2

Integers

2

Texting aliens.

Mathematics is said to be the language that we could use to communicate with aliens. How would this work?

How could we use maths to discover other forms of intelligent life in the universe? Our number system is based on tens (mainly because we have ten fingers), but we cannot assume that an alien number system would be the same. It is believed that the best way to send a message would be to use prime numbers. Prime numbers, such as 2, 3, 5 and 7, have only two factors: 1 and the number itself. This property means that prime numbers will be the same in any number system. In 1974, the Arecibo telescope in Puerto Rico (pictured here) broadcast a message into a star cluster 21 000 light years away. The message consisted of 1679 'bits' of data, which can be arranged into 73 lines of 23 characters (73 and 23 are prime numbers). No answer has been detected yet; this is not surprising given the distance it will have to travel. Later in this chapter you can learn about another way prime numbers are used to send information.

Forum

If you had the opportunity to send the first message to an alien species, what would you say?

Our number system is based on multiplying and dividing by 10; however, sometimes we count by 2, 7, 60, 360 and 365. What do we count using these numbers?

Why learn this?

Understanding relationships between numbers allows us to work with them confidently and efficiently, often without the need for a calculator. A knowledge of factors, multiples and prime numbers is a good foundation for our study of many other areas of mathematics. Negative numbers are an important set of numbers that we will also consider in this chapter. Temperatures, elevations, goal differences and money owed are a few examples of the uses of negative numbers.

After completing this chapter you will be able to:

- find the lowest common multiple of a group of numbers
- find the highest common factor of a group of numbers
- use divisibility tests to assist in finding factors
- identify prime and composite numbers
- find the prime factors of a number
- use positive and negative integers to represent quantities
- compare and order integers
- add and subtract integers.

Recall 2

Prepare for this chapter by attempting the following questions. If you have difficulty with a question, go to Pearson Places and download the Recall Worksheet from Pearson Reader.

PP Worksheet R2.1

1 Copy and complete these within 3 minutes.

(a)	$6 \times 7 =$	$6 \times 6 =$	$6 \times 4 =$	$6 \times 11 =$	$6 \times 8 =$
(b)	$7 \times 11 =$	$7 \times 7 =$	$7 \times 5 =$	$7 \times 2 =$	$7 \times 3 =$
(c)	$8 \times 7 =$	$8 \times 6 =$	$8 \times 4 =$	$8 \times 10 =$	$8 \times 8 =$
(d)	$9 \times 12 =$	$9 \times 3 =$	$9 \times 5 =$	$9 \times 11 =$	$9 \times 8 =$
(e)	$12 \times 7 =$	$12 \times 6 =$	$12 \times 12 =$	$12 \times 9 =$	$12 \times 11 =$

PP Worksheet R2.2

2 **(a)** List all the digits with which an even number can end.

(b) List all the digits with which an odd number can end.

PP Worksheet R2.3

3 Copy and complete each of the following by writing a $<$ (less than) or $>$ (greater than) sign between the given values.

(a) 10 ______ 7 **(b)** 3 ______ 6 **(c)** 2 ______ 0 **(d)** 0 ______ 5

PP Worksheet R2.4

4 Calculate:

(a) $3 + 8 + 12$ **(b)** $22 + 19 - 7$ **(c)** $22 - 9 + 87 - 35$

(d) $18 - 9 - 4$ **(e)** $72 - 39 + 14$ **(f)** $51 + 43 - 11 - 7$

PP Worksheet R2.5

5 Write the following temperatures in order from coldest to warmest.

(a) 15°C, 7°C, 0°C, -4°C, 21°C, -11°C

(b) 5°C, -3°C, 10°C, -25°C, 32°C, -14°C

PP Worksheet R2.6

6 Write the following in expanded form, then evaluate.

(a) 7^2 **(b)** 3^4 **(c)** 2^6 **(d)** 1^9

PP Worksheet R2.7

7 Calculate the following.

(a) $3^2 \times 5^2$ **(b)** $4^3 \div 2^3$ **(c)** $8^2 + 6^2$ **(d)** $9^2 - 7^2$

Key Words

common factor	factor	positive
common multiple	Highest Common Factor (HCF)	prime factor
composite number	integers	prime number
coprime	loss	profit
deposit	Lowest Common Multiple (LCM)	withdrawal
divisibility	multiple	
divisible	negative	

Multiples, factors and divisibility

2.1

Multiples and factors

The numbers 1, 2, 3, 4, 5, ... are called the whole numbers, or the counting numbers. (Any time we use '...' in mathematics, we are saying the pattern is infinite, or goes on forever.)

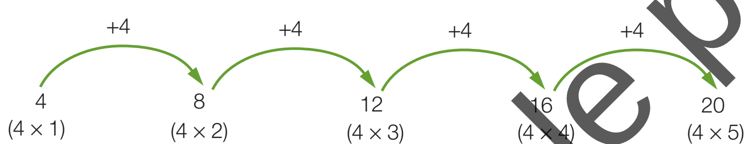
We find the **multiples** of a whole number by multiplying it by another whole number.

For example, the multiples of 7 are:

	1×7	2×7	3×7	4×7	5×7	...
Multiples of 7	7	14	21	28	35	...

Another way to create a list of multiples of a number is to start at the number and add it repeatedly.

For example, the multiples of 4 are:



The first in the sequence of multiples of a number is always the number itself. We can see from the above table and sequence that the first multiple of 7 is 7 (1×7), and the first multiple of 4 is 4 (1×4).

A **factor** is a number that divides exactly into another number.

'Exactly' means that there is no remainder left after the division.

You can think of the process of finding factors as the reverse of finding multiples.

By reversing (flipping) the above table, we can see some factors:

	7	14	21	28	35	...
Some factors	1, 7	2, 7	3, 7	4, 7	5,7	...

This means that the factors of 7 are 1 and 7, some factors of 14 are 2 and 7 etc.

It is often important to find *all* the factors that a number has. We can see from the table that 28 has factors of 4 and 7, because 4 and 7 multiply to give 28.

However, 28 has other factors as well:

$28 = 4 \times 7$

and $28 = 2 \times 14$

and $28 = 1 \times 28$

So, 28 has a total of six factors: 1, 2, 4, 7, 14 and 28.

Worked Example 1 WE1

Find all the factors of each of the following numbers.

(a) 12 (b) 110

Thinking		Working
(a)	**1** Write down the pairs of numbers that multiply to give the original number. The number will always be divisible by 1, so write 1 × original number as the first pair, then consider whether there are pairs beginning with 2, 3 etc.	(a) $1 \times 12 = 12$ $2 \times 6 = 12$ $3 \times 4 = 12$
	2 List the factors from smallest to largest.	Factors of 12: 1, 2, 3, 4, 6, 12.
(b)	**1** Write down the pairs of numbers that multiply to give the original number. The number will always be divisible by 1, so write 1 × original number as the first pair, then consider whether there are pairs beginning with 2, 3 etc.	(b) $1 \times 110 = 110$ $2 \times 55 = 110$ $5 \times 22 = 110$ $10 \times 11 = 110$
	2 List the factors from smallest to largest.	Factors of 110: 1, 2, 5, 10, 11, 22, 55, 110.

Sometimes, two of the same factor are multiplied to give the original number. For example, $7 \times 7 = 49$. We include 7 only once in the list of factors for 49. If we reach such a pair, this also tells us we have finished finding the pairs of numbers.

Divisibility

Another way of considering factors and multiples is to talk about **divisibility**. A larger number is **divisible** by a smaller number if dividing by the smaller number gives an exact whole number answer with no remainder. The following sentences all refer to the same idea.

Two factors of 35 are 5 and 7.
35 is divisible by 5 and 7.
Both 5 and 7 go into 35 exactly, without any remainder.
5 multiplied by 7 gives 35.
35 is a multiple of 5 and also a multiple of 7.

A good knowledge of factors and multiples will help us determine which numbers are divisible by others. For larger numbers, we can use some tests that enable us to determine whether one number is divisible by another. These tests are summarised in the following table.

A number is divisible by …	If it passes this divisibility test
2	The last digit is an even number (0, 2, 4, 6 or 8).
3	The sum of the digits is divisible by 3.
4	The number formed by the last two digits is divisible by 4.
5	The last digit is 0 or 5.
6	The number is even (divisible by 2) and also divisible by 3.
8	The number formed by the last 3 digits is divisible by 8.
9	The sum of the digits is divisible by 9.
10	The last digit is 0.

Worked Example 2

WE2

Determine which of the numbers 75, 98, 110 and 132 are divisible by each of the following.

(a) 3 **(b)** 4 **(c)** 5 **(d)** 6

Thinking	Working
(a) 1 Add up the digits in each of the numbers. If the sum of the digits is divisible by 3, the number is divisible by 3.	(a) 75: 7 + 5 = 12 ✓ 98: 9 + 8 = 17 ✗ 110: 1 + 1 + 0 = 2 ✗ 132: 1 + 3 + 2 = 6 ✓
2 State the answer for each number considered.	75 and 132 are divisible by 3. 98 and 110 are not divisible by 3.
(b) 1 Look at the number formed by the last two digits. If that number is divisible by 4, then the whole number is divisible by 4.	(b) 75 ✗ 98 ✗ 110 ✗ 132 ✓
2 State the answer for each number considered.	132 is divisible by 4. 75, 98 and 110 are not divisible by 4.
(c) 1 Is the last digit 5 or 0?	(c) 75 ✓ 98 ✗ 110 ✓ 132 ✗
2 State the answer for each number considered.	75 and 110 are divisible by 5. 98 and 132 are not divisible by 5.
(d) 1 Write down the even numbers (these are divisible by 2). Add up the digits in each of these numbers and see whether the number is divisible by 3.	(d) Using the working from (a): 98: 17 ✗ 110: 2 ✗ 132: 6 ✓
2 State the answer for each number considered.	132 is divisible by 6. 75, 98 and 110 are not divisible by 6.

Multiples of a whole number are found by multiplying it by another whole number.

A factor is a number that divides exactly into another number.

Divisibility tests can help find the factors of a whole number.

Common multiples

A **common multiple** of two numbers is a number that both of them divide into exactly. Changing the multiple table from the start of the section slightly, we get:

	1 and 7	2 and 7	3 and 7	4 and 7	5 and 7	...
A common multiple	7	14	21	28	35	...

This table only gives one common multiple for each pair of numbers. There is an infinite number of others. The **Lowest Common Multiple (LCM)** of two numbers is the *smallest* number that both of the numbers divide into exactly. The common multiples of 2 and 7 are 14, 28, 42, 56,… The LCM of 2 and 7 is 14. There is no highest common multiple.

Worked Example 3

WE3

Find the lowest common multiple (LCM) of the following set of numbers, by first listing the multiples of each: 4 and 6.

Thinking	Working
1 List the first few multiples of the first number.	4: 4, 8, 12, 16, 20, 24, …
2 List the first few multiples of the second number.	6: 6, 12, 18, 24, 30, 36, …
3 Circle the first number that appears in both lists. This is the LCM.	LCM of 4 and 6 is 12.

Common factors

A **common factor** of two numbers is a number that divides exactly into both of them. Common factors should not be confused with common multiples. Consider the following.

	7 and 14	4 and 20	9 and 15	8 and 40	12 and 18
Common factors	1, 7	1, 2, 4	1, 3	1, 2, 4, 8	1, 2, 3, 6

1 will always be a common factor of any set of numbers. Sometimes it's important for us to find the **Highest Common Factor (HCF)** of two numbers. From the above table, we can see that the HCF of 7 and 14 is 7, the HCF of 9 and 15 is 3, the HCF of 12 and 18 is 6 etc.

If the smaller number in the pair is a factor of the larger number, the smaller number is the HCF. For example, the HCF of 4 and 20 is 4 and the HCF of 8 and 40 is 8. The HCF of a pair of numbers cannot be bigger than the smaller number of the pair.

Worked Example 4 WE4

Find the highest common factor (HCF) of the following pairs of numbers, by first listing the factors of each number: 12 and 18.

Thinking	Working
1 List all factors of the first number.	12: (1), (2), (3), 4, (6), 12
List all factors of the second number.	18: (1), (2), (3), (6), 9, 18
2 Circle the factors appearing in both lists. These are the common factors.	
3 Select the largest number that appears in both lists. This is the HCF.	HCF of 12 and 18 is 6.

The lowest common multiple (LCM) of two numbers is the smallest number that both of the numbers divide into exactly.

The highest common factor (HCF) of two numbers is the largest number that divides exactly into both of the numbers. The highest common factor is also known as the Greatest Common Divisor (GCD).

2.1 Multiples, factors and divisibility

Navigator

Q1 Columns 1–3, Q2, Q3 Columns 1 & 2, Q4 Columns 1–3, Q5, Q6, Q7, Q9, Q10, Q12, Q13, Q14, Q15, Q18, Q23	Q1 Columns 2 & 3, Q2, Q3 Columns 2 & 3, Q4 Columns 2–4, Q6, Q7, Q8, Q9, Q10, Q11, Q12, Q13, Q14, Q15, Q17, Q18, Q19, Q20(a), Q23, Q24	Q1 Columns 3 & 4, Q2, Q3 Column 3, Q4 Columns 3 & 4, Q6, Q7, Q8, Q9, Q10, Q11, Q12, Q13, Q15, Q16, Q17, Q18, Q19, Q20, Q21, Q22, Q24, Q25

Answers page 627

Fluency

1 Find all the factors of each of the following numbers. WE1

(a) 18 (b) 16 (c) 23 (d) 24
(e) 20 (f) 35 (g) 36 (h) 42
(i) 53 (j) 60 (k) 77 (l) 84

2 Determine which of the numbers 92, 108, 245 and 3100 are divisible by each of the following. WE2

(a) 3 (b) 4 (c) 5 (d) 8 (e) 9

WE3

3 Find the lowest common multiple (LCM) of the following sets of numbers, by first listing the multiples of each.

(a) 2 and 5 (b) 3 and 9 (c) 5 and 25

(d) 5 and 6 (e) 4 and 7 (f) 8 and 12

(g) 7 and 9 (h) 10 and 12 (i) 6 and 11

(j) 9 and 12 (k) 20 and 50 (l) 8 and 14

(m) 3, 4 and 5 (n) 2, 25 and 50 (o) 20, 50 and 60

WE4

4 Find the highest common factor (HCF) of the following pairs of numbers, by first listing the factors of each number.

(a) 10 and 15 (b) 8 and 24 (c) 5 and 12 (d) 26 and 36

(e) 11 and 33 (f) 28 and 70 (g) 44 and 22 (h) 10 and 30

(i) 40 and 70 (j) 32 and 60 (k) 35 and 70 (l) 42 and 48

A factor of a number can't be larger than the number itself.

5 (a) The lowest common multiple of 8 and 1 is:

A 8 B 16 C 24 D 80

(b) Which of the following is a factor of 34?

A 4 B 12 C 17 D 68

6 (a) A number divisible by 2, 3 and 5 is:

A 6 B 15 C 60 D 65

(b) Which pair of numbers are both divisible by 4?

A 38 and 42 B 38 and 52 C 38 and 60 D 52 and 60

Understanding

7 (a) Which one of the following numbers is not a multiple of 8?

A 4 B 24 C 72 D 88

(b) Which of the following is not a factor of 42?

A 1 B 6 C 21 D 84

8 State TRUE or FALSE for the following.

(a) 346 is a multiple of 3. (b) 872 is divisible by 6.

(c) 2 is a factor of 348. (d) 52 is a multiple of 4.

(e) 854 is divisible by 9. (f) 3 is a factor of 56 902.

9 For each group of numbers, find **(i)** the LCM and **(ii)** the HCF.

(a) 4, 6 and 10 (b) 6, 8 and 12

(c) 8, 12 and 16 (d) 10, 25 and 40

10 Complete the following sentences by using the words 'multiple', 'factor' or 'divisible'.

(a) 32 is a multiple of 8 because it is ____________ by 8.

(b) 6 is a ____________ of 54, so 54 is a multiple of 6.

(c) 72 is divisible by 9, so that makes it a ____________ of 9.

(d) 4 is a factor of 60, so 60 is ____________ by 4.

11 **(a)** If 24 lollies are placed into bags so that each bag contains the same number, how many lollies can be in each bag? List all possible answers.

(b) If 36 lollies are placed into bags so that each bag contains the same number, how many lollies can be in each bag? List all possible answers.

12 Mrs Williams wants to arrange the seating in the hall for the Year 7s. There must be the same number of chairs in each row. She wants the students to take up all the seats in a row. There are 96 students.

(a) How many rows could there be, and how many seats are in each row? Give all possible combinations, including impractical ones.

(b) Mrs Williams would like the arrangement to be as 'square' as possible. Which arrangement is best for this?

13 Mr Rasheed is putting his students into groups to work on a project. Students must be in groups of 3 or 4. He has 26 students in his class. Find the two different ways Mr Rasheed can divide up his class.

14 The smallest number divisible by 3, 4 and 5 is:

A 12 **B** 24 **C** 30 **D** 60

15 If two events occur at different time intervals, the lowest common multiple (LCM) of the two time intervals is the point when the two events coincide, or occur together. Use this information to answer the following question.

In a city lighting display, one set of lights flashes every 25 seconds and the other set flashes every minute. If they are turned on at the same time, write down the next three times when the two sets of lights flash together.

16 **(a)** Find the lowest number greater than 50 that is divisible by 7.

(b) Find the lowest number greater than 100 that is divisible by 11.

(c) Find the first common multiple of 2 and 7 that is greater than 100.

(d) Find the first common multiple of 2, 5 and 7 that is greater than 200.

Reasoning

17 Peter power-walked around an oval while Mei-ling jogged. They started and finished at the same time. They started on the same spot and went in the same direction, keeping up a constant speed for 1 hour. Peter walked 8 laps and Mei-ling jogged 24 laps in the hour.

(a) How many times did Mei-ling pass Peter?

(b) How many times did Mei-ling pass Peter exactly on the spot where they started?

(c) At the beginning of which laps did Mei-ling pass Peter exactly on the spot where they started?

18 **(a)** Copy the following table and do the divisibility tests on the numbers in the left column. Circle the number if the original number is divisible by it. The first one has been done for you.

100 000	②	3	④	⑤	6	⑧	9	⑩
202 008	2	3	4	5	6	8	9	10
12 121 212	2	3	4	5	6	8	9	10
300 300 300	2	3	4	5	6	8	9	10
7 500	2	3	4	5	6	8	9	10
900 090	2	3	4	5	6	8	9	10
123 456 789	2	3	4	5	6	8	9	10

(b) Complete the following.

(i) If a number is divisible by 4 it will also be divisible by ______.

(ii) If a number is divisible by 9 it will also be divisible by ______.

(c) Explain your answers to **(b)**.

19 The test to determine whether a number is divisible by 6 is to test whether it is divisible by 2 *and* 3. Explain why the test works.

20 A *perfect* number is a number for which the sum of its factors (excluding itself) equals the number. The first perfect number is 6, as $1 + 2 + 3 = 6$.

(a) What is the next perfect number? It is less than 40.

(b) The next perfect number is between 490 and 510. See if you can find it.

21 An *abundant* number is a number for which the sum of its factors is greater than two times the number itself. The first abundant number is 12, as $1 + 2 + 3 + 4 + 6 + 12 = 28$, which is greater than 2×12. Find the next 2 abundant numbers. (Both are less than 40.)

22 **(a)** How can you always find a common multiple of a pair of numbers?

(b) How can you check if this number is the lowest common multiple?

Open-ended

23 Darren is designing a box for 60 identical chocolates to be placed in rows.

(a) Draw three ways Darren could arrange the chocolates in the box.

(b) Which of your arrangements do you think is the most practical for a chocolate box? Explain your answer.

24 Zena is five years of age and Sam is less than 90 years old. Sam's age is a multiple of three and is also a multiple of Zena's age. Find three possible ages Sam could be.

25 Is it possible to find the highest common multiple of two or more numbers? Explain your answer.

Primes and composites

2.2

A number that has exactly two factors, itself and 1, is a **prime number**.

A number that has more than two factors is called a **composite number**.

The number 1 is a special number. It is neither prime nor composite.

The number 7 is a prime number as its factors are 1 and 7. The number 8 is a composite number as its factors are 1, 2, 4 and 8.

Two numbers are said to be **coprime** if their highest common factor is 1.

The sieve of Eratosthenes

Eratosthenes was a Greek mathematician who lived from 276 BCE to 195 BCE. He was the first person to calculate a value for the circumference of the Earth. Another thing he was famous for was his 'sieve'.

See if you can reproduce what he did.

Copy the table and follow the instructions.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Step 1 Cross out the number 1.

Step 2 Go to the next number, which is 2, and circle it. Then, cross out all of the other multiples of 2.

Step 3 Go to the next number that isn't crossed out. This should be 3. Circle it. Then, cross out all of the other multiples of 3.

Step 4 Go to the next number that isn't crossed out, circle it, then cross out all of its multiples.

Step 5 Repeat for the next number that isn't crossed out. Keep repeating until there is no 'next number'.

Step 6 Write the factors of each of the circled numbers. What types of numbers are these?

Step 7 Write the factors of any five of the crossed out numbers, except for 1.

Step 8 Which type of number—circled or crossed out—has more factors? Explain why.

2.2 Primes and composites

Navigator

Answers page 628

Q1, Q2, Q3, Q5, Q6, Q7, Q8, Q10, Q11, Q13, Q14, Q17	Q1, Q2, Q3, Q4, Q6, Q7, Q8, Q9, Q10, Q11, Q12, Q13, Q14, Q17, Q18	Q1, Q3, Q4, Q6, Q7, Q8, Q9, Q10, Q12, Q13, Q14, Q15, Q16, Q17, Q18

Use 'The sieve of Eratosthenes' on the previous page to help you answer Questions **1–6**.

Fluency

1 Write the prime numbers between 1 and 20.

2 How many single-digit prime numbers are there? List them.

3 List all the primes between 20 and 60.

4 **(a)** The first prime number after 50 is:

A 51 **B** 53 **C** 55 **D** 57

(b) A number coprime with 18 is:

A 9 **B** 21 **C** 24 **D** 25

Understanding

To show that a number is composite, you only need to show that one of the divisibility tests works.

5 **(a)** What is the next prime number after 60?

(b) What is the next composite number after 60?

(c) What are the two odd composite numbers less than 20?

(d) What is the largest prime number less than 50?

6 Write TRUE or FALSE for each of the following statements.

(a) 21 is prime. **(b)** 38 is composite.

(c) 59 is prime. **(d)** 49 is prime.

(e) 5 and 7 are coprime. **(f)** 5 and 6 are coprime.

(g) All even numbers greater than 2 are composite.

(h) All odd numbers are primes.

7 Name a divisibility test that shows that the following numbers are composites.

(a) 410 **(b)** 621 **(c)** 9909

(d) 4 516 803 **(e)** 87 912 404 **(f)** 2 871 025

8 Are the following pairs of numbers coprime? Give reasons for your answer.

(a) 9 and 17 **(b)** 8 and 11 **(c)** 13 and 52 **(d)** 27, 63

Reasoning

9 Explain why any pair of prime numbers is coprime.

10 2 is the only even prime number. Explain why.

11 Explain why it is easy to tell that 4 567 278 is a composite number.

12 Explain why 2 and 3 are the only two consecutive prime numbers.

13 What is the smallest difference between any two consecutive composite numbers?

14 **(a)** Find the numbers closest to 100 that are coprime with 100.

(b) Find the numbers closest to 36 that are coprime with 36.

15 Will a prime number always be coprime with any other whole number? Explain your answer.

16 If one number is a multiple of another number and both numbers are greater than 1, explain why they cannot be coprime.

Open-ended

17 A conjecture is a mathematical statement that is believed to be true, but has not yet been proven. Goldbach's conjecture (named after the mathematician Christian Goldbach) states that 'every even number greater than 2 can be written as the sum of two primes'. Choose 10 even numbers, and use them to demonstrate Goldbach's conjecture.

18 A pair of 'Sophie Germain primes' (named after the mathematician) is a pair of prime numbers where one number is exactly one more than double the other number. For example, 11 and 23 are Sophie Germain primes, because $11 \times 2 + 1 = 23$. Find two more pairs of Sophie Germain primes.

Outside the Square Puzzle

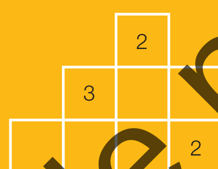
Gold digger 1

It's the final day of the 16th annual gold-digging competition. Carmen, your partner for the competition, has almost worked out where the gold is located. She has marked on two separate maps for you the places next to where the gold lies. If a number 3 is in a box, then it means that there are 3 pieces of gold in adjacent squares, either horizontally, vertically or diagonally. (Adjacent squares share an edge or a corner.) None of the already numbered squares contains a piece of gold, and no square contains more than one piece of gold.

Your task is to find exactly where the gold lies, so your team can get to it first and win the competition. There is only one possible solution for each map.

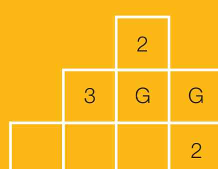
Basic techniques

Look for an easy opening:

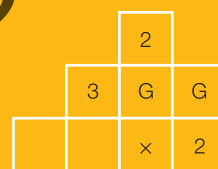


The top square (numbered 2) is only touching 2 other empty squares, so both of these must contain pieces of gold. Mark these squares with a 'G' to signify this.

Go back to other squares:



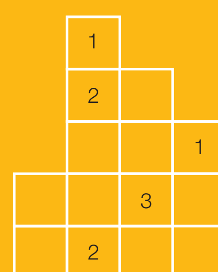
Now, look at the 2 in the bottom right-hand corner. It is already next to 2 pieces of gold, so the other square it is touching is empty. Mark this square with a cross.



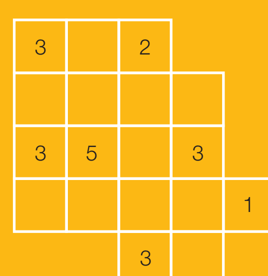
Now, look at the 3, and you can see that it is already next to one piece of gold, and is only touching two other empty squares, so both of these must also contain gold.

Now, copy the following maps and find the gold.

(a)



(b)



Investigation

Heat beads and ice blocks

Equipment required: 1 brain, 10 red and 10 blue counters (or any other pair of colours)

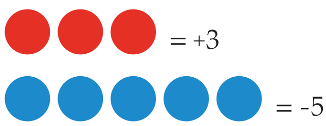
The Big Question

Positive and negative integers can be represented by different coloured counters. How can we use them to show integer addition and subtraction?

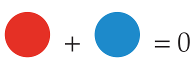
Engage

We will use the red counters to represent positive integers and the blue counters to represent negative integers.

For example:



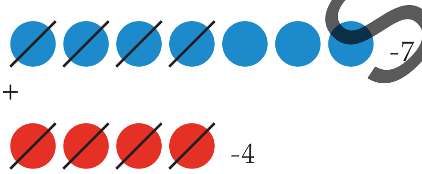
Because they are opposites, 1 blue counter will cancel out, or 'annihilate', 1 red counter. In other words, 1 blue + 1 red = 0. We can think of this as 1 'heat bead' (red) cancelling out 1 'ice block' (blue).



We can write this as a number sentence: 1 + -1 = 0.

We can use combinations of red and blue counters to represent integers, by cancelling red and blue pairs.

For example:



= -3

(We have shown here that 4 red and 4 blue pairs cancelled each other.)

1 Place 3 red counters on the table to represent +3. Now, place 2 blue counters underneath them.

What number is represented now (remembering that 2 blues will cancel 2 reds)?

Complete the number sentence: +3 + -2 = ______ .

2 Use your counters to do the following. Draw diagrams to show each addition, cancelling red and blue pairs.

(a) +3 + -4

(b) +2 + -5

(c) +4 + -6

3 Use your counters to do the following. Draw diagrams to show each addition, cancelling red and blue pairs.

(a) -4 + +1

(b) -7 + +3

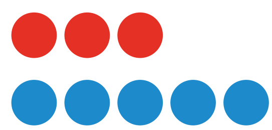
(c) -9 + +2

Explore

4 So far, we have used the counters to model addition. Modelling subtraction can be a little more challenging.

> **Strategy options**
> - Draw a diagram.
> - Act it out.

(a) What number is shown by this collection of counters?



(b) If you removed the red counters, what number would be shown now?

Complete the number sentence:
-2 – (+3) = ______

5 Notice that in order to perform this subtraction, we had to have 3 red counters present, so that we could show the subtraction by removing them. They were balanced or 'annihilated' by 3 of the blue counters, so they had no effect on the starting number.

Use your counters to do the following subtractions. You may need to add pairs of red and blue counters in order to have enough to remove.

(a) -4 – +2

(b) -1 – +3

(c) -2 – +5

6 Use your counters to do the following subtractions. Again, you may need to add pairs of counters at the beginning.

(a) +3 – +4

(b) +2 – +5

(c) +4 – +6

7 Looking at the counters diagram in Question **4**, what number would be shown if you removed all of the blue counters? Write this as a subtraction.

8 Use your counters to do the following subtractions.

(a) -4 – -1

(b) -7 – -3

(c) -9 – -2

9 Use your counters to do the following subtractions.

(a) -2 – -4

(b) -5 – -1

(c) -3 – -7

10 Use your counters to do the following subtractions.

(a) +2 – -4

(b) +5 – -1

(c) +3 – -7

Explain

11 (a) What do you notice about the answers to Question **2** and Question **6**? What conclusion can you draw from this?

(b) What do you notice about the answers to Question **3** and Question **8**? What conclusion can you draw from this?

(c) What do you notice about the answers to Question **9** and Question **10**? Why is this the case?

12 Why was it necessary to use counters of both colours to represent your starting number in Questions **5** and **6**, and **9** and **10**?

13 Helen wants to use her red and blue counters to show +3 – -5. She has lined up 3 red counters to show +3.

She has no blue counters to remove. Explain how Helen can include 5 blue counters in her representation of +3.

Elaborate

14 For each of the following, choose an integer between -10 and +10, and use counters of one or both colours to represent it. (For example, you might choose -3, which you could show with 3 blue counters, or 2 red and 5 blue counters.) Then, add or subtract a second number of counters to show the operation described. Write a number sentence for the operation you have modelled, and draw a diagram of the counters.

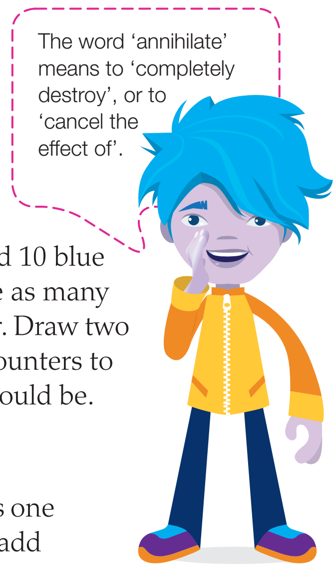
(a) adding a positive integer to a negative integer

(b) adding a negative integer to a positive integer

(c) subtracting a positive integer from a negative integer

(d) subtracting a negative integer from a positive integer

15 Explain why this method of using counters to show integer addition and subtraction is called the 'annihilation method'.



16 Sam has made a number using some of his 10 red and 10 blue counters. He has used twice as many of one colour than the other. Draw two different arrangements of counters to show what Sam's number could be.

Evaluate

17 Using counters in this way is one method of learning how to add and subtract negative numbers. Another method is to use a number line, which is presented elsewhere in this chapter.

Do you find one method easier to work with than the other? Which one? Why do you think this is?

Extend

18 Can you use your counters to show multiplication?

(a) Start by showing 2 × -4 and 3 × -3. It may be useful to remember that multiplication means 'groups of'.

(b) Can you show -4 × -2 or -3 × -3? How is this method of using counters limited here?

Chapter review 2

D.I.Y. Summary

Key Words

common factor	factor	positive
common multiple	Highest Common Factor (HCF)	prime factor
composite number	integers	prime number
coprime	loss	profit
deposit	Lowest Common Multiple (LCM)	withdrawal
divisibility	multiple	
divisible	negative	

1 A ___________ of 6 is 18. The ___________ of 6 and 4 is 12.

2 The ___________ are all of the positive and negative whole numbers, and zero, which is neither positive nor negative.

3 1, 2, 3, 6, 9 and 18 are the ___________s of 18. The ___________ of 18 and 27 is 9.

4 A number that is not ___________ by any numbers other than 1 and itself is called a ___________.

5 A number with more than two factors is called a ___________.

6 When you put money into a bank account, you are making a ___________.

7 The addition of two negative numbers will always give a ___________ answer.

8 If you sell something for less money than you bought it for, you have made a ___________.

9 Every whole number greater than 1 can be written as the unique product of its ___________s.

10 You make a ___________ when you take money out of your bank account.

11 If you sell something for more than you bought it for, you have made a ___________.

12 Two numbers are ___________ if their highest common factor is 1.

Fluency

1 Find the LCM of: Ex. 2.1

(a) 9 and 6 (b) 9 and 12 (c) 10 and 15

2 List all the factors of: Ex. 2.1

(a) 36 (b) 48 (c) 51 (d) 100

3 Find the HCF of: Ex. 2.1

(a) 24 and 56 (b) 18 and 72 (c) 45 and 80

4 Copy the following table and do the divisibility tests. Circle the number if the original number is divisible by it. Ex. 2.1

5301	2	3	4	5	6	9	10
10 000	2	3	4	5	6	9	10
333 333	2	3	4	5	6	9	10
31 700	2	3	4	5	6	9	10
43 521 820	2	3	4	5	6	9	10

5 State whether each of the following numbers is a prime number or a composite number, and explain why. Ex 2.2

(a) 5 (b) 16 (c) 77 (d) 276 350

6 By drawing a factor tree or using the 'repeated division' method, express each number as a product of its prime factors. Ex 2.3

(a) 24 (b) 30 (c) 88 (d) 200

7 Use prime factors to find the HCF of: Ex 2.3

(a) 27 and 36 (b) 72 and 96 (c) 108 and 240

8 Write an integer to represent the following. Ex 2.4

(a) 14 degrees below zero (b) an altitude of 200 metres

9 State the opposite of: Ex 2.4

(a) north 5 km (b) adding 27

10 Write < or > between the following pairs of numbers to make a true statement. Ex 2.4

(a) -52 _______ 25 (b) 19 _______ -20

11 Arrange the following numbers in ascending order. Ex 2.4

(a) -7, 12, 0, -9, 7 (b) 4, -4000, 40, 400

12 Calculate: Ex 2.5

(a) +16 + 2 (b) -3 + 18 (c) -15 + 5 (d) +9 – 3

(e) +1 – 5 (f) +16 – 8 (g) +7 – 12 (h) -14 – 18

13 Calculate: Ex 2.6

(a) +7 + (-10) (b) +9 + (-6) (c) -11 – (-4) (d) -4 – (-4)

(e) -12 – (-5) (f) +5 + (-3) (g) -8 – (-5) (h) -5 + (-7)

14 Rewrite the following with a single sign between the integers, then evaluate. Ex 2.7

(a) 9 – (+11) (b) -3 + (+10) (c) -10 – (-21) (d) 8 + (-12)

(e) -4 – (-41) (f) -14 + (+28) (g) -5 – (+8) – (-2) (h) 4 + (-9) + (+2)

Understanding

15 Use the words 'multiple', 'factor' and 'divisible' to complete the following sentences.

(a) 45 is ___________ by 9, so that makes it a ___________ of 9.

(b) 8 is a ___________ of 56, so 56 is a ___________ of 8.

(c) 27 is ___________ by 3, so that makes it a ___________ of 3.

16 If 96 lollies are to be divided into packets so that each packet contains the same number, how many lollies can be in each packet? Give all possible combinations.

17 Use the symmetry of the number line to help you calculate the following.

(a) -31 + 19 (b) -54 – 27 (c) -22 + (-38) (d) -9 – (61)

18 Describe the number line journey you could follow to find the value of:

(a) +3 + (-8) (b) -6 – (+5)

19 For each pair of numbers, state whether or not they are coprime. If not, explain why.

(a) 11 and 27 (b) 51 and 63 (c) 14 and 35 (d) 24 and 55

20 In the game of indoor cricket, 3 runs are subtracted from a team's score every time a wicket is taken. Here is what happened in the first over of a game.

Ball 1: 2 Runs
Ball 2: Wicket
Ball 3: 1 Run
Ball 4: Wicket
Ball 5: 1 Run
Ball 6: 4 Runs

What was the score at the end of the over?

21 There are 84 male and 108 female guests at an official dinner. The dinner organiser wants to have an equal number of male and female guests at each table.

(a) Use prime factors to find the HCF of 84 and 108, and so find the number of tables required.

(b) Use your answer from **(a)** to find the number of male and female guests at each table.

22 Find the first common multiple of 2, 7 and 9 that is greater than 500.

23 (a) What is the first prime number after 70?

(b) What is the first composite number after 70?

24 Michelle made deposits of $210, $25, $45 and $66 into her bank account during one month, and withdrawals of $35, $56, $214 and $102 during the same period.

(a) At the end of the period, had her balance increased or decreased?

(b) By how much had it increased or decreased?

25 Joanna and Petra are on two different ferris wheels, both rotating clockwise. The first wheel takes 25 seconds to make a rotation and the other takes 30 seconds. If Joanna and Petra were both at eye level at the bottom of each of their ferris wheels when they start turning, how many seconds will pass until they are again both at the bottom at eye level?

26 Miners in a copper mine are working 900 m underground. They get in a lift and travel a further 250 m down. What depth are they working at now? Write your working and answer using negative integers.

Reasoning

27 Copy the following and write <, = or > to make true statements.

(a) 4 – (-7) _____ 4 + 7

(b) -3 + 2 _____ 3 – 2

(c) 5 + (-3) _____ 5 – (+3)

(d) -8 – 9 _____ -8 + 9

28 If you know that a number is divisible by 8, what other numbers do you also know it is divisible by?

29 A number between 900 and 1000 has four prime factors: 2, 5, 7 and one other factor. What is the number, and what is the missing factor?

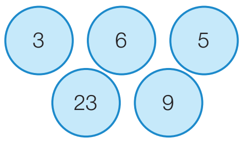
NAPLAN practice 2

Numeracy: Non-calculator

1 Alicia is standing at -4 on a large number line placed on the floor. She walks 11 steps in the positive direction. At which number is she standing now?

A -15 **B** -7 **C** 7 **D** 15

2



The sum of the composite numbers in the group shown above is:

A 8 **B** 15 **C** 18 **D** 31

3 A maintenance worker in a city office building gets in a lift in the 3rd basement level (3 floors below ground level) and goes up 11 levels. What floor does he get out on?

4 The number 42 written as a product of prime factors is:

A 21×2 **B** $2 \times 3 \times 7$ **C** 1×42 **D** 6×7

5 On a sunny winter's day in Moscow, the temperature at midday was 3°C. By midnight it had dropped to -9°C. The integer that represents this change is:

A -12 **B** -3 **C** 3 **D** 12

Numeracy: Calculator allowed

6 A train passes a town every week and every 10 days an aeroplane flies overhead. If the train and the plane were observed to be at the town on a certain day, after how many more days will both appear again at the same time?

A 10 **B** 11 **C** 50 **D** 70

7 Vin has $260 in his bank account. During one month, he makes the following transactions.

Deposit: $55
Withdrawal: -$75
Withdrawal: -$33
Deposit: $85
Withdrawal: -$27

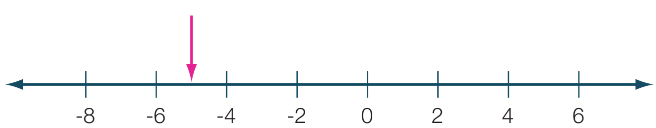
How much does Vin have in his account at the end of the month?

8 The ages in years of three people are 65, 39 and 52. The highest common factor of the three ages is:

A 1 **B** 13 **C** 39 **D** 165

9 The arrow is pointing to an integer on the number line.

What number is at this position? _____



Mixed review A

Fluency

1 Write these integers in ascending order. Ex. 2.4

8, 17, -10, 0, -25, 32, -48

2 Write the following in index form. Ex. 1.2

(a) 9 squared (b) $7 \times 7 \times 7 \times 7$ (c) 4 cubed

3 List all numbers divisible by both 8 and 6 that are less than 100. Ex. 2.1

4 Write an integer to represent the following. Ex. 2.4

(a) a bank withdrawal of $570 (b) a win by 5 points

5 Write < or > between the following pairs of numbers to make a true statement. Ex. 2.4

(a) -27 _____ 14 (b) 0 _____ -35

6 Calculate: Ex. 1.3

(a) $8000 \div 200$ (b) $1200 \div 4$ (c) $45\,000 \div 90$

(d) 30×120 (e) 400×1500 (f) 2000×5000

7 Use a mental strategy to calculate the following. Ex. 1.1

(a) $4 \times 17 \times 5$ (b) $183 + 220$ (c) 42×19

(d) 36×11 (e) $169 + 71$ (f) $5 \times 24 \times 8$

8 Evaluate: Ex. 1.5

(a) $6 \times 4 \div 2 \times 6$ (b) $5 + 6 \times 7$ (c) $18 + 12 - 7 + 6$

(d) $2 + 5 \times 9$ (e) $18 \div 6 - 3$ (f) $8 \times (15 - 5)$

9 Calculate the following. Ex. 2.5, 2.6

(a) $-9 + 7$ (b) $5 + (-8)$ (c) $-3 - (-7)$ (d) $-6 - 11$

10 Estimate the answers to the following by rounding to the first digit. Ex. 1.4

(a) 17×93 (b) 46×281 (c) 337×240

(d) $953 \div 11$ (e) $8195 \div 237$ (f) $12\,495 \div 5400$

11 Arrange the following numbers in ascending order (from smallest to largest). Ex. 2.4

(a) 5, 0, -15, 10, -5 (b) -300, 3, 0, -30, 3000

12 Find the lowest common multiple of: Ex. 2.1

(a) 8 and 12 (b) 12 and 16

13 Find the highest common factor of: Ex. 2.1

(a) 36 and 27 (b) 64 and 72

14 List the factors of each of these numbers and state whether each number is prime or composite. Ex. 2.2

(a) 18 (b) 23 (c) 44 (d) 79

15 Write each number as the product of its prime factors in index form. Ex. 2.3

(a) 63 (b) 48 (c) 72 (d) 120

16 Simplify the following by writing a single sign between the values, then calculate the answer.

Ex. 2.7

(a) -6 + (-7) (b) 4 – (-11) (c) -2 + (+7)

(d) 22 + (-9) (e) -5 – (-10) (f) 18 + (+3)

Understanding

17 A submarine 110 m below the surface of the water rises 80 m, then dives 150 m. What depth is it at now?

18 Which of the following numbers are:

(a) prime

(b) perfect squares

(c) powers of 2?

3, 7, 9, 24, 11, 16, 19, 43, 32, 28, 13, 8, 2, 25

19 What is the first perfect cube that is divisible by both 3 and 4?

20 Calculate an approximate answer for the following by rounding to the first digit, then state whether the actual answer will be higher or lower.

(a) 256×37 (b) 1379×24 (c) $5498 \div 46$

21 The Royal Easter Show runs for 7 days. The total attendance at this year's show was 62 982.

(a) Approximately how many people per day was this? Use rounding to a convenient multiple of 1000 to calculate your answer.

(b) If each person paid an average ticket price of $12, use your answer from **(a)** to calculate how much money the show organisers made from ticket sales. Use some mental or written strategies to calculate your answer.

Reasoning

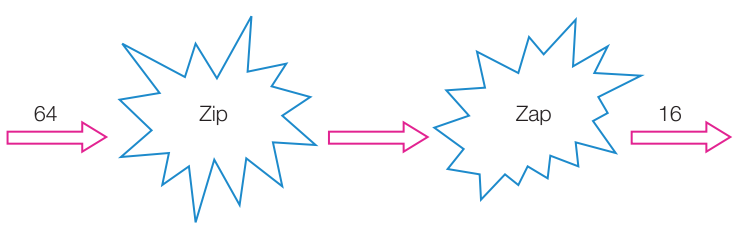
22 On Monday, Kiran withdrew $100 from his bank account at an ATM. On Tuesday, he used his account to pay his $85 phone bill online. On Wednesday, he deposited $250. On Friday, he withdrew another $60 from the account.

(a) By the end of the week, did Kiran have more or less money in his bank account?

(b) How much more or less?

23 The number 64 passes through two 'magic clouds' Zip and Zap and emerges as the number 16.

Which of the following can describe what Zip and Zap did to the number passing through?



A Zip: square root, Zap: square B Zip: cube root, Zap: square

C Zip: square root, Zap: nothing D Zip: nothing, Zap: cube root