the placement of the number 3 if it was in 3 ones or 3 thousands by adding zeroes. Have them demonstrate using the cards. Have students create a list of numbers you request using the cards. They can do this in pairs. Ask them to say the numbers to each other, including the 'and'.

Year 4 Vocabulary

2D shape	cubic unit	fact family	metre	quotient	sum
3D object	cylinder	factor	millennium	ray	survey
acute angle	data	fraction	millilitre	reasonable	symbol
algebraic expression	decade	geometric	millimetre	rectangular prism	symmetry
angle	decimal	gram	mixed number	rectangular pyramid	tally chart
area	degrees Celsius	greater than	multiple	regroup	tenth
array	denominator	hundred chart	multiplication	regrouping	tessellation
ascending order	descending order	hundredth	numerator	remainder	thermometer
bar graph	difference	hundred thousand	obtuse angle	repeated addition	thousand
benchmark fraction	digit	improper fraction	outcome	results	triangular prism
bridging	division	inverse operations	partitioning	right angle	variable
calculate	doubles	kilogram	pattern	rule	Venn diagram
capacity	edge	kilometre	perimeter	scale	vertex
centimetre	elapsed time	leap year	perpendicular	simplest form	vertices
century	equality	length	pie graph	sphere	volume
compass	equation	less than	place-value blocks	splitting	whole numbers
compatible numbers	equivalent fraction	litre	place-value chart	square pyramid	word form
cone	expanded form	location	predict	square unit	
cube	face	mass	product	standard form	

Materials

The following table lists the concrete materials you may need to use during the program for this year. To assist you in pre-preparation, the brackets indicate how many of each item you may need throughout the year, based on a class size of 25 students.

- ☐ atlas
- ☐ blocks
- ☐ bundling sticks (500)
- ☐ calculators (25)
- ☐ calendars (25)
- ☐ cardboard
- ☐ centimetre cubes
- ☐ centimetre grid paper
- ☐ clock faces or play clocks (25)
- ☐ coloured pencils
- ☐ compasses (25)
- ☐ construction paper
- ☐ counters in several colours (1 000 of at least 2 colours)
- ☐ cubes (such as Unifix)
- ☐ demonstration analogue clock face (1)
- ☐ demonstration digital clock face (1)
- ☐ dice (50)
- ☐ drinking straws (500)
- ☐ examples of Australian coins and notes
- ☐ felt pens
- ☐ geoboards (25)
- ☐ fraction strips (50)
- ☐ hundred charts
- ☐ index cards (200)
- ☐ large map of Australia
- ☐ magazines/newspapers
- ☐ measuring equipment
- ☐ metre sticks (15)
- ☐ money (real and play)
- ☐ nets for 3D objects
- ☐ number cards 0–20 (25 sets)
- ☐ number cubes numbered 1–6 (10)
- ☐ number lines
- ☐ number tiles 0–9 (25 sets)
- ☐ paper (or cloth) bags to hold tiles (100)
- ☐ paperclips (large and small)
- ☐ pattern blocks
- ☐ pipecleaners
- ☐ place-value blocks
- ☐ place-value charts
- ☐ rubber bands
- ☐ rulers (25)
- ☐ scissors (25)
- ☐ spinners (25)
- ☐ sticky tape
- ☐ string
- ☐ thermometers (25)
- ☐ tracing paper
- ☐ two-colour counters (500)
- ☐ water
- ☐ water bottles

ISBN 978 1 4425 3018 8

The Multi-age Classroom

enVisionMATHS is designed to allow for differentiation at all stages and provides resources that allow the planning of pathways for different ages and levels within the same classroom.

- **Skills Trace** in the Teacher Resource Booklet gives advice on which lessons from the preceding and following year levels relate to the topic.
- **Topics** starting with general knowledge and vocabulary make students think about the maths and the 'why' of the strategies, making real-life connections.
- Open-ended **problem solving**, catering to a variety of skill levels, is provided in every lesson.
- Topic-based **Activity Zone Cards** present differentiated learning activities:
  - **Investigations Cards**—progressively more advanced inquiry activities
  - **Games Cards**—provided at two different levels
  - **Minds Cards**—maths computation strategies and quizzes with sets of questions increasing in complexity
  - **Digital Cards**—allow students to work at their own pace with Tools4Maths.
- **Differentiated Worksheets** on the Teacher Resource DVD provide three levels of activity to cover replays, practices and challenges.
- **Individual needs** activity ideas and strategies
- **ESL needs** strategies and activity ideas
- **Error intervention** tips
- **Extension** ideas

enVisionMATHS lends itself to an inquiry-based classroom through the topic opener pages and investigations that encourage cross-curricular and interdisciplinary study. The abundance of separate resources such as Activity Zone cards, Visual Learning Animations (VLAs), Differentiated Worksheets and Tools4Maths can then be used with different age and ability groups as required. The Teacher Resource DVD, which covers three year level components, can provide the means to plan for this. To aid in planning, the program provides for year 4:

- Pre- and Post-assessments for years 3, 4 and 5
- VLBs and planning documents for years 3, 4 and 5
- Differentiated Worksheets for years 3, 4 and 5
- colour-coded strands throughout the program
- easy-to-use sections of text in the Teacher Resource Booklets that are short but colourful and informative.

Sample Pathways

In a multi-age classroom, the teacher could organise the class to reflect a guided reading session with students rotating around activities, including teacher focus groups. The teacher could show the younger students the VLA and instruct them, while older students do an Investigation Card or other Activity Zone activity. Then when the younger students are set to work on student books, the older students watch the VLA.

Some sample pathways are shown on the following pages.

The Multi-age Classroom

Year 3/4 Classroom

Year 4 Topic 1 ✪ Number and Place Value ✪ Lesson 3: Comparing and Ordering Whole Numbers

Skills Trace

Find a year 3 lesson that complements the year 4 lesson according to the ability mix within the class.

Looking Back ✪ Year 3 Lessons

Topic 1: Number and Place Value

- 1.1 Reading and Writing Hundreds
- 1.2 Reading and Writing Numbers to 1 000
- 1.3 Building Numbers Beyond 1 000
- 1.5 Using Clues to Identify Numbers
- 1.6 Using Place Value to Add and Subtract
- 1.7 Ordering Three Numbers
- 1.8 Comparing Numbers
- 1.9 Rounding Whole Numbers

Year 4 ✪ Topic 1 Lessons

- 1.1 Reading and Writing Thousands
- 1.2 Reading and Writing Larger Numbers
- 1.3 Comparing and Ordering Whole Numbers
- 1.4 Rounding Whole Numbers

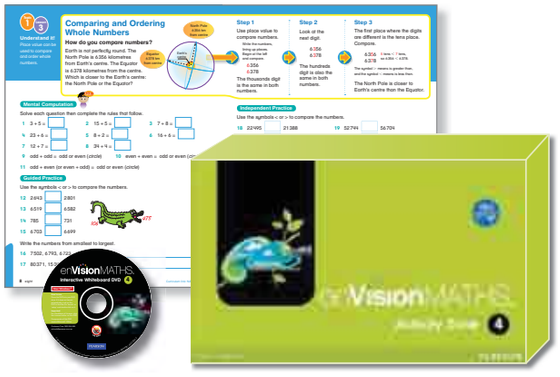
Looking Ahead ✪ Year 5 Lessons

Topic 1: Number and Place Value

- 1.1 Representing Thousands in Different Ways
- 1.2 Representing Millions in Different Ways
- 1.3 Comparing and Ordering Whole Numbers

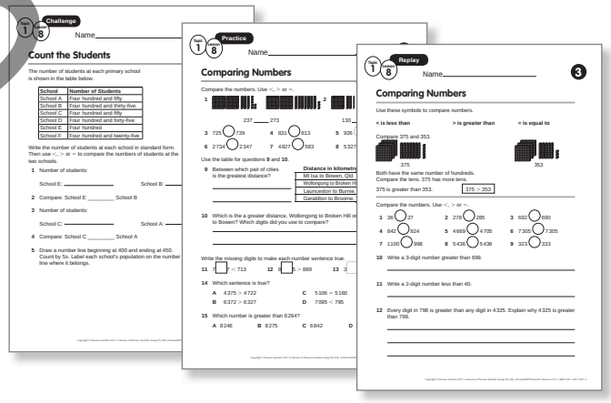
VLB/VLA Groups

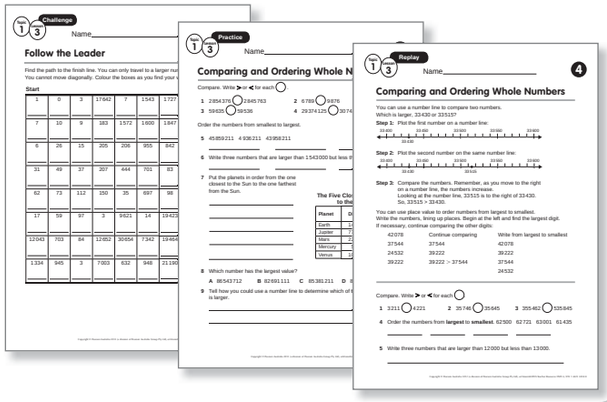
Show year 3 VLA 1.8 to the whole class, then break into ability/age groups to use year 4 VLA 1.3, SAB year 4 lesson page 1.3, SAB year 3 lesson page 1.8, and year 3 and year 4 topic 1 Activity Zone cards, with or without supervision as appropriate.



Differentiated Worksheets

For extra class practice or homework: Replay, Practice or Challenge worksheets for year 3 lesson 1.8 or year 4 lesson 1.3 as appropriate.





Year 4/5 Classroom

Year 4 Topic 5 ✪ Angles and Shapes ✪ Lesson 5: Identifying 2D Shapes in Other Shapes

Skills Trace

Find a year 5 lesson that complements the year 4 lesson according to the ability mix within the class.

Looking Back ✪ Year 3 Lessons

Topic 6: Angles and Shapes

- 6.1 Understanding Lines and Line Segments
- 6.2 Introducing Angles
- 6.3 Identifying Polygons
- 6.4 Making Generalisations About Polygons
- 6.5 Identifying 3D Objects
- 6.6 Identifying Faces on 3D Objects
- 6.7 Relating 2D Shapes and 3D Objects

Year 4 ✪ Topic 6 Lessons

- 5.1 Describing Angles
- 5.2 Relating 2D Shapes and 3D Objects
- 5.3 Describing and Classifying Objects
- 5.4 Acting Out to Reason About 3D Objects
- 5.5 Identifying 2D Shapes in Other Shapes

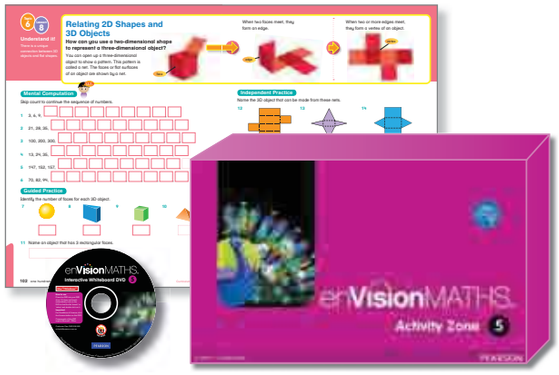
Looking Ahead ✪ Year 5 Lessons

Topic 6: Angles and Shapes

- 6.1 Understanding Lines and Angles
- 6.2 Measuring Angles
- 6.3 Classifying Triangles
- 6.4 Classifying Quadrilaterals
- 6.5 Making and Testing Generalisations about Rectangles
- 6.6 Identifying 3D Objects
- 6.7 Understanding Polyhedrons
- 6.8 Relating 2D Shapes and 3D Objects
- 6.10 Viewing Perspectives of 3D Objects

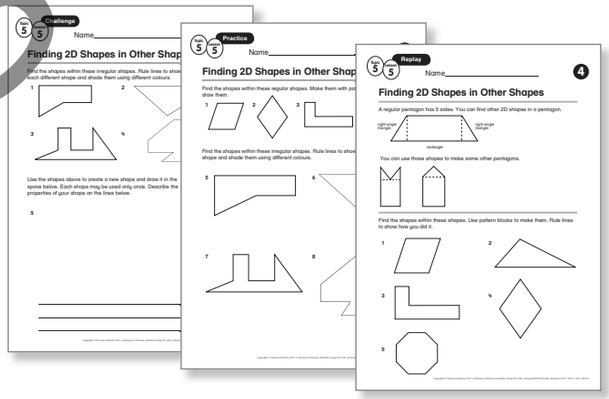
VLB/VLA Groups

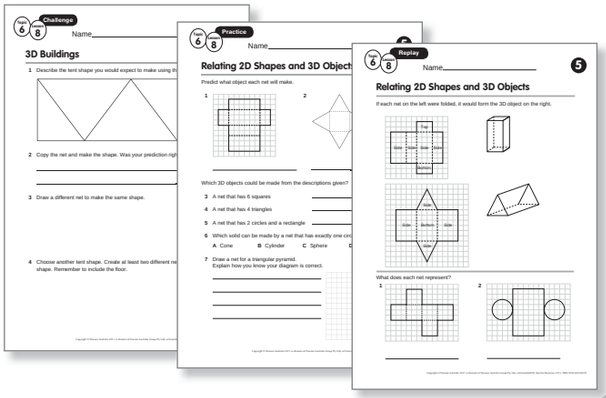
Show year 4 VLA 5.5 to the whole class, then break into ability/age groups to use year 5 Video topic 6, SAB year 4 lesson page 5.5, SAB year 5 lesson page 6.8, year 4 topic 5 Activity Zone cards and year 5 topic 6 Activity Zone cards, with or without supervision as appropriate.



Differentiated Worksheets

For extra class practice or homework: Replay, Practice or Challenge worksheets for year 4 lesson 5.5 or year 5 lesson 6.8 as appropriate.





Teacher Resource Booklet

A teacher starts a topic using their set of **enVisionMATHS** Teacher Resource Booklets to assist in planning and assessment. The Maths Background section in the Teacher Resource Booklet enhances the professional development of generalist teachers and their implementation of each topic.

Each booklet contains reduced-sized copies of all components for the topic so that when working away from school, teachers need only take a lightweight booklet with them for their planning.

There is one Teacher Resource Booklet for each year 4 topic.

Skills Trace and Suggested Teaching Sequence

Looking Back: year 3 Lessons: signposts to earlier to lessons that have dealt with year 4 topics or which provide an introduction or good basis for the year 4 topic. These lessons may be useful for extra reinforcement of the topic basics.

Looking Ahead: year 5 Lessons: signposts to later lessons that deal with year 4 topics. These lessons may be useful for more able students who require further challenges.

Topic 12 Data, Graphs and Probability

Skills Trace

Looking Back ✪ Year 3 Lessons

Topic 12: Data, Graphs and Chance

- 12.1 Organising Data
- 12.2 Posing Questions to Collect Data
- 12.3 Reading Picture Graphs and Bar Graphs
- 12.4 Making Bar Graphs
- 12.5 Using Tables to Construct Bar Graphs
- 12.6 Using Tables and Graphs to Draw Conclusions
- 12.7 Predicting and Comparing Outcomes
- 12.8 Comparing Outcomes with Predictions
- 12.9 Finding Combinations

Year 4 ✪ Topic 12 Lessons

- 12.1 Using Surveys
- 12.2 Organising Data
- 12.3 Reading Pie Graphs
- 12.4 Making a Graph
- 12.5 Understanding Venn Diagrams
- 12.6 Using Reasoning to Draw Conclusions
- 12.7 Predicting and Comparing Outcomes
- 12.8 Identifying Related Events
- 12.9 Identifying Independent Events

Looking Ahead ✪ Year 5 Lessons

Topic 12: Data, Graphs and Chance

- 12.1 Displaying Data From Surveys
- 12.2 Selecting Samples for Surveys
- 12.3 Interpreting Graphs
- 13.4 Making and Interpreting Bar and Picture Graphs
- 12.5 Using Line Graphs
- 12.6 Interpreting Line Graphs

Suggested Teaching Sequence

Topic

- Teacher planning (using pages 2–11 of this booklet)
- Use Topic Opener to introduce topic (page 2)
- Pre-assessment given to students (pages 31–33)

Lessons

- Introduce each lesson by setting the purpose
- Make connections to students' previous learning (connect)
- Watch the Visual Learning Animation (VLA) and show students the Visual Learning Bridge (VLB) on the IWB or at the top of the relevant lesson page in the Student Activity Book
- Whole-class teaching focus consolidating the concept and including problem solving
- Students complete activities in the Student Activity Book
- Small-group work with students who may need further instruction (error intervention, extension); rest of class work in groups on differentiated learning centre activities from the Activity Zone that are appropriate to their level (refer to pages 6–10 of this booklet), recording their findings in the Maths Thinking Skills Book
- Whole-class reflection
- Students record reflections in the Maths Thinking Skills Book
- Differentiated worksheets used for extra practice at home or school

Assessment

- Ongoing and throughout using pages 30–36, including post-assessment (pages 34–36), observations and recorded work in the Maths Thinking Skills Book

Note

Blue text = suggested question/language for teachers to use

Pink text = answer/solution

Suggested Teaching Sequence: a best-practice example for the use of **enVisionMATHS** in the classroom. Teachers are free to adapt this sequence to best suit their classes.

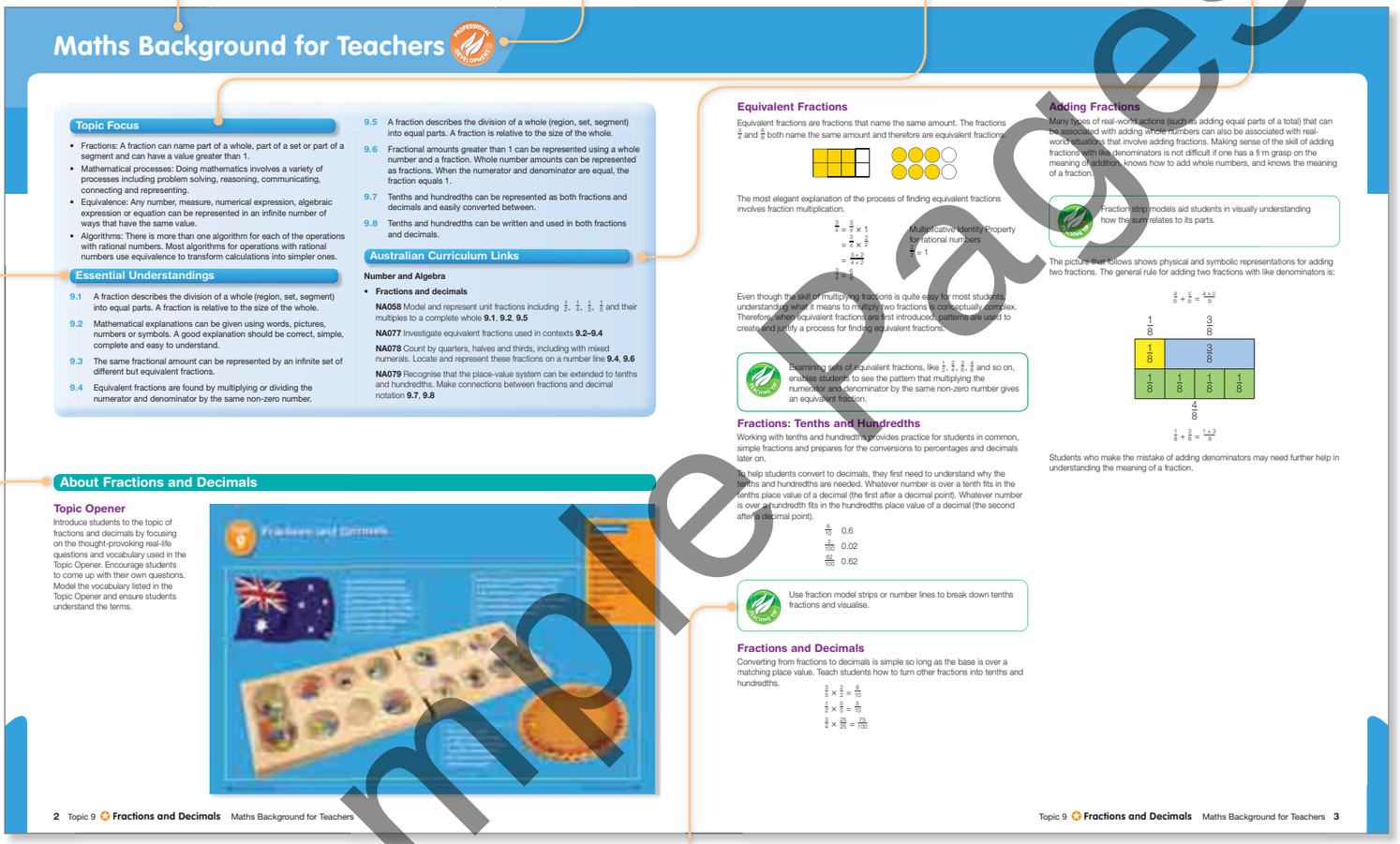
Maths Background for Teachers

Maths Background for Teachers: topic-focused information designed as professional development for teachers, to fill in gaps in knowledge or to provide teaching tips for the topic.

Professional development logo: indicates material designed for professional development.

Topic Focus: a reminder of the general focus for this topic.

AC Links: a quick reference relating each lesson to the relevant content description of the Australian Curriculum: Mathematics.



Maths Background for Teachers

Topic Focus

- Fractions: A fraction can name part of a whole, part of a set or part of a segment and can have a value greater than 1.
- Mathematical processes: Doing mathematics involves a variety of processes including problem solving, reasoning, communicating, connecting and representing.
- Equivalence: Any number, measure, numerical expression, algebraic expression or equation can be represented in an infinite number of ways that have the same value.
- Algorithms: There is more than one algorithm for each of the operations with rational numbers. Most algorithms for operations with rational numbers use equivalence to transform calculations into simpler ones.

Essential Understandings

9.1 A fraction describes the division of a whole (region, set, segment) into equal parts. A fraction is relative to the size of the whole.

9.2 Mathematical explanations can be given using words, pictures, numbers or symbols. A good explanation should be correct, simple, complete and easy to understand.

9.3 The same fractional amount can be represented by an infinite set of different but equivalent fractions.

9.4 Equivalent fractions are found by multiplying or dividing the numerator and denominator by the same non-zero number.

9.5 A fraction describes the division of a whole (region, set, segment) into equal parts. A fraction is relative to the size of the whole.

9.6 Fractional amounts greater than 1 can be represented using a whole number and a fraction. Whole number amounts can be represented as fractions. When the numerator and denominator are equal, the fraction equals 1.

9.7 Tenths and hundredths can be represented as both fractions and decimals and easily converted between.

9.8 Tenths and hundredths can be written and used in both fractions and decimals.

Australian Curriculum Links

Number and Algebra

- **Fractions and decimals**

NA058 Model and represent unit fractions including $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{3}$, $\frac{1}{5}$ and their multiples to a complete whole **9.1**, **9.2**, **9.5**

NA077 Investigate equivalent fractions used in contexts **9.2–9.4**

NA078 Count by quarters, halves and thirds, including with mixed numerals. Locate and represent these fractions on a number line **9.4**, **9.6**

NA079 Recognise that the place-value system can be extended to tenths and hundredths. Make connections between fractions and decimal notation **9.7**, **9.8**

About Fractions and Decimals

Topic Opener

Introduce students to the topic of fractions and decimals by focusing on the thought-provoking real-life questions and vocabulary used in the Topic Opener. Encourage students to come up with their own questions. Model the vocabulary listed in the Topic Opener and ensure students understand the terms.

Equivalent Fractions

Equivalent fractions are fractions that name the same amount. The fractions $\frac{1}{4}$ and $\frac{2}{8}$ both name the same amount and therefore are equivalent fractions.

The most elegant explanation of the process of finding equivalent fractions involves fraction multiplication.

$\frac{a}{b} = \frac{a}{b} \times 1$
$= \frac{a}{b} \times \frac{c}{c}$
$= \frac{a \times c}{b \times c}$
$= \frac{ac}{bc}$

Multiplicative Identity Property for rational numbers
$\frac{c}{c} = 1$

Even though the skill of multiplying fractions is quite easy for most students, understanding what it means to multiply two fractions is conceptually complex. Therefore, when equivalent fractions are first introduced, patterns are used to create and justify a process for finding equivalent fractions.

Examining sets of equivalent fractions, like $\frac{1}{2}$, $\frac{2}{4}$, $\frac{3}{6}$, $\frac{4}{8}$ and so on, enables students to see the pattern that multiplying the numerator and denominator by the same non-zero number gives an equivalent fraction.

Fractions: Tenths and Hundredths

Working with tenths and hundredths provides practice for students in common, simple fractions and prepares for the conversions to percentages and decimals later on.

To help students convert to decimals, they first need to understand why the tenths and hundredths are needed. Whatever number is over a tenth fits in the tenths place value of a decimal (the first after a decimal point). Whatever number is over a hundredth fits in the hundredths place value of a decimal (the second after a decimal point).

$\frac{6}{10}$ 0.6
$\frac{2}{100}$ 0.02
$\frac{62}{100}$ 0.62

Use fraction model strips or number lines to break down tenths fractions and visualise.

Fractions and Decimals

Converting from fractions to decimals is simple so long as the base is over a matching place value. Teach students how to turn other fractions into tenths and hundredths.

$\frac{3}{5} \times \frac{2}{2} = \frac{6}{10}$
$\frac{1}{2} \times \frac{5}{5} = \frac{5}{10}$
$\frac{2}{4} \times \frac{25}{25} = \frac{75}{100}$

Adding Fractions

Many types of real-world actions (such as adding equal parts of a total) that can be associated with adding whole numbers can also be associated with real-world equations that involve adding fractions. Making sense of the skill of adding fractions with like denominators is not difficult if one has a firm grasp on the meaning of addition, knows how to add whole numbers, and knows the meaning of a fraction.

Fraction strip models aid students in visually understanding how the sum relates to its parts.

The picture that follows shows physical and symbolic representations for adding two fractions. The general rule for adding two fractions with like denominators is:

$\frac{a}{b} + \frac{c}{b} = \frac{a+c}{b}$

$\frac{1}{10} + \frac{3}{10} = \frac{1+3}{10}$

Students who make the mistake of adding denominators may need further help in understanding the meaning of a fraction.

2 Topic 9 Fractions and Decimals Maths Background for Teachers

Topic 9 Fractions and Decimals Maths Background for Teachers 3

About [the topic]: further information about the topic for both teachers and students, including suggested teaching strategies.

Teaching Tip logo: indicates specific quick teaching tips.

Essential Understandings: the specific understandings related to each lesson in this topic.

'Research says that teaching for understanding results in better performance that lasts longer' (Pesek & Kirshner, 2000).

Maths Language/Meeting Individual Needs

Vocabulary: a list of the specific new and review vocabulary that should be used within this topic, and suggestions for encouraging students to connect with this vocabulary.

ESL: strategies for encouraging effective vocabulary understandings for ESL students.

Additional Needs: strategies to suit additional needs students, emerging-level students and extending-level students.

Maths Language

Vocabulary

Language of Shapes and 3D Objects

Help students become familiar with Topic 5 terms as they relate to angles and shapes. The following terms will all be encountered in this topic.

Review Vocabulary	*New Vocabulary*
3D object	object
sphere	net
cylinder	edge
cone	face
cube	vertex
	vertices
	rectangular prism
	triangular prism
	rectangular pyramid
	square pyramid
	ray
	angle
	right angle
	acute angle
	obtuse angle
	perpendicular

Connection to Everyday Vocabulary

Making real-life connections to vocabulary can strengthen students' understanding of mathematical terms.

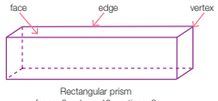
3D Objects All Around

Examples of 3D objects can be seen all around. The pyramids of Egypt are a very large example of one type of 3D object, while an ice-cream cone is an example of a 3D object you can hold in your hand.

Vocabulary Activities

Identifying 3D Objects

Give students drawings of various 3D objects and have them count the faces, edges and vertices of each and label an example of each term. Then have them identify what type of object it is.



Rectangular prism
faces: 6; edges: 12; vertices: 8

Meeting Individual Needs

ESL

Considerations for ESL Students

Repeated oral language practice with the names of 2D shapes, the way they go together and their properties will help English learners remember and understand the shapes.

- **Beginning** Write the words for the different shapes explored in this unit on index cards. Draw matching pictures on separate index cards. Have students match the words with the pictures.
- **Intermediate** Have students do the above activity and say the words aloud.
- **Advanced** Give students just the name of the shape or angle and have them draw them. And then vice versa: give them a picture of the shape and have them define it.

ESL Activity: Identifying 3D Shapes from Their 2D-Shape Prism Base

Use with Lesson 5.5

10–15 minutes

- To help students understand that a prism is based on a 2D shape that has been 'stretched' (and that is why they are named accordingly), show them examples of a:
  - rectangle
  - square
  - triangle
  - circle.

  These can be drawn or shown as a net.
- Then show examples of a:
  - rectangular-based prism
  - cube (though call it a square-based prism for familiarity and then later introduce special terms)
  - triangular-based prism (use the same shape triangle as the 2D example)
  - cylinder (though call it a circle-based prism, even though this is strictly not its name).

Additional Needs

Considerations for Additional Needs Students

- Some students may understand 3D objects better if architectural elements are used to reinforce their recognition of shapes.
- Have students research and find two different 3D objects used for buildings.

Additional Needs Activity: Triangles and Side Lengths

Use with Lesson 5.1

10–15 minutes

Materials

drinking straws
scissors
glue
sheets of construction paper (per pair)

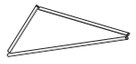
- Write the following sentences on the board. Ask students to copy each description onto a separate sheet of construction paper.

  A triangle with all sides the same length is an equilateral triangle.

  A triangle with at least two sides the same length is an isosceles triangle.

  A triangle with no sides the same length is a scalene triangle.

Have students cut straws to create the three types of triangles. Students should glue each triangle onto the paper that describes it.



A triangle with no sides the same length is a scalene triangle.

Emerging-level

Considerations for Emerging-level Students

- Provide links to familiar contexts. Some students may have difficulty recognising and/or identifying the various 3D objects. Providing a variety of real-world examples of 3D objects may help.
- Some students may have trouble applying skills learnt. Bring a wide range of picture books and magazines into the classroom. Have students look through them for examples of 3D objects.

Emerging-level Activity: Net Recognition

Use with Lesson 5.4

10–15 minutes

Materials

nets for cone, cube, cylinder, rectangular prism, rectangular pyramid, square pyramid and triangular prism

- Provide a variety of unidentified nets for different 3D objects and have each pair of students choose two different patterns.
- As a pair, students study the patterns to determine how many faces there are and which of the faces are bases.
- Then have each student write a sentence to describe each 3D object. [Sample sentence: My object has a square base and four triangles for faces.]
- Finally have students identify each 3D object based on its net.

Extending-level

Considerations for Extending-level Students

Extend the concepts of 2D shapes by having students draw them, and come up with their own ideas.

Extending-level Activity: Creating Shapes

Use with Lesson 5.5

10–15 minutes

Instruct students:

- Work in groups of four. Each person does a step and then passes to the left.
- Draw a shape of any type that you can come up with. Use only straight lines and at least some regular angles. Use a maximum of eight sides.
- Pass to the left.
- Looking at your classmate's picture, try to find a shape within it using at least two sides of their shape.
- Draw the shape inside the first shape.
- Pass to the left again.
- Repeat; find another shape either from the outside lines OR joining to the second person's lines. Draw the shape.
- Pass to the left again.
- Draw any remaining shapes you can identify.
- Pass to the left; this should be back to the person who drew the original shape.
- Cut them all out.
- Are any shapes similar to the pictures the other groups drew? What does this tell you about the shapes that are found within other shapes?

Students have the opportunity to discuss the numerous ways of breaking up a shape. When irregular shapes are used, the number of ways is increased substantially; however, common shapes will still likely show up. Discuss this in light of Lesson 5.5.

Activity icons (clock and student numbers): suggestions for how activities could be structured and how much time might be needed for each one.

'Research says there are a number of effective strategies for teaching math to English learners. These include strategies to enhance reading, writing, speaking and listening' (Cuevas et al. 1986).

'Research says that students learn best when instruction is designed to accommodate diverse types of learner' (Fillmore and Meyer, 1996).

Activity Zone

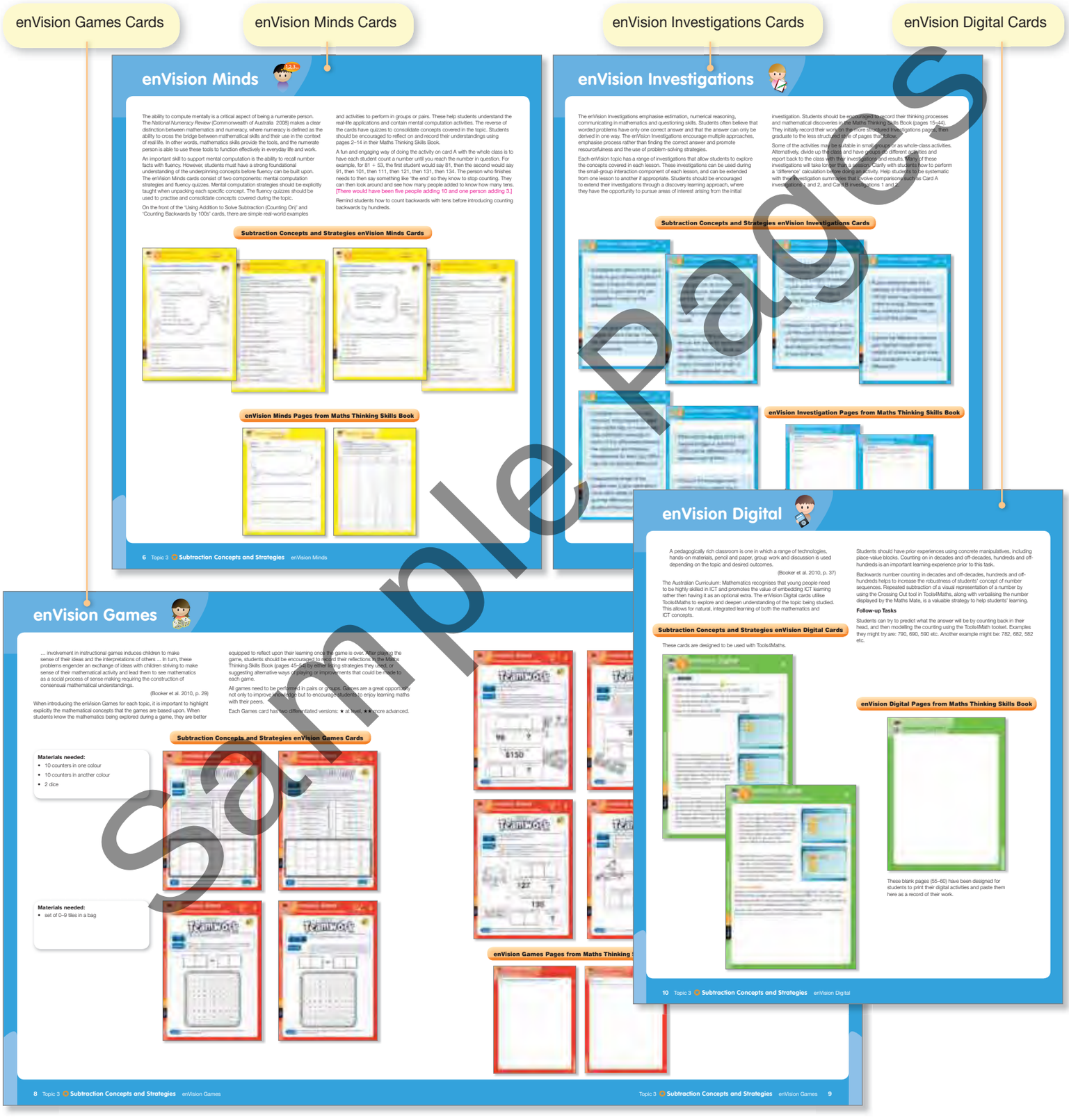
Each Activity Zone card is reproduced in the Teacher Resource Booklet for ease of planning, together with the relevant Maths Thinking Skills Book pages.

enVision Games Cards

enVision Minds Cards

enVision Investigations Cards

enVision Digital Cards



Lesson Page

Each Student Activity Book lesson page is reproduced in full in the relevant Teacher Resource booklet, with annotations to simplify lesson planning.

Topic and lesson identifier

Understanding the Concept icon: indicates text where the main lesson concept is outlined and explained.

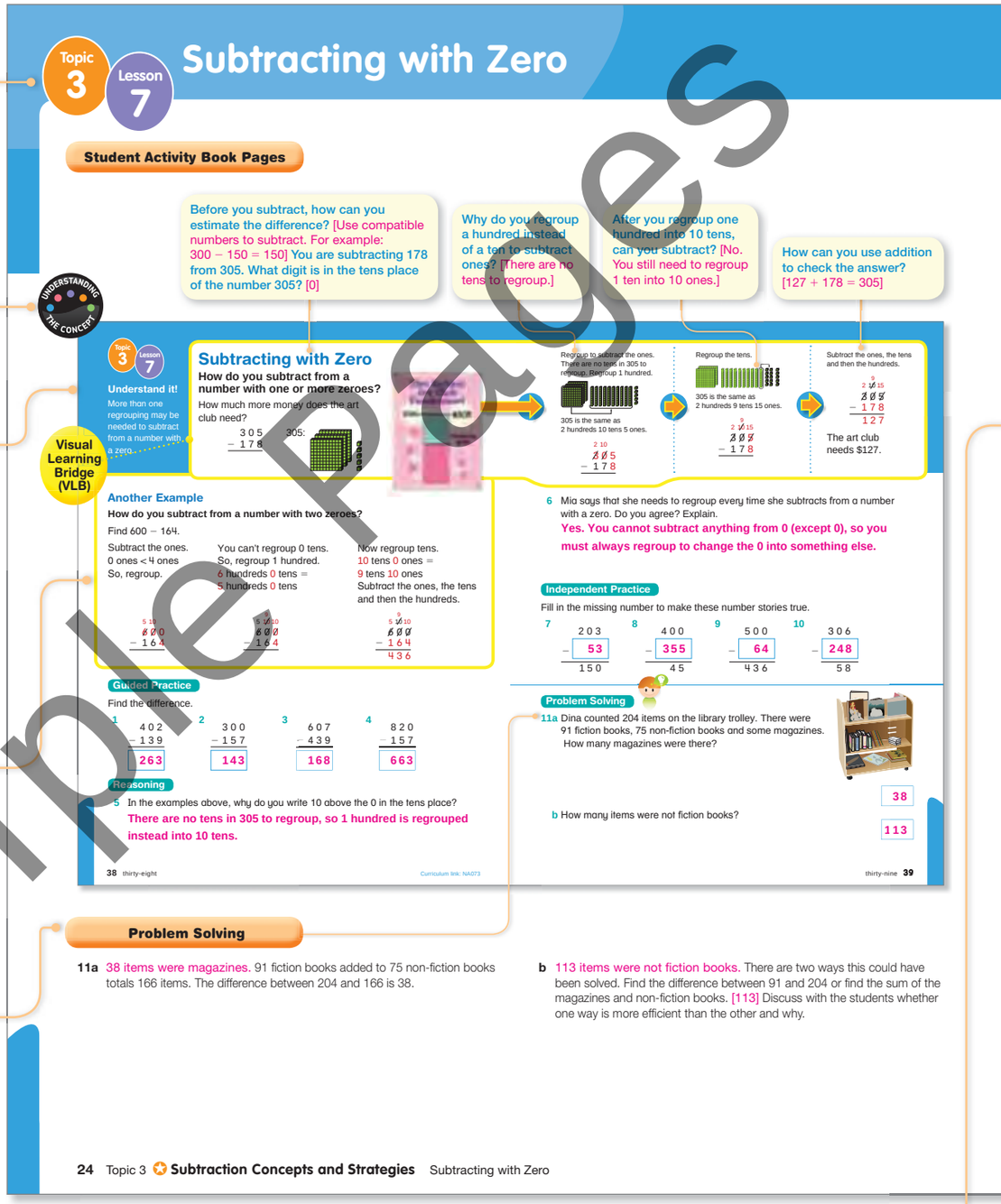
Student Activity Book page: The relevant SAB page is reproduced in full here (with answers shown in pink). This allows teachers to plan lessons easily without needing to access copies of all program components.

Visual Learning Bridge (VLB): A VLB is shown at the start of the lesson in the Student Activity Book and also appears here in the Teacher Resource Booklet along with guiding questions in blue type and expected sample answers in pink. The VLB is a pictorial, step-by-step bridge between the interactive learning activity and the lesson exercises. It helps students focus on one idea at a time as well as see connections within a sequence of ideas. This is especially helpful for visual learners and English-language learners. Visual Learning Animations (VLAs) can be used to present the Visual Learning Bridge digitally with animation. These VLAs are on the IWB DVD.

Problem solving: problem-solving activities related to each lesson. These can be set as individual student tasks or done as whole-class activities. These activities are based on the following problem-solving process:

- Read and Understand: What am I trying to find? What do I know?
- Plan and Solve: What strategy or strategies should I try? Can I show the problem? How will I solve the problem? What is the answer?
- Look Back and Check: Did I check my work? Is my answer reasonable?

Expanded answers and teaching notes are provided for each problem-solving question.



Topic 3 Lesson 7 **Subtracting with Zero**

Student Activity Book Pages

Before you subtract, how can you estimate the difference? [Use compatible numbers to subtract. For example: 300 − 150 = 150] You are subtracting 178 from 305. What digit is in the tens place of the number 305? [0]

Why do you regroup a hundred instead of a ten to subtract ones? [There are no tens to regroup.]

After you regroup one hundred into 10 tens, can you subtract? [No. You still need to regroup 1 ten into 10 ones.]

How can you use addition to check the answer? [127 + 178 = 305]

UNDERSTANDING THE CONCEPT

Visual Learning Bridge (VLB)

Subtracting with Zero

How do you subtract from a number with one or more zeroes?

How much more money does the art club need?

305 − 178

305:

Regroup to subtract the ones. There are no tens in 305 to regroup. Regroup 1 hundred.

305 is the same as 2 hundreds 10 tens 5 ones.

Regroup the tens.

305 is the same as 2 hundreds 9 tens 15 ones.

Subtract the ones, the tens and then the hundreds.

The art club needs $127.

Another Example

How do you subtract from a number with two zeroes?

Find 600 − 164.

Subtract the ones. 0 ones < 4 ones So, regroup.

You can't regroup 0 tens. So, regroup 1 hundred. 6 hundreds 0 tens = 5 hundreds 0 tens

Now regroup tens. 10 tens 0 ones = 9 tens 10 ones Subtract the ones, the tens and then the hundreds.

Understand it!

More than one regrouping may be needed to subtract from a number with a zero.

6 Mia says that she needs to regroup every time she subtracts from a number with a zero. Do you agree? Explain.

Yes. You cannot subtract anything from 0 (except 0), so you must always regroup to change the 0 into something else.

Guided Practice

Find the difference.

1 402 − 139 = 263

2 300 − 157 = 143

3 607 − 439 = 168

4 820 − 157 = 663

Independent Practice

Fill in the missing number to make these number stories true.

7 203 − 53 = 150

8 400 − 355 = 45

9 500 − 64 = 436

10 306 − 248 = 58

Reasoning

5 In the examples above, why do you write 10 above the 0 in the tens place?

There are no tens in 305 to regroup, so 1 hundred is regrouped instead in 10 tens.

Problem Solving

11a Dina counted 204 items on the library trolley. There were 91 fiction books, 75 non-fiction books and some magazines. How many magazines were there?

38

b How many items were not fiction books?

113

38 thirty-eight

Curriculum link: NA073

thirty-nine 39

Problem Solving

11a 38 items were magazines. 91 fiction books added to 75 non-fiction books totals 166 items. The difference between 204 and 166 is 38.

b 113 items were not fiction books. There are two ways this could have been solved. Find the difference between 91 and 204 or find the sum of the magazines and non-fiction books. [113] Discuss with the students whether one way is more efficient than the other and why.

24 Topic 3 ✪ Subtraction Concepts and Strategies Subtracting with Zero

Maths Background for Teachers: a brief, straightforward, lesson-specific masterclass for teachers.

Topic Focus: clarifies and explains the overall mathematical focus for the lesson.

Quick and Easy Lesson Overview: at-a-glance overview of the lesson including the objective, the essential understanding, the new vocabulary and any materials required.

Topic 3 Lesson 7

Topic Focus

There is more than one algorithm for each of the operations with rational numbers. Most algorithms for operations with rational numbers, using both mental maths and paper and pencil, use equivalence to transform calculations into simpler ones.

Quick and Easy Lesson Overview

Objective

Students subtract 3-digit numbers using paper-and-pencil methods and use subtraction to solve problems.

Essential Understanding

Place-value relationships can help simplify subtracting across zero.

Vocabulary

regroup

Maths Background for Teachers

Subtracting across zeroes follows the same basic steps of normal subtraction. However, this special case requires more than one regrouping since it is impossible to regroup zero units as a whole number of smaller units.

To find 600 − 217, write each number in expanded form then regroup until the digit in each place is larger than the number you are subtracting from.

600 = 500 + 100
= 500 + 90 + 10
= 5 hundreds + 9 tens + 10 ones
217 = 200 + 10 + 7
= 2 hundreds + 9 tens + 7 ones

Then rewrite the exercise and subtract.

600	5 hundreds	9 tens	10 ones
− 217	− 2 hundreds	− 1 ten	− 7 ones
383	3 hundreds	8 tens	3 ones

Set the Purpose

In this lesson you will learn how to subtract from a number with one or more zeroes.

Connect

When have you needed to regroup when subtracting 3-digit numbers?

Explore the Concept

You need to save $50 for your holiday. You currently have $23. How much more do you need? [$27] Help students subtract across zeroes to work it out.

Error Intervention

If students are getting answers that are exactly 10 or 100 off from the actual answer, ask: How do you regroup 1 hundred as 10 tens in 300? [2 hundreds, 10 tens and 0 ones.] How do you regroup 1 ten as 10 ones in 300? [2 hundreds, 9 tens, 10 ones.] (Substitute 300 for any hundred number the students are having trouble with.)

Extension

Challenge students in pairs to consider the following questions: 403 − 187 or 423 − 187. Do you think one of these examples is harder to subtract than the other? Explain.

Small-group Interaction

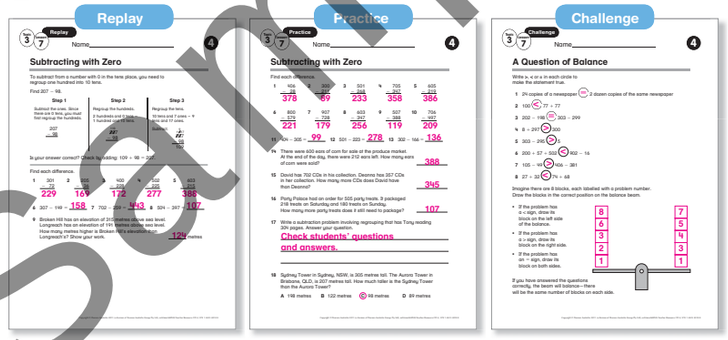
Work with a small group who may need further instruction, practice or extension. Use blocks, counters and other concrete materials; review the VLA with students; make further connections to real life; or look at one of the Investigations together. Other students work in groups on learning centre activities from the Activity Zone (Minds, Investigations, Games and Digital activity cards; see pages 6–11 of this booklet). Students will record their findings in their Maths Thinking Skills Book.

Reflection

In this lesson you learnt how to subtract from a number with one or more zeroes. Explain what has gone wrong if you subtract 127 from 500 and get 283.

Differentiated Worksheets

Provide spare paper to help students completing the Replay worksheet. Encourage the students attempting the Challenge worksheet to explain their working and to check their answers.



Set the Purpose: tells students explicitly what they will learn in this lesson.

Connect: engages learners by connecting the new lesson to the students' prior experiences.

Explore the Concept: engaging ideas for concept-based student activities.

Error Intervention: simple and immediate intervention strategies if students are making consistent errors.

Extension: concept-based extension activities either to reinforce the mathematical concept or to provide an additional challenge.

Small Group Interaction: guidance on how to incorporate small group interaction in the classroom. **enVisionMATHS** allows for this while providing ongoing activities for the rest of the class.

Reflection: guided reflection questions for each lesson.

Differentiated Worksheets: Replay, Practice and Challenge worksheets provide differentiated activities related to the lesson. These are reproduced here in full to allow for easy lesson planning, together with brief notes for the teacher.

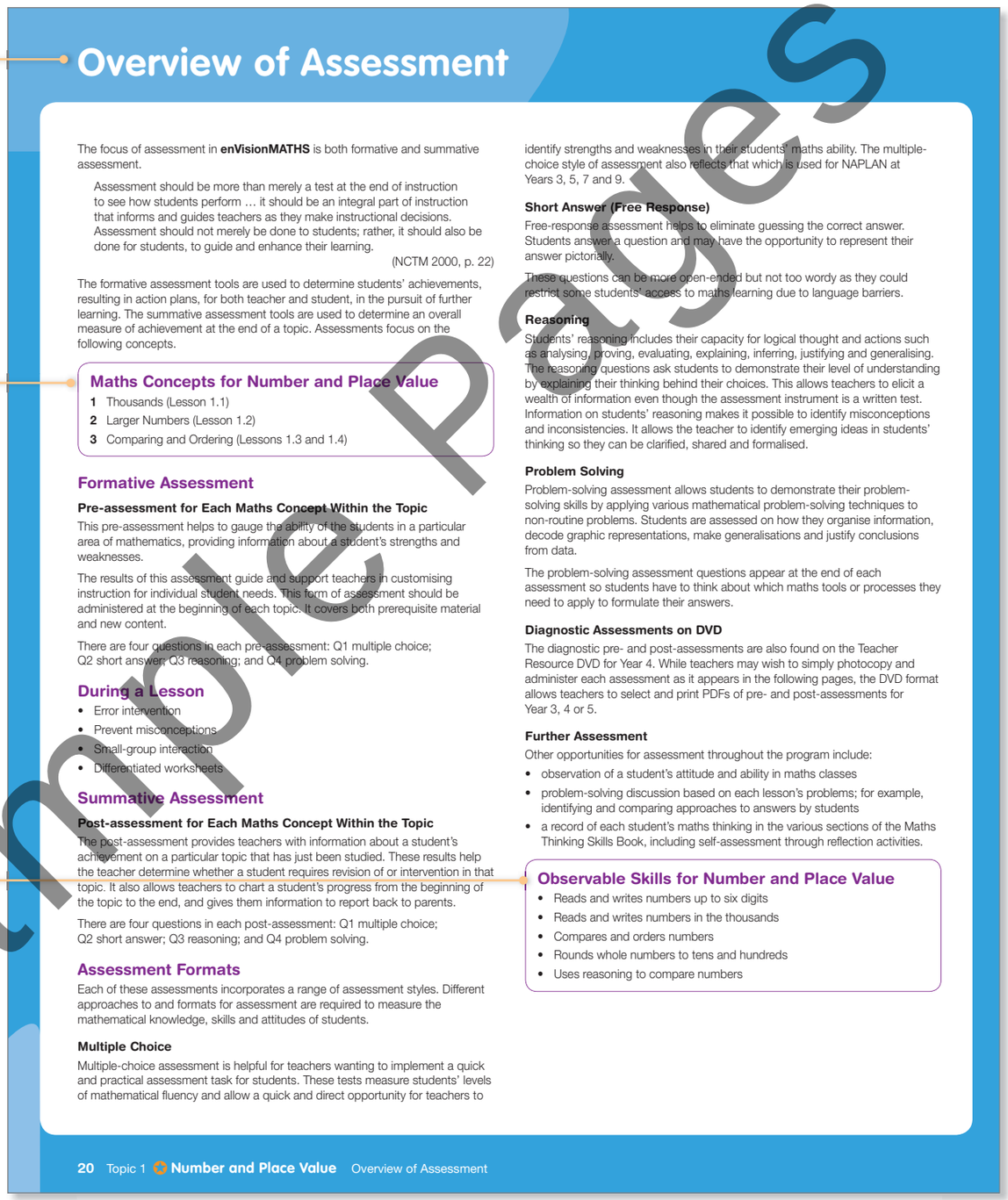
'Research says it's best to teach new content by connecting it to prior knowledge with a sustained focus over time' (Empson, 2003).

Assessment

Overview of Assessment: an outline of the different assessments possible within **enVisionMATHS**.

Mathematics Concepts: concepts addressed throughout the topic that form the basis of the diagnostic assessment. There are three to five concepts per topic.

Observable skills: specific to each lesson.



Overview of Assessment

The focus of assessment in **enVisionMATHS** is both formative and summative assessment.

> Assessment should be more than merely a test at the end of instruction to see how students perform … it should be an integral part of instruction that informs and guides teachers as they make instructional decisions. Assessment should not merely be done to students; rather, it should also be done for students, to guide and enhance their learning.
>
> (NCTM 2000, p. 22)

The formative assessment tools are used to determine students' achievements, resulting in action plans, for both teacher and student, in the pursuit of further learning. The summative assessment tools are used to determine an overall measure of achievement at the end of a topic. Assessments focus on the following concepts.

Maths Concepts for Number and Place Value

1 Thousands (Lesson 1.1)
2 Larger Numbers (Lesson 1.2)
3 Comparing and Ordering (Lessons 1.3 and 1.4)

Formative Assessment

Pre-assessment for Each Maths Concept Within the Topic

This pre-assessment helps to gauge the ability of the students in a particular area of mathematics, providing information about a student's strengths and weaknesses.

The results of this assessment guide and support teachers in customising instruction for individual student needs. This form of assessment should be administered at the beginning of each topic. It covers both prerequisite material and new content.

There are four questions in each pre-assessment: Q1 multiple choice; Q2 short answer; Q3 reasoning; and Q4 problem solving.

During a Lesson

- Error intervention
- Prevent misconceptions
- Small-group interaction
- Differentiated worksheets

Summative Assessment

Post-assessment for Each Maths Concept Within the Topic

The post-assessment provides teachers with information about a student's achievement on a particular topic that has just been studied. These results help the teacher determine whether a student requires revision of or intervention in that topic. It also allows teachers to chart a student's progress from the beginning of the topic to the end, and gives them information to report back to parents.

There are four questions in each post-assessment: Q1 multiple choice; Q2 short answer; Q3 reasoning; and Q4 problem solving.

Assessment Formats

Each of these assessments incorporates a range of assessment styles. Different approaches to and formats for assessment are required to measure the mathematical knowledge, skills and attitudes of students.

Multiple Choice

Multiple-choice assessment is helpful for teachers wanting to implement a quick and practical assessment task for students. These tests measure students' levels of mathematical fluency and allow a quick and direct opportunity for teachers to identify strengths and weaknesses in their students' maths ability. The multiple-choice style of assessment also reflects that which is used for NAPLAN at Years 3, 5, 7 and 9.

Short Answer (Free Response)

Free-response assessment helps to eliminate guessing the correct answer. Students answer a question and may have the opportunity to represent their answer pictorially.

These questions can be more open-ended but not too wordy as they could restrict some students' access to maths learning due to language barriers.

Reasoning

Students' reasoning includes their capacity for logical thought and actions such as analysing, proving, evaluating, explaining, inferring, justifying and generalising. The reasoning questions ask students to demonstrate their level of understanding by explaining their thinking behind their choices. This allows teachers to elicit a wealth of information even though the assessment instrument is a written test. Information on students' reasoning makes it possible to identify misconceptions and inconsistencies. It allows the teacher to identify emerging ideas in students' thinking so they can be clarified, shared and formalised.

Problem Solving

Problem-solving assessment allows students to demonstrate their problem-solving skills by applying various mathematical problem-solving techniques to non-routine problems. Students are assessed on how they organise information, decode graphic representations, make generalisations and justify conclusions from data.

The problem-solving assessment questions appear at the end of each assessment so students have to think about which maths tools or processes they need to apply to formulate their answers.

Diagnostic Assessments on DVD

The diagnostic pre- and post-assessments are also found on the Teacher Resource DVD for Year 4. While teachers may wish to simply photocopy and administer each assessment as it appears in the following pages, the DVD format allows teachers to select and print PDFs of pre- and post-assessments for Year 3, 4 or 5.

Further Assessment

Other opportunities for assessment throughout the program include:

- observation of a student's attitude and ability in maths classes
- problem-solving discussion based on each lesson's problems; for example, identifying and comparing approaches to answers by students
- a record of each student's maths thinking in the various sections of the Maths Thinking Skills Book, including self-assessment through reflection activities.

Observable Skills for Number and Place Value

- Reads and writes numbers up to six digits
- Reads and writes numbers in the thousands
- Compares and orders numbers
- Rounds whole numbers to tens and hundreds
- Uses reasoning to compare numbers

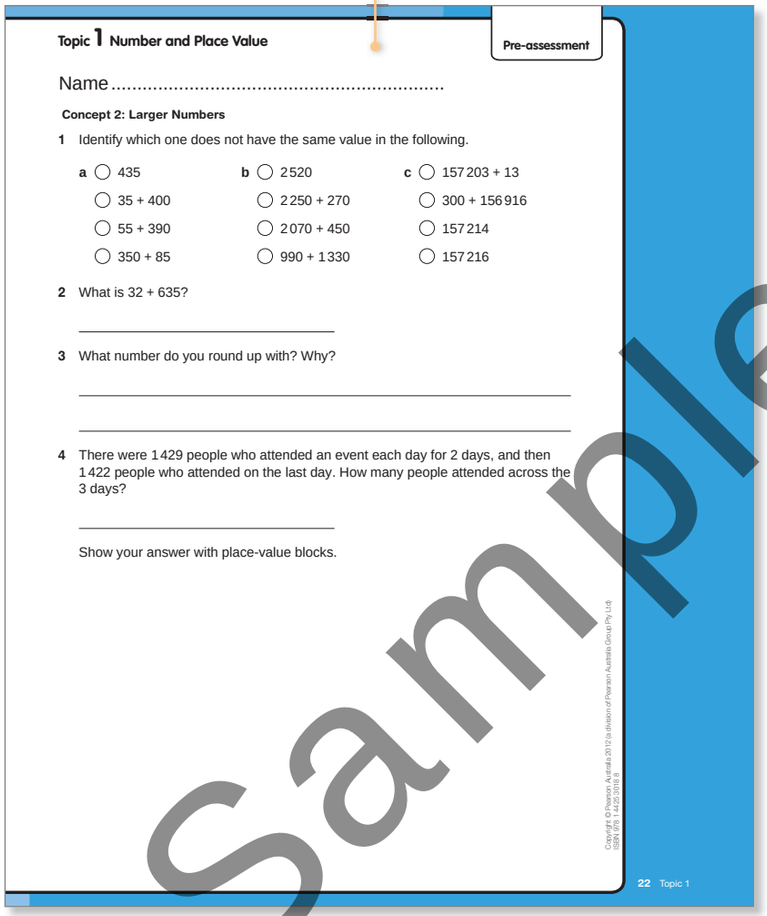
'Research says that ongoing assessment prevents misconceptions and provides valuable information to guide data-driven instruction' (Vye et al., 1998).

Diagnostic Pre-assessment: copyable concept-based test to be provided to students at the start of a topic. There is a Pre-assessment for each concept related to the topic.

These assessments have been designed and placed on the page to facilitate copying. They are also available on the Teacher Resource DVD.

Diagnostic Post-assessment: copyable concept-based test to assess understanding and proficiency at the end of a topic. There is one Post-assessment for each concept related to the topic.

These assessments have been designed and placed on the page to facilitate copying. They are also available on the Teacher Resource DVD.



Topic 1 Number and Place Value

Pre-assessment

Name .....

Concept 2: Larger Numbers

1 Identify which one does not have the same value in the following.

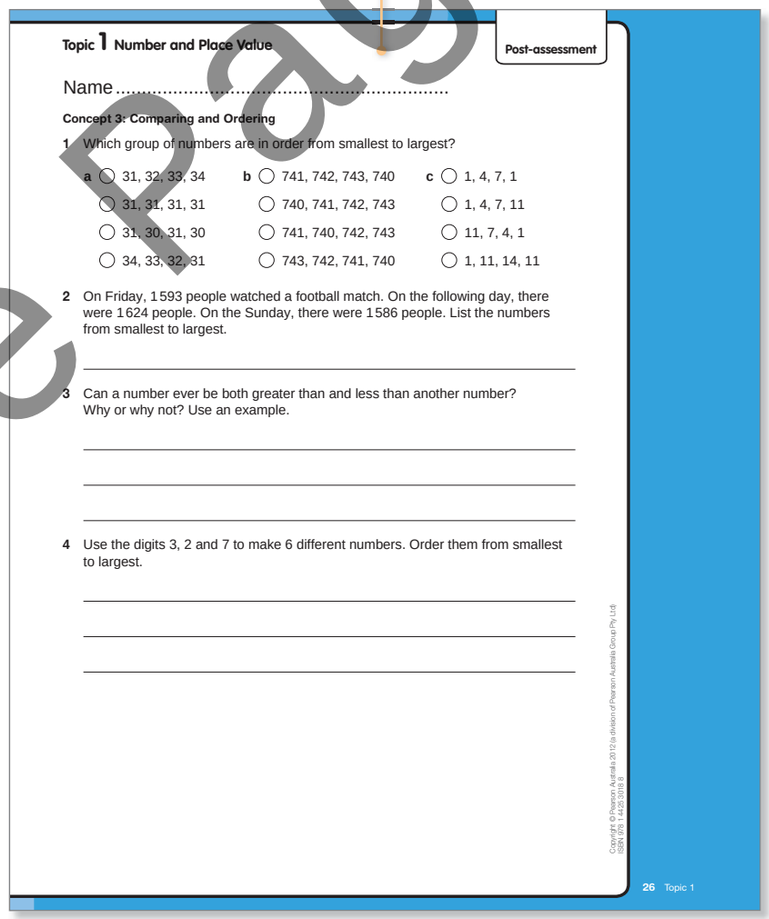
a	b	c
◯ 435	◯ 2520	◯ 157 203 + 13
◯ 35 + 400	◯ 2250 + 270	◯ 300 + 156 916
◯ 55 + 390	◯ 2070 + 450	◯ 157 214
◯ 350 + 85	◯ 990 + 1330	◯ 157 216

2 What is 32 + 635?

3 What number do you round up with? Why?

4 There were 1429 people who attended an event each day for 2 days, and then 1422 people who attended on the last day. How many people attended across the 3 days?

Show your answer with place-value blocks.



Topic 1 Number and Place Value

Post-assessment

Name .....

Concept 3: Comparing and Ordering

1 Which group of numbers are in order from smallest to largest?

a	b	c
◯ 31, 32, 33, 34	◯ 741, 742, 743, 740	◯ 1, 4, 7, 1
◯ 31, 31, 31, 31	◯ 740, 741, 742, 743	◯ 1, 4, 7, 11
◯ 31, 30, 31, 30	◯ 741, 740, 742, 743	◯ 11, 7, 4, 1
◯ 34, 33, 32, 31	◯ 743, 742, 741, 740	◯ 1, 11, 14, 11

2 On Friday, 1593 people watched a football match. On the following day, there were 1624 people. On the Sunday, there were 1586 people. List the numbers from smallest to largest.

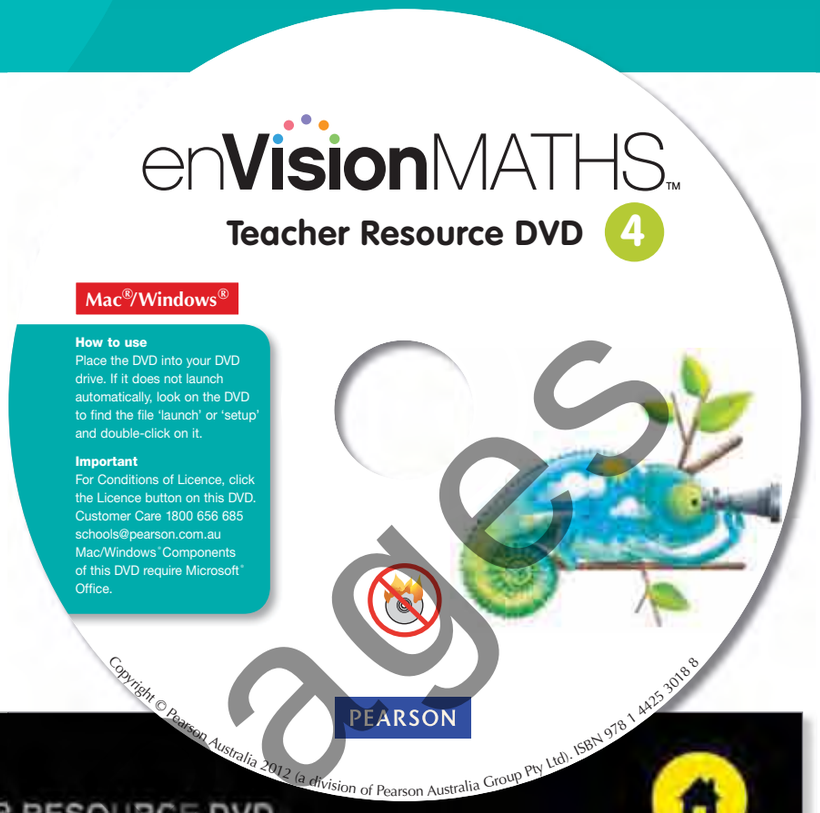
3 Can a number ever be both greater than and less than another number? Why or why not? Use an example.

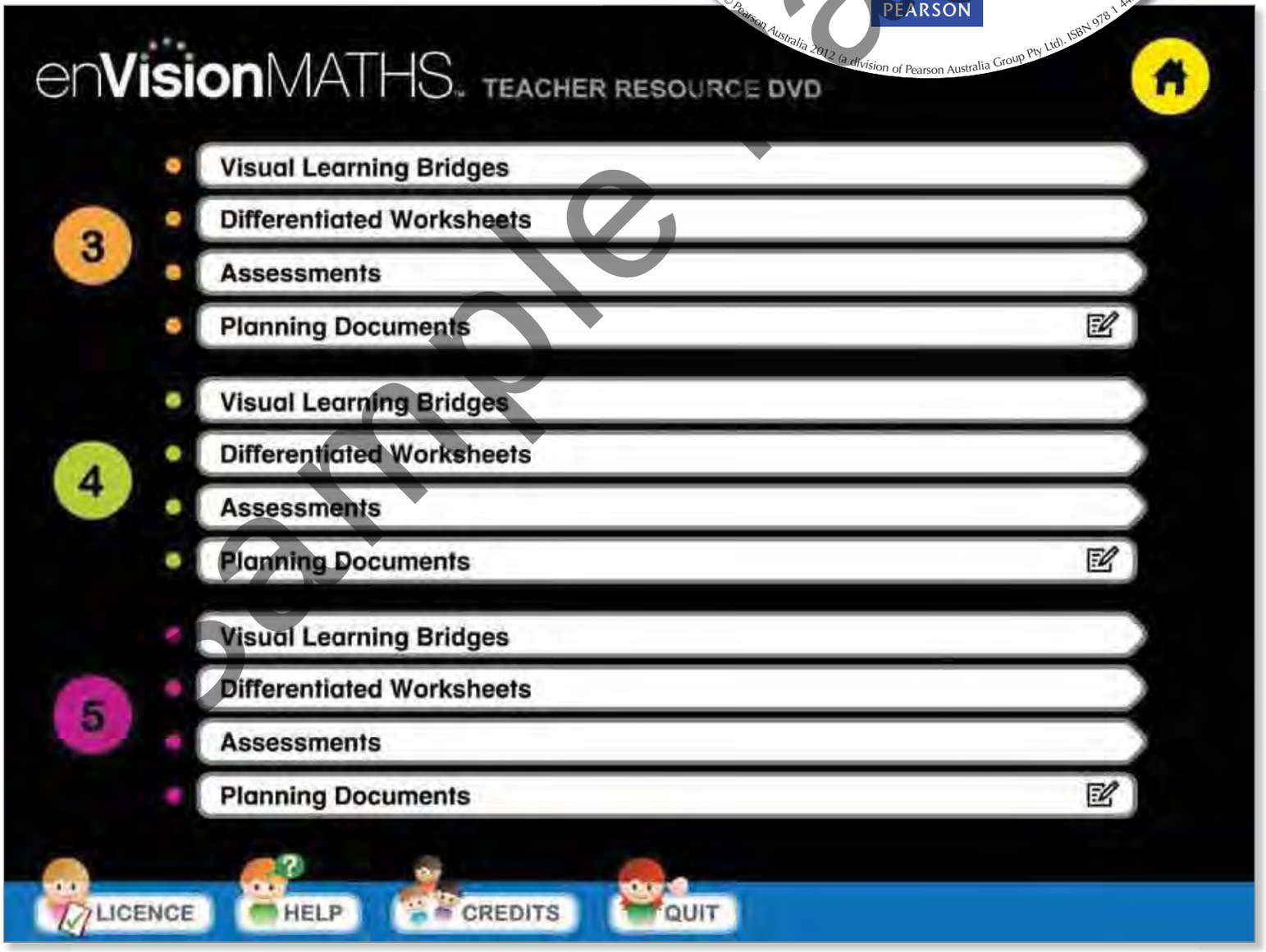
4 Use the digits 3, 2 and 7 to make 6 different numbers. Order them from smallest to largest.

Teacher Resource DVD

The Teacher Resource DVD is found at the back of each Overview and Implementation Guide. For year 4, the DVD includes planning and recording documents, VLBs, diagnostic assessments and Differentiated Worksheets for years 3, 4 and 5. This allows for differentiation in a multi-age or multi-ability classroom.

Teachers search by year level for the component they need. The VLBs, Diagnostic Pre- and Post-assessments and Differentiated Worksheets are in PDF format, allowing teachers to print these in sufficient quantities for their class. The planning and recording documents are Microsoft® Excel® spreadsheets that can be edited and tailored to teachers' requirements.





Interactive Whiteboard DVD

The Interactive Whiteboard DVD for year 4 contains the Visual Learning Animations and Bridges for year 4, as well as Tools4Maths.

Teachers introduce each lesson to students through the Visual Learning Animation (VLA) on this DVD. Each lesson then begins with a focus on conceptual understanding through use of the Visual Learning Bridge provided on this DVD and in the Student Activity Books.

Tools4Maths can be used in any number of ways: alongside the Digital Cards in the Activity Zone, as a tool for further explanation or enrichment in whole-class, small-group or individual work, or as individual or group extension activities.

The enVision Digital Cards provide structured formats for students to use Tools4Maths. Students can then explore each of the 11 digital tool sets on their own. The tools themselves are neither structured in any order nor related to specific activities, and so provide any number of opportunities for discovery within different mathematics contexts.

Student Activity Book

Students practise the mathematical concept in their Student Activity Book (SAB). This SAB is also a reference book where older students can independently teach themselves using the Visual Learning Bridges.

Topic Opener Pages

Topic Identifier

Topic Openers: interesting investigations and facts introducing the topic. These are a great way to start a topic without launching straight in to the conceptual mathematics.

Vocabulary: lists of the new vocabulary required for each unit.

Lesson Pages

Topic and Lesson Identifier: allows students to easily locate and identify specific lessons.

Understand it!: shows the basic understanding that will be mastered by the lesson.

Visual Learning Bridge (VLB): the visual representation of the mathematical concept being addressed in each lesson. This VLB should form the starting point of the lesson and can be referred to at anytime throughout the lesson to reinforce the concept.

Independent Practice: designed as practice for students once they have mastered the basic concepts. As some students will reach this level earlier than others, this practice section can be used for differentiated learning in the classroom.

Topic 7 Lesson 1

Understand it!
Divide to find the number of equal groups and the number in each group.

Using Diagrams to Divide

When do you divide?

A museum wants to display a collection of 24 gems on four shelves, placing the same number of gems on each shelf. How many gems will be on each shelf?

Choose an Operation Think about sharing. Divide to find the number in each group.

What You Show

Think of sharing the gems equally among the 4 shelves. How many gems are on each shelf?

What You Write

$24 \div 4 = 6$

Each shelf should have 6 gems.

Mental Computation

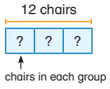
Use repeated subtraction to find the quotient.

1 $40 \div 8 =$ ☐ 2 $70 \div 10 =$ ☐ 3 $36 \div 6 =$ ☐

Guided Practice

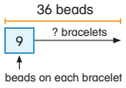
Complete the diagrams to help you divide. Draw pictures to help if you need to.

4 Kevin is arranging 12 chairs in 3 equal groups. How many chairs are in each group?



☐

5 Megan has bracelets made up of 36 beads. Each bracelet has 9 beads. How many bracelets does she have?



☐

Reasoning

6 Explain how you could use repeated addition to check the answer to the example at the top of the page.

Independent Practice

Draw pictures to help you divide.

7 The photographer put 18 people into 3 rows. How many people are in each row?

☐

8 Rick is putting 14 drawings into 2 art binders. How many drawings are in each binder?

☐

Problem Solving

9a Jake has a collection of 48 football cards. He wants to stick them in a scrapbook in equal rows. What are his options? Draw diagrams to explain.

b How many options would he have if he had 24 cards? ☐

Curriculum link: NA076, NA082

Reasoning: provides the opportunity for students to reinforce their understandings of concepts by explaining their reasoning.

Guided Practice: allows students to practise and work through the mathematical concepts with guidance from teacher, calculator use etc.

Mental Computation: provides the opportunity for mental maths practice directly related to the lesson.

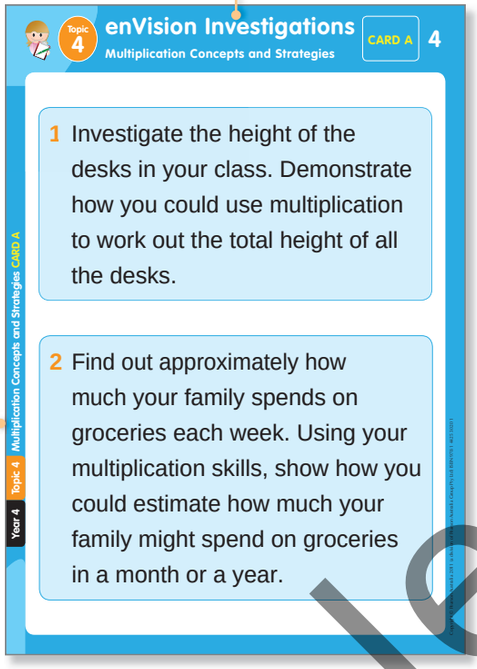
The design of the questions allows for working-out space. Students should be encouraged to write or draw their maths thinking in these spaces.

Problem Solving: presents an open-ended problem related to the overall mathematical concept. This allows the concept to be practised and reinforced by interesting and challenging investigations.

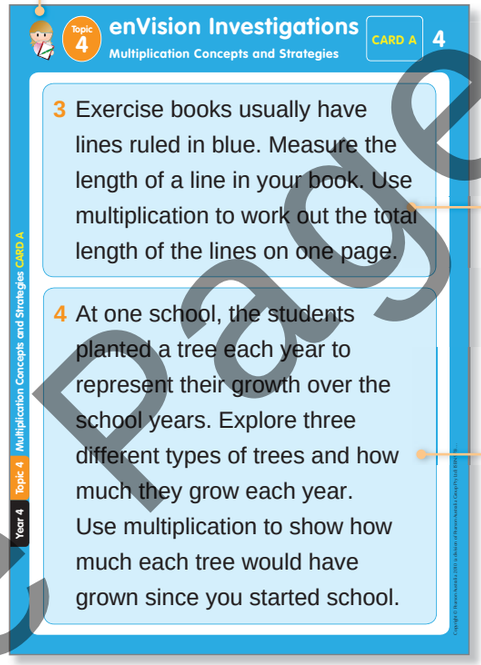
Activity Zone

The class splits into ability groups to use the Activity Zone. This class set of investigations, mental computation, games and digital cards supports the learning of each topic and differentiates the curriculum. Providing ready-made resources that are topic-linked and differentiated saves time for teachers, engages students and is sound pedagogy across Australia. Students record their Activity Zone work in the Maths Thinking Skills Book.

enVision Investigations Cards



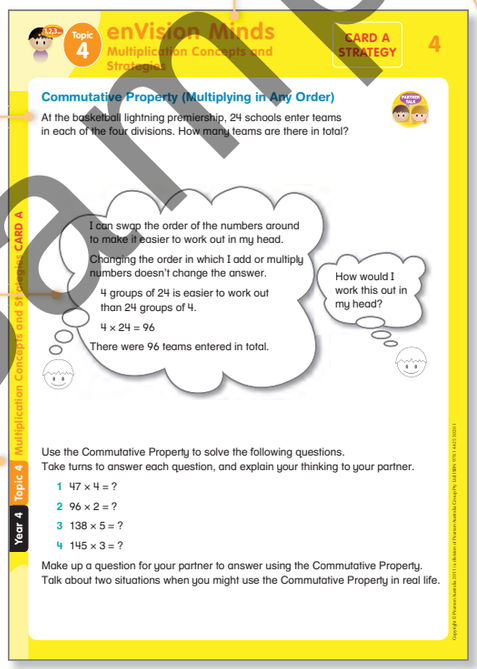
Placement of card identifiers: down the side and along the top for easy identification.



Investigation icon: identifies card as an investigation.

Question boxes: clearly identify the different questions.

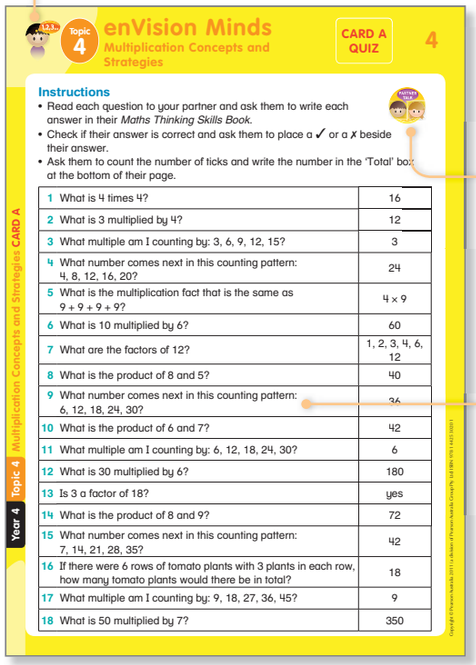
enVision Minds Cards



Strategy: presents the situation and problem.

Thought balloons: identifies what needs to be thought about.

Placement of card identifiers: down the side and along the top for easy identification.



enVision Minds icon: identifies card as a mental computation card.

Partner Talk: indicates clearly if a partner is needed.

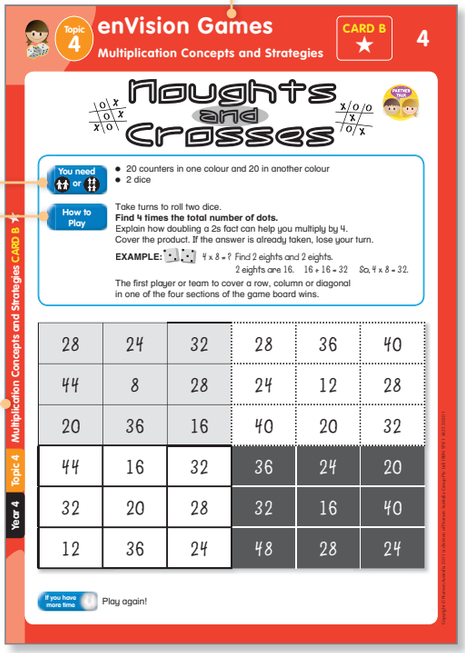
Quiz: quick questions on the topic designed to be used by students in pairs.

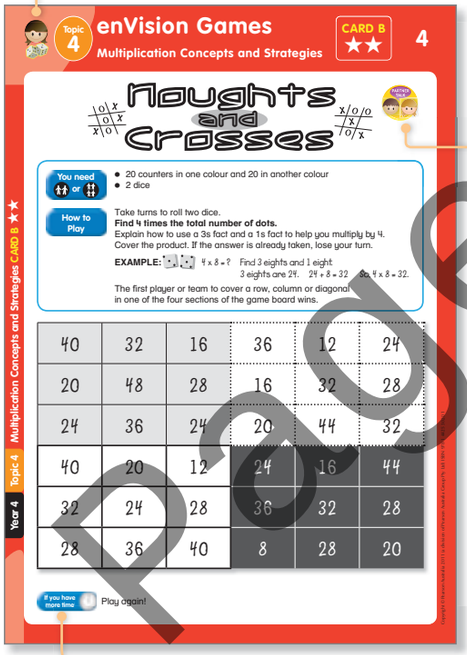
enVision Games Cards

You Need icon: shows at a glance what you need, and how many students can play at any one time.

How to Play icon: immediately identifiable instructions.

Placement of card identifiers: down the side and along the top for easy identification.





enVision Games icon: identifies card as part of enVision Games.

Partner Talk: indicates clearly if a partner is needed.

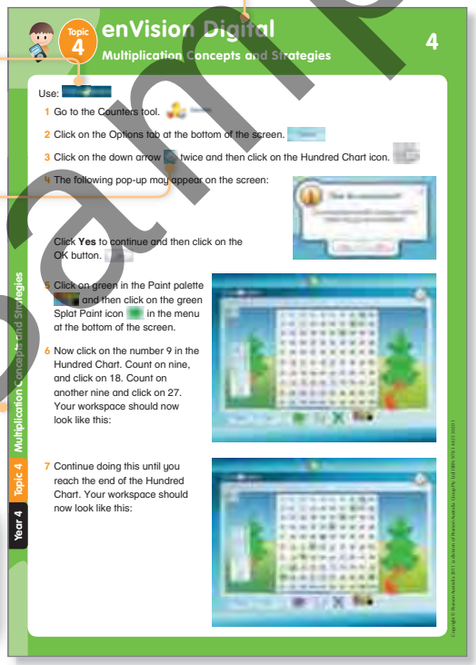
If You Have More Time icon: extension activities.

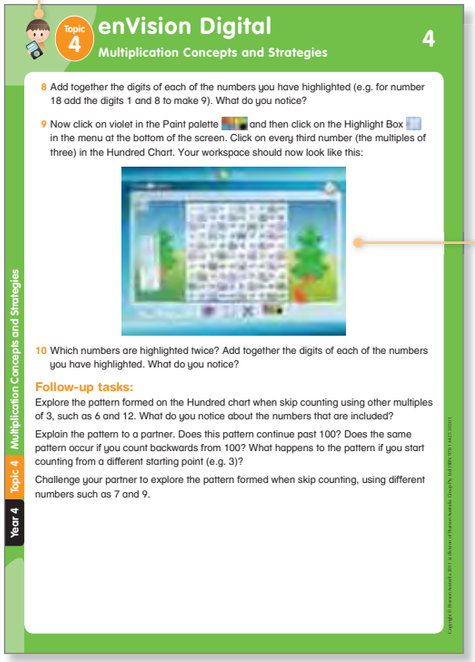
enVision Digital Cards

Tools4Maths icon: indicates the use of Tools4Maths.

Visual representation of Tools4Maths tools and icons.

Placement of card identifiers: down the side and along the top for easy identification.





enVision Digital icon: identifies card as a digital activity card.

Screen shots show students what their work should look like at each stage.

Maths Thinking Skills Book

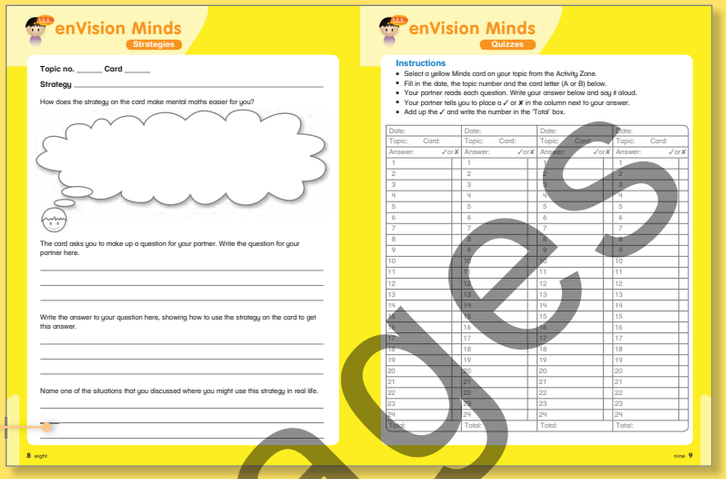
The Maths Thinking Skills Book acts as a student record/ self-reflection journal for each year of the program. There is one book suitable for use in years 3, 4, 5 or 6 and another book suitable for use in years F, 1 or 2.

In these books, students are able to write and draw their own maths thinking. The pages are divided into different colours/sections corresponding to the Activity Zone cards, reflection questions and language presented in each topic.

The completed Maths Thinking Skills Book can form a record of that student's maths learning for the year, and can then be used as a self-reflection tool for students and to inform parents and teachers.

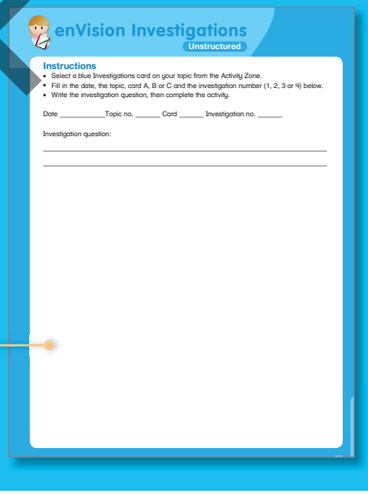
enVision Minds: Students record their results from the enVision Minds strategies and quizzes on these pages.

enVision Minds



enVision Investigations: Students record their working out for their chosen investigation in these pages. The first few blue pages have questions to guide students, but as their investigation skills improve, students are able to work on the blank blue pages, setting out their answers in their own way.

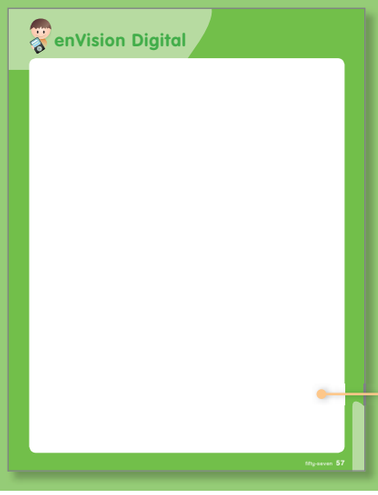
enVision Investigations



enVision Games: Students can record the strategies they use to play the games in these pages. They can also add their own ideas to an enVision game and record the ideas on these pages.

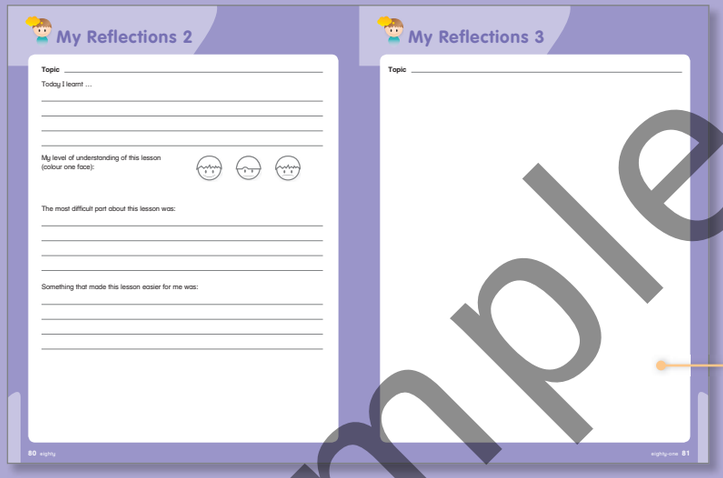
enVision Games

enVision Digital



enVision Digital: Students can print out the work they have done on the Digital Cards and paste it into these pages.

My Reflections



My Reflections: Students can record their reflections on their maths learning on these pages. These reflections may be the result of an end-of-lesson reflection activity or a class discussion.

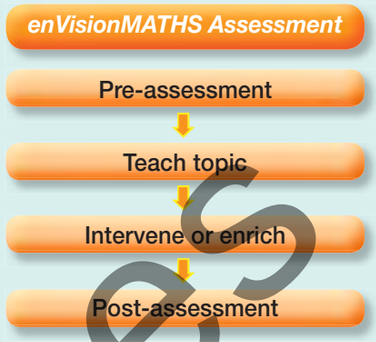
My Maths Language

My Maths Language: As students learn new maths language, they can record this vocabulary, together with any diagrams, on this page.

Overview of Assessment

> Assessment should be more than merely a test at the end of instruction to see how students perform … it should be an integral part of instruction that informs and guides teachers as they make instructional decisions. Assessment should not merely be done to students; rather it should also be done *for* students, to guide and enhance their learning.
>
> NCTM, 2000, p. 22

The focus of assessment in **enVisionMATHS** is both formative and summative assessment. The formative assessment tools are used to determine students' achievements, resulting in action plans (for both you and the student) in the pursuit of further learning. The summative assessment tools are used to determine an overall measure of achievement at the end of a topic.



The following assessment tools are available for **enVisionMATHS** topics in year 4.

Formative Assessment

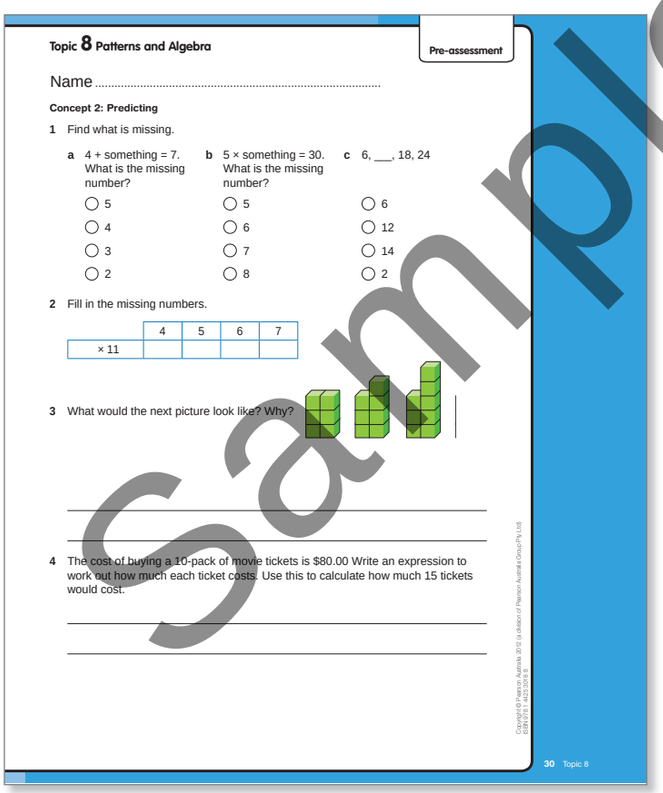
Diagnostic Pre-assessment for each maths concept within the topic

This Pre-assessment helps to gauge the ability of students in a particular area of mathematics, providing information about their strengths and weaknesses. The results of this assessment will guide and support teachers in customising instruction for individual student needs. This form of assessment should be administered at the beginning of each topic. It covers both prerequisite material and new content.

Summative Assessment

Diagnostic Post-assessment for each maths concept within the topic

This Post-assessment provides teachers with information about students' achievement on a particular topic that has just been studied. These results help determine whether an individual student requires revision of, or intervention on, their topic. It also allows teachers to chart a students' progress from the beginning of the topic to the end and gives them information to report back to parents.



Topic 8 Patterns and Algebra — Pre-assessment

Name .....

Concept 2: Predicting

1 Find what is missing.

a 4 + something = 7. What is the missing number?
○ 5 ○ 4 ○ 3 ○ 2

b 5 × something = 30. What is the missing number?
○ 5 ○ 6 ○ 7 ○ 8

c 6, ___, 18, 24
○ 6 ○ 12 ○ 14 ○ 2

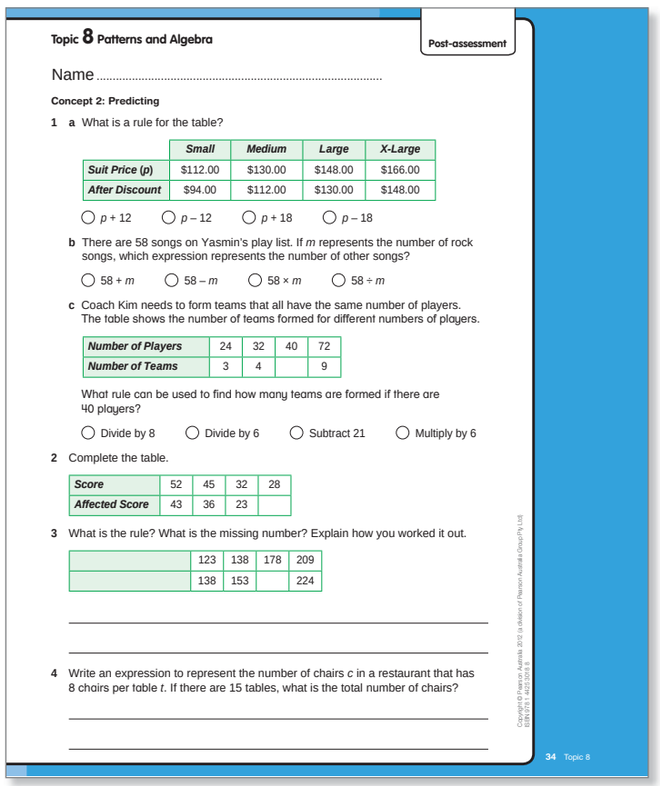
2 Fill in the missing numbers.

	4	5	6	7
× 11				

3 What would the next picture look like? Why?

4 The cost of buying a 10-pack of movie tickets is $80.00 Write an expression to work out how much each ticket costs. Use this to calculate how much 15 tickets would cost.

30 Topic 8



Topic 8 Patterns and Algebra — Post-assessment

Name .....

Concept 2: Predicting

1 a What is a rule for the table?

	Small	Medium	Large	X-Large
Suit Price (p)	$112.00	$130.00	$148.00	$166.00
After Discount	$94.00	$112.00	$130.00	$148.00

○ $p + 12$ ○ $p - 12$ ○ $p + 18$ ○ $p - 18$

b There are 58 songs on Yasmin's play list. If m represents the number of rock songs, which expression represents the number of other songs?
○ $58 + m$ ○ $58 - m$ ○ $58 \times m$ ○ $58 \div m$

c Coach Kim needs to form teams that all have the same number of players. The table shows the number of teams formed for different numbers of players.

Number of Players	24	32	40	72
Number of Teams	3	4		9

What rule can be used to find how many teams are formed if there are 40 players?
○ Divide by 8 ○ Divide by 6 ○ Subtract 21 ○ Multiply by 6

2 Complete the table.

Score	52	45	32	28
Affected Score	43	36	23	

3 What is the rule? What is the missing number? Explain how you worked it out.

	123	138	178	209
	138	153		224

4 Write an expression to represent the number of chairs c in a restaurant that has 8 chairs per table t. If there are 15 tables, what is the total number of chairs?

34 Topic 8

The diagnostic assessments for year 4 are found in the year 4 Teacher Resource Booklets, and also on the year 4 Teacher Resource DVD. While teachers may wish to simply photocopy and administer each assessment as it appears in the Teacher Resource Booklet, the DVD format allows teachers to select and print appropriate Pre- and Post-assessments for any year 4 concept. Assessments for years 3 and 5 are also provided on the year 4 DVD to allow for differentiated tests to suit individual students. Answers are provided at the back of each Teacher Resource Booklet.

Concepts for Assessment

The following table lists the maths concepts used in the Diagnostic Pre-assessments and Post-assessments for year 4 level.

Topic	Concept
1 Number and Place Value	**1** Thousands **2** Larger Numbers **3** Comparing and Ordering
2 Addition Concepts and Strategies	**1** Mental Maths **2** Models for Adding **3** Addition of Larger Numbers **4** Diagrams for Adding
3 Subtraction Concepts and Strategies	**1** Models and Hundred Charts **2** Subtracting 2- and 3-digit Numbers **3** Subtracting with Zeroes **4** Word and Money Problems
4 Multiplication Concepts and Strategies	**1** Using Arrays to Multiply **2** Multiplication Mentally **3** Multiply with Any Number **4** Multiply with 10s
5 Angles and Shapes	**1** Angles **2** 2D and 3D Objects **3** Recognising Shapes
6 Location and Transformation	**1** Compass Use **2** Understanding Scale Diagrams **3** Symmetry and Tessellations
7 Division Concepts and Strategies	**1** Using Diagrams and Models to Divide **2** Inverse Operations **3** Division Mentally **4** Division and Quotients
8 Patterns and Algebra	**1** Patterns **2** Predicting **3** Solving Patterns **4** Equality and Number Sentences
9 Fractions and Decimals	**1** Recognising Numerators and Denominators **2** Simplifying and Equivalent Fractions **3** Improper Fractions **4** Decimals
10 Time and Temperature	**1** Calculating with Time **2** Elapsed Time **3** Temperature
11 Length, Area, Capacity, Mass and Volume	**1** Units **2** Length Around **3** Area **4** Mass and Volume
12 Data, Graphs and Probability	**1** Bar Graphs and Pie Charts **2** Predicting and Recognising Data **3** Related and Independent Events

Overview of Assessment

Assessment Formats

Each of the diagnostic assessments incorporates a range of assessment styles. Different approaches to, and formats for, assessment are required to measure the mathematical knowledge, skills and attitudes of students.

Multiple-choice (Question 1)

Multiple-choice assessment is helpful in implementing a quick and practical assessment task for students. These tests measure students' levels of mathematical fluency and allow for a quick and direct opportunity for identifying strengths and weaknesses in students' maths ability. The multiple-choice style of assessment also reflects that which is used for NAPLAN at years 3, 5, 7 and 9.

Short Answer (free-response) (Question 2)

Free-response assessment helps to eliminate students guessing the correct answer. Students answer a question and may have the opportunity to represent their answer pictorially.

These questions can be more open-ended but should not be too wordy as they could restrict some students' access to maths learning due to language barriers.

Reasoning (Question 3)

Included in each Diagnostic Assessment is an open-ended question designed to measure students' reasoning ability. Students' reasoning includes their capacity for logical thought and actions such as analysing, proving, evaluating, explaining, inferring, justifying and generalising. The reasoning questions ask students to demonstrate their level of understanding by explaining the thinking behind their choices. This allows teachers to elicit a wealth of information about students' reasoning, making it possible to identify misconceptions and inconsistencies. It allows teachers to identify emerging ideas in students' thinking so they can be clarified, shared and formalised.

Problem Solving (Question 4)

Problem-solving assessment allows students to demonstrate their problem-solving skills by applying various mathematical problem-solving techniques to non-routine problems. Students are assessed on how they organise information, decode graphic representations, make generalisations and justify conclusions from data.

The problem-solving assessment questions appear together at the end so students are challenged to think about which maths tools or processes they need to apply to formulate their answers.

During a Lesson

Topic 8 Lesson 3

Topic Focus

Relationships can be described and generalisations made for mathematical situations that have numbers or objects that repeat in predictable ways. For some relationships, mathematical expressions and equations can be used to describe how members of one set are related to members of a second set.

Quick and Easy Lesson Overview

Objective

Students will extend patterns of cubes or tiles.

Essential Understanding

Some sequences of geometric objects change in predictable ways that can be described using a mathematical rule.

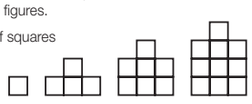
Materials

grid paper cubes or blocks

Maths Background for Teachers

Some geometric patterns are growing patterns. In the pattern below, for example, the first figure is a square. Then rows of squares are added repeatedly to form the subsequent figures.

As each row is added, the total number of squares increases. So there is a numerical aspect to this pattern that can be recorded in a table as follows.



Rows	1	2	3	4
Squares	1	4	7	10

The numbers in the second row of this table form a number sequence. A rule for this sequence is 'add 3'. By continuing the sequence, it is possible to find how many squares are in subsequent figures without actually drawing them.
So a figure with five rows would have 13 squares, a figure with six rows would have 16 squares and so on.

In this lesson students will use tables like the one above to relate geometric patterns and number patterns.

Set the Purpose

In this lesson you will learn to continue a geometric pattern and then use that pattern to complete a table of number pairs.

Connect

Have you ever made towers with blocks? How do you make storeys?
[Stack blocks on top of other blocks.]

Explore the Concept

Stella begins a tile pattern with a yellow tile and will put five blue tiles after each yellow tile. If she uses 20 blue tiles, then how many yellow tiles will she use? (Hint: Draw a picture.)

Provide grid paper and yellow and blue crayons or felt pens so that students can sketch Stella's pattern.

[Four yellow tiles.]

Error Intervention

If students have trouble knowing how to start and recognise a pattern, then ask questions to help them get started: **What do you have to find? How many _____ are in what you are looking at? How many _____ are next in the pattern?**

Extension

Ask students what pattern they see in the towers image below.

[Where there is one more storey, there is one more cube.] Draw the next two patterns and develop a table to represent this.

Small-group Interaction

Work with a small group who may need further instruction, practice or extension. Use blocks, counters and other concrete materials; review the VLA with students; make further connections to real life; or look at one of the Investigations together. Other students work in groups on learning centre activities from the Activity Zone (Minds, Investigations, Games and Digital activity cards; see pages 6–11 of this booklet). Students will record their findings in their Maths Thinking Skills Book.

Reflection

In this lesson you learnt how to continue a geometric pattern and then use that pattern to complete a table of number pairs. How would you describe a geometric pattern?

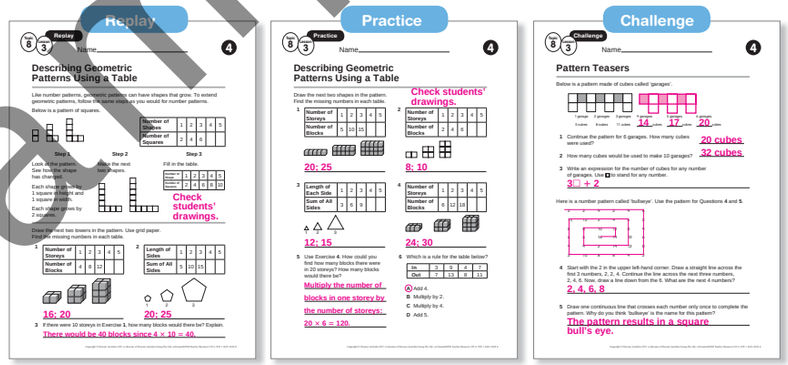
Error Intervention: Prevents misconceptions.

Small Group Interaction: Results are recorded to use in assessment as required.

Differentiated Worksheets: Can be used in class or as homework to assist in assessing students' understanding.

Differentiated Worksheets

Along with the differentiated worksheets, provide access for students to use place-value blocks, hundred charts and open number lines. Allow students to explore patterns by modelling them.



Overview of Assessment

Problem Solving

Students demonstrate their problem-solving skills by applying various mathematical techniques to non-routine problems. Students are assessed on how they organise information, decode graphic representations, make generalisations and justify conclusions from data.

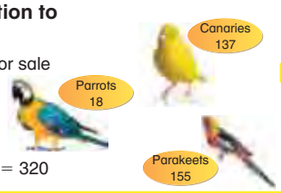
Topic 2 Lesson 4

Understand it!
The sum of three or more numbers can be found using the same rules as for two numbers.

Adding Three or More Numbers

How can you use addition to solve problems?

Different kinds of birds are for sale at a pet shop. How many birds are for sale in total?



Find 137 + 155 + 18.

Estimate: 140 + 160 + 20 = 320

Step 1
Line up ones, tens and hundreds.

```
 137
 155
+ 18
```

Step 2
Add the ones. Regroup.

```
  2
 137
 155
+ 18
   0
```

Step 3
Add the tens. Regroup.

```
 1 2
 137
 155
+ 18
 10
```

Step 4
Add the hundreds.

```
 1 2
 137
 155
+ 18
 310
```

310 birds are for sale in total.

Guided Practice

Find the sum of these numbers.

1	2	3	4	5	6
36 + 47 + 35 = ☐	247 + 362 + 49 = ☐	273 + 82 + 125 = ☐	59 + 56 + 302 + 24 = ☐	46 + 24 + 9 + 7 = ☐	385 + 97 + 34 = ☐

Reasoning

7 Meg said that 95 + 76 + 86 is greater than 300. Explain why her answer is not reasonable.

Look at the example at the top of the page.

8 Why is there a 2 above the tens place in Step 2?

9 How can you tell that 310 birds is a reasonable answer?

Curriculum link: NA073

Independent Practice

Find the sum of these numbers.

10	11	12	13	14
64 + 42 + 88 = ☐	307 + 37 + 234 = ☐	602 + 125 + 231 = ☐	246 + 54 + 233 + 205 = ☐	303 + 128 + 63 + 149 = ☐

Problem Solving

15a Three friends collected a total of 300 coins to raise money to buy new sports equipment for their school. Each collected a different number of coins. How many coins might each have collected?

b What if they each collected about the same number of coins? How many coins would they each have collected?

Problem Solving

Answers and discussion of Problem-solving questions are shown on the corresponding Teacher Resource Booklet page.

Topic 2 Lesson 4 Adding Three or More Numbers

Student Activity Book Pages

When we estimated the sum, why did we round to the nearest ten? [Since 18 is a 2-digit number it makes sense to round all the numbers to the nearest ten.] Is there another way we could have esimated? [Yes; sample answers: 100 + 200 + 20 = 320 or 140 + 150 + 20 = 310.] How do the estimated sums compare? [All estimates should be reasonably close.]

Prevent Misconceptions

When helping students to practise rounding, make sure they understand that rounding to the nearest place will give a better estimate.

In Step 3, why are there two small numbers above the main numbers? [They show that 20 ones were regrouped as 2 tens and that 10 tens were regrouped as 1 hundred.] In Step 4, why is no regrouping needed? [There are fewer than 10 hundreds.]

UNDERSTANDING THE CONCEPT

Visual Learning Bridge (VLB)

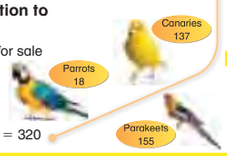
Topic 2 Lesson 4

Understand it!
The sum of three or more numbers can be found using the same rules as for two numbers.

Adding Three or More Numbers

How can you use addition to solve problems?

Different kinds of birds are for sale at a pet shop. How many birds are for sale in total?



Find 137 + 155 + 18.

Estimate: 140 + 160 + 20 = 320

Step 1 Line up ones, tens and hundreds.

$$\begin{array}{r} 137 \\ 155 \\ +\ 18 \end{array}$$

Step 2 Add the ones. Regroup.

$$\begin{array}{r} {\scriptstyle 2} \\ 137 \\ 155 \\ +\ 18 \\ \hline 0 \end{array}$$

Step 3 Add the tens. Regroup.

$$\begin{array}{r} {\scriptstyle 1\ 2} \\ 137 \\ 155 \\ +\ 18 \\ \hline 10 \end{array}$$

Step 4 Add the hundreds.

$$\begin{array}{r} {\scriptstyle 1\ 2} \\ 137 \\ 155 \\ +\ 18 \\ \hline 310 \end{array}$$

310 birds are for sale in total.

Guided Practice

Find the sum of these numbers.

1. $\begin{array}{r} 36 \\ 47 \\ +\ 35 \\ \hline 118 \end{array}$
2. $\begin{array}{r} 247 \\ 362 \\ +\ 49 \\ \hline 658 \end{array}$
3. $\begin{array}{r} 273 \\ 82 \\ +\ 125 \\ \hline 480 \end{array}$
4. $\begin{array}{r} 59 \\ 56 \\ 302 \\ +\ 24 \\ \hline 441 \end{array}$
5. $\begin{array}{r} 46 \\ 24 \\ 9 \\ +\ 7 \\ \hline 86 \end{array}$
6. $\begin{array}{r} 385 \\ 97 \\ +\ 34 \\ \hline 516 \end{array}$

Reasoning

7 Meg said that 95 + 76 + 86 is greater than 300. Explain why her answer is not reasonable.

Answers will vary. Rounding up to estimate gives 100 + 80 + 90 = 270, which is less than 300, so Meg's answer is not reasonable.

Look at the example at the top of the page.

8 Why is there a 2 above the tens place in Step 2?

The ones added to 20 and were regrouped to 2 tens.

9 How can you tell that 310 birds is a reasonable answer?

Answers will vary. The estimate was 320, and 310 is reasonably close to this.

Independent Practice

Find the sum of these numbers.

10. $\begin{array}{r} 64 \\ 42 \\ +\ 88 \\ \hline 194 \end{array}$
11. $\begin{array}{r} 307 \\ 37 \\ +\ 234 \\ \hline 578 \end{array}$
12. $\begin{array}{r} 602 \\ 125 \\ +\ 231 \\ \hline 958 \end{array}$
13. $\begin{array}{r} 246 \\ 54 \\ 233 \\ +\ 205 \\ \hline 738 \end{array}$
14. $\begin{array}{r} 303 \\ 128 \\ 63 \\ +\ 149 \\ \hline 643 \end{array}$

Problem Solving

15a Three friends collected a total of 300 coins to raise money to buy new sports equipment for their school. Each collected a different number of coins. How many coins might each have collected?

Answers will vary.

b What if they each collected about the same number of coins? How many coins would they each have collected?

300 divided by 3 is 100, so if each friend collected about the same number then each would have collected about 100.

20 twenty — Curriculum link: NA073

twenty-one 21

Problem Solving

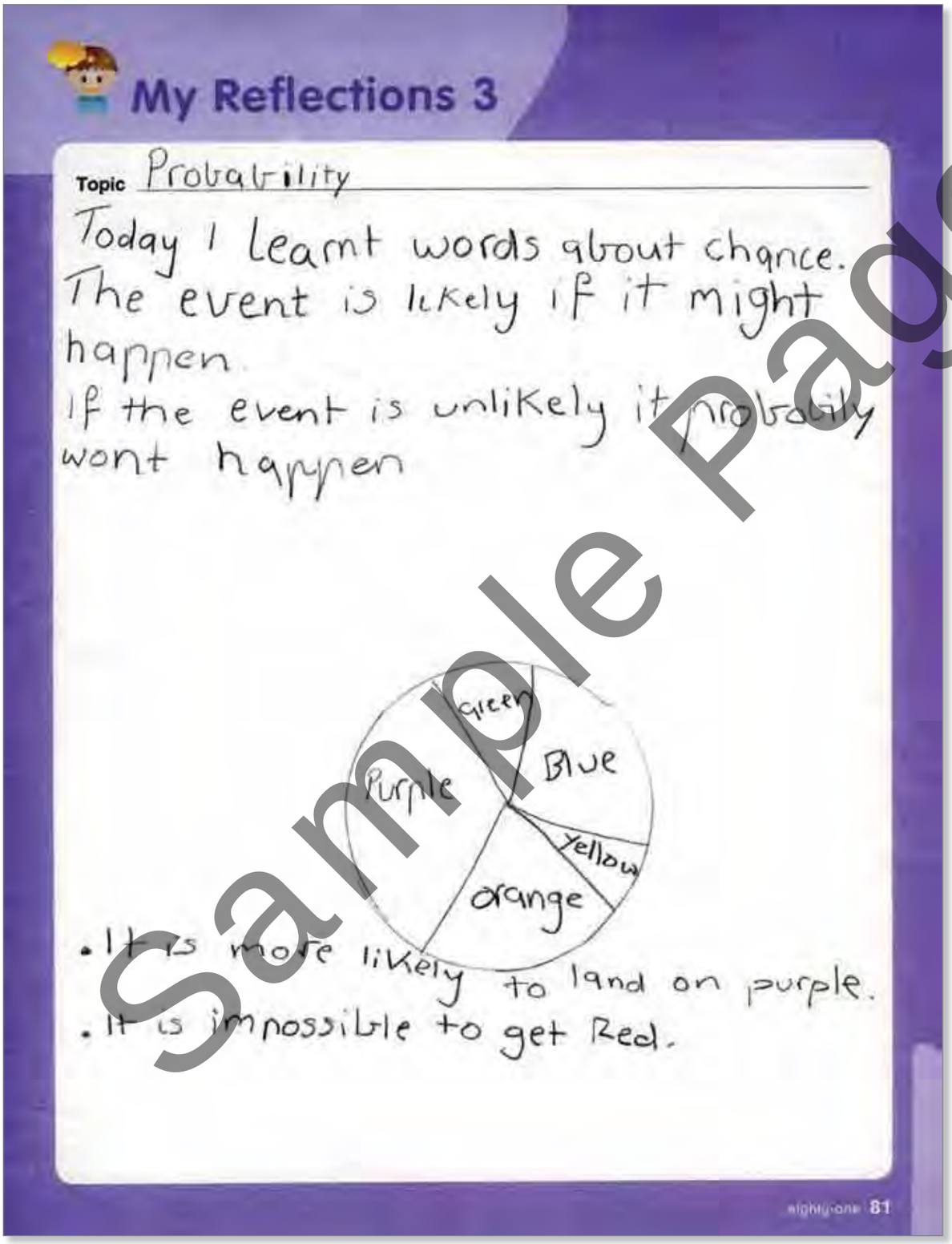
15a There are many possibilities; responses will vary and show students' level of thinking. How did students go about solving this problem and how did they record their responses. Who used a systematic approach? For example:

Friend 1	Friend 2	Friend 3
11	91	198
12	92	196
13	93	194

b The average of 300 is 100; so if each friend collected about the same number of coins, then each would have to have collected about 100. Did students start from here, then use number sense to work out some possibilities?

Self-assessment: Maths Thinking Skills Book

A record of each student's maths thinking can be recorded in the various sections of the Maths Thinking Skills Book. This includes self-assessment through reflection activities that are suggested in each lesson topic.



My Reflections 3

Topic Probability

Today I learnt words about chance.
The event is likely if it might happen.
If the event is unlikely it probably wont happen

- It is more likely to land on purple.
- It is impossible to get Red.

eighty-one 81

Assessment Recording

enVisionMATHS Year 4 Class Record

Class:

	Topic 1 Number and Place Value						
	Thousands		Larger Numbers		Comparing and Ordering		
Student Name	Pre	Post	Pre	Post	Pre	Post	Observations/Notes
1							
2							
3							
4							
5							
6							
7							
8							
9							
10							
11							
12							
13							
14							
15							
16							
17							
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19							
20							
21							
22							
23							
24							
25							
26							
27							
28							
29							
30							

Class and Student Record templates are provided in the planning documents on the year 4 Teacher Resource DVD.

These are Microsoft® Excel® documents that can be tailored for individual class needs. They have been arranged by **enVisionMATHS** topic and assessment concept, with space for teachers to record notes or add formulae, as appropriate.

Topic name

Assessment concepts

Student names: once a student's name is entered in topic 1, the name will automatically appear in the Class Record for all other year 4 topics.

enVisionMATHS Year 4 Student Record

Student Name: **Class:**

Topic 1 Number and Place Value

Observable Skills:
Reads and writes numbers up to six digits
Reads and writes numbers in the thousands
Compares and orders numbers
Rounds whole numbers to tens and hundreds
Uses reasoning to compare numbers

Observations/Notes

	Pre-Assessment	Post-Assessment
Thousands		
Larger Numbers		
Comparing and Ordering		

Other Observations

Topic name

Observable skills for the topic

Assessment concepts

Scope and Sequence

Australian Curriculum Reference	enVisionMATHS Scope and Sequence		
	Foundation	Year 1	Year 2
	Number and Algebra		
Number and place value	**3.1** Counting and Writing 1, 2 and 3 **3.2** Counting and Writing 4 and 5 **3.3** Identifying One More Than **3.4** Reading and Writing 0 **3.5** Counting and Writing 6 and 7 **3.6** Counting and Writing 8 and 9 **3.7** Counting and Writing 10 **3.8** Ordering Numbers on a Number Line **3.9** Comparing Numbers to 10 **5.1** Adding Groups **5.2** Making Numbers in Different Ways **5.3** Adding to 4 and 5 **5.4** Adding to 6 and 7 **5.5** Adding to 8 and 9 **5.6** Adding to 10 **5.7** Using Objects to Add **9.1** Revising 10 **9.2** Counting and Writing 11 and 12 **9.3** Counting and Writing 13, 14 and 15 **9.4** Counting and Writing 16 and 17 **9.5** Counting and Writing 18 and 19 **9.6** Counting and Writing 20 **10.1** Identifying and Creating Groups **10.2** Identifying More Than One Group **10.3** Sharing Objects **10.4** Sharing Equally **12.1** Skip Counting by 5s **12.2** Skip Counting by 2s **12.3** Counting Groups of 10 **12.4** Counting Backwards **12.5** Looking for a Pattern **12.6** Understanding Ordinal Numbers **13.1** Acting Out Subtraction Stories **13.2** Understanding Separation Stories **13.3** Understanding Take Away Stories **13.4** Understanding Comparison Stories	**1.1** Counting and Writing Numbers 0 to 5 **1.2** Counting and Writing Numbers 6 to 10 **1.3** Counting and Writing Numbers 10, 11, 12 **1.4** Counting and Writing Numbers 13 to 19 **1.5** Counting and Writing Numbers to 20 **1.6** Ordering Numbers Using a Number Line **1.7** Counting Beyond 20 **1.8** Comparing Two Numbers **1.9** Ordering Three Numbers **2.1** Drawing Addition Stories **2.2** Adding to 6 and 7 **2.3** Adding to 8 and 9 **2.4** Using the Plus Sign **2.5** Writing Addition Number Sentences **2.6** More Addition Number Sentences **2.7** Making Numbers in Different Ways **4.1** Representing Numbers on a Ten Frame **4.2** Understanding Parts of 10 **4.3** Using Numbers Made With 10s **4.4** Using Numbers With 10s and 1s **4.5** Counting With 10s and 1s **4.6** Writing Numbers in Different Ways **5.1** Counting On **5.2** Adding 1 More **5.3** Adding Doubles **5.4** Using Near Doubles to Add **5.5** Adding More Near Doubles **5.6** Showing Addition Facts on a Ten Frame **5.7** Making 10 on a Ten Frame **5.8** Making 10 to Add 9 **5.9** Adding Three Numbers **5.10** Adding With 0 **8.1** Using the Minus Sign **8.2** Finding Difference **8.3** Writing Subtraction Sentences **8.4** Understanding Comparison Stories **8.5** Subtracting with 0, 1 and 2	**1.1** Counting and Writing Numbers 11 to 20 **1.2** Counting by 10s **1.3** Using Models for 10s and 1s **1.4** Reading and Writing Numbers to 99 **1.5** Using Models to Compare Numbers **1.6** Identifying Before, After and Between **1.7** Ordering Numbers **1.8** Ordering Numbers on a Hundred Chart **1.9** Making 100 **1.10** Counting by 10s Past 100 **2.1** Writing Addition Number Sentences **2.2** Understanding Addition Stories **2.3** Understanding More Addition Stories **2.4** Adding in Any Order **2.5** Adding 0, 1 and 2 **2.6** Adding Using Doubles **2.7** Adding Using Near Doubles **2.8** More Adding in Any Order **2.9** Making 10 to Add Two Numbers **2.10** Adding Three Numbers **3.1** Identifying Missing Parts **3.2** Writing Subtraction Number Sentences **3.3** Using Separation Stories **3.4** Using Comparison Stories **3.5** Subtracting 0, 1 and 2 **3.6** Drawing Subtraction Stories **3.7** Using Doubles Facts to Subtract **3.8** Using Addition to 10 to Subtract **4.3** Skip Counting **4.4** Understanding Odd and Even Numbers **4.5** Understanding Ordinal Numbers **11.1** Multiplication as Repeated Addition **11.2** Writing Multiplication Stories **11.3** Building Arrays Using Counters **11.4** Building Arrays Using Pictures **11.5** Understanding Division as Sharing **11.6** Writing Division Stories

Scope and Sequence

enVisionMATHS Scope and Sequence

Number and Algebra

Year 3	Year 4	Year 5	Year 6
1.1 Reading and Writing Hundreds	**1.1** Reading and Writing Thousands	**1.1** Representing Thousands in Different Ways	**1.1** Representing Millions in Different Ways
1.2 Reading and Writing Numbers to 1 000	**1.2** Reading and Writing Larger Numbers	**1.2** Representing Millions in Different Ways	**1.2** Comparing and Ordering Whole Numbers
1.3 Building Numbers Beyond 1 000	**1.3** Comparing and Ordering Whole Numbers	**1.3** Comparing and Ordering Whole Numbers	**1.3** Understanding Positive and Negative Numbers
1.4 Understanding Odd and Even Numbers	**1.4** Rounding Whole Numbers	**2.1** Using Mental Maths to Find Missing Parts	**1.7** Multiplying and Dividing by 10, 100 and 1 000
1.5 Using Clues to Identify Numbers	**2.1** Using Mental Maths to Add	**2.2** Rounding and Estimating Whole Numbers	**2.1** Understanding Commutative and Associative Properties of Addition
1.6 Using Place Value to Add and Subtract	**2.2** Using Models to Add 3-Digit Numbers	**2.3** Using Mental Strategies to Add and Subtract	**2.2** Understanding Multiplication Properties
1.7 Ordering Three Numbers	**2.3** Adding Whole Numbers	**2.4** Adding and Subtracting Large Numbers	**2.3** Using the Distributive Property
1.8 Comparing Numbers	**2.4** Adding Three or More Numbers	**3.6** Asking Questions to Solve Multiple-Step Problems	**2.4** Using Mental Maths to Apply Properties of Operations
1.9 Rounding Whole Numbers	**2.5** Using Diagrams to Connect Addition and Subtraction	**4.1** Finding Factors Using Multiplication	**2.5** Understanding Properties and Relationships Between Operations
2.1 Using Mental Maths to Make 10	**3.1** Using Models to Subtract 2-Digit Numbers	**4.2** Understanding Multiplication Properties	**2.6** Finding Elapsed Time
2.2 Adding Tens to a 2-Digit Number	**3.2** Subtracting on a Hundred Chart	**4.3** Multiplying with Three Factors	**2.7** Using Mental Maths to Multiply
2.3 Adding Tens	**3.3** Subtracting 2-Digit Numbers	**4.4** Multiplying 2-Digit by 1-Digit Numbers	**2.8** Using Mental Maths to Multiply and Divide
2.4 Adding Tens and Ones	**3.4** Using Models to Subtract 3-Digit Numbers	**4.5** Using Mental Maths to Multiply	**2.9** Dividing with 1 and 0
2.5 Using Models to Add	**3.5** Subtracting 3-Digit Numbers	**4.6** Identifying Missing or Extra Information	**2.10** Relating Factors and Divisibility
2.6 Add 9 by Adding 10	**3.7** Subtracting With Zero	**5.1** Understanding Factors	**2.12** Using Operations to Solve Problems
2.7 Adding Larger Numbers	**3.8** Subtracting With More Than One Zero	**5.2** Using Patterns to Divide	**3.1** Looking for Patterns in a Decimal Chart
2.8 Adding More Than Two Numbers	**3.9** Solving Subtraction Problems in Two Steps	**5.3** Dividing 3-Digit by 1-Digit Numbers	**6.1** Understanding Prime and Composite Numbers
3.1 Identifying the Missing Part	**4.1** Using Arrays to Multiply	**5.4** Dividing Using Zeroes	**6.2** Identifying Prime Factors
3.3 Reasons to Subtract	**4.2** Using Arrays to Multiply with 4 and 8	**5.5** Understanding Remainders	**6.3** Finding Common and Greatest Common Factors
3.4 Using Addition to 20 to Subtract	**4.3** Using Arrays to Multiply with 3, 6 and 9	**5.6** Dividing with Remainders	**6.4** Finding Greatest Common Factors
3.5 Subtracting Tens	**4.4** Multiplying in Any Order	**5.7** Connecting Models with Symbols	**6.6** Drawing a Diagram to Solve an Equation
3.6 Sorting Tens on a Hundred Chart	**4.5** Solving Multiplication Problems in Two Steps	**5.8** Checking for Reasonableness in Division Problems	**6.7** Understanding Properties of Equality
3.7 Subtracting on a Hundred Chart	**4.6** Using Mental Maths to Multiply		**6.8** Solving Addition and Subtraction Equations
3.8 Estimating Differences	**4.7** Recording Multiplication in Different Ways		**6.10** Identifying Square and Triangular Numbers
5.1 Representing Multiplication	**4.8** Multiplying 2-Digit by 1-Digit Numbers		**8.6** Finding the Percentage of a Number
5.2 Writing Multiplication Sentences	**4.9** Multiplying by Multiples of 10 and 100		**8.7** Checking for Reasonableness in Percentage Problems
5.3 Using Arrays to Show Multiplication	**7.1** Using Diagrams to Divide		
5.4 Solving Multiplication Problems	**7.2** Using a Table to Show Division		
5.5 Using Number Patterns to Multiply	**7.3** Using Materials to Divide 2-Digit Numbers		
8.1 Understanding Division as Sharing	**7.4** Relating Multiplication and Division Facts		
8.2 Writing Division Stories			
8.3 Relating Division to Multiplication			
8.4 Representing Money Values Using Division			
9.5 Identifying Patterns for Multiples			
13.1 Using Mental Strategies to Subtract			
13.2 Using a Number Line to Subtract 2-Digit Numbers			
13.3 Using Models to Subtract 2-Digit Numbers			

Scope and Sequence

Australian Curriculum Reference	enVisionMATHS Scope and Sequence		
	Foundation	Year 1	Year 2
Number and Algebra			
Number and place value (cont.)		**8.6** Finding Missing Parts of 8 **8.7** Finding Missing Parts of 6 and 7 **8.8** Finding Missing Parts of 10 **8.9** Writing More Subtraction Sentences **9.1** Making Numbers to 100 **9.2** Identifying One More, One Less **9.3** Counting to 100 **9.4** Comparing Numbers **9.5** Identifying Before, After and Between **9.6** Ordering Numbers on a Number Line **12.1** Identifying Groups **12.2** Using Groups of 2, 5 and 10 **12.3** Understanding Division as Sharing **13.5** Counting Patterns of 2, 5 and 10 **13.6** Understanding Odd and Even Numbers **13.7** Identifying Patterns on a Hundred Chart	**11.7** Relating Multiplication and Division **12.1** Adding Groups of Tens **12.2** Counting 10 More Than, 10 Less than **12.3** Comparing Numbers **12.4** Understanding Place Value up to 1 000 **12.5** Building 1 000 **12.6** Using Numbers to 1 000 **12.7** Ordering Numbers to 1 000
Fractions and decimals		**10.1** Folding In Half **10.2** Making Equal Parts **10.3** Identifying Half of a Group **10.4** Using Different Attributes to Find Half **10.5** Making a Whole	**7.1** Identifying Parts of Collections **7.2** Drawing Pictures of Fractions **7.3** Identifying Equal Parts **7.4** Naming One Part of a Whole **7.5** Naming Fractions of Collections

enVisionMATHS Scope and Sequence

Year 3	Year 4	Year 5	Year 6
Number and Algebra			
13.4 Using Written Methods to Subtract 2-Digit Numbers **13.5** Using Other Mental Strategies to Subtract **13.6** Using Addition to Check Subtraction **13.7** Solving Two-step Problems	**7.5** Using Inverse Operations **7.6** Using Multiplication Facts to Find Division Facts **7.7** Using Mental Maths to Divide **7.8** Dividing Multiples of 10, 100 and 1 000 **7.9** Dividing With 2-Digit Numbers **7.10** Dividing With Remainders **8.1** Looking for a Pattern **8.2** Looking for a Pattern to Solve a Problem **8.5** Working Backwards to Solve Problems **8.7** Finding Equality		
11.1 Dividing a Whole into Equal Parts **11.2** Writing Fractions of a Whole **11.3** Naming Groups Using Fractions **11.4** Finding Equivalent Fractions **11.5** Finding a Fraction of a Group **11.6** Explaining Fractions	**9.1** Using Fractions to Name Parts of Wholes or Sets **9.2** Writing a Maths Explanation **9.3** Finding Equivalent Fractions **9.4** Finding More Equivalent Fractions **9.5** Defining Fractions **9.6** Using Improper Fractions and Mixed Numbers **9.7** Representing Tenths and Hundredths as Decimals **9.8** Writing Fractions and Decimals	**1.4** Representing Decimals in Different Ways **1.5** Comparing Decimals **1.6** Comparing and Ordering Decimals **1.7** Ordering Decimals on a Number Line **1.8** Writing Fractions as Decimals—Tenths **1.9** Writing Fractions as Decimals—Hundredths **1.10** Ordering Fractions on a Number Line **1.11** Ordering Fractions and Decimals on a Number Line **1.12** Extending Beyond Hundredths **3.1** Writing Fractions and Decimals **3.2** Using Number Lines to Represent Fractions and Decimals **3.3** Comparing Fractions **3.4** Finding Equivalent Fractions **3.5** Ordering Mixed Numbers and Decimals on a Number Line **3.6** Asking Questions to Solve Multiple-Step Problems	**1.4** Using Decimals **1.5** Understanding Decimal Place Value **1.6** Comparing and Ordering Decimals **1.7** Multiplying and Dividing by 10, 100 and 1 000 **3.2** Estimating Sums and Differences **3.3** Multiplying Decimals by 10, 100 or 1 000 **3.4** Multiplying Decimals **3.5** Dividing Decimals by 10, 100 or 1 000 **3.6** Using Models and Number Lines to Add and Subtract Decimals **3.7** Adding and Subtracting Decimals **3.8** Multiplying a Whole Number by a Decimal **3.9** Dividing a Decimal by a Whole Number **5.1** Expressing a Fraction as a Whole Number **5.2** Representing Fractions in Different Ways **5.3** Using Fractions to Represent Division **5.4** Using Models to Compare Fractions **5.5** Finding Equivalent Fractions

Scope and Sequence

Australian Curriculum Reference	enVisionMATHS Scope and Sequence			
	Foundation	Year 1	Year 2	
Number and Algebra				
Fractions and decimals (cont.)				
Money and financial mathematics		**4.7** Identifying Features and Values of Coins **4.8** Recognising the Values of Coins **13.5** Counting Patterns of 2, 5 and 10	**9.1** Counting Collections of Coins **9.2** Showing the Same Amount in Different Ways **9.3** Making Larger Amounts **9.4** Estimating Sums **9.5** Recognising and Counting Notes and Coins **9.6** Using Money	
Patterns and algebra	**1.1** Classifying Shapes and Objects **1.2** Sorting by One Attribute **1.3** Sorting in Different Ways **1.4** Sorting by More Than One Attribute **1.5** Sorting Using Logical Reasoning **8.1** Identifying Sound and Other Patterns **8.2** Identifying Colour Patterns	**13.1** Describing Patterns **13.2** Using Patterns to Predict **13.3** Extending Shape Patterns **13.4** Using More Patterns to Predict **13.7** Identifying Patterns on a Hundred Chart	**1.10** Counting by 10s Past 100 **2.2** Understanding Addition Stories **2.3** Understanding More Addition Stories **2.5** Adding 0, 1 and 2 **2.6** Adding Using Doubles **2.7** Adding Using Near Doubles **2.8** More Adding in Any Order	

enVisionMATHS Scope and Sequence

Year 3	Year 4	Year 5	Year 6
Number and Algebra			
		7.1 Using Models to Add Fractions **7.2** Using Models to Subtract Fractions **7.3** Adding and Subtracting Fractions with Like Denominators **7.4** Adding Mixed Numbers with Like Denominators	**5.6** Writing Fractions in Simplest Form **5.7** Ordering Fractions **5.8** Writing Equivalent Fractions and Decimals **5.9** Understanding Improper and Mixed Number Fractions **5.10** Comparing and Ordering Fractions and Mixed Number Fractions **7.1** Adding and Subtracting Fractions with Like Denominators **7.2** Adding Fractions with Related Denominators **7.3** Subtracting Fractions with Related Denominators **7.4** Adding and Subtracting Fractions with Related Denominators **7.5** Adding Mixed Numbers with Related Denominators **7.6** Subtracting Mixed Numbers **8.1** Understanding Percentage **8.2** Using Models to Represent Percentage **8.3** Relating Fractions, Decimals and Percentage **8.4** Expressing Percentages Greater Than 100 and Less Than 1 **8.5** Estimating Percentage **8.6** Finding the Percentage of a Number
2.4 Adding Tens and Ones **8.4** Representing Money Values Using Division	**3.10** Calculating Change from Money	**10.4** Solving Money-Related Problems	**8.5** Estimating Percentage **8.6** Finding the Percentage of a Number **8.7** Checking for Reasonableness in Percentage Problems
3.2 Drawing Subtraction Stories **9.1** Continuing a Pattern **9.2** Finding a Rule for a Pattern **9.3** Using Tables to Identify Patterns **9.4** Extending Tables **9.6** Writing to Explain How to Use Patterns **9.7** Using Addition and Subtraction Expressions	**2.5** Using Diagrams to Connect Addition and Subtraction **3.5** Subtracting 3-Digit Numbers **3.6** Drawing a Diagram to Solve Subtraction Problems **4.2** Using Arrays to Multiply with 4 and 8 **4.3** Using Arrays to Multiply with 3, 6 and 9	**4.2** Understanding Multiplication Properties **4.3** Multiplying with Three Factors **4.4** Multiplying 2-Digit by 1-Digit Numbers **4.5** Using Mental Maths to Multiply **4.6** Identifying Missing or Extra Information	**2.7** Using Mental Maths to Multiply **2.8** Using Mental Maths to Multiply and Divide **2.9** Dividing with 1 and 0 **2.11** Ordering Operations **3.1** Looking for Patterns in a Decimal Chart **6.5** Writing Algebraic Expressions

Scope and Sequence

Australian Curriculum Reference	enVisionMATHS Scope and Sequence		
	Foundation	Year 1	Year 2
Number and Algebra			
Patterns and algebra (cont.)	**8.3** Identifying Shape Patterns **8.4** Comparing Patterns **8.5** Creating Patterns		**2.9** Making 10 to Add Two Numbers **2.10** Adding Three Numbers **3.2** Writing Subtraction Number Sentences **3.3** Using Separation Stories **3.4** Using Comparison Stories **3.5** Subtracting 0, 1 and 2 **3.6** Drawing Subtraction Stories **3.7** Using Doubles Facts to Subtract **3.8** Using Addition to 10 to Subtract **4.1** Revising Shape Patterns **4.2** Identifying Number Patterns **4.3** Skip Counting **9.4** Estimating Sums **12.2** Counting 10 More Than, 10 Less than **12.6** Using Numbers to 1 000 **12.7** Ordering Numbers to 1 000
Measurement and Geometry			
Using units of measurement	**4.1** Comparing and Ordering Size **4.2** Comparing Length **4.3** Ordering by Length **4.4** Ordering Many Objects by Length **4.5** Measuring Length with Informal Units **4.6** Measuring Length with Cubes **4.7** Comparing Capacity **4.8** Comparing Mass **7.1** Reasoning to Find Times of the Day **7.2** Ordering Events **7.3** Understanding More Time, Less Time	**3.1** Comparing and Ordering Length **3.2** Measuring Length with Informal Units **3.3** Estimating and Measuring Length **3.4** Comparing and Ordering Capacity **3.5** Comparing and Ordering Mass **7.1** Estimating and Ordering Time Lengths **7.2** Understanding Times of Events **7.3** Understanding Hour and Minute Hands **7.4** Telling and Writing Time to the Hour **7.5** Telling and Writing Time to Half an Hour	**5.1** Estimating Time **5.2** Telling and Writing Time to the Half Hour **5.3** Telling and Writing Time to the Quarter Hour **5.4** Understanding Months and Seasons **5.5** Using a Calendar **8.1** Exploring Length **8.2** Reasoning About Measurement **8.3** Estimating and Comparing Length Using Metres **8.4** Understanding Length Around

enVisionMATHS Scope and Sequence

Year 3	Year 4	Year 5	Year 6
Number and Algebra			
	4.4 Multiplying in Any Order **4.5** Solving Multiplication Problems in Two Steps **4.8** Multiplying 2-Digit by 1-Digit Numbers **4.9** Multiplying by Multiples of 10 and 100 **7.1** Using Diagrams to Divide **7.2** Using a Table to Show Division **7.3** Using Materials to Divide 2-Digit Numbers **7.6** Using Multiplication Facts to Find Division Facts **7.7** Using Mental Maths to Divide **7.8** Dividing Multiples of 10, 100 and 1 000 **7.9** Dividing With 2-Digit Numbers **7.10** Dividing With Remainders **8.1** Looking for a Pattern **8.2** Looking for a Pattern to Solve a Problem **8.3** Describing Geometric Patterns Using a Table **8.4** Using Expressions to Find Missing Numbers **8.5** Working Backwards to Solve Problems **8.6** Using Multiplication and Division Expressions **8.7** Finding Equality **8.8** Identifying Equal or Unequal Number Sentences **10.6** Understanding Temperature	**4.7** Using Equivalent Number Sentences **5.2** Using Patterns to Divide **5.3** Dividing 3-Digit by 1-Digit Numbers **5.4** Dividing Using Zeroes **5.5** Understanding Remainders **5.6** Dividing with Remainders **5.8** Checking for Reasonableness in Division Problems **7.1** Using Models to Add Fractions **7.2** Using Models to Subtract Fractions **7.3** Adding and Subtracting Fractions with Like Denominators **7.4** Adding Mixed Numbers with Like Denominators **10.1** Using Patterns to Show Relationships **10.2** Making a Table to Show Patterns **10.3** Working Backwards to Solve Problems	**6.6** Drawing a Diagram to Solve an Equation **6.7** Understanding Properties of Equality **6.8** Solving Addition and Subtraction Equations **6.9** Creating Number Sequences Using Whole Numbers, Fractions and Decimals
Measurement and Geometry			
4.1 Using Different Tools to Measure **4.2** Estimating and Measuring with Centimetres **4.3** Estimating and Measuring with Centimetres and Metres **4.4** Estimating and Measuring with Metres and Kilometres **4.5** Estimating and Measuring with Millilitres and Litres **4.6** Estimating and Measuring with Grams and Kilograms **4.7** Understanding Perimeter **4.8** Identifying Different Shapes with the Same Perimeter	**10.1** Telling Time to the Minute **10.2** Relating Units of Time **10.3** Comparing Units of Time **10.4** Finding Elapsed Time **10.5** Working Backwards to Solve Time Problems **10.6** Understanding Temperature **11.1** Measuring with Formal Units of Length **11.2** Finding Perimeter **11.3** Identifying Different Shapes with Same Perimeter **11.4** Understanding Area	**8.1** Finding Perimeter **8.2** Identifying Different Shapes with the Same Perimeter **8.3** Finding Area of Rectangles and Squares **8.4** Exploring Perimeter and Area of Rectangles **8.5** Solving Problems with Perimeter and Area **8.6** Measuring Units of Capacity **8.7** Understanding Units of Mass **8.8** Measuring Volume **11.1** Finding Elapsed Time	**2.6** Finding Elapsed Time **9.1** Finding Perimeter **9.2** Converting Measurements **9.3** Identifying Decimals in the Metric System **9.4** Finding Area of Irregular Shapes **9.5** Finding Surface Area **9.6** Finding Volume of a Prism **9.7** Finding Volume and Capacity of Rectangular Prisms

Scope and Sequence

Australian Curriculum Reference	enVisionMATHS Scope and Sequence		
	Foundation	Year 1	Year 2
Measurement and Geometry			
Using units of measurement (cont.)	**7.4** Understanding Days of the Week **7.5** Yesterday, Today and Tomorrow **7.6** Identifying Numbers on a Clock **7.7** Telling Time on the Hour **7.8** Understanding Hot and Cold **7.9** Understanding the Seasons		**8.5** Understanding Area **8.6** Estimating and Comparing Capacity Using Litres **8.7** Measuring Mass **8.8** Estimating and Comparing Mass Using Kilograms **8.9** Understanding Attributes of Objects
Shape	**6.1** Recognising 2D Shapes **6.2** Describing Properties of Shapes **6.3** Making Pictures Using Shapes **6.4** Identifying 3D Objects	**6.1** Understanding Properties of 2D Shapes **6.2** Identifying 2D Shapes **6.3** Understanding Orientation of Shapes **6.4** Identifying 3D Objects **6.5** Sorting 3D Objects	**6.1** Identifying Shapes and Objects Using Reasoning **6.2** Describing Features of Polygons **6.3** Identifying Features of 3D Objects
Location and transformation	**2.1** Identifying Inside and Outside **2.2** Identifying Over, Under and On **2.3** Identifying Top, Middle and Bottom **2.4** Identifying In Front Of, Between and Behind **2.5** Identifying Near and Far **2.6** Identifying Left and Right **2.7** Describing Position and Movement	**7.6** Giving and Following Directions	**6.4** Moving Shapes: Flip, Slide, Turn **6.5** Understanding Half and Quarter Turns **6.6** Using Maps of Familiar Locations
Geometric reasoning			

enVisionMATHS Scope and Sequence

Measurement and Geometry

Year 3	Year 4	Year 5	Year 6
10.1 Telling Time to the Nearest Five Minutes **10.2** Telling Time to the Minute **10.3** Reading Timetables **10.4** Converting Units of Time **10.5** Understanding Elapsed Time Using a Calendar **10.6** Solving Multi-step Time Problems	**11.5** Estimating and Measuring Area **11.6** Estimating and Measuring with Millilitres and Litres **11.7** Estimating and Measuring with Grams and Kilograms **11.8** Introducing Volume **11.9** Enlarging and Reducing Measurements **11.10** Reading Scales	**11.2** Solving Problems Using Elapsed Time **11.3** Solving More Problems Using Elapsed Time **11.4** Reading 24-Hour Time **11.5** Reading Schedules	
6.3 Identifying Polygons **6.4** Making Generalisations About Polygons **6.5** Identifying 3D Objects **6.6** Identifying Faces on 3D Objects **6.7** Relating 2D Shapes and 3D Objects	**5.2** Relating 2D Shapes and 3D Objects **5.3** Describing and Classifying Objects **5.4** Acting Out to Reason About 3D Objects **5.5** Identifying 2D Shapes in Other Shapes	**6.5** Making and Testing Generalisations about Rectangles **6.6** Identifying 3D Objects **6.7** Understanding Polyhedrons **6.8** Relating 2D Shapes and 3D Objects **6.9** Relating 2D Shapes and 3D Objects Using Nets **6.10** Viewing Perspectives of 3D Objects **9.6** Understanding Pentominoes **9.7** Using Pentominoes to Solve Problems	**4.3** Using Properties of Shapes to Solve Problems **4.4** Constructing Simple Prisms and Pyramids **4.6** Identifying Properties of Different Quadrilaterals **4.7** Identifying Properties of Circles **10.1** Viewing Perspectives of 3D Objects **10.2** Using Objects to Solve a Simpler Problem
7.1 Giving and Following Directions **7.2** Using Ordered Pairs on a Grid **7.3** Identifying Lines of Symmetry **7.4** Creating and Using Tangrams	**6.1** Using Compass Directions **6.2** Using Compass Points to Describe Location **6.3** Understanding Scale Drawings **6.4** Drawing Shapes with Lines of Symmetry **6.5** Understanding Tessellating Shapes	**6.5** Making and Testing Generalisations about Rectangles **6.11** Enlarging and Reducing Shapes **9.1** Moving Shapes: Flip, Slide, Turn **9.2** Moving Congruent Shapes: Flip, Slide, Turn **9.3** Translating Shapes **9.4** Reflecting Shapes **9.5** Rotating Shapes **9.6** Understanding Pentominoes **9.7** Using Pentominoes to Solve Problems **9.8** Creating and Using Tessellations **9.9** Identifying Lines of Symmetry **9.10** Understanding Symmetry **9.11** Using Maps to Describe and Interpret Location	**4.3** Using Properties of Shapes to Solve Problems **10.1** Viewing Perspectives of 3D Objects **10.2** Using Objects to Solve a Simpler Problem **10.3** Investigating Translations, Reflections and Rotations **10.4** Locating and Plotting Coordinates **10.5** Introducing the Cartesian Plane
6.1 Understanding Lines and Line Segments **6.2** Introducing Angles	**5.1** Describing Angles	**6.1** Understanding Lines and Angles **6.2** Measuring Angles **6.3** Classifying Triangles **6.4** Classifying Quadrilaterals	**4.1** Measuring and Drawing Angles **4.2** Measuring and Predicting Angles **4.5** Classifying Triangles **4.6** Identifying Properties of Different Quadrilaterals

Scope and Sequence

Australian Curriculum Reference	enVisionMATHS Scope and Sequence		
	Foundation	Year 1	Year 2
Statistics and Probability			
Chance		**11.4** Identifying What Will, Won't or Might Happen **11.5** Identifying More Likely **11.6** Identifying Certain or Impossible	**10.6** Identifying Likely and Unlikely **10.7** Identifying Certain, Possible and Impossible **10.8** Understanding Probability
Data representation and interpretation	**11.1** Collecting Data **11.2** Understanding Data **11.3** Representing Data on a Picture Graph **11.4** Making a Graph	**11.1** Collecting Data **11.2** Making Graphs with Objects **11.3** Understanding Data	**10.1** Using Tally Marks **10.2** Making a Picture Graph to Show Data **10.3** Making a Bar Graph to Show Data **10.4** Graphing to Show Data **10.5** Interpreting Graphs **10.8** Understanding Probability

enVisionMATHS Scope and Sequence

Year 3	Year 4	Year 5	Year 6
Statistics and Probability			
12.7 Predicting and Comparing Outcomes **12.8** Comparing Outcomes with Predictions **12.9** Finding Combinations	**12.7** Predicting and Comparing Outcomes **12.8** Identifying Related Events **12.9** Identifying Independent Events	**12.8** Understanding Averages **12.9** Using Tree Diagrams to Present Outcomes **12.10** Making an Organised List to Record Outcomes **12.11** Exploring Probability	**12.1** Conducting Chance Experiments **12.2** Counting Methods **12.3** Using Statistics to Make Predictions **12.4** Making a List to Record Outcomes **12.5** Comparing Outcomes with Predictions **12.6** Using Fractions, Decimals and Percentages to Describe Probability
12.1 Organising Data **12.2** Posing Questions to Collect Data **12.3** Reading Picture Graphs and Bar Graphs **12.4** Making Bar Graphs **12.5** Using Tables to Construct Bar Graphs **12.6** Using Tables and Graphs to Draw Conclusions	**12.1** Using Surveys **12.2** Organising Data **12.3** Reading Pie Graphs **12.4** Making a Graph **12.5** Understanding Venn Diagrams **12.6** Using Reasoning to Draw Conclusions	**12.1** Displaying Data from Surveys **12.2** Selecting Samples for Surveys **12.3** Interpreting Graphs **12.4** Making and Interpreting Bar and Picture Graphs **12.5** Using Line Graphs **12.6** Interpreting Line Graphs **12.7** Organising Data Using a Dot Plot **12.10** Making an Organised List to Record Outcomes	**11.1** Comparing Graphs **11.2** Presenting Multiple Sets of Data on One Graph **11.3** Labelling Pie Graphs **11.4** Representing and Interpreting Data on a Line Graph **11.5** Making a Graph to Represent Data **11.6** Understanding Stem and Leaf Plots **11.7** Using Statistics to Draw Conclusions **11.8** Interpreting Secondary Data **12.2** Counting Methods

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