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Ready-Ed Publications

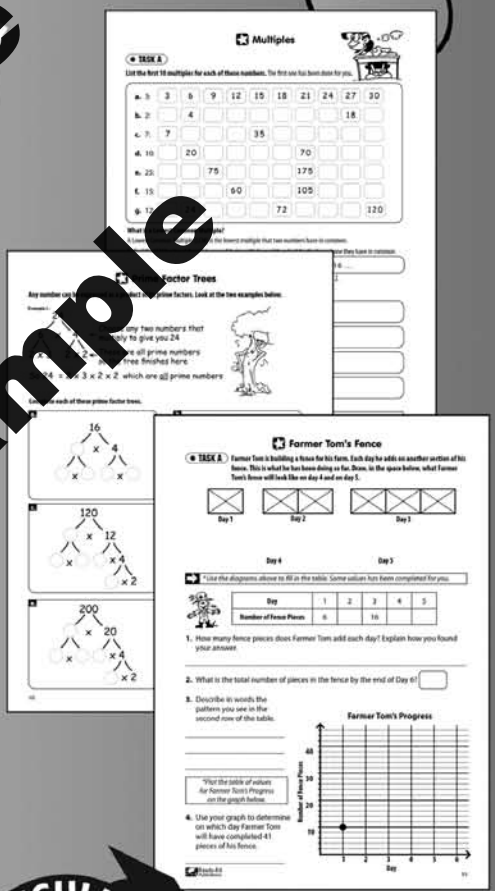
Number Strategies

PHOTOCOPY MASTERS

Working On Algebra

Aligned with the Australian National Curriculum

Ages 10-12 years



CURRICULUM LINKED

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Teachers' Notes

This resource is focused on the Number and Algebra Strand of the Australian Curriculum for students in Year 5 ,Year 6 and Year 7, aged between 10 and 12 years old.

Each section provides students with the opportunity to explore a key area of their algebraic understanding, often with the opportunity to explore their real life contexts or extend their exploration further.

While students are to be encouraged to use their mental arithmetic skills at every opportunity, many tasks will be made more efficient with the use of a calculator. When it is advisable to use a calculator you will see this symbol.

The section entitled "Groups of Numbers" exposes students to various categories of numbers, their uses in calculations and their real life applications. The section on Conjectures allows students to really exercise their mathematical justification.

The section entitled "Number Patterns" exposes students to a large variety of different patterns, from the basic to the famous. Students will learn about some famous Mathematicians and explore some of the fascinating patterns that exist within these famous number sequences.

The section entitled "Rules, Table and Graphs" is a more advanced section which builds towards students being able to move fluidly between the three different representations of functions. Each task is set within a real life context, many of which would be familiar to students. Once mastered, students will have an excellent basis for future function work.

Each section is also prefaced by a Teacher Notes page, explaining the idea and purpose behind each activity. Included here are methods to extend the activities or modify the activities based on the level of individual student ability.

The majority of activities are scaffolded into two sections. Task A builds up the general skills to be mastered, usually enabling students competence in a given skill or an understanding of the basic number sequence. Task B explores the skill further with a more in-depth investigation or consideration and often extends the concept further.

Most activities contain a Challenge at the bottom of the page. These challenges range from Individual Challenges, through to Research and Small Group Challenges. Each of these are designed to complement the activity page, yet extend beyond the material. They are designed to engage student interest and appreciation for Mathematics as well as exposing students to the idea that Mathematics can be a creative and investigative pursuit. Challenges can be included in the lesson of the day, or used as a stand-alone lesson when time permits. Many can be set as homework or assignment tasks over a longer period of time. Research tasks do tend to include the use of internet resources and it is advisable that computer resources are organized in advance.

It is hoped that *Working On Algebra* will be used to help guide teachers in their teaching strategies and methods of presentation. While some activities are designed to be extra practice for students, many others can be used to present and teach students new concepts.

National Curriculum Links

Describe, continue and create patterns with fractions, decimals and whole numbers resulting from addition and subtraction (ACMNA107)

Continue and create sequences involving whole numbers, fractions and decimals. Describe the rule used to create the sequence (ACMNA133)

Identify and describe properties of prime, composite, square and triangular numbers (ACMNA122)

Identify and describe factors and multiples of whole numbers and use them to solve problems (ACMNA098)

Given coordinates, plot points on the Cartesian plane, and find coordinates for a given point (ACMNA178)

Teachers' Notes

Number Properties

Groups of Numbers
This activity exposes students to different types of numbers and their definitions. Task C can be extended into an assignment or a small group task over a few lessons.

Multiples
This activity revises the concept of multiples and provides students with a method to determine a Lowest Common Multiple. The mini quiz task is a useful strategy to employ to determine whether students have understood a topic and can used often to engage student participation and knowledge.

Factors
This activity revises the concept of factors and provides students with a method to determine the Highest Common Factor. Task C is suitable as an extension activity and is probably best attempted by your more able students.

Prime Factor Trees
Understanding that all numbers can be broken down into the product of their prime factors is an important concept and this method provides students with an easy and visual approach to find these prime factors.

Can it be divided by this number?
Task A encourages students to think about how they know if whether a number can be divided (without a remainder) by a particular one digit number. Task B then introduces students to a simple divisibility test to determine whether a number can be divided by 3 without leaving a remainder. Task C can be treated as a research project or for the more able students, as an investigative task.

Divisibility Tests
This task should be attempted after "Can it be divided by this number?". This task encourages students to explore other divisibility tests with the more difficult tests left for Task C and those more capable students.

Factors and Their Prime Factors
This activity is best attempted after "Prime Factor Trees". This activity helps reinforce the idea that once two factors are known, all factors can be known for any number. Task C provides those more curious and capable students with an opportunity to research the real life and important applications of prime numbers.

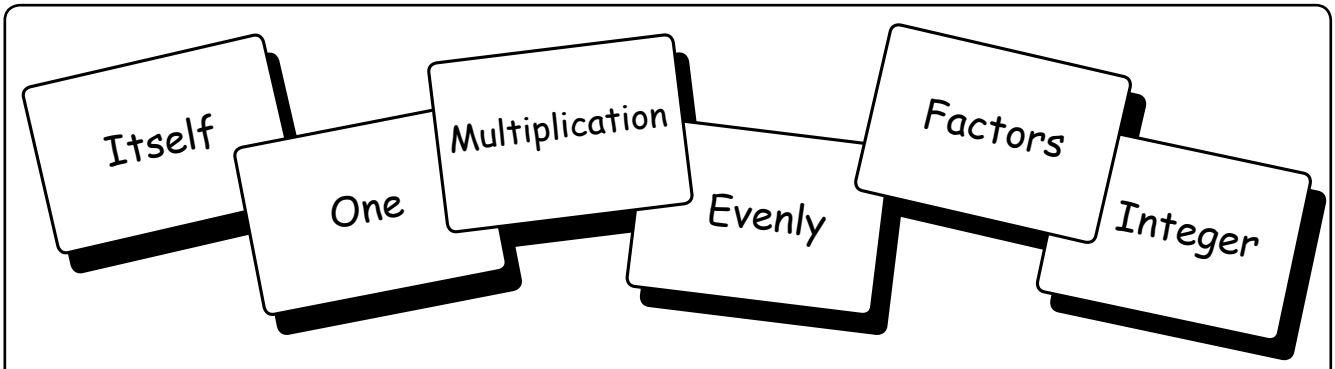
Easy Calculation Using Primes
This activity is best completed after "Prime Factor Trees". Students are encourage to multiply number by first determining their prime factors. The early concepts of indices and manipulation for multiplication come into play here.

Conjectures 1
Students are asked to consider various mathematical statements, called conjectures, and determine whether they are either true or false. This provides students with an opportunity to think about how they know what they know. Task C can extend this concept further by considering geometrical statements.

Conjectures 2
This activity is best completed after "Conjectures 1". These conjectures are more advanced and tie in many of the concepts covered in the previous 9 activities.

Groups of Numbers

✳ TASK A **Choose a word from the list to make each of the following definitions true.**
Some words can be used more than once.



a. Prime Number: A number that is only divisible by _____ and _____.

b. Composite Number: A number that has _____ other than itself and one.

c. Factors: The numbers that can be divided _____ into another number.

d. Multiples: The numbers that are created when we multiply one integer (whole number) by another _____. These can be thought of as the _____ tables of a number.

e. Square Numbers: The number created when we multiply a number by _____.

✳ TASK B **Write down the next ten numbers for each set of numbers:**

a. Prime Numbers: 2, 3, 5, ___ , ___ , ___ , ___ , ___ , ___ , ___ , ___ , ___ , ___ .

b. Composite Numbers: 4, 6, 8, ___ , ___ , ___ , ___ , ___ , ___ , ___ , ___ , ___ , ___ .

c. Square Numbers: 1, 4, 9, ___ , ___ , ___ , ___ , ___ , ___ , ___ , ___ , ___ , ___ .

d. Cube Numbers: 1, 8, 27, ___ , ___ , ___ , ___ , ___ , ___ , ___ , ___ , ___ , ___ .

e. Multiples of 5: 5, 10 , 15, ___ , ___ , ___ , ___ , ___ , ___ , ___ , ___ , ___ , ___ .

✳ TASK C: RESEARCH CHALLENGE

In small groups of 3 or 4 students research the Sieve of Eratosthenes.
Who invented it? What is it used for? Display your findings on a poster with clear explanations and examples of how the sieve works.

Teachers' Notes

Number Patterns

Complete the Pattern 1
Task A encourages students to look for patterns within each number sequence. Each pattern involves the addition or subtraction of numbers only. Task B enables students to describe the patterns they see and to extend the patterns further. Task C provides students with an opportunity to explore an interesting number pattern. Calculators can be used for this activity.

Complete the Pattern 2
Similar to "Complete the Pattern 1", except here students will find that each pattern involves the multiplication or division of numbers. Task C is a fun pattern which is as it's name suggests. You "look" and see one 1, so the next number is 11, then you look and seen two 1s, so the next number is 21 and so on. Calculators can be used for this activity.

Complete the Pattern 3
This task is best completed after the first two "Complete the Patterns" and is an opportunity for students to test their knowledge and skills. Pascal's Triangle 1
This task exposes students to this famous triangle and number sequence. Students are encouraged to explore further patterns within the triangle and to research Blaise Pascal's place in history. This activity is more difficult because students need to investigate each sequence very carefully. You may like to have students work in small groups to discuss the patterns they see.

Square Numbers
This activity enables students to explore square numbers and gain a visual understanding for their name. Task B combines square numbers with other number patterns and some may be a little tricky. Task C provides the students with an opportunity to explore where square numbers are used and a chance to learn about this important Mathematician.

Cube Numbers
This activity is best completed after "Square Numbers" and explores the same concepts, this time about cubic numbers. Task C should keep students busy with their investigations and is the beginnings of one of the most puzzling Mathematical problems of all time, Fermat's Last Theorem.

Triangular Numbers
Triangular numbers are a great number sequence that can be easily visualized. This activity exposes students to this sequence with further explorations.

Fibonacci Numbers
This activity is a rich task, inviting students to explore the Fibonacci sequence and a few of the many patterns to be found within the sequence itself. In Task B students have the opportunity to explore the Golden Ratio and a class discussion of where the Golden Ratio can be found in "real life' might spark an extra interest and enthusiasm from students. Task C is a great opportunity for students to extend their understanding of the history of Mathematics.

Pascal's Triangle 1
This task exposes students to this famous triangle and number sequence. Students are encouraged to explore further patterns within the triangle and to research Blaise Pascal's place in history.

Pascal's Triangle 2
This task is best completed after "Pascal's Triangle 1" and "Fibonacci Numbers". Exploring some of the patterns to be found by adding numbers in Pascal's Triangle allows students to explore the triangle further.

Pascal's Triangle 3
A fun and important application of Pascal's Triangle is in the use of combination theory. Here students can explore the connection between choosing groups of objects and what each row of the triangle represents. Task C allows for further practice of these concepts.

Complete The Pattern 1

✱ TASK A **Look carefully at each of these number patterns and find the next three numbers in the sequence. The first one has been done for you.**

a. 7, 11, 15, 19, 23 , 27 , 31 . *We add 4 each time.*

b. 200, 190, 180, 170, _____, _____, _____.

c. 30, 37, 44, 51, _____, _____, _____.

d. 30 000, 28 200, 26 400, _____, _____, _____.

e. $\frac{1}{12}$, $\frac{1}{6}$, $\frac{1}{4}$, _____, _____, _____ ***Hint:*** *Make all the fractions have the same denominator first before completing the pattern.*

f. 40, 43, 41, 44, 42, _____, _____, _____ *Look closely, this one is tricky!*

✱ TASK B **Complete the numbers in the sequences below and describe the pattern you notice .**

a. 80, 75, 70, 65, ___, ___, ___, ___,

Describe the pattern: _____

b. 1, 3, 6, 15, ___, ___, ___, ___,

Describe the pattern: _____

c. 320, 370, 420, 470, ___, ___, ___, ___,

Describe the pattern: _____

d. $\frac{1}{20}$, $\frac{1}{10}$, $\frac{3}{20}$, ___, ___ , ___, $\frac{7}{20}$ ***Hint:*** *Make all the fractions have the same denominator first before completing the pattern.*

Describe the pattern: _____

✱ TASK C: PERSONAL CHALLENGE

Explain what's happening in this number sequence:

$1 + 3 = 4$ $4 + 5 = 9$ $9 + 7 = 16$ $16 + 9 = 25$

What are the next 10 calculations in this sequence?

Do you think this sequence continues to work forever?

Teachers' Notes

Tables and Graphs

Plotting Points
This activity serves as an introduction to the first quadrant of the Cartesian Plane. Students are encouraged to view plotting coordinates as performing a translation.

Join the Dots
Building on the Plotting Points activity, this activity allows students to follow a series of translation instructions while improving their understanding of plotting points.

Tables and Coordinates
Crucial to future understanding of function work, students are made aware through this activity that a table of values is a more efficient method of recording a series of coordinates.

Plotting Tables of Values
In this activity students practice the skill of turning a table of values into a graph of plotted coordinates. The scale on each axis is already established and it is important to discuss with students the reason for using different scales.

Graphs and Coordinates
Students are encouraged to fluidly transform a graph of coordinates into a table of values. The emphasis is on ensuring that the independent, or horizontal variable, is arranged in the table in ascending orders.

Match Them Up!
A great task to test student understanding after completing the previous tasks. Students need to carefully analyse the graphs, tables and rules and, as the title suggests, match them up.

Marble Clusters
This task begins with an easy visual and requires students to think about the connections between a table of values and the shape of a graph.

Farmer Tom's Fence
A task which explores another linear pattern and enables students the opportunity to compare and contrast and explain how they have determined their answers.

Writing a Short Story
Students are encouraged to compare and contrast different linear patterns and to extend the pattern beyond the available table of values. Describing linear patterns in terms of slope is an important foundational concept.

Roast Dinner
Another familiar problem and linear relationship, students can explore and compare various graphs, exposing them to concepts of slope. Students are encouraged to create their own table of values and you might like to hold a class discussion before students embark on Task C.

Social Media
A very relevant linear relationship, students can explore how mathematical relationships exist even in their spare time! In a familiar context, students also have the opportunity to add their own experiences to the task.

Handshakes and Kisses
The familiar handshakes problem is explored here in more detail and extended by considering the related problem of exchanging kisses! Students are encouraged to explore the connection between the two problems and to consider the shape of curved graphs.

Super Savings
A great task to get students thinking about how even small amounts of savings can go a long way in the long term. Another opportunity to explore curved graphs and to consider a financial application. Students also have the opportunity to add their own experiences and create their own pattern.

Plotting Points

To plot a point on a grid, or the Cartesian Plane, we consider the Left/Right point (or coordinate) and the Up/Down point (or coordinate).
For example, if we are given the point (3, 5) this means we plot the coordinate 3 right, and 5 up, from the origin, or the (0, 0) point.

✱ TASK A

Plot each of the following coordinates on the grid below and be sure to write each letter next to its coordinate. The first one has been done for you.

A: (2, 6)

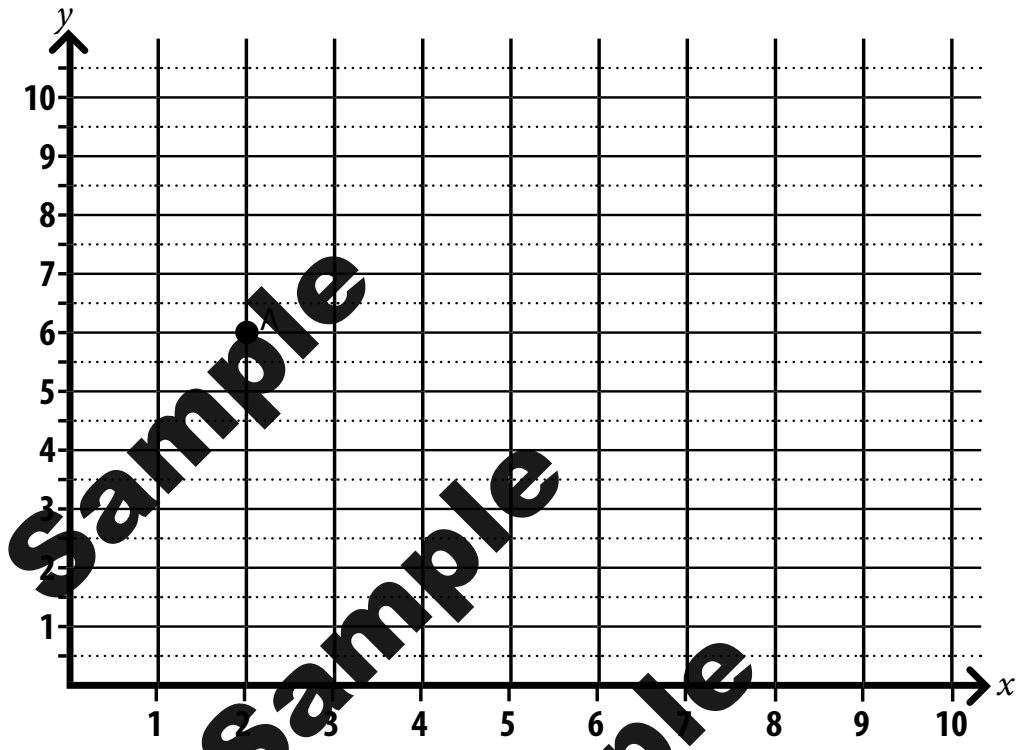
B: (5, 1)

C: (8, 7)

D: (1, 5)

E: (3, 9)

F: (4, 3)



✱ TASK B

For each of the points on the graph below, write their coordinates. Remember you write the horizontal, or x coordinate first, and the vertical, or y coordinate second. The first one has been done for you.

A: (2, 7)

B:

C:

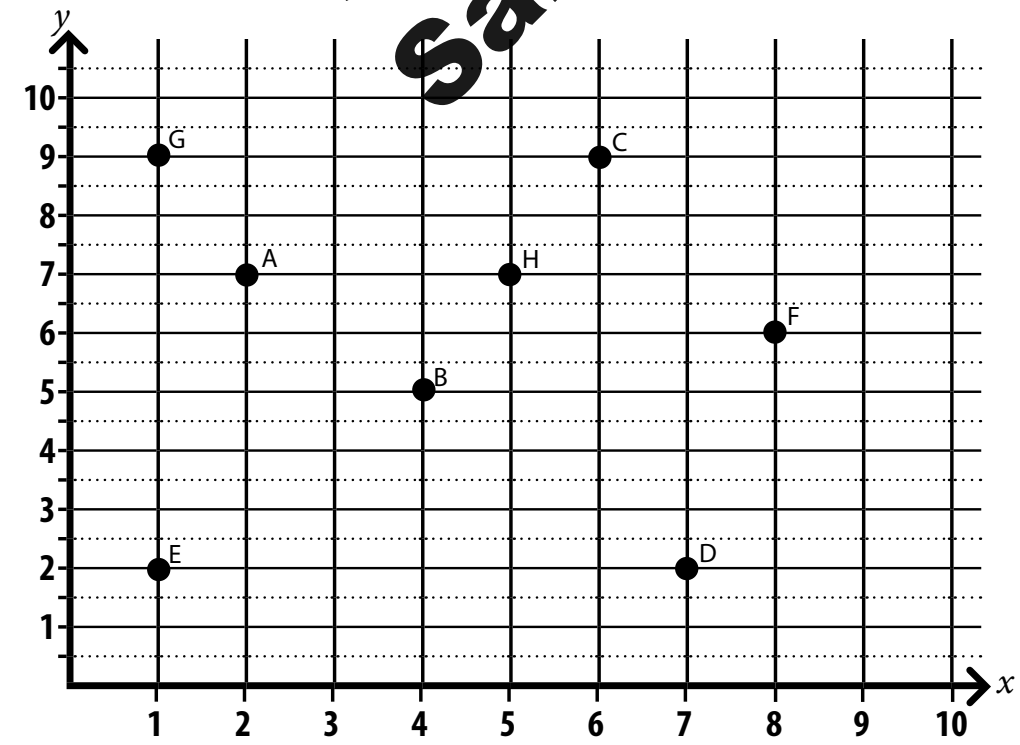
D:

E:

F:

G:

H:



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